Engineering Failure Analysis 18 (2011) 2084–2092

Contents lists available at ScienceDirect

Engineering Failure Analysis

journal homepage: www.elsevier.com/locate/engfailanal

Risk evaluation in failure mode and effects analysis of aircraft turbine rotor blades using Dempster–Shafer evidence theory under uncertainty

Jianping Yang [a,b], Hong-Zhong Huang [a,*], Li-Ping He [a], Shun-Peng Zhu [a], Dunwei Wen [c]

[a] School of Mechatronics Engineering, University of Electronic Science and Technology of China, Chengdu, Sichuan 611731, PR China
[b] Unit 95389 of People's Liberation Army, Guilin, Guangxi 541007, PR China
[c] School of Computing & Information Systems, Athabasca University, 1 University Drive, Athabasca, AB, Canada T9S 3A3

ARTICLE INFO

Article history:
Received 4 April 2011
Received in revised form 14 June 2011
Accepted 15 June 2011
Available online 30 June 2011

Keywords:
Failure mode and effects analysis
D–S evidence theory
Aircraft turbine rotor blade
Risk priority number

ABSTRACT

Rotor blades are the major components of an aircraft turbine. Their reliability seriously affects the overall aircraft turbine security. Failure mode and effects analysis (FMEA), especially, the risk priority order of failure modes, is essential in the design process. The risk priority number (RPN) has been extensively used to determine the risk priority order of failure modes. When multiple experts give different risk evaluations to one failure mode, which may be imprecise and uncertain, the traditional RPN is not a sufficient tool for risk evaluation. In this paper, the modified Dempster–Shafer (D–S) is adopted to aggregate the different evaluation information by considering multiple experts' evaluation opinions, failure modes and three risk factors respectively. A simplified discernment frame is proposed according to the practical application. Moreover, the mean value of the new RPN is used to determine the risk priority order of multiple failure modes. Finally, this method is used to deal with the risk priority evaluation of the failure modes of rotor blades of an aircraft turbine under multiple sources of different and uncertain evaluation information. The consequence of this method is rational and efficient.

1. Introduction

Rotor blades are the key rotating components of an aircraft turbine, which play an important role in the task of energy conversion. This includes compressor rotor blades and turbo rotor blades. Since they are the thin-form, components moving in high-speed rotation, under the severe load conditions in complex work environments, rotor blades are one of the components of the highest failure rates in aircraft turbines. Meanwhile, with the development of the aviation industry, the Thrust-Weight Ratio (TWR) of aircraft turbines has become higher and higher. The stress level of rotor blades is greatly increasing as well. Moreover, their safety plays an important role in the aircraft turbine security. In order to improve their safety and reliability, failure mode and effects analysis (FMEA) is prerequisite in their design.

FMEA is an analysis technique for defining, identifying and eliminating known and/or potential failures, problems, errors from systems, design, processes, and/or services before they reach the customers [1,2]. FMEA can facilitate the identification of potential failures in the design or process of products or systems. This can help designers adjust the existing programs, increase compensating provisions, employ the recommended actions to reduce the likelihood of failures, decrease the probability of failure rates and avoid hazardous accidents. FMEA has been extensively used in a number of industrial products, including structures operating in power, aeronautics and astronautics. FMEA can identify each possible failure mode and determine the effect of each failure. However, there can be multiple failure modes and their risks and effects are different.

* Corresponding author. Tel.: +86 28 6183 0248; fax: +86 28 6183 0227.
E-mail address: hzhuang@uestc.edu.cn (H.-Z. Huang).

1350-6307/$ - see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.engfailanal.2011.06.014

Table 1
Suggested criteria of rating for occurrence of a failure in FMEA [4,13,14].

Rating	Probability of occurrence	Possible failure rate
10	Extremely high: failure almost inevitable	$\geqslant$1/2
9	Very high	1/3
8	Repeated failures	1/8
7	High	1/20
6	Moderately high	1/80
5	Moderate	1/400
4	Relatively low	1/2000
3	Low	1/15000
2	Remote	1/150000
1	Nearly impossible	$\leqslant$1/1500000

Table 2
Suggested criteria of rating for severity of a failure in FMEA [4,13,14].

Rating	Effect	Severity of effect
10	Hazardous without warning	Highest severity ranking of a failure mode, occurring without warning and consequence is hazardous
9	Hazardous with warning	Higher severity ranking of a failure mode, occurring with warning, consequence is hazardous
8	Extreme	Operation of system or product is broken down without compromising safe
7	Major	Operation of system or product may be continued but performance of system or product is affected
6	Significant	Operation of system or product is continued and performance of system or product is degraded
5	Moderate	Performance of system or product is affected seriously and the maintenance is needed
4	Low	Performance of system or product is small affected and the maintenance may not be needed
3	Minor	System performance and satisfaction with minor effect
2	Very minor	System performance and satisfaction with slight effect
1	None	No effect

Table 3
Suggested criteria of rating for detection of a failure in FMEA [4,13,14].

Rating	Detection	Criteria
10	Absolute uncertainty	Potential occurring of failure mode cannot be detected in concept, design and process FMEA/mechanism and subsequent failure mode
9	Very remote	The possibility of detecting the potential occurring of failure mode is very remote/mechanism and subsequent failure mode
8	Remote	The possibility of detecting the potential occurring of failure mode is remote/mechanism and subsequent failure mode
7	Very low	The possibility of detecting the potential occurring of failure mode is very low/mechanism and subsequent failure mode
6	Low	The possibility of detecting the potential occurring of failure mode is low/mechanism and subsequent failure mode
5	Moderate	The possibility of detecting the potential occurring of failure mode is moderate/mechanism and subsequent failure mode
4	Moderately high	The possibility of detecting the potential occurring of failure mode is moderately high/mechanism and subsequent failure mode
3	High	The possibility of detecting the potential occurring of failure mode is high/mechanism and subsequent failure mode
2	Very high	The possibility of detecting the potential occurring of failure mode is very high/mechanism and subsequent failure mode
1	Almost certain	The potential occurring of failure mode will be detect/mechanism and subsequent failure mode

Consequently, the ability to prioritize risks is important; the risk priority number (RPN) offers one way to rank failure modes. In general, RPN includes three factors: the severity of a failure effect (S), which is a numerical subjective estimation of the degree of severity of the consequence of a occurred failure, the probability of the occurrence of a failure mode (O), which is a numerical subjective estimation of the likelihood of the cause, and the probability of a failure being detected (D), which is a numerical subjective estimation of the effectiveness of the controls preventing or detecting the cause or failure mode. Each factor uses a numeric scale (rating) from 1 to 10, as expressed in Tables 1–3. These ratings are then multiplied to obtain RPN, expressed as

$$\mathrm{RPN} = S \times O \times D \tag{1}$$

Because of its extensive application, considerable research has been conducted to investigate the use of the RPN in FMEA. Chang and Cheng [3] identified some limitations of the traditional RPN and proposed a method which utilizes fuzzy ordered weighted averaging and evaluation averaging (DEMATEl) to rank the risk of failure modes. The method was illustrated by the thin film transistor liquid crystal display (TFT-LCD). Sankar and Prabhu [4] proposed the modified approach for prioritization of failures. This method defines the new ratings from 1 to 1000 to represent possible severity-occurrence-detection states, and provides a much more sensitive ranking process to quantitate risk factors. The failure modes having a higher rating are given higher priorities. Chang [5] proposed a more general RPN method considering situation parameters and relationships between the components of a system with respect to its type (direct/indirect) and severity. The method combined the ordered weighted geometric averaging (OWGA) operator and the decision-making trial and evaluation laboratory (DEMATEL) approach. To support real application, Wang et al. [6] regarded the three risk factors as fuzzy variables and evaluated their risks using fuzzy linguistic terms and fuzzy ratings. The fuzzy RPN was defined as a fuzzy weighted geometric mean of the risk factors and could be exactly solved by using alpha level sets and the fuzzy extension principle. Wang's method was compared with other various fuzzy improvements using numerical example [6]. Guimarães and Lapa [7] defined the concept of a pure fuzzy logic system where the fuzzy rule base consists of a collection of fuzzy IF–THEN rules. The fuzzy inference engine used these fuzzy IF–THEN rules to determine a mapping from fuzzy sets in the input universe of discourse to fuzzy sets in the output universe of discourse based on fuzzy logic principles. Fuzzy RPN is compared with traditional RPN using the example of standard four-loop pressurized water reactor (PWR) containment cooling system (CCS). Tay and Lim [8] proposed the use of weighted fuzzy production rules in fuzzy inference system of FMEA. This allowed a global weight to be attached to each IF–THEN rule. A fuzzy IF–THEN rule with global weight has been expressed. Cassanelli et al. [9] proposed the concept of failure analysis-assisted FMEA, as a logical warning for proper FMEA. These concepts use of the field data from the second year forces a more robust review FMEA. Stamatis [10] developed a multi-attribute failure mode analysis (MAFMA) based on the analytic hierarchy process (AHP) technique. It considers four different factors O, S, D, and expected cost as decision attributes, possible causes of failure as decision alternatives, and the selection of cause of failure as decision goal. Narayanagounder and Gurusami [11] used the risk priority code (RPC) to prioritize failure modes for cases when two or more failure modes had the same RPN. A new method was proposed to prioritize failure modes when there was a disagreement in ranking scale for severity, occurrence and detection. An Analysis of Variance (ANOVA) was used to compare means of RPN values. SPSS (Statistical Package for the Social Sciences) statistical analysis package was used to analyze the data. Considering the subjective and qualitative nature of information and interdependences among various failure modes, Sharma et al. [12] proposed a fuzzy logic approach based fuzzy decision support system (FDSS) to rank the risk priority order of the failure modes. The frequency of each risk factor was represented as members of a fuzzy set. The risk level of the failure mode was evaluated by an inference system. Sharma's method can handle vague and qualitative information in risk evaluation.

Based on the above review, the limitations of the traditional RPN have been investigated. There is, however, a little attention paid to the case where multiple experts provide imprecise or uncertain risk evaluations for a given failure mode. In recent years, the D–S evidence theory has been employed to quantitate the imprecision and uncertainty in reliability and failure analysis. Simon et al. [19] used the D–S evidence theory and Bayesian networks tools to treat the incomplete or inconsistent data in reliability studies. This method can extract the greatest amount of information form the available data. Limbourg and Savié [20] used the applicability of D–S evidence theory which merges interval-based and probabilistic uncertainty modeling on a fault tree analysis from the automotive area. Bae et al. [21] proposed the cost effective algorithm to alleviate the computational cost and made D–S evidence theory useful in the structural engineering design based on uncertainty quantification analysis. The cost effective algorithm employed optimization and approximation techniques to evaluate efficiently the belief and plausibility functions without sacrificing the accuracy of resulting measurements. The techniques were demonstrated on composite material structures and an airframe wing aeroelastic design problem. Popescu et al. [22] used the Dempster–Shafer (D–S) evidence theory to accommodate the imprecise or vague input data and showed how a false-negative pattern can be observed. This illustrates that D–S evidence theory is a clear advantage over binary assignments in representing vagueness. In order to deal with the imprecision and uncertainty in FMEA, Chin et al. [13] proposed a group-based evidential reasoning approach to deal with different opinions and knowledge from multiple experts. The method included assessing risk factors using belief structures, synthesizing individual belief structures into group belief structures, aggregating the group belief structures into overall belief structures, converting the overall belief structures into expected risk scores and ranking the expected risk scores using the minimax regret approach (MRA). Inspired by this work, this paper uses the modified D–S evidence theory to aggregate the different opinions which may be imprecise and uncertain. The method is used to deal with the risk priority evaluation of failure modes of rotor blades of an aircraft turbine.

The rest of this paper is organized as follows. In Section 2, the D–S evidence theory is briefly introduced. In Section 3, new RPN model using modified D–S evidence theory is proposed. Section 4 uses the novel model for risk priority evaluation of failure modes of rotor blades of an aircraft turbine. The final section makes the conclusions.

2. D–S evidence theory

The evidence theory is developed and presented by Shafer [15] based on Dempster's work [16] on milestones on the upper and lower bounds of belief assignment to the hypothesis, also called Dempster–Shafer (D–S) evidence theory. The D–S evidence theory adopts the belief interval to describe the uncertainty of the hypothesis. Moreover, the method can deal with the information of multiple sources which may be imprecise, uncertain and incomplete.

2.1. The frame of discernment

Let Θ be the set of N elements which is a finite nonempty exhaustive set of mutually exclusive possibilities. Θ is defined as the frame of discernment. The power set of Θ is all the possible subsets, noted as 2^{Θ}. There are 2^N elements in the 2^{Θ}. For example, if $\Theta = \{1, 2, 3\}$ and $N = 3$, the power set is $2^{\Theta} = \{\emptyset, 1, 2, 3, 12, 13, 23, 123\}$, where $\emptyset$ denotes the empty set. The D–S evidence theory starts by defining the frame of discernment.

2.2. The basic belief assignment (BBA)

The basic belief assignment is a primitive from of evidence theory, which is denoted by $m(X)$. The function $m(X)$ is a mapping: $m(X)$: $2^{\Theta} \to [0, 1]$, and satisfies the following conditions:

$$m(\emptyset) = 0 \tag{2}$$

$$\sum_{X \in 2^{\Theta}} m(X) = 1 \tag{3}$$

$$0 \leqslant m(X) \leqslant 1 \quad X \in 2^{\Theta} \tag{4}$$

$m(X)$ expresses the precise probability in which the evidence corresponds to m supports proposition X. X may not only be a single possible event, but could be a set of multiple possible events.

2.3. Belief function (Bel)

A belief function is often defined by the basic probability assignment function which is represented by $Bel(X)$.

$$Bel(X) = \sum_{Y \subseteq X} m(Y) \tag{5}$$

$Bel(X)$ represents the total amount of probability that must be distributed among elements of X. It reflects inevitability, signifying the total degree of belief of X, and constitutes a lower limit function on the probability of X [17]. Obviously,

$$Bel(\emptyset) = 0 \tag{6}$$

$$Bel(\Theta) = 1 \tag{7}$$

Shafer shows that for natural number $n, X_k \subseteq \Theta$

$$Bel(X_1 \cup X_2 \cup \cdots X_n) \geqslant \sum_i Bel(X_i) - \sum_{i>j} Bel(X_i \cap X_j) + \cdots + (-1)^n Bel(X_1 \cap X_2 \cap \cdots X_n) \tag{8}$$

where $i, j, k = 1, 2, \cdots, n$.

2.4. Plausibility function (pl)

A Plausibility function (Pl) is defined as the following:

$$Pl(X) = 1 - Bel(\overline{X}) = \sum_{Y \cap X \neq \emptyset} m(Y) \tag{9}$$

where $\overline{X}$ is the negation of a hypothesis X. $Pl(X)$ measures the maximal amount of probability that can be distributed among the elements in X. It describes the total degree of belief related to X and constitutes an upper limit function on the probability of X [17].

[$Bel(X)$, $Pl(X)$] is the posteriori confidence interval which expresses the uncertainty of X. When the ignorance to proposition X is decreased, the length of interval is diminished. The relationship between $Bel(X)$ and $Pl(X)$ is illustrated in Fig. 1.

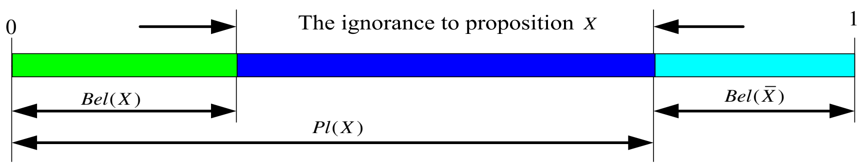

Fig. 1. Belief function and Plausibility function.

2.5. Dempster's combination rule

The D–S evidence theory can aggregate multiple sources of evidence through the combination rule. Dempster's combination rule is expressed as follows. Given two basic probability assignment functions $m_i(X)$ and $m_j(Y)$, the Dempster's combination rule can be defined by

$$m(C) = (m_i \oplus m_j)(C) = \begin{cases} 0, & C = \varnothing \\ \frac{\sum_{X \cap Y = C, \forall X, Y \subseteq \Theta} m_i(X) \times m_j(Y)}{1 - \sum_{X \cap Y = \varnothing, \forall X, Y \subseteq \Theta} m_i(X) \times m_j(Y)}, & C \neq \varnothing \end{cases} \tag{10}$$

where $m(C)$ denotes the BBA of c that is supported by ith (jth) evidence.

3. Risk priority number model using D–S evidence theory

As briefly introduced above, D–S evidence theory employs the confidence interval to describe the uncertainty of hypothesis and can deal with the incomplete, imprecise and uncertain information of a system. Moreover, the new aggregated belief assignment can be attained through a combination of multiple sources of evidence using combination rule. Furthermore, owing to the flexibility of the basic axioms in evidence theory, no further assumptions are needed to quantitate the uncertain information of the system [18]. In this section, the modified D–S evidence theory is used to deal with and model the difference and uncertainty of evaluation information received from multiple experts in FMEA. The method aggregates evaluation information of multiple experts about risk factors. The evaluation consequence of each expert with respect to each risk factor of each failure mode is regarded as a new evidence body.

3.1. The structure to frame of discernment

There are three risk factors: occurrence (O), severity (S) and detection (D) included in the RPN. Because the three risk factors are independent of each other, three discernment frames are needed and denoted by Θ_O, Θ_S and Θ_D. In this paper, according to Tables 1–3, the discernment frame:

$$\Theta_i = (1, 2, 3, 4, 5, 6, 7, 8, 9, 10) \quad i = O, S, D \tag{11}$$

In FMEA, the traditional risk factor can only take one single value. Because of their different expertise and backgrounds, different experts may give out different opinions for the same risk factor. There may be several values for one risk factor. Suppose there are L experts in a TEMA group and N failure modes: $(E_1, \cdots, E_L)$ and $(F_1, \ldots, F_N)$. Consequently, every failure mode has three discernment frames. Θ_i^n is used to express the discernment frame of the nth failure mode to the ith risk factor,

$$\Theta_i^n = (1, 2, 3, 4, 5, 6, 7, 8, 9, 10) \quad i = O, S, D; \quad n = 1, 2, \ldots, N \tag{12}$$

Obviously, the number of discernment frames is $3N$. When the risk factors are analyzed by experts, there are rarely large differences in the ranks of risk factor given by experts. Moreover, every rank of risk factor has the distinct selected criterion. In order for simplification and engineering application, the discernment frames can be modified. In practice, the discernment frame can be simplified to

$$\Theta_i^n = \left(\min X|_{X \subseteq \Theta_i^n}, \min X|_{X \subseteq \Theta_i^n} + 1, \cdots, \max X|_{X \subseteq \Theta_i^n}\right) \tag{13}$$

where $\min X|_{X \subseteq \Theta_i^n}$ means the minimum of the rank of the nth failure mode to the ith risk factor in the evaluations of L experts. Similarly, $\max X|_{X \subseteq \Theta_i^n}$, it means the maximum of the rank of the nth failure mode to the ith risk factor in the evaluations of L experts. Moreover, the following can be attained

$$1 \leqslant \min X|_{X \subseteq \Theta_i^n} \leqslant \max X|_{X \subseteq \Theta_i^n} \leqslant 10 \tag{14}$$

3.2. The modified belief function and combination rule

The basic belief assignment rating of a risk factor for a failure mode is assigned by different experts. Different experts may make different decisions given the same risk factor according to their domain knowledge and backgrounds. Each expert makes his/her own risk evaluation of the evidence based on his/her own criteria [14]. In order to differentiate the level and professional background of experts, the weight of an expert should be considered. For the three risk factors and L experts, the weight can be expressed as the following matrix

$$\boldsymbol{\omega} = \begin{pmatrix} \omega_{O1} & \cdots & \omega_{OL} \\ \omega_{S1} & \ddots & \omega_{SL} \\ \omega_{D1} & \cdots & \omega_{DL} \end{pmatrix} \tag{15}$$

where ω_{ij} is the relative weight on the importance of jth expert to ith risk factor and is normalized, so that

$$0 \leqslant \omega_{ij} \leqslant 1 \quad i = O, S, D \tag{16}$$

The larger ω_{ij} is, the higher the importance of the jth expert to ith risk factor is. If there is no sufficient reason or knowledge to distinguish the diversity among the experts in their evaluation level, the weight of the experts should be equal.

Considering the weight, the new BBA is denoted as $\bar{m}_{ij}^{n}(\cdot)$

$$\bar{m}_{ij}^{n}(C) = \omega_{ij} \times m_{ij}^{n}(C) \quad C \subset \Theta_i^n,\ C \neq \Theta_i^n \tag{17}$$

$$\bar{m}_{ij}^{n}(\Theta_i^n) = 1 - \sum_{B \subset \Theta_i^n} \omega_{ij} \times m_{ij}^{n}(B) \quad \mathrm{B} \neq \Theta_i^n \tag{18}$$

where $i = O, S, D$, $j = 1, 2, \ldots, L$, L is the number of experts, $n = 1, 2, \ldots, N$, N is the number of failure modes.

The new combination rule of D–S evidence theory can be transformed as

$$m_{i,jg}^{n}(C) = (\bar{m}_{ij}^{n} \oplus \bar{m}_{ig}^{n})(C) = \begin{cases} 0, & C = \varnothing \\ \dfrac{\sum_{X \cap Y = C, \forall X,Y \subseteq \Theta_i^n} (\omega_{ij} \cdot m_{ij}^{n}(X)) \times (\omega_{ig} \cdot m_{ig}^{n}(Y))}{1 - \sum_{X \cap Y = \varnothing, \forall X,Y \subseteq \Theta_i^n} (\omega_{ij} \cdot m_{ij}^{n}(X)) \times (\omega_{ig} \cdot m_{ig}^{n}(Y))}, & C \neq \varnothing \end{cases} \tag{19}$$

Dempster's combination rule can be generalized to more than two experts, as shown in Eq. (19). The final result represents the synthetic effects of all sources of evidence.

$$M_i^n = M_{i1}^n \oplus M_{i2}^n \oplus \cdots \oplus M_{iL}^n = \left(\left(\left(M_{i1}^n \oplus M_{i2}^n\right) \oplus \cdots\right) \oplus M_{iL}^n\right) \tag{20}$$

3.3. Risk priority number

Using Eq. (19), the novel brief assignment function of three risk factors can be attained through fusing the risk evaluation consequences of multiple experts. This new BBA can be regarded as a degree of belief in these ratings. Because BBA satisfies the axiom of additivity, the belief degrees can be the probability of rating of the risk factors. The three risk factors can be considered as discrete random variables. The RPN is a function of the discrete random variables. Consequently, the RPN is a discrete random variable with several different ratings and the corresponding probabilities. Suppose RPN has several ratings $\left(\mathrm{RPN}_n^1, \ldots, \mathrm{RPN}_n^q\right)$ with respective probabilities $\left(P(\mathrm{RPN}_n^1), \ldots, P(\mathrm{RPN}_n^q)\right)$ for nth failure mode through Eq. (19) using random theory. In order to compare the overall risk of each failure mode, the mean value of RPN is needed, which can be defined as following:

$$\mathrm{MVRPN}_n = E(\mathrm{RPN}_n) = \sum_q \left(\mathrm{RPN}_n^q\right) \cdot P\left(\mathrm{RPN}_n^q\right) \tag{21}$$

4. Application analysis of the proposed approach to the rotor blades of an aircraft turbine

In this section, the proposed method is used on an FMEA case of rotor blades for an aircraft turbine. These rotor blades include two different subsystems: the compressor rotor blades and the turbo rotor blades. The different failure modes which could result in an accident with undesired consequences of each of the subsystems are investigated. The major failure mode of the turbo rotor blades is fracture and can be caused by different factors, including high-cycle intrigue, low-cycle intrigue and creep-fatigue interaction. There are nine potential failure modes in the FMEA of the turbo rotor blades. The eight failure modes are considered in the FMEA of the compressor rotor blades. The effects of each failure mode on the system are studied. For each of the failure modes, the system is investigated for any alarms of conditions monitoring arrangement according to the practical engineering background. There are 17 failure modes in total which were recognized and are delineated together with their effects and alarms on the system in Table 4.

In the FMEA of rotor blades of the aircraft turbine, there are three experts, each gives a different risk evaluation. Each expert evaluates the failure modes and identifies the rating information of the three risk factors. Table 5 expresses the evaluation consequences of the three experts on the 17 failure modes with respect to the three major risk factors.

In this application, suppose the weight of expert is equal, and ω_{ij} is equal to 1. The detailed computing of the first failure mode can be expressed as follows.

According to Section 3, the discernment frames can be constructed respectively: $\Theta_O^1 = (3,4),\ \Theta_S^1 = (7),\ \Theta_D^1 = (2)$. For the risk factors: O, the modified brief function can be attained using Eq. (17): $\bar{m}_{O1}^1(3) = 0.4, \bar{m}_{O1}^1(4) = 0.6;\ \bar{m}_{O2}^1(3) = 0.9,\ \bar{m}_{O2}^1(4) = 0.1;\ \bar{m}_{O3}^1(3) = 0.8,\ \bar{m}_{O3}^1(4) = 0.2$. Secondly, the combined brief function can be obtained using Eq. (19) between Expert 1 and Expert 2:

$$m_{O,12}^1(3) = \bar{m}_{O1}^1(X) \oplus \bar{m}_{O2}^1(Y) = \frac{\sum_{X \cap Y = 3, \forall X,Y \subseteq \Theta_O^1} \bar{m}_{O1}^1(X) \times \bar{m}_{O2}^1(Y)}{1 - \sum_{X \cap Y = \varnothing, \forall X,Y \subseteq \Theta_O^1} \bar{m}_{O1}^1(X) \times \bar{m}_{O2}^1(Y)} = \frac{0.4 \times 0.9}{1 - (0.4 \times 0.1 + 0.6 \times 0.9)} = 0.857$$

Table 4
FMEA for the rotor blades of an aircraft turbine [23].

Item	Component	Failure mode	Major reason to the failure	Failure effect	Alarm
1	Compressor rotor blades	Deformation	Low yield strength caused by the improper material and heat treatment technology, low blade strength caused by over-temperature, high centrifugal stress caused by engine overspeed	Replace blade	Yes
2	Compressor rotor blades	Crack	The thermal fatigue, corrosion pit and blade wound by foreign objects, improper material	Replace blade	Yes
3	Compressor rotor blades	Fracture	The improper material, crack, corrosion, low intensity caused by improper heat treatment, high local stress	Damage the engine, endanger the flight safety	No
4	Compressor rotor blades	Corrosion	The oxygen and impurities in high-temperature gas, imperfect surface of blade,	Replace blade	Yes
5	Compressor rotor blades	Blade tip wear	The improper material, low intensity caused by improper heat treatment, horizontal high-frequency alternating load, vertical low-frequency centrifugal load,	Replace blade and engine casing	Yes
6	Compressor rotor blades	Deflection	The low blade strength caused by over-temperature, high centrifugal stress caused by engine overspeed	Replace blade	Yes
7	Compressor rotor blades	Guideway crack	The improper material, the thermal tress and centrifugal stress,	Replace blade	No
8	Compressor rotor blades	Injured by foreign objects	Foreign objects of Inhalation of inlet channel	Replace blade	No
9	Turbo rotor blades	High-cycle intrigue fracture	The torsional resonance caused by design and technology factors	Broke the engine, endanger the flight safety	No
10	Turbo rotor blades	Low-cycle intrigue fracture	Low intensity caused by improper heat treatment	Damage the engine, endanger the flight safety	No
11	Turbo rotor blades	Intergranular fracture	The blades is over-heating because the engine is over-temperature caused by unreasonable use	Damage the engine, endanger the flight safety	No
12	Turbo rotor blades	Creep-fatigue fracture	The failure resistance is reduced because recrystallization of local part of blades occurs and additional stress increases caused by assembly	Damage the engine, endanger the flight safety	No
13	Turbo rotor blades	Fatigue-creep fracture	The surface coating is desquamated by the thermal tress and centrifugal stress	Replace blade	No
14	Turbo rotor blades	The fracture of combining high cycle and low cycle fatigue	The reason is poorly design, high local stress, the large gap of blade crown and high vibration stress	Damage the engine, endanger the flight safety	No
15	Turbo rotor blades	Crack	High thermal stress, the thermal fatigue, corrosion pit and blade wound by foreign objects and	Replace blade	Yes
16	Turbo rotor blades	Corrosion	The oxygen and impurities in high-temperature gas, the loss of corrosion resistant materials	Replace blade	Yes
17	Turbo rotor blades	Deformation	Low yield strength caused by the improper material and heat treatment technology, low blade strength caused by over-temperature, high centrifugal stress caused by engine overspeed	Replace blade	Yes

$$m^1_{O,12}(4) = \bar{m}^1_{O1}(X) \oplus \bar{m}^1_{O2}(Y) = \frac{\sum_{X\cap Y=4,\forall X,Y\subseteq\Theta^1_O} \bar{m}^1_{O1}(X)\times\bar{m}^1_{O2}(Y)}{1-\sum_{X\cap Y=\emptyset,\forall X,Y\subseteq\Theta^1_O} \bar{m}^1_{O1}(X)\times\bar{m}^1_{O2}(Y)} = \frac{0.6\times 0.1}{1-(0.4\times 0.1+0.6\times 0.9)} = 0.143$$

Then, the final consequence can be attained through fusing Expert 3 using Eq. (19)

$$m_O(3) = \bar{m}^1_{O,12}(X) \oplus \bar{m}^1_{O3}(Y) = \frac{\sum_{X\cap Y=3,\forall X,Y\subseteq\Theta^1_O} \bar{m}^1_{O,12}(X)\times\bar{m}^1_{O3}(Y)}{1-\sum_{X\cap Y=\emptyset,\forall X,Y\subseteq\Theta^1_O} \bar{m}^1_{O,12}(X)\times\bar{m}^1_{O3}(Y)} = \frac{0.857\times 0.8}{1-(0.857\times 0.2+0.143\times 0.8)} = 0.960$$

$$m_O(4) = \bar{m}^1_{O,12}(X) \oplus \bar{m}^1_{O3}(Y) = \frac{\sum_{X\cap Y=4,\forall X,Y\subseteq\Theta^1_O} \bar{m}^1_{O,12}(X)\times\bar{m}^1_{O3}(Y)}{1-\sum_{X\cap Y=\emptyset,\forall X,Y\subseteq\Theta^1_O} \bar{m}^1_{O,12}(X)\times\bar{m}^1_{O3}(Y)} = \frac{0.20\times 0.143}{1-(0.857\times 0.2+0.143\times 0.8)} = 0.040$$

Consequently, the discrete random variable: $O(3, 4)$ and the respective probability (0.960, 0.040) can be attained. It is the same to S, D, $S(7, 100\%)$ and $O(2, 100\%)$. Because S and D have only one value each respectively and the corresponding probability is equal to 1, S and D can be regarded as constant variables. According to random theory, the mean value of RPN can be attained using Eq. (21) as:

Table 5
Evaluation information on 17 failure modes by three experts.

Item	Rating of risk factor								
	Expert 1			Expert 2			Expert 3		
	O	*S*	*D*	*O*	*S*	*D*	*O*	*S*	*D*
1	3:40% 4:60%	7	2	3:90% 4:10%	7	2	3:80% 4:20%	7	2
2	2	8	4	2	8:70% 9:30%	4	2	8	4
3	1	10	3	1	10	3	1	10	3
4	1	6:80% 7:20%	3	1	6	3:70% 2:30%	1	6	3
5	1	3	2:50% 1:50%	1	3	1:70% 2:30%	1	3:60% 2:40%	1
6	2	6	5	2	6	5	2	6	5
7	1	7	3	1	7	3	1	7	3
8	3	5:60% 6:40%	1	3	5:80% 6:20%	1	3	5:80% 7:20%	1
9	2:90% 1:10%	10:60% 9:40%	4	2:75% 1:25%	10:90% 9:10%	4	2:80% 1:20%	10:90% 9:10%	4
10	1	10	6	1	10	6	1	10	6
11	1	10	5	1	10	5	1	10	5
12	1	10	6:60% 5:40%	1	10	5:80% 4:20%	1	10	6:70% 5:30%
13	1	10	5:80% 4:20%	1	10	5	1	10	5
14	1	10	6	1	10	6:80% 7:20%	1	10	6
15	2	7:95% 6:5%	3	2	7	3	2	7	3:70% 4:30%
16	2:90% 1:10%	4	3	2:75% 1:25%	4	3	2:80% 1:20%	4	3:80% 2:20%
17	2	5:90% 6:10%	3	2	5:90% 6:10%	3	2	5:60% 6:40%	3

Table 6
The mean value of risk priority number of 17 failure modes.

Component	Compressor rotor blades								Turbo rotor blades								
Failure mode	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
MVRPN	42.56	64	30	18	4.17	60	21	15	78.92	60	50	50	60	60	42	23.88	50.90

$$\mathrm{MVRPN}_1 = E(\mathrm{RPN}_1) = \sum_{q=1}^{2} \left(\mathrm{RPN}_1^q\right) \cdot P\left(\mathrm{RPN}_1^q\right) = 3 \times 0.960 \times 7 \times 2 + 4 \times 0.040 \times 7 \times 2 = 42.56$$

Similar to other failure modes, the mean value of the RPN can be attained and expressed in Table 6.

As clearly shown in Table 6, Failure mode 2 has the largest RPN in the failure modes of compressor rotor blades, followed by failure modes 6, 1, 3, 7, 4, 8 and 5. Failure mode 5 is apparently the least overall risk. The same is seen for turbo rotor blades. Failure mode 9 has the largest RPN in the failure modes of compressor rotor blades, followed by failure mode 10, 13, 14, 17, 11, 12, 15 and 16. Failure mode 16 is apparently the least overall risk. When the RPN among the different failure modes is the same, these failure modes should be given the same risk attention. For example, failure mode 10, 13, 14, each have the same RPN equal to 60. Since the design of the aircraft turbine rotor blades has historical data, the risk ranking consequence of failure mode is compared with historical data. The results are consistent with the practical engineering background and demonstrate that the proposed method is rational and efficient.

5. Conclusions

In this paper, D–S evidence theory is adopted to aggregate the risk evaluation information of multiple experts, which may be inconsistent, imprecise and uncertain. A modified evidence theory is proposed for dealing with different opinions of multiple experts, multiple failure modes and three risk factors in RPN analysis of FMEA of aircraft turbine rotor blades. In this method, simplified discernment frames are provided according to our practical engineering application. Meanwhile, the fused three risk factors are regarded as the discrete random variables. Consequently, the RPN is a function of the discrete random variable. The mean value of RPN is used for the risk priority ranking of failure modes. The proposed method is demonstrated by an application of risk priority ranking of failure modes in FMEA of aircraft turbine rotor blades. The different evaluation information of three experts about eight failure modes is aggregated. The mean value of RPN is attained. The risk ranking of the failure modes are consistent with the practical engineering background.

Acknowledgments

This research was partially supported by the Specialized Research Fund for the Doctoral Program of Higher Education of China under the Contract Number 20090185110019. The authors wish to express their thanks to Victor Ontiveros at University of Maryland for his help in writing this paper.

References

[1] Vliegen HJW, Mal HH. Rational decision making: structuring of design meetings. IEEE Trans Eng Manage 1990;37(3):185–91.
[2] Linton JD. Facing the challenges of service automation: an enabler for e-commerce and productivity gain in traditional services. IEEE Trans Eng Manage 2003;50(4):478–84.
[3] Chang KH, Cheng CH. Evaluating the risk of failure using the fuzzy OWA and DEMATEL method. J Intell Manuf. doi: 10.1007/s10845-009-0266-x.
[4] Sankar NR, Prabhu BS. Modified approach for prioritization of failures in a system failure mode and effects analysis. Int J Qual Reliab Manage 2001;18(3):324–35.
[5] Chang KH. Evaluate the orderings of risk for failure problems using a more general RPN methodology. Microelectron Reliab 2009;49:1586–96.
[6] Wang YM, Chin KS, Poon GKK, Yang JB. Risk evaluation in failure mode and effects analysis using fuzzy weighted geometric mean. Expert Syst Appl 2009:1195–207.
[7] Guimarães ACF, Lapa CMF. Fuzzy inference to risk assessment on nuclear engineering systems. Appl Soft Comput 2007(7):17–28.
[8] Tay KM, Lim CP. Fuzzy FMEA with a guided rules reduction system for prioritization of failures. Int J Qual Reliab Manage 2006;23(8):1047–66.
[9] Cassanelli G, Mura G, Fantini F, et al. Failure analysis-assisted FMEA. Microelectron Reliab 2006;46(9–11):1795–9.
[10] Stamatis DH. Failure mode and effects analysis—FMEA from theory to execution. New York: Quality Press; 1995.
[11] Narayanagounder S, Gurusami K. A new approach for prioritization of failure modes in design FMEA using ANOVA. Proc World Acad Sci Eng Technol 2009;37:524–31.
[12] Sharma RK, Kumar D, Kumar P1. J Inst Engrs (India) Mech Eng Div 2007;88:39–44.
[13] Chin KS, Wang YM, Poon GKK, Yang JB. Failure mode and effects analysis using a group-based evidential reasoning approach. Comput Oper Res 2009;36:1768–79.
[14] Xu K, Tang LC, Xie M, Ho SL, Zhu ML. Fuzzy assessment of FMEA for engine systems. Reliab Eng Syst Saf 2002;75:17–29.
[15] Shafer G. A mathematical theory of evidence. Princeton, NJ: Princeton University Press; 1976.
[16] Dempster AP. Upper and lower probabilities Induced by a multivalued mapping. Ann Statist 1967;28:325–39.
[17] Ayoun A, Smets P. Data association in multi-target detection using the transferable belief model. Int J Intell Syst 2001;16(10):1167–82.
[18] Bae H, Grandhi RV, Canfield RA. Epistemic uncertainty quantification techniques including evidence theory for large-scale structures. Comput Struct 2004;82(13–14):1101–12.
[19] Simon C, Weber P, Levrat E. Bayesian networks and evidence theory to model complex systems reliability. J Comput 2007;2(1):33–43.
[20] Limbourg P, Savié R. Fault tree analysis in an early design stage using the Dempster–Shafer theory of evidence. In: Aven and vinnem, editors. Risk, reliability and societal safety. ISBN: 978-0-415-44786-7.
[21] Bae HR, Grandhi RV, Canfield RA. An approximation approach for uncertainty quantification using evidence theory. Reliab Eng Syst Saf 2004;86(3):215–25.
[22] Popescu DE, Lonea M, Zmaranda D. Some aspects about vagueness & imprecision in computer network fault-tree analysis. Int J Comput Commun Control 2010;v(4):558–66.
[23] Zhong BD. Failure and lesson of turbo rotation blades for aero-engines. J Mater Eng 2003;Z1:30–3.