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Yan-Feng LI
Hong-Zhong HUANG
Yu LIU
Ningcong XIAO
Haiqing LI

A NEW FAULT TREE ANALYSIS METHOD: FUZZY DYNAMIC FAULT TREE ANALYSIS

NOWA METODA ANALIZY DRZEWA USZKODZEŃ: ROZMYTA ANALIZA DYNAMICZNEGO DRZEWA USZKODZEŃ

Fault tree analysis (FTA) is a widely used reliability assessment tool for large and complex engineering systems. The conventional fault tree analysis method, which contains AND, OR, and Voting gates, etc., can efficiently build an analytical model to represent combinations of component failures that cause the failure of a system. However, due to its limited modeling capability, we may confront difficulties when modeling dynamic systems which involve complicated dynamic characteristics such as sequence dependency and functional dependency. Markov-based dynamic fault tree analysis (DFTA) extends the static FTA by introducing additional gates to model such complicated interactions among events. In many circumstances, it is quite difficult to obtain an accurate system reliability estimate due to limited data. To overcome this issue, a fuzzy dynamic fault tree model is put forth to assess system reliability. To obtain the membership function of the fuzzy probability for the top event of the studied fault trees, the extension principle is employed to calculate the associated membership function via a pair of parametric programming problems. Finally, a case study is presented to demonstrate the application of the proposed approach for the hydraulic system of a CNC machining centre.

Keywords: *Fault tree analysis, Dynamic fault tree, Fuzzy number, Fuzzy Markov model, Parametric programming.*

Analiza drzewa uszkodzeń (FTA) znajduje szerokie zastosowanie jako narzędzie oceny niezawodności dużych i złożonych systemów inżynierskich. Tradycyjna metoda analizy drzewa uszkodzeń z bramkami logicznymi typu AND, OR, k-z-n, itd. pozwala na sprawne konstruowanie modeli analitycznych reprezentujących kombinacje uszkodzeń elementarnych składowych systemu, które prowadzą do awarii systemu jako całości. Jednakże ograniczone możliwości modelowania jakie daje ta metoda mogą prowadzić do trudności przy modelowaniu systemów dynamicznych posiadających złożone charakterystyki dynamiczne, takie jak zależność sekwencyjna czy zależność funkcjonalna. Analiza dynamicznych drzew uszkodzeń (DFTA) oparta na metodzie Markowa stanowi rozszerzenie tradycyjnej FTA. Wprowadza ona dodatkowe bramki, pozwalając na modelowanie wspomnianych wyżej złożonych interakcji między zdarzeniami. W wielu okolicznościach, ograniczone dane nie pozwalają na otrzymanie dokładnej oceny niezawodności systemu. By rozwiązać ten problem, zaproponowano zastosowanie rozmytego modelu dynamicznego drzewa uszkodzeń do oceny niezawodności systemu. Aby otrzymać funkcję przynależności rozmytego prawdopodobieństwa wystąpienia zdarzenia szczytowego badanego drzewa uszkodzeń, obliczono, na podstawie pary problemów programowania parametrycznego, skojarzoną funkcję przynależności wykorzystując zasadę rozszerzenia. Na zakończenie przedstawiono studium przypadku, w którym proponowane podejście zastosowano do analizy systemu hydraulicznego centrum obróbkowego CNC.

Słowa kluczowe: *Analiza drzewa uszkodzeń, dynamiczne drzewo uszkodzeń, liczba rozmyta, rozmyty model Markowa, programowanie parametryczne.*

Introduction

Fault tree analysis (FTA) is a logical and graphic method being widely used to evaluate the reliability of complex engineering systems from both qualitative and quantitative perspectives. Fault tree provides a graphical representation of combinations of component failures leading to an undesired system failure [22, 25].

Fault tree (FT) was first introduced in 1961 by H. A. Watson of Bell telephone laboratories in connection with a U.S. Air Force contract to study the minuteman missile launch control system [5]. In the 1965 safety symposium, sponsored by the University of Washington and the Boeing company, several papers were presented, which expounded the virtues of fault-tree analysis [21]. Since then, a variety of methods for modeling and evaluating the complex system reliability via FTA have been reported [1, 2, 4, 6].

However, in many situations, the behaviour of components in a complex system and their interactions, such as failure priority, sequentially dependent failures, functional dependent failures, and dynamic redundancy management, cannot be adequately addressed by traditional FT due to its limited modeling capability. Several approaches have been proposed to overcome these difficulties. Dugan et al. [9, 10, 16] introduced a modularization method to identify the independent sub-trees with dynamic gates. Markov models were used to solve these dynamic fault trees. Amari et al. [3] proposed a numerical integration technique to solve dynamic gates without converting them to Markov models. By using the probability distribution and conditional probability distribution of the basic events, this method can accurately assess the fault tree behaviour with dynamic gates and repeated basic events. Bobbio et al. [7-8] proposed a Bayesian network-based method to further reduce the complexity of solving dynamic FTs based on a state-space approach.

The aforementioned state-based approaches are capable of evaluating the reliability of complex systems either qualitatively or quantitatively. We can obtain the accurate failure probability value of the top event by using these state-based fault tree analysis methods. When performing a system reliability evaluation, both operation states and failure states of components are generally assumed to be known. That is to say, components are assumed in either functioning state or failed state, and the probability for being in each individual state can be determined in advance. Actually, this is not always the case in many real engineering systems due to two main reasons:

(1) The states of components/systems often deteriorate over time, so failures of these components/systems may not occur at a certain point in time. Sometimes, we cannot exactly identify the state of a component or a system due to various kinds of uncertain factors, such as inaccurate measurements and human errors. In addition, ambiguity of system and component behaviour, and the dynamic operating environment of a system introduces additional difficulties in estimating the exact failure probabilities of basic events.

(2) Obtaining large and accurate failure data is costly, difficult, or even impossible for many real and complex systems. This is true especially for those systems with components whose failure rates are very low and/or with new designs. In these situations, it is not realistic or possible to represent the component failure behaviour using crisp values.

Fuzzy set theory proposed by Zadeh [23-24] has shown to be a useful methodology to cope with these cases where subjective judgement or estimation of an individual plays a vital and useful role in dealing with the ambiguity or uncertainty. Many papers have been published to incorporate the fuzzy set theory into the fault tree analysis for reliability analysis. Tanaka et al. [20] proposed an enumeration approach to estimate the cut sets of FT for which the trapezoidal fuzzy number is used to represent failure probabilities of events. Singer [19] used triangular fuzzy numbers to substitute the exact probability value as a representation of failure probabilities for basic events and top events. Based on the extension principle, Misra and Weber [17], Liang and Wang [13], described the fuzzy arithmetic operations for fault tree analysis. To avoid uncertainty in probabilistic risk assessment, Singer [19], Lai et al. [12] and Sawyer [18] introduced fuzzy set theory into safety and reliability modeling process. Based on posbist reliability theory, posbist fault tree analysis of coherent systems was proposed by Huang et al. [11]. In their approach, event failure behaviour is characterized in the context of possibilistic measures, and the structure function of the posbist fault tree of a coherent system is defined.

In this paper, triangular fuzzy membership functions are used to describe the vagueness of quantification of failure probability for basic events. Based on a fuzzy transition rate matrix, a fuzzy Markov model is introduced to capture dynamic behaviour of systems. Finally, a numerical example is provided to illustrate the application of the proposed method.

2. Dynamic fault tree

2.1. Dynamic gates

A major disadvantage of the traditional FTA is its inability to capture sequence dependencies in the system while still allowing an analytic solution [9-10]. To overcome this difficulty, Dugan proposed a new reliability analysis method called Dynamic Fault Tree Analysis (DFTA) by introducing several dynamic gates to describe the dynamic behaviours of these systems [9, 10, 16]. There are four major basic dynamic gates which will be elaborated in follows.

2.1.1. Priority-AND gate (PAND gate)

The PAND gate has two inputs, A and B, both of which could be a basic event or the output of other logic gates. The PAND gate reaches a failure state if all its input components have failed in a pre-assigned order (generally from left to right in a graphical notation).

2.1.2. Functional dependency gate (FDEP gate)

The FDEP gate frequently includes one trigger input (either a basic event or a output of another gate) and one or more dependent events. The dependent events are functionally relying on the trigger event. When the trigger event occurs, the dependent basic events are forced to occur.

2.1.3. Sequence enforcing gate (SEQ gate)

The SEQ gate forces its inputs to fail in a particular order. It never happens if the failure sequence takes place in different orders. While the SEQ gate allows events to occur only in a pre-specified order and state that a different failure sequence can never take place, the PAND gate does not impose such a strong assumption: it simply detects the failure order and a failure triggered upon the match with the order.

2.1.4. Spare gates

The spares often include one principal component that can be substituted by one or more backups that have the same function with the principal one. The Spare gate is classified into three types of backups, i.e., Cold Spare (CSP), Warm Spare (WSP) and Hot Spare (HSP). Suppose λ being the failure rate of a component, and then the failure rate alters to $\alpha\lambda$ while being used as a spare. If $\alpha = 0$, the spare is a CSP and $\alpha = 1$, the spare is a HSP, otherwise it's a WSP for the case where $0<\alpha<1$.

The four different categories of dynamic gates are enumerated in Table 1 with their input information, failure criteria, and corresponding symbols.

2.2. Markov model

In a dynamic fault tree, the occurrence of a top event depends not only on the combination of component failures, but also on the sequence of occurrences of these events. Thus, the Markov model has been used as a quantitative method to model the failure process and evaluate system reliability.

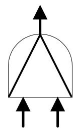
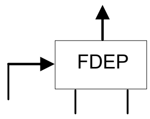
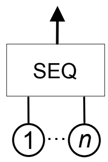
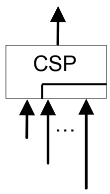
Let T be an infinite real set, $t \in T$, then $\{X(t), t \in T\}$ is called a stochastic process. For each $t \in T$, $X(t)$ is a random variable. A typical continuous stochastic process $\{X(t), t \in T\}$ is called a Markov process if its conditional probability satisfies the relation

$$\begin{aligned} &P\{X(t_n) = x_n \mid X(t_1) = x_1, X(t_2) = x_2, \cdots, X(t_{n-1}) = x_{n-1}\} \\ &= P\{X(t_n) = x_n \mid X(t_{n-1}) = x_{n-1}\} \end{aligned} \tag{1}$$

where $x_i \in S$, S is the state space of the stochastic process and $t_1 < t_2 < \cdots < t_{n-1} < t_n$. This memory-less characteristic means that the probability of this stochastic process being in state x_i at time t_i depends only on the state at time t_{i-1} and is independent of the state at time $t_i (i = 1, 2, \cdots, n-2)$.

The occurrence of component failure is a frequent stochastic process which can be represented by certain types of probability distribution functions. In a dynamic system, the failure process of the system can be represented by a Markov process. Suppose that the system has *n* states $s_i (i = 1, 2, \cdots, n)$, $s_i \in S$, and S is the state space of the Markov process $\{S(t),\ t \geq 0\}$. The transition rate from state i to state j is denoted by $\lambda_{i,j}$. Then the failure process of the system can be represented by a transition diagram shown in Fig. 1.

Table 1. Dynamic gates and failure mechanism

Dynamic gate	Input events information	Failure criteria	Symbol
PAND Gate	The PAND Gate has two inputs, A and B, both of which can be basic events or outputs of other logic gates.	The output of this gate is true if both inputs have occurred, and A occurred before B.	
FDEP Gate	The FDEP Gate has a trigger event and multiple dependent basic events.	If the trigger event occurs, all the dependent events occur subsequently, and the output becomes true.	
SEQ Gate	The SEQ Gate has multiple inputs	The sequence-enforcing gate (SEQ) forces its inputs to occur in a particular order. If all the inputs occur, the output is true.	
Spare Gate	The Spare has one primary input and one or more alternate inputs	If the primary unit fails, the first alternate component begins to function, till all the replacements fail, the output becomes true.	

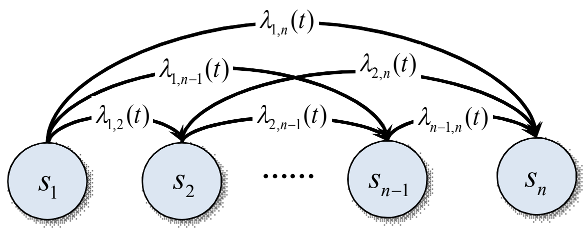


Fig. 1. A sample state transition diagram

Let $P_i(t),\ i=1,2,\cdots,n$ be the probability of the system being on state $s_i(i=1,2,\cdots,n)$, the differential equations for the aforementioned dynamic process take the form as follows:

$$\begin{cases} \dfrac{dp_1(t)}{dt} = -p_1(t)\sum\limits_{j=2}^{n}\lambda_{1,j} \\ \dfrac{dp_i(t)}{dt} = \sum\limits_{j=1}^{i-1} p_j(t)\lambda_{j,i} - \sum\limits_{j=i+1}^{n} p_i(t)\lambda_{i,j},\ \ 1<i<n,\ t\geq 0 \\ \dfrac{dp_n(t)}{dt} = \sum\limits_{j=1}^{n-1} p_j(t)\lambda_{j,n} \end{cases} \tag{2}$$

Solving the equations with the initial condition:

$$\begin{cases} P_1(0)=1, \\ P_i(0)=0, i=2,\cdots,n \end{cases}$$

we can obtain the probability value $P_n(t)$, which is also the failure probability of the top event corresponding to the Markov state transition diagram.

3. Fuzzy set theory

3.1. Fuzzy set and fuzzy number

During the evaluation of reliability of complex engineering systems, there exist two kinds of uncertainties, i.e. aleatory uncertainty and epistemic uncertainty. Zadeh [23-24] proposed a set of systematic mathematical theories, namely fuzzy set theory, which can deal with fuzzy characteristics of uncertainty in real engineering systems.

Given a universal set U, for a set $\tilde{A}$, and for each u, there exits a real number $\mu_{\tilde{A}}(u)\in[0,1]$ that corresponds to u, which represents the degree of u belonging to $\tilde{A}$. We call the set $\tilde{A}$ a fuzzy set, and the value $\mu_{\tilde{A}}(u)$ the membership degree of u to $\tilde{A}$.

$$\mu_{\tilde{A}}:\ U\to[0,1]$$
$$u\mapsto\mu_{\tilde{A}}(u)$$

For a fuzzy set $\tilde{A}$, it becomes a fuzzy number if it is a normal as well as a convex fuzzy set. Triangular fuzzy number, normal fuzzy number, and trapezoidal fuzzy number are among the mostly used fuzzy numbers.

A typical triangular fuzzy number $\tilde{A}(a,b,c)$ can be defined by its membership function as follows. Its graphic representation is shown in Fig. 2.

$$\mu_{\tilde{A}} = \begin{cases} \dfrac{x-a}{b-a}, & a\leq x<b \\ 1\ \ , & x=b \\ \dfrac{x-c}{b-c}, & b<x\leq c \\ 0\ \ , & \text{otherwise} \end{cases} \tag{3}$$

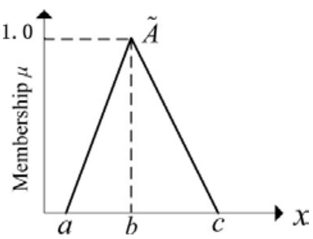


Fig. 2. Membership function of the triangular fuzzy number

3.2. Extension principle

The concept of fuzzy set and fuzzy number proposed by Zadeh [23-24] provides a means to represent and quantify fuzzy information. Besides, Zadeh introduced the extension principle for fuzzy operations between fuzzy numbers.

Given $\tilde{X}_i(i=1,2,\cdots,n)$ are fuzzy numbers corresponding to universal set $R_i(i=1,2,\cdots,n)$, respectively. $x_i\in R_i(i=1,2,\cdots,n)$ are the variables associated with each fuzzy number $\tilde{X}_i(i=1,2,\cdots,n)$. $f(x_1, x_2,...,x_n)$ is a function that maps the variables $x_i(i=1,2,\cdots,n)$ to a variable $y(y\in R)$. Then we can induce (generate) a fuzzy number

$\tilde{Y}$ from fuzzy numbers $\tilde{X}_i (i=1,2,\cdots,n)$ by function $f(x_1, x_2,...,x_n)$. The membership function of $\tilde{Y}$ can be obtained by the extension principle shown as follow:

$$u_{\tilde{Y}}(y) = \sup_{\substack{x_i \in R_i (i=1,2,\cdots,n) \\ y=f(x_1,x_2,\cdots,x_n)}} \min(u_{\tilde{X}_1}(x_1), u_{\tilde{X}_2}(x_2), \cdots, u_{\tilde{X}_n}(x_n)) \quad (4)$$

According to the extension principle, the interval of the α-cut of the fuzzy number $\tilde{Y}$ is given by:

$$\begin{aligned} \tilde{Y}_\alpha(y) &= [\min_{1 \le i \le n} f(x; \mu_{\tilde{x}_i}(x_i) \ge \alpha), \max_{1 \le i \le n} f(x; \mu_{\tilde{x}_i}(x_i) \ge \alpha)] \\ &= [\tilde{Y}_\alpha{}^L, \tilde{Y}_\alpha{}^U] \end{aligned} \quad (5)$$

Thus, the lower and upper bounds of $\tilde{Y}$ can be obtained by following a pair of parametric programming problems

$$\begin{aligned} &\tilde{Y}_a^L = \min f(x_1, x_2, \cdots, x_n) \\ &\text{subject to} \\ &\tilde{x}_{1a}^L \le x_1 \le \tilde{x}_{1a}^U \\ &\tilde{x}_{2a}^L \le x_2 \le \tilde{x}_{2a}^U \\ &\quad \vdots \\ &\tilde{x}_{na}^L \le x_n \le \tilde{x}_{na}^U \end{aligned} \quad (6)$$

$$\begin{aligned} &\tilde{Y}_a^U = \max f(x_1, x_2, \cdots, x_n) \\ &\text{subject to} \\ &\tilde{x}_{1a}^L \le x_1 \le \tilde{x}_{1a}^U \\ &\tilde{x}_{2a}^L \le x_2 \le \tilde{x}_{2a}^U \\ &\quad \vdots \\ &\tilde{x}_{na}^L \le x_n \le \tilde{x}_{na}^U \end{aligned} \quad (7)$$

We can easily obtain the intervals at different α-cut levels by the extension principle.

4. Fuzzy dynamic fault tree (FDFT)

Combining the Markov model with the fuzzy set theory, we propose a reliability analysis method called Fuzzy Dynamic Fault Tree (FDFT) to estimate the reliability of systems having dynamic characteristics and fuzzy uncertainty.

The fuzzy Markov model corresponds to the Dynamic Fault Tree (DFT) with *n* states $s_i, 1 \le i \le n$, and the crisp state transition rate $\lambda_{i,j}$ of Markov model is replaced by fuzzy state transition rate $\tilde{\lambda}_{i,j}$ due to difficulty of estimating accurate values. Then the fuzzy state transition rate matrix $\tilde{A}$ is given as follows:

$$\tilde{A} = \left(\tilde{\lambda}_{i,j}\right) = \begin{pmatrix} \tilde{\lambda}_{1,1} & \tilde{\lambda}_{1,2} & \cdots & \tilde{\lambda}_{1,n} \\ \tilde{\lambda}_{2,1} & \tilde{\lambda}_{2,2} & \cdots & \tilde{\lambda}_{2,n} \\ \vdots & \vdots & \cdots & \vdots \\ \tilde{\lambda}_{n,1} & \tilde{\lambda}_{n,2} & \cdots & \tilde{\lambda}_{n,n} \end{pmatrix} \quad (8)$$

Thus, the state transition diagram is given as Fig. 3.

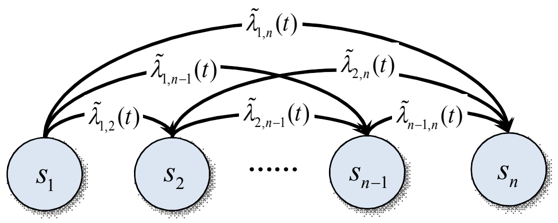


Fig. 3 The fuzzy state transition diagram for a non-repairable system

As a result, the differential equations with the fuzzy transition rate take the form of:

$$\begin{cases} \dfrac{d\tilde{p}_1(t)}{dt} = -\tilde{p}_1(t) \sum\limits_{j=2}^{n} \tilde{\lambda}_{1,j} \\ \dfrac{d\tilde{p}_i(t)}{dt} = \sum\limits_{j=1}^{i-1} \tilde{p}_j(t)\tilde{\lambda}_{j,i} - \sum\limits_{j=i+1}^{n} \tilde{p}_i(t)\tilde{\lambda}_{i,j}, \quad 1 < i < n, \ t \ge 0 \\ \dfrac{d\tilde{p}_n(t)}{dt} = \sum\limits_{j=1}^{n-1} \tilde{p}_j(t)\tilde{\lambda}_{j,n} \end{cases} \quad (9)$$

To solve the differential equations, the Laplace-Stieltjes transform can be used with the initial condition: $\tilde{p}_1(0) = 1, \tilde{p}_i(0) = 0 (i \ne 1)$.

The corresponding linear equations take the form as follows:

$$\begin{cases} s\tilde{p}_1(s) - 1 = -\tilde{p}_1(s) \sum\limits_{i=2}^{n} \tilde{\lambda}_{1,i} \\ s\tilde{p}_i(s) = \sum\limits_{j=1}^{i-1} \tilde{p}_j(s)\tilde{\lambda}_{j,i} - \sum\limits_{j=i+1}^{n} \tilde{p}_i(s)\tilde{\lambda}_{i,j}, \quad 1 < i < n \\ s\tilde{p}_n(s) = \sum\limits_{j=1}^{n-1} \tilde{p}_j(s)\tilde{\lambda}_{j,n} \end{cases} \quad (10)$$

After solving the linear equations to obtain $\tilde{p}_n(s)$, and the inverse Laplace-Stieltjes transform can be used to solve $\tilde{p}_n(t)$. Then the interval of the probability of $\tilde{p}_n(t)$ can be solved by the extension principle mentioned in Section 3.

5. Reliability analysis for CNC via FDFT

5.1. Brief introduction of CNC hydraulic systems

A complete hydraulic system is mainly composed of five parts: power components, control components, executive components, auxiliary components, and hydraulic oil. The studied system here is composed of four circuits, i.e. spindle balancing circuit, spindle releasing circuit, C axle clamping and releasing circuit, and D axle clamping and releasing circuit. In this section the Spindle balancing circuit is analysed by using FDFT model. The configuration of the spindle is shown in Fig. 4. It is composed of a tank, three filters, one hydraulic gear pump, three shut-off valves, check valve, one pressure relief valve, two pressure gauges, one pressure relay, two overflow valves, one cylinder, and one power accumulator.

Its operation principle can be described as follows. The oil is pumped from the tank to the main oil line through the control valves aforementioned. When the pressure value exceeds 140 bar, the oil will flow back to the tank through the overflow valve to reduce the pressure down to 140 bar. The pressure will drop to 65 bar through the pressure relief valve. The power accumulator can be accumulated when the pressure is over 50 bar. The operation of the pump will be blackout by the pressure delay when the pressure of the main circuit is over 60 bar. Afterwards, it will start again until the pressure value drops below 55 bar.

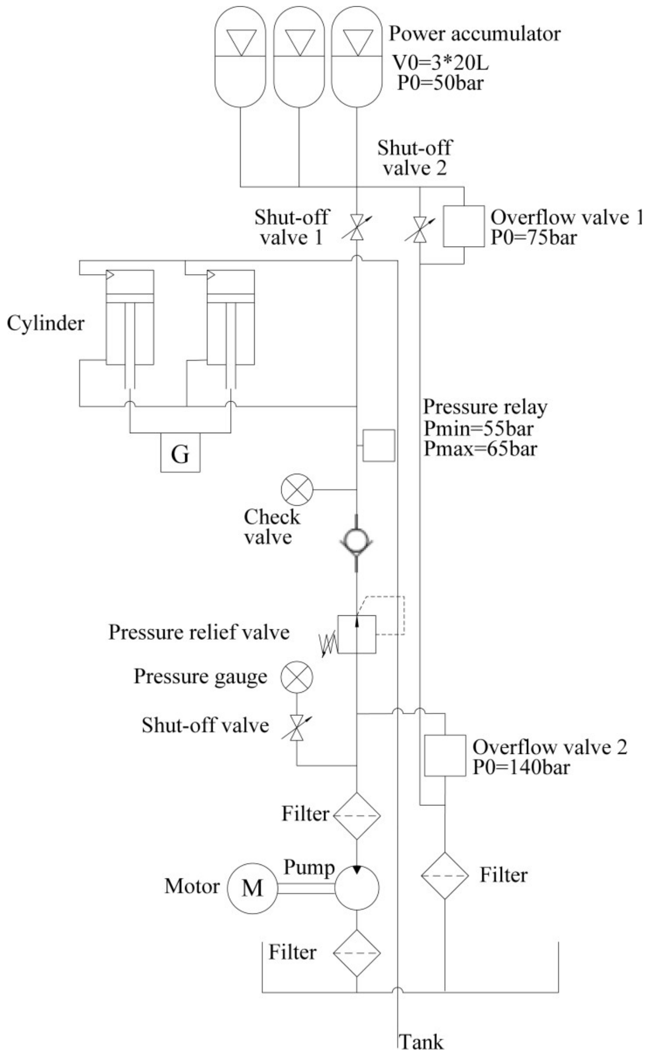


Fig. 4 Spindle carrier balance circuit of CNC machining center

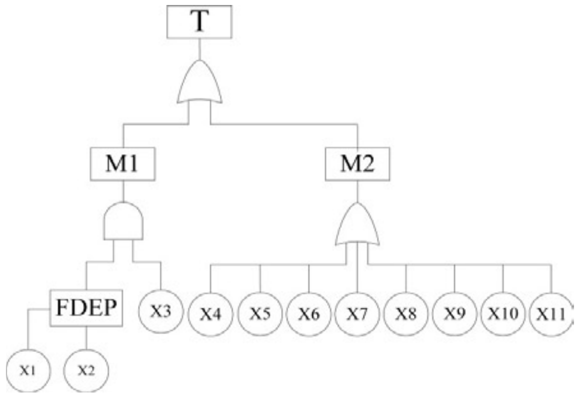


Fig. 5 Dynamic fault tree of the hydraulic system

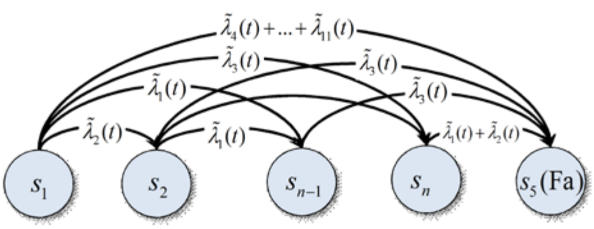


Fig. 6 State transition diagram of the spindle circuit of hydraulic system`

5.2. Construction of dynamic fault tree of spindle balance circuit

The event of the insufficiency of pressure in the balance circuit is considered as the top event in the following analysis. The filter failure and pressure gauge failure are ignored due to their low failure rate. The basic events are enumerated as follows.

x_1: pressure relay failure; x_2: pump failure; x_3: power accumulator failure; x_4: tank failure; x_5: check valve failure; x_6: cylinder failure; x_7: shut-off valve 1 failure; x_8: shut-off valve 2 failure; x_9: pressure relief valve failure; x_{10}: overflow valve 1 failure; x_{11}: overflow valve 2 failure. The dynamic fault tree is shown in Fig. 5.

5.3. Quantitative assessment of FDFT

According to the FDFT shown in Section 5.2 and the basic failure data shown in Table 2, the reliability of the spindle in hydraulic system is analysed based on FDFT. The failure rate of each component is represented by triangular fuzzy numbers shown in Table 2. The DFT is transformed into the fuzzy Markov model shown in Fig. 6.

The corresponding fuzzy state transition rate matrix is as follows:

$$\tilde{A}=\begin{pmatrix} -\sum_{i=1}^{11}\tilde{\lambda}_i & \tilde{\lambda}_2 & \tilde{\lambda}_1 & \tilde{\lambda}_3 & \sum_{i=4}^{11}\tilde{\lambda}_i \\ 0 & -\tilde{\lambda}_1-\tilde{\lambda}_3 & \tilde{\lambda}_1 & 0 & \tilde{\lambda}_3 \\ 0 & 0 & -\tilde{\lambda}_3 & 0 & \tilde{\lambda}_3 \\ 0 & 0 & 0 & -\tilde{\lambda}_1-\tilde{\lambda}_2 & \tilde{\lambda}_1+\tilde{\lambda}_2 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

The dynamic differential equations take the form:

$$\begin{cases} \dfrac{d\tilde{p}_1(t)}{dt}=-\tilde{p}_1(t)\sum_{i=1}^{11}\tilde{\lambda}_i \\ \dfrac{d\tilde{p}_2(t)}{dt}=\tilde{p}_1(t)\tilde{\lambda}_2-\tilde{p}_2(t)(\tilde{\lambda}_1+\tilde{\lambda}_3) \\ \dfrac{d\tilde{p}_3(t)}{dt}=\tilde{p}_1(t)\tilde{\lambda}_1+\tilde{p}_2(t)\tilde{\lambda}_1-\tilde{p}_3(t)\tilde{\lambda}_3 & t\geq 0 \\ \dfrac{d\tilde{p}_4(t)}{dt}=\tilde{p}_1(t)\tilde{\lambda}_3-\tilde{p}_4(t)(\tilde{\lambda}_1+\tilde{\lambda}_2) \\ \dfrac{d\tilde{p}_5(t)}{dt}=\tilde{p}_1(t)\sum_{i=4}^{11}\tilde{\lambda}_i+(\tilde{p}_2(t)+\tilde{p}_3(t))\tilde{\lambda}_3+\tilde{p}_4(t)(\tilde{\lambda}_1+\tilde{\lambda}_2) \end{cases}$$

Table 2. Basic event and failure rate

Basic event	Fuzzy failure rate $\tilde{\lambda}(t)$ ($\times 10^{-6}$)	Basic event	Fuzzy failure rate $\tilde{\lambda}(t)$ ($\times 10^{-6}$)
x_1	(0.0425,0.0500,0.0575)	x_7	(7.2250,8.5000,9.7750)
x_2	(11.4750,13.5000,15.5250)	x_8	(7.2250,8.5000,9.7750)
x_3	(6.1200,7.2000,8.2800)	x_9	(1.8190,2.1400,2.4610)
x_4	(1.2750,1.5000,1.7250)	x_{10}	(4.8450,5.7000,6.5550)
x_5	(4.2500,5.0000,5.7500)	x_{11}	(4.8450,5.7000,6.5550)
x_6	(0.0068,0.0080,0.0092)		

with the initial conditions $\tilde{p}_1(0)=1$ and $\tilde{p}_i(0)=0\ (1<i\le 5)$. The Laplace-Stieltjes transform can be used to solve the differential equations. After the transform, the differential equations become a set of linear equations as follows:

$$\begin{cases} s\tilde{p}_1(s)-1=-\tilde{p}_1(s)\sum_{i=1}^{11}\tilde{\lambda}_i \\ s\tilde{p}_2(s)=\tilde{p}_1(s)\tilde{\lambda}_2-\tilde{p}_2(s)(\tilde{\lambda}_1+\tilde{\lambda}_3) \\ s\tilde{p}_3(s)=\tilde{p}_1(s)\tilde{\lambda}_1+\tilde{p}_2(s)\tilde{\lambda}_1-\tilde{p}_3(s)\tilde{\lambda}_3 \\ s\tilde{p}_4(s)=\tilde{p}_1(s)\tilde{\lambda}_3-\tilde{p}_4(s)(\tilde{\lambda}_1+\tilde{\lambda}_2) \\ s\tilde{p}_5(s)=\tilde{p}_1(s)\sum_{i=4}^{11}\tilde{\lambda}_i+(\tilde{p}_2(s)+\tilde{p}_3(s))\tilde{\lambda}_3+\tilde{p}_4(s)(\tilde{\lambda}_1+\tilde{\lambda}_2) \end{cases}$$

Then, the fuzzy probability of state s_5 can be obtained as

$$\tilde{p}_5(s)=\frac{1}{s}-\frac{\tilde{\lambda}_3}{(\tilde{\lambda}_3+\sum_{i=4}^{11}\tilde{\lambda}_i)\times(s+\tilde{\lambda}_1+\tilde{\lambda}_2)} -\frac{\tilde{\lambda}_1+\tilde{\lambda}_2}{(\tilde{\lambda}_1+\tilde{\lambda}_2+\sum_{i=4}^{11}\tilde{\lambda}_i)\times(s+\tilde{\lambda}_3)} +\frac{(\tilde{\lambda}_1\times\tilde{\lambda}_3+\tilde{\lambda}_2\times\tilde{\lambda}_3-(\sum_{i=4}^{11}\tilde{\lambda}_i)^2}{(\tilde{\lambda}_3+\sum_{i=4}^{11}\tilde{\lambda}_i)\times(\tilde{\lambda}_1+\tilde{\lambda}_2+\sum_{i=4}^{11}\tilde{\lambda}_i)\times(s+\sum_{i=1}^{11}\tilde{\lambda}_i)}$$

By taking the inverse Laplace-Stieltjes transform, the fuzzy probability of state s_5 can be derived as a function of t.

$$\tilde{p}_5(t)=1-\frac{\tilde{\lambda}_3}{\tilde{\lambda}_3+\sum_{i=4}^{11}\tilde{\lambda}_i}\times\exp(-(\tilde{\lambda}_1+\tilde{\lambda}_2)\times t) -\frac{\tilde{\lambda}_1+\tilde{\lambda}_2}{\tilde{\lambda}_1+\tilde{\lambda}_2+\sum_{i=4}^{11}\tilde{\lambda}_i}\times\exp(-\tilde{\lambda}_3\times \mathrm{t}) +\frac{(\tilde{\lambda}_1\times\tilde{\lambda}_3+\tilde{\lambda}_2\times\tilde{\lambda}_3-(\sum_{i=4}^{11}\tilde{\lambda}_i)^2)\times\exp(-(\tilde{\lambda}_1+\tilde{\lambda}_2+\tilde{\lambda}_3+\sum_{i=4}^{11}\tilde{\lambda}_i)\times \mathrm{t})}{(\tilde{\lambda}_3+\sum_{i=4}^{11}\tilde{\lambda}_i)\times(\tilde{\lambda}_1+\tilde{\lambda}_2+\sum_{i=4}^{11}\tilde{\lambda}_i)}$$

Using Eqs. (4)-(7), the membership of fuzzy probability of state s_5 at different cut level α can be calculated as shown in Figs.7-9.

The membership function of fuzzy failure probability presented in Fig.7 is obtained by Eqs. (6) and (7) at different α-cut level at time $t=5000$ hours. The median value of fuzzy failure probability is 0.1631 at time $t=5000$ hours. The membership functions of fuzzy failure probability at time $t=10000$ and $t=15000$ are obtained by the same way with the median value 0.2892 and 0.3875 shown in Fig.8 and Fig.9

The reliability curve is presented in Fig.10. The blue one is obtained by the fuzzy failure rates corresponding to α =0. The red one which falls into the blue one is obtained by the crisp failure rates corresponding to α =1.

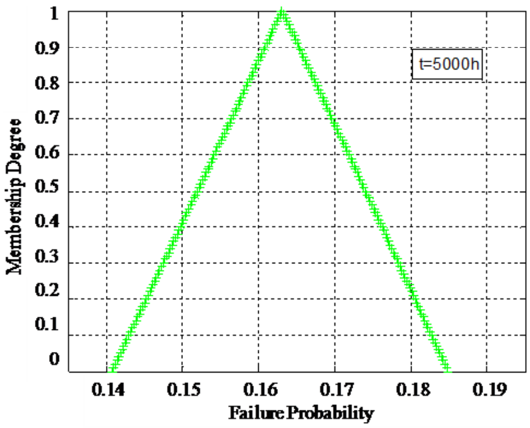


Fig.7 The membership of $\tilde{p}_5$ *at* $t=5000$ h

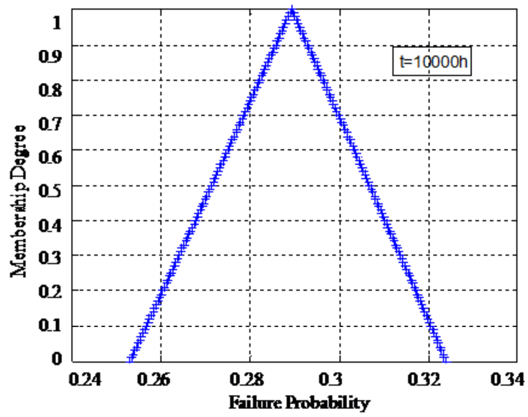


Fig.8 The membership of $\tilde{p}_5$ *at* $t=10000$ h

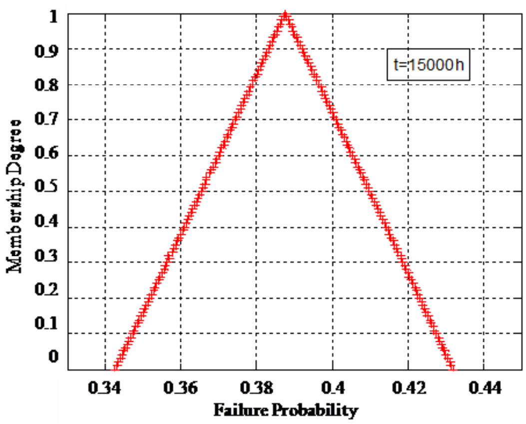


Fig.9 The membership of $\tilde{p}_5$ *at* $t=15000$ h

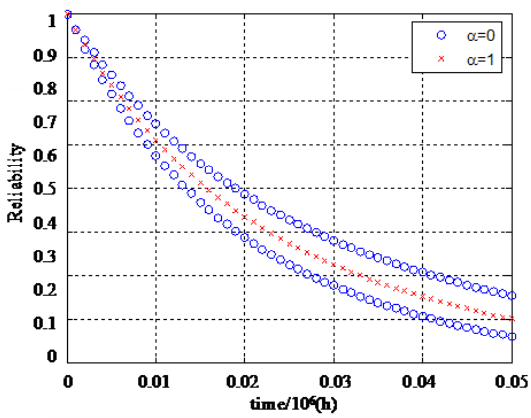


Fig.10 The fuzzy reliability of the system with α = 0 and α = 1

6. Conclusion

This paper applied the fuzzy Markov model to evaluate the reliability of a complex mechanical system used in CNC machine centre. When modeling of the fault tree, the dynamic logic gates are used to capture the dynamic behaviour in the system. The Markov model and Laplace-Stieltjes transform are used to solve the fault tree model. Fuzzy set theory is shown to be quite effective in quantifying the information uncertainty in the system under study. Triangular fuzzy numbers and the extension principle are used to solve the Markov model with fuzzy uncertainty. The result shows that the proposed method is a promising approach to reliability analysis.

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Yan-Feng LI, Ph.D. candidate
Prof. Hong-Zhong HUANG, Ph.D.
Associate Prof. Yu LIU, Ph.D.
Ningcong XIAO, Ph.D. candidate
Associate Prof. Haiqing LI, Ph.D.
School of Mechanical, Electronic, and Industrial Engineering
University of Electronic Science and Technology of China
No. 2006, Xiyuan Avenue, West Hi-Tech Zone, Chengdu, Sichuan, 611731 P. R. China
E-mail: hzhuang@uestc.edu.cn