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Evidential Networks for Fault Tree Analysis with Imprecise Knowledge

Jianping Yang,[1,2] **Hong-Zhong Huang,**[1,*] **Yu Liu**[1] **and Yan-Feng Li**[1]

[1] School of Mechatronics Engineering, University of Electronic Science and Technology of China, Chengdu, Sichuan, China
[2] Unit 95389 of People's Liberation Army, Guilin, Guangxi, China

Abstract. Fault tree analysis (FTA), as one of the powerful tools in reliability engineering, has been widely used to enhance system quality attributes. In most fault tree analyses, precise values are adopted to represent the probabilities of occurrence of those events. Due to the lack of sufficient data or imprecision of existing data at the early stage of product design, it is often difficult to accurately estimate the failure rates of individual events or the probabilities of occurrence of the events. Therefore, such imprecision and uncertainty need to be taken into account in reliability analysis. In this paper, the evidential networks (EN) are employed to quantify and propagate the aforementioned uncertainty and imprecision in fault tree analysis. The detailed conversion processes of some logic gates to EN are described in fault tree (FT). The figures of the logic gates and the converted equivalent EN, together with the associated truth tables and the conditional belief mass tables, are also presented in this work. The new epistemic importance is proposed to describe the effect of ignorance degree of event. The fault tree of an aircraft engine damaged by oil filter plugs is presented to demonstrate the proposed method.

Keywords. Dempster-Shafer (D-S) evidence theory, fault tree analysis, imprecise probability, evidential networks.

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1 Introduction

High reliability and safety are desired by a great multitude for advanced engineered systems and products. With the increasing requirements on system complexity and performance, these quality attributes become even more important. In order to enhance the quality attributes of a system, many methods have been developed and proposed [1–3], and FTA is the one among these methods. FTA was first introduced to evaluate the Minuteman I Intercontinental Ballistic Missile (ICVM) Launch Control System by Watson at Bell Laboratories in 1962. It is a methodology to systematically describe the causes leading to a top event at different levels of detail down to the component's level [4]. A FT is a logical graph which consists of events and logic gates. Due to its extensive applications, great efforts have been made to investigate the FTA method in the literature. Contini [4] proposed a method to analyze large coherent fault trees in the cases where working memory is not sufficient to construct the Binary Decision Diagrams (BDD). Considering the dependencies among fault events in FTA, Dugan proposed the dynamic fault tree (DFA) [31]. Zineb [5] proposed an approach to filter the faults. This method has solved two problems. They are related to the filtering of false alarms, and the reduction in the size of the ambiguity of fault isolation related to the fault occurrence. Moreover, a set of new dynamic gates have been defined to translate the new dependencies and relationships. When a FT is extremely large and complex, the conventional BDD technology is infeasible. In order to solve this problem, Cristina [6] proposed a reduction process by using the information provided by a set of the most relevant minimal cut sets of the model. This method allows controlling the degree of reduction, and therefore impacts the simplification of the final quantification results. Liang [7] applied FTA to an underwater dry maintenance cabin. The weakest links of the system were identified and an effective preventive maintenance strategy was determined. In order to improve the prediction of the potential risk of coal and gas outburst events during the underground mining of thick and deep Chinese coal seams, Zhang [8] proposed a method coupling FTA and artificial neural network (ANN) models. The dominant influence on the potential occurrence of in-situ coal and gas outburst events mining was identified. The outline of constructing and applying a generic FTA model was presented based on the coal seam gas factors and the geological conditions that exist within the Huabei coalfield. Renjith [9] outlined the estimation of the probability of release of chlorine from storage and filling facility of chlor-alkali industry using FTA. An attempt had also been made to obtain the probability of chlorine release using expert elicitation and

* **Corresponding author:** Hong-Zhong Huang, School of Mechatronics Engineering, University of Electronic Science and Technology of China, Chengdu, Sichuan, 611731, China;
E-mail: hzhuang@uestc.edu.cn.

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proven fuzzy logic technique for Indian conditions. Sensitivity analysis has been performed to evaluate the percentage of each basic event that contributes to chlorine release. Two-dimensional fuzzy FTA (TDFFTA) has been proposed to balance the hesitation factor involved in expert elicitation. One can take the tolerances of the probability values of hazards and the estimation uncertainty of the system component failure rates or the probabilities of undesired event occurrence due to the lack of sufficient data. Ayhan [3] proposed a new FTA method based on the fuzzy set theory and applied to the spread mooring system. Choi [10] proposed a method to quantify the two sources of uncertainties based on a Monte Carlo simulation technique and estimate the probability of the discarded minimal cut sets and the sum of disjoint products approach complemented by the correction factor approach (CFA). The method provides a tool to accurately quantify the two uncertainties and estimate the top event probability and importance measures of large coherent fault trees. Remenyte-Prescott [11] proposed an alternative approach to overcome the limitation in the size of the final binary decision diagram (BDD) in FTA. The method constructed from each of the gate types were built and merged to represent a parent gate.

The aforementioned review indicates the limitations of the conventional FTA. However, little attention has been focused on the situation where the data is insufficient and imprecise, especially in early stage of product design. This may lead to the difficulty of estimating the failure rates of individual events or the probabilities of occurrence of events accurately [28–30]. Imprecision and uncertainty must be considered in the quantification of the probability of event occurrence. Carreras [12] encoded inherent uncertainty in input data by modeling such data in terms of intervals. Appropriate interval arithmetic was used to propagate the data standard FTs to generate the output distributions which reflect the uncertainty of the input data. Popescu [13] used the Dempster-Shafer (D-S) evidence theory to accommodate the imprecise or vague input data and showed how a pattern of false-negative can be observed. It is illustrated that the D-S evidence theory has a clear advantage over binary assignments in representing vagueness. Limbourg [14] used the D-S evidence theory which merges interval-based and probabilistic uncertainty modeling on a FTA in the automotive area. In this paper, the EN is employed to quantify and propagate the imprecision and uncertainty in FTA. The conversion process from FT to EN is introduced in details. The FT of aircraft engines damaged by oil filter plugs is analyzed and compared with the results from the EN.

The remainder of this paper is organized as follows. In Section 2, the D-S evidence theory and EN are briefly introduced. The conversion process from FT to EN is provided in Section 3. Section 4 describes the conversion procedure from FT to EN. The importance measure is represented in Section 5. Section 6 uses the FT of the aircraft engines damaged by oil filter plugs to illustrate the process of conversion. The conclusion is provided at last.

2 A Brief Introduction to D-S Evidence Theory and Evidential Networks

In this section, the basic concepts of Dempster-Shafer (D-S) evidence theory and evidential networks are briefly reviewed, and related functions, notations, and reasoning mechanism are introduced.

2.1 D-S Evidence Theory

The evidence theory, also called Dempster-Shafer evidence theory, is developed and expanded by Shafer [15] based on the innovative work of Dempster on the upper and lower bounds of belief assignment to hypothesis in ref. [16]. The D-S evidence theory may be interpreted as a generalization of probability theory where probabilities can be distributed to sets as opposed to mutually exclusive singletons [17]. It can distinguish the ignorance and uncertainty to the hypothesis by adopting the belief interval. Due to the flexibility of the basic axioms in evidence theory, no further assumptions are needed for quantifying the uncertain information of system [18].

2.1.1 The Frame of Discernment

The D-S evidence theory starts with defining the frame of discernment (FD). The FD is a finite nonempty exhaustive set of mutually exclusive possibilities, denoted by Θ, which includes all the elementary proposition of the problem:

$$\Theta = \{q_1, q_2, \ldots, q_n\}\,. \tag{1}$$

The power set of Θ consists of all the possible subsets, noted as 2^Θ. There are 2^n elements in the 2^Θ:

$$\begin{aligned}2^\Theta =&\{\emptyset,\{q_1\},\ldots,\{q_n\},\{q_1,q_2\},\{q_1,q_3\},\ldots,\{q_1,q_n\},\ldots,\\&\{q_{n-1},q_n\},\{q_1,q_2,q_3\},\ldots,\{q_1,q_2,\ldots,q_n\}\}\,.\end{aligned} \tag{2}$$

For example, if $\Theta = \{\{up\},\{down\}\}$ and $n = 2$, the power set is $2^\Theta = \{\emptyset,\{up\},\{down\},\{up,down\}\}$, where $\emptyset$ denotes the empty set.

2.1.2 The Basic Belief Assignment (BBA)

The basic belief assignment is a primitive of evidence theory, which is denoted by $m(A)$. The function $m(A)$ is a mapping: $m(A) : 2^{\Theta} \to [0, 1]$, and satisfies the following conditions:

$$m(\emptyset) = 0\,, \tag{3}$$

$$\sum_{A \in 2^{\Theta}} m(A) = 1\,, \tag{4}$$

$$0 \leqslant m(A) \leqslant 1 \quad A \in 2^{\Theta}\,, \tag{5}$$

where $m(A)$ indicates the precise probability in which the evidence corresponds to m supports proposition X. Other functions are evolved and ratiocinated based on the BBA which is a basic function in D-S evidence theory.

2.1.3 Belief Function (Bel)

A belief function is a mapping: $\mathrm{Bel} : 2^{\Theta} \to [0, 1]$

$$\mathrm{Bel}(A) = \sum_{B \subseteq A} m(B)\,, \tag{6}$$

where $\mathrm{Bel}(A)$ represents the total amount of probability that must be distributed among elements of A. It reflects the inevitability, signifies the total degree of belief of A, and constitutes a lower limit function on the probability of A [19]. For example:

$$\Theta = \{q_1, q_2, q_3\},$$
$$\mathrm{Bel}(\{q_1, q_2\}) = m(\{q_1\}) + m(\{q_2\}) + m(\{q_1, q_2\})\,.$$

$\mathrm{Bel}(A)$ can be obtained by BBA. Symmetrically, BBA can be obtained by $\mathrm{Bel}(A)$ through the Möbius transformation" [20]:

$$m(A) = \sum_{B \subseteq A} (-1)^{|A-B|} \mathrm{Bel}(B)\,, \tag{7}$$

where $A,\ B \in 2^{\Theta}$, and $|\cdot|$ denotes the cardinality function.

2.1.4 Plausibility Function (Pl)

A plausibility function (Pl) is a mapping: $\mathrm{Pl} : 2^{\Theta} \to [0, 1]$. It is defined as follows

$$\mathrm{Pl}(A) = 1 - \mathrm{Bel}(\overline{A}) = \sum_{A \cap B \neq \emptyset} m(B)\,, \tag{8}$$

where $\overline{A}$ is the negation of a hypothesis A. $\mathrm{Pl}(A)$ measures the maximal amount of probability that can be distributed among the elements in A. It describes the total degree of belief related to A and constitutes an upper limit function on the probability of A [19]. For example, $\Theta = \{q_1,\ q_2,\ q_3\}$, $\mathrm{Pl}(\{q_1, q_2, q_3\}) = m(\{q_1\}) + m(\{q_2\}) + m(\{q_3\}) + m(\{q_1, q_2\}) + m(\{q_1, q_3\}) + m(\{q_2, q_3\}) + m(\{q_1, q_2, q_3\})$.

In an analogous way, the basic probability assignment function can lead to the plausibility function using the following formula [20]:

$$m(A) = \sum_{B \subseteq A} (-1)^{|A|-|B|+1} \mathrm{Pl}(\overline{B})\,. \tag{9}$$

Moreover, the relationship of the belief function, plausibility function, and probability of hypothesis can be described as:

$$\mathrm{Bel}(A) \leqslant P(A) \leqslant \mathrm{Pl}(A)\,, \tag{10}$$

where $P(A)$ is the probability of the hypothesis A.

$[\mathrm{Bel}(A), \mathrm{Pl}(A)]$ is the posteriori confidence interval which expresses the uncertainty of A. When the ignorance to proposition X is decreased, the length of interval is diminished.

Probability interval $[\underline{P(A)}, \overline{P(\mathrm{A})}]$ is often regarded as a measure for modeling uncertainty. Probability interval can be directly transformed into the posteriori confidence interval [21]:

$$\underline{P(A)} = \mathrm{Bel}(A)\,, \tag{11}$$

$$\overline{P(\mathrm{A})} = \mathrm{Pl}(A)\,. \tag{12}$$

2.2 Evidential Networks

Evidential networks were originally proposed by Simon [21]. It is a directed acyclic graphs (DAG) which can deal with the aleatory, epistemic uncertainties in reliability engineering. It represents the conditional dependencies between variables in a description space integrating uncertainty as belief masses [24].

An EN is a DAG $G = ((N, A), M)$. (N, A) represents the graph, N is a set of nodes, A is a set of arcs, and M expresses the set of belief distributions that are distributed to each node [23]. For a root node, its priori belief mass table is defined. When a node is not a root node, its belief mass distribution is defined by a conditional belief mass table given the relations between the node and its parents [21].

Each conditional belief mass table defines the relation between the belief masses on the frame of variable discernment of each parent's nodes and the belief masses of the discernment frame of the child node [22]. Simultaneously, to compute the marginal belief mass distributions of each node in evidential networks, the inference algorithm is proposed by Simon [21]. This algorithm updates the marginal belief mass distributions on each node according to the additional evidence introduced into the evidential networks. More details can be found in refs [21, 23, 24].

3 Evidential Networks for Fault Tree Analysis

FTA is a logical and diagrammatic method to evaluate the probability of an accident resulting from sequences and combinations of faults and failure events in the processing of system design. It can be regarded as a special case of event tree analysis [27]. Conventionally, the precise value is adopted to represent the probabilities of the events. Due to the lack of sufficient data and imprecise knowledge in early stage of product design, it is often difficult to estimate accurately the failure rates of individual events or the probabilities of occurrence of events. The uncertainty needs to be taken into account [12]. Evidential networks can deal with aleatory and epistemic uncertainties in reliability engineering. Consequently, the conversion algorithm from FT to evidential networks should be detailed.

The basic assumption of the standard FTA is that events have binary states. In evidential networks, the corresponding frame of discernment is described as the following:

$$\Theta = \{\{up\}, \{down\}\}\,, \tag{13}$$

$$2^{\Theta} = \{\emptyset, \{up\}, \{down\}, \{up, down\}\}\,, \tag{14}$$

where $\{up\}$ means the occurrence of the event, $\{down\}$ means the non-occurrence of the event.

If the basic belief assignment of $\{up, down\}$ is equal to zero, FTA is a conventional probability reasoning method. If the basic belief assignment of $\{up, down\}$ is nonzero, the conventional method based on the probability theory cannot deal with this situation. This characterizes the ignorance on the real state of the event, and the event may be in the state $\{up\}$ or $\{down\}$. Using Eqs. (6), (8), (11), and (12), the posteriori confidence interval of the occurrence of event is gained. This describes the uncertainty of occurrence of an event in FT.

A FT includes two types of elements, event and logic gate. A logic gate represents the relation and causes among the events. The AND/OR gates are the two main gates. The AND gate denotes that an output event occurs if and only if all the input events occur. The OR gate delineates that an output event occurs if at least one of the input events occurs. Simon [21] shows the conversion of an AND/OR gate into equivalent nodes in an EN. For more details, readers are referred to [21].

In some situations, AND/OR cannot completely represent the relations and causes among the events, so other gates are adopted to model the FT. Consequently, the conversion of other gates to equivalent nodes in an EN should be investigated. In the remainder of this section, the conversions of EXCLUSIVE OR gate, EXCLUSIVE NOR gate, NOT OR gate, NOT AND gate, and Inhibit gate to EN are provided, respectively.

3.1 EXCLUSIVE OR Gate

EXCLUSIVE OR gate denotes that there is no output unless one and only one of the input events occurs. The input of this gate may be basic events or intermediate events. For example, there are two fault modes in electronic systems: the open-circuit fault and the short-circuit fault. These two fault modes cannot appear simultaneously. Consequently, the EXCLUSIVE OR gate must express the relation between the fault modes.

Due to the uncertainty of event occurrence probability, the conventional EXCLUSIVE OR gate cannot manipulate this situation. Evidential networks can be used to address this issue. Through reasoning, the EXCLUSIVE OR gate and its equivalent evidential networks model are presented in Figure 1. The truth tables of EXCLUSIVE OR gate in EN is shown in Table 1. Table 2 represents the conditional belief mass table. Ci (i =1, 2) denotes the state of the event, and E_1 corresponds to the state of the event and to the output of the gate. C1 and C2 are the inputs to the EXCLUSIVE OR gate. Bel($\mathrm{E}_1 = up$) is the lower limit belief function to the occurrence of the event and Pl($\mathrm{E}_1 = up$) is the upper limit plausibility function to the occurrence of the event. The logical algebraic expression of EXCLUSIVE OR gate is as follows:

$$\mathrm{E}_1 = \left(\mathrm{C1} \cap \overline{\mathrm{C2}}\right) \cup \left(\overline{\mathrm{C1}} \cap \mathrm{C2}\right)\,. \tag{15}$$

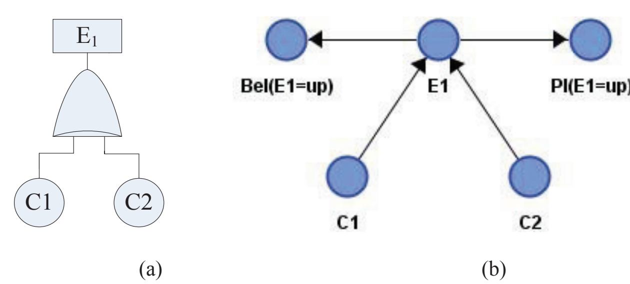


Figure 1. EXCLUSIVE OR gate and its equivalent evidential network.

EXCLUSIVE OR Gate	{Up}	{Down}	{Up, Down}
{Up}	{Down}	{Up}	{Up, Down}
{Down}	{Up}	{Down}	{Up, Down}
{Up, Down}	{Up, Down}	{Up, Down}	{Up, Down}

Table 1. Truth table of an EXCLUSIVE OR gate.

C1	C2	E_1		
		{Up}	{Down}	{Up, Down}
{Up}	{Up}	0	1	0
{Down}	{Up}	1	0	0
{Up, Down}	{Up}	0	0	1
{Up}	{Down}	1	0	0
{Down}	{Down}	0	1	0
{Up, Down}	{Down}	0	0	1
{Up}	{Up, Down}	0	0	1
{Down}	{Up, Down}	0	0	1
{Up, Down}	{Up, Down}	0	0	1

Table 2. Conditional belief mass table of an EXCLUSIVE OR gate.

3.2 EXCLUSIVE NOR Gate

EXCLUSIVE NOR gate is a combination EXCLUSIVE OR gate followed by a NOT gate. It denotes that there is output if the input events occur and do not occur simultaneously. When one and one of the input events occurs, there is no output. The input events may be basic and intermediate events. The output may be intermediate event or top event. The output only has one element.

In the conventional FT, the occurrence probability of input event of an EXCLUSIVE NOR gate does not consider the imprecise probability. In order to deal with this situation, the EN is adopted. The EXCLUSIVE NOR gate and its corresponding EN model are presented in Figure 2. The truth table of the EXCLUSIVE NOR gate in EN is shown in Table 3. Table 4 represents the conditional belief mass table. $Ci\,(i\ =1,\ 2)$ denotes the state of the event, and E_1 corresponds to the state of the event. If the state of one of the input events is {Up, Down}, the state of output event, E_1, is {Up, Down}. The logical algebraic expression of EXCLUSIVE NOR gate is following:

$$E_1 = \left(\overline{C1 \cap C2}\right) \cup (C1 \cap C2)\ . \tag{16}$$

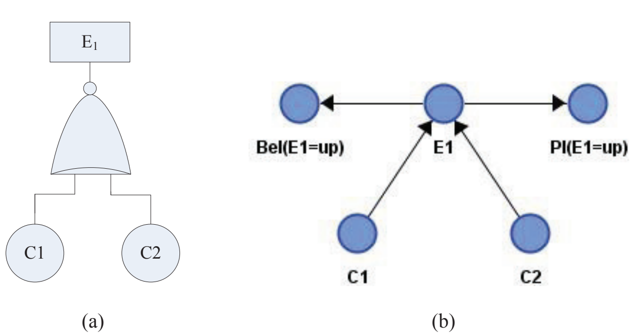


Figure 2. EXCLUSIVE NOR gate and its equivalent evidential network.

EXCLUSIVE NOR Gate	{Up}	{Down}	{Up, Down}
{Up}	{Up}	{Down}	{Up, Down}
{Down}	{Down}	{Up}	{Up, Down}
{Up, Down}	{Up, Down}	{Up, Down}	{Up, Down}

Table 3. Truth table of an EXCLUSIVE NOR gate.

C1	C2	E_1		
		{Up}	{Down}	{Up, Down}
{Up}	{Up}	1	0	0
{Down}	{Up}	0	1	0
{Up, Down}	{Up}	0	0	1
{Up}	{Down}	0	1	0
{Down}	{Down}	1	0	0
{Up, Down}	{Down}	0	0	1
{Up}	{Up, Down}	0	0	1
{Down}	{Up, Down}	0	0	1
{Up, Down}	{Up, Down}	0	0	1

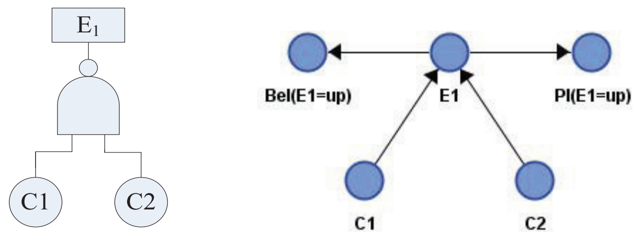
Table 4. Conditional belief mass table of an EXCLUSIVE NOR gate.

3.3 NOT AND Gate

NOT AND gate is a combination of AND gate and NOT gate. It denotes that an output event does not occur if and only if all the input events occur. The input events may be the basic events, intermediate events, or a combination of the basic events and intermediate events. NOT AND gate may have two or more than two input events and one output. The output event could be the intermediate events or the top event.

Taking into account the uncertainty of occurrence of the event, the conventional FTA has some limitations and EN can be adopted to deal with this situation. The conversion of the NOT AND gate to EN is shown in Figure 3. In Figure 3, the two input events are considered. The truth table of a NOT AND gate is presented in Table 5. If C1 and C2 occur, then the output event E_1 does not occur. When the state of C2 contains epistemic uncertainty and thus the event could be either {Up} or {Down}, the state of the output event E_1 could also be either {Up} or {Down}. The conditional belief mass table of a NOT AND gate is presented in Table 6 for the case with only two input events. When there are n input events, the logical algebraic expression of NOT AND gate is as follows:

$$E_1 = \overline{C1 \cap C2 \cap \cdots \cap Cn}\,. \tag{17}$$



(a) (b)

Figure 3. NOT AND gate and its equivalent evidential network.

NOT AND GATE	{Up}	{Down}	{Up, Down}
{Up}	{Down}	{Up}	{Up, Down}
{Down}	{Up}	{Up}	{Up}
{Up, Down}	{Up, Down}	{Up}	{Up, Down}

Table 5. Truth table of a NOT AND gate.

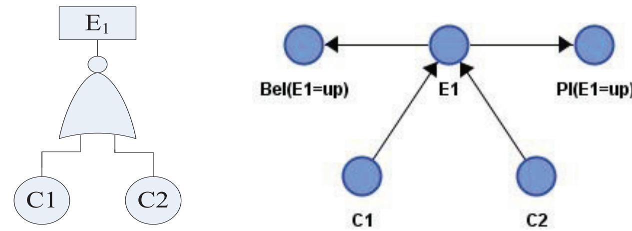
C1	C2	E_1		
		{Up}	{Down}	{Up, Down}
{Up}	{Up}	0	1	0
{Down}	{Up}	1	0	0
{Up, Down}	{Up}	0	0	1
{Up}	{Down}	1	0	0
{Down}	{Down}	1	0	0
{Up, Down}	{Down}	1	0	0
{Up}	{Up, Down}	0	0	1
{Down}	{Up, Down}	1	0	0
{Up, Down}	{Up, Down}	0	0	1

Table 6. Conditional belief mass table of a NOT AND gate.

3.4 NOT OR Gate

NOT OR gate is a combination of an OR gate and a NOT gate. It denotes that an output event occurs if and only if all the input events do not occur. The input events may be the basic events, intermediate events or a combination of these events. NOT OR gate may have two or more than two input events and one output. The output event could be either the intermediate events or the top event. The same as the analysis of the NOT AND gate, the conversion of the NOT OR gate to EN is presented in Figure 4. The truth table of a NOT OR gate and the conditional belief mass table of a NOT OR gate are expressed in Table 7 and Table 8, respectively. When there are n input events, the logical algebraic expression of NOT OR gate is as follows:

$$E_1 = \overline{C1 \cup C2 \cup \cdots \cup Cn}\,. \tag{18}$$



(a) (b)

Figure 4. NOT OR gate and its equivalent evidential network.

NOT OR GATE	{Up}	{Down}	{Up, Down}
{Up}	{Down}	{Down}	{Down}
{Down}	{Down}	{Up}	{Up, Down}
{Up, Down}	{Down}	{Up, Down}	{Up, Down}

Table 7. Truth table of a NOT OR gate.

C1	C2	E_1		
		{Up}	{Down}	{Up, Down}
{Up}	{Up}	0	1	0
{Down}	{Up}	0	1	0
{Up, Down}	{Up}	0	1	0
{Up}	{Down}	0	1	0
{Down}	{Down}	1	0	0
{Up, Down}	{Down}	0	0	1
{Up}	{Up, Down}	0	1	0
{Down}	{Up, Down}	0	0	1
{Up, Down}	{Up, Down}	0	0	1

Table 8. Conditional belief mass table of a NOT OR gate.

3.5 Inhibit Gate

Inhibit gate denotes that the output event occurs if all input events and an additional conditional event occur. The input of this gate may be basic events, intermediate events or a combination of these events. The output of this gate may be intermediate event or top event. However, the conditional event is special and independent. Inhibit gate may have multiple input events and multiple conditional events. The inhibit gate is logically equal to an "AND" gate when the conditional events are regarded as the input events. The truth table of this gate is equal to "AND" gate. The conversion of the inhibit gate to EN is presented in Figure 5 which includes the two input events C1 and C2, and one conditional event A. The corresponding belief mass table is represented in Table 9. When there are n input events and k conditional events, the logical algebraic expression of

C1	C2	A	E_1		
			{Up}	{Down}	{Up, Down}
{Up}	{Up}	{Up}	1	0	0
{Down}	{Up}	{Up}	0	1	0
{Up, Down}	{Up}	{Up}	0	0	1
{Up}	{Down}	{Down}	0	1	0
{Down}	{Down}	{Down}	0	1	0
{Up, Down}	{Down}	{Down}	0	1	0
{Up}	{Up, Down}	{Up, Down}	0	0	1
{Down}	{Up, Down}	{Up, Down}	0	0	1
{Up, Down}	{Up, Down}	{Up, Down}	0	0	1

Table 9. Conditional belief mass table of an inhibit gate.

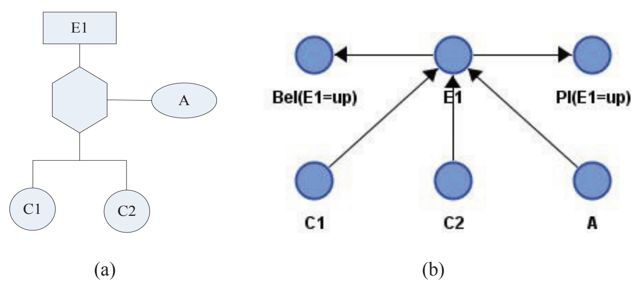


Figure 5. Inhibit gate and its equivalent evidential network.

inhibit gate is as follows:

$$E_1 = (C1 \cap C2 \cap \cdots \cap Cn) \cap (A1 \cap A2 \cap \cdots \cap Ak)\,. \quad (19)$$

4 The Description of Conversion Procedure from FT to EN

FT is a logical block diagram to describe the relationship of events. It adopts the events, logical gates, and transfer symbols to analyze the event causality form top-down. According to conversion method of the logic gates, FT can be mapped to EN models. The basic event, intermediate event, and top event in FT correspond to the parent node, the child node and the most top-level child node in EN respectively. The logical relationship among the events in FT corresponds to the arc in EN. FT is constructed from top-down, while EN is constructed from down-top. The detailed conversion procedure is described in the following steps:

1) Determine every event (basic event, intermediate event, and top event) and their logical relationship (logic gates) in FT;
2) Construct the truth table and conditional belief mass table of every logic gate according to Section 3;
3) Construct the node sets of EN. Each basic event in FT is converted into parent node in EN. According to Section 3, every logic gate is converted into child node. For the basic event which is repeated in EN, only a child node is used to represent the basic event Also, the one arc is only increased from parent node to child node;
4) Construct the arc between nodes in EN. According to Step 2 and Sept 3, the arc is constructed between parent child and child node.
5) Assign BPA to every child node corresponding to the basic event. Through the computation, the BPA of every child node can be attained.

Every node in EN is corresponding to the event in FT. The logical relationship of event in FT is reflected by the conditional BPA. Through the above steps, the FT can be converted into the EN which can deal with the imprecise probability.

5 Importance Measure

In FT, the effect of state change of every basic event to the state change of top event is different. The occurrence of some of the basic events will cause the occurrence of top event. Importance measure is adopted to describe the different effect of basic events. Importance measure is an indication of the contribution of the occurrence of basic event to the occurrence of the top event. Through the importance measure, the contribution ranking of effect of basic events to the top event can be attained. This is significant in identifying the weak parts in system design, improving the system design and reliability, providing guidance for preventive maintenance, and reasonable allocation of maintenance resources. Because of the different structures of FT, different logical relationships of event, and the different contributions of event, the unified importance measure can not be constructed. The two common importance measures, namely probability importance and structure importance, will be introduced in EN. Probability importance expressed the degree of change of incidence of top event when the state of basic event is from $\{down\}$ to $\{up\}$. In EN, the computation of probability importance is same to the conventional FT. The only difference is on using BPA in EN instead of the incidence in FT. The structure importance describes the important degree of the location of event in FT and is not related to the incidence of all of events. In EN, the computation of structure importance is the same as the one in the conventional FT.

Because the epistemic state {Up, Down} of event is considered in EN, the contribution of the occurrence of epistemic state of every basic event to the occurrence of epistemic state of top event must be evaluated. In order to describe the situation, the new importance measure, epistemic importance, can be defined as:

$$\begin{aligned} I_i^{un} &= 2m(E = \{\text{Up,Down}\}\,|Ci = \{\text{Up,down}\}) \\ &\quad - m(E = \{\text{Up,down}\}\,|Ci = \{\text{Down}\}) \\ &\quad - m(E = \{\text{Up,Down}\}\,|Ci = \{\text{up}\}), \end{aligned} \tag{20}$$

where $Ci, 1 \leqslant i \leqslant m$, represents the basic event, I_i^{un} expresses the epistemic importance of basic event, and Ci, E represents the top event.

Epistemic importance expresses the degree of change of belief of epistemic state of top event results from the state of basic event changing from certainty state to epistemic state. The larger this factor, the greater the impact on the epistemic state of the basic event to the top event. The epistemic importance is not related to the incidence of this event; however, it is related to its location in FT and the epistemic state of other basic event. Using Eq. (20), the epistemic importance of all of the basic events can be attained. This will be significant in decreasing the ignorance of event and constructing experimental Analysis.

6 An Example

The oil subsystem is one of the important parts of aero-engines. The fault of this system can lead to the damage of aero-engines and affect the safety of aircraft. In order to improve the system safety, reliability analysis of the oil subsystem is necessary. After a preliminary analysis, there are two major faults which seriously endanger the safety of aero-engines. One fault is aircraft engine damage resulting from oil starvation caused by the low inlet pressure of oil subsystem. Instructions are not given by oil pressure indicator, and warning signal is not generated by oil pressure warning system. The other fault is similar aircraft engine damage which happens when the oil filter is plugged and the oil pressure indicator does not give instructions [32].

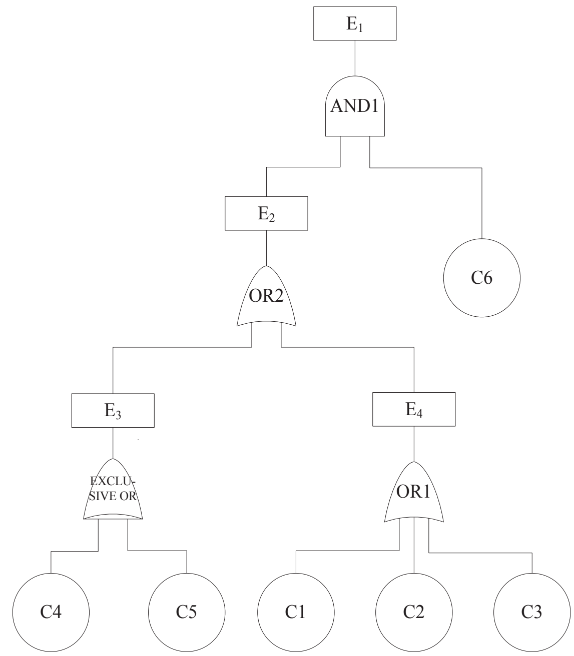


Figure 6. The fault tree of the aero-engine damage caused by the oil filter plug.

	C1	C2	C3	C4	C5	C6
P(Ci)	0.120	0.080	0.100	0.025	0.030	0.012

Table 10. Probabilities of the basic events.

In this paper, the second fault is analyzed. The FT of the aircraft engine damage caused by the oil filter plug is provided in Figure 6. The top event is E_1. The intermediate events and basic event are denoted by E_i (i =2, 3, 4) and Ci (i =1, 2, 3, 4, 5, 6) respectively. The probabilities of the basic events are expressed in Table 10. Meanwhile, these events represent the following practical situations:

E1: The aero-engine damage caused by oil filter plug;

E2: The warring light does not shine when the oil pressure difference >50 mpa/m^2;

E3: The fault of power supply II;

E4: The fault of oil pressure difference device in the oil pressure warning system;

C1: The fault of differential pressure switches of oil filter;

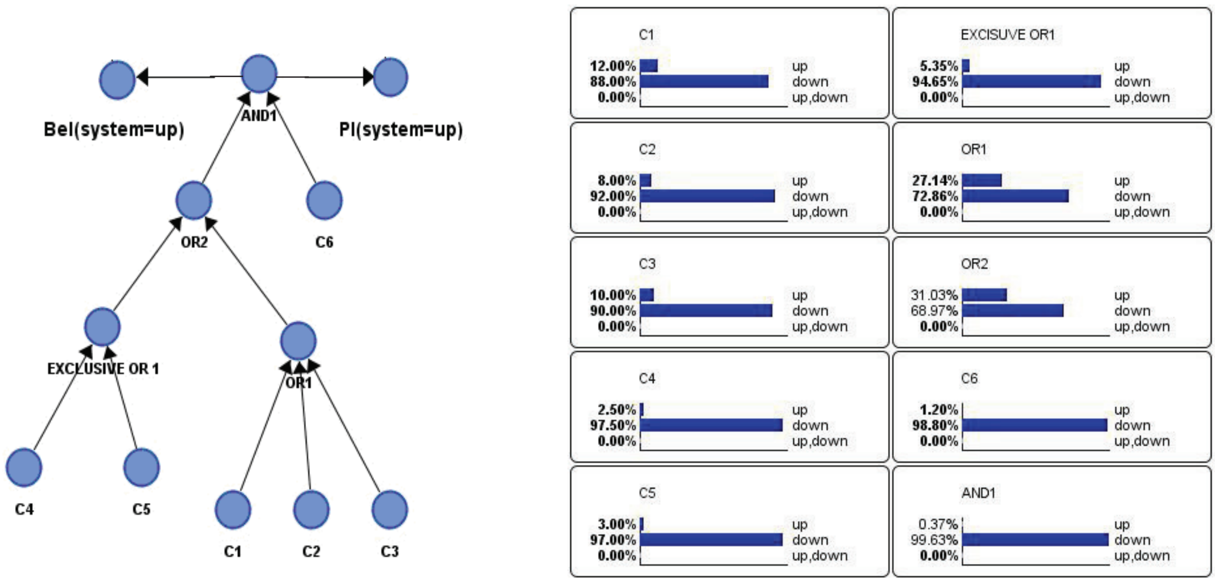
C2: The fault of wire component;

C3: The fault of oil catheter;

C4: The open-circuit fault of power supply II;

C5: The short-circuit fault of power supply II;

C6: The oil filter plug.



(a) (b)

Figure 7. Evidential network to modeling fault tree and basic belief assignments without imprecise input.

Using the conventional fault tree analysis method, the minimal cut sets can be attained. All of the minimal cut sets $K_i, 1 \leqslant i \leqslant 5$ are listed: $K_1 = \mathrm{C1C6}$, $K_2 = \mathrm{C2C6}$, $K_3 = \mathrm{C3C6}$, $K_4 = \mathrm{C4\overline{C5}C6}$, and $K_5 = \mathrm{\overline{C4}C5C6}$. Because the minimal cutset is not disjoint, the sum of disjoint product should be used to attain sum of disjoint of minimal cutset in the computation of occurrence probability of top event. The computation process is as follows:

$$\begin{aligned} T &= K_1 \cup K_2 \cup K_3 \cup K_4 \cup K_5 \\ &= K_1 + \overline{K_1}\,(K_2 \cup K_3 \cup K_4 \cup K_5) \\ &= \mathrm{C1C6} + \overline{\mathrm{C1C6}} \\ &\quad \times \left(\mathrm{C2C6} \cup \mathrm{C3C6} \cup \mathrm{C4\overline{C5}C6} \cup \mathrm{\overline{C4}C5C6}\right) \\ &= \mathrm{C1C6} + \left(\overline{\mathrm{C1}} \cup \overline{\mathrm{C6}}\right) \\ &\quad \times \left(\mathrm{C2C6} \cup \mathrm{C3C6} \cup \mathrm{C4\overline{C5}C6} \cup \mathrm{\overline{C4}C5C6}\right) \\ &= \mathrm{C1C6} + \mathrm{\overline{C1}C2C6} + \overline{\mathrm{\overline{C1}C2C6}} \\ &\quad \times \left(\mathrm{\overline{C1}C3C6} \cup \mathrm{\overline{C1}C4\overline{C5}C6} \cup \mathrm{\overline{C1}\,\overline{C4}C5C6}\right) \\ &= \mathrm{C1C6} + \mathrm{\overline{C1}C2C6} + \mathrm{\overline{C1}\,\overline{C2}C3C6} \\ &\quad + \mathrm{\overline{C1}\,\overline{C2}\,\overline{C3}C4\overline{C5}C6} + \mathrm{\overline{C1}\,\overline{C2}\,\overline{C3}\,\overline{C4}C5C6}\,. \end{aligned}$$

Consequently,

$$\begin{aligned} P\mathrm{system} = \{up\}) = P(T) &= P(\mathrm{C1C6}) + P(\mathrm{\overline{C1}C2C6}) \\ &\quad + P(\mathrm{\overline{C1}\,\overline{C2}C3C6}) \\ &\quad + P(\mathrm{\overline{C1}\,\overline{C2}\,\overline{C3}C4\overline{C5}C6}) \\ &\quad + P(\mathrm{\overline{C1}\,\overline{C2}\,\overline{C3}\,\overline{C4}C5C6}) \\ &= 0.37 \times 10^{-2}\,. \end{aligned}$$

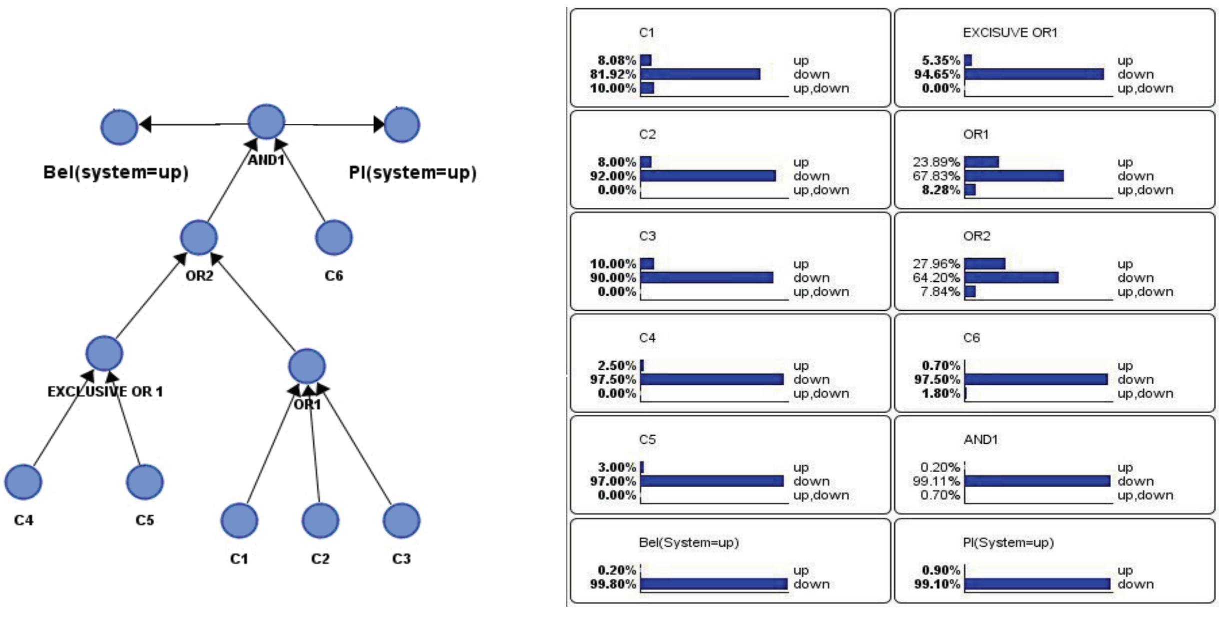
According to Section 4, the ENs can be mapped from FT. Its structure defined in Bayesialab [25] is depicted in Figure 7 (a).

Firstly, without imprecision on the failure rates, belief mass $m_{Ci}\{up{,}down\} = 0\ (i = 1, 2, 3, 4, 5, 6)$ expresses that the evidential networks being degraded into Bayesian networks and $\mathrm{Bel}(Ci = \{up\}) = P(Ci = \{up\}) = \mathrm{Pl}(Ci = \{up\})$. Bobbio has claimed that FT can be directly mapped into Bayesian networks [26]. After computation, the consequence of evidential networks is equal to that of FT. Figure 7 (b) shows that the evidential networks compute the exact value of probability of the top event.

$$\begin{aligned} \mathrm{Bel}\,(\mathrm{system} = \{up\}) &= P\,(\mathrm{system} = \{up\}) \\ &= \mathrm{Pl}\,(\mathrm{system} = \{up\}) \\ &= 0.37 \times 10^{-2}. \end{aligned}$$

In FTA, at an early design stage, it is often difficult to accurately estimate the probability of occurrence of events because the available data are insufficient or imprecise. Therefore, imprecise probabilities should be considered. In this example, the probability of C1, the fault of differential pressure switches of oil filter, is in an interval of $[0.808 \times 10^{-1}, 1.808 \times 10^{-1}]$. A priori belief mass distribution defining the event state is obtained via Eqs. (7), (9), (11) and (12):

$$\begin{aligned} m_{\mathrm{C1}}\{up\} &= 0.808 \times 10^{-1}, \\ m_{\mathrm{C1}}\{down\} &= 8.192 \times 10^{-1}, \\ m_{\mathrm{C1}}\{up, down\} &= 1.0 \times 10^{-1}\,. \end{aligned}$$



(a) (b)

Figure 8. Evidential network for modeling FT and basic belief assignments with $m(\mathrm{C1} = \{up, down\}) = 0.1$ and $m(\mathrm{C6} = \{up, down\}) = 0.018$.

The interval probability of C6, the oil filter plug, is $[0.7 \times 10^{-2}, 0.25 \times 10^{-1}]$. A priori belief mass distribution defining event state is also obtained by Eq. (7), (9), (11), (12):

$$m_{\mathrm{C6}}\{up\} = 0.7 \times 10^{-2},$$
$$m_{\mathrm{C6}}\{down\} = 9.75 \times 10^{-1},$$
$$m_{\mathrm{C6}}\{up, down\} = 0.18 \times \times 10^{-1}\,.$$

Figure 8 (b) shows the consequence using the evidential networks. The probability of the top event is in the interval $[\underline{P}(\mathrm{E_1}), \overline{P}(\mathrm{E_1})] = [0.20 \times 10^{-2}, 0.9 \times 10^{-2}]$. Comparing to the consequence obtained in the previous section, the bounding property in Eq. (10) is verified. Moreover, the imprecision probabilities of the events are propagated through the evidential networks shown in Figure 8 (a). This demonstrates that the evidential networks can delineate the propagation of the imprecise probability.

As shown in this section, evidential networks can deal with both aleatory and epistemic uncertainties in FTA. Imprecise probabilities on basic events can be propagated through the ENs. The imprecision is regarded as a priori belief masses, and imprecise results can be obtained.

The three importance measures are computed respectively and expressed in Table 11.

Based on the results shown in Table 11, the probability importance of C6 has the largest value. C6 is also the most important event, while C1, C2 and C3 are relatively unimportant. The structure importance of C6 has the largest value and C4 and C5 have the smallest values. For epistemic importance, C6 is the largest. These consequences will be significant for improving design, fault diagnosis, and providing guidance for preventive maintenance of the oil subsystem of aero-engines.

7 Summary & Conclusions

FTA is a very popular and widely used method in reliability analysis in early stages of product design. In most cases, the precise probabilities are adopted to delineate the probability of occurrence of the events. Due to the lack of sufficient data as well as imprecise data in early stage of product design, it is often difficult to accurately estimate the failure rates of individual events or the probabilities of events. Consequently, the imprecision and uncer-

Basic event	Probability importance	Structure importance	Epistemic importance
C1	0.04%	6.25%	1.69%
C2	0.04%	6.25%	1.51%
C3	0.04%	6.25%	1.54%
C4	0.037%	0.01%	1.46%
C5	0.037%	0.01%	1.46%
C6	27.96%	97.35%	63.76%

Table 11. The importance measures of basic events with imprecise probabilities.

tainty needs to be quantified. EN can deal with aleatory, epistemic, and imprecise uncertainties in the reliability engineering. Moreover, the EN can model the propagation of uncertainty in reliability analysis of the system. In this study, the EN has been employed to quantify and propagate the uncertainty and imprecision of the FTA. The conversion process from FT to EN has been developed. The detailed conversion process for some logic gates to EN has been deduced. Each of the truth tables and the conditional belief mass tables of these gates have been illustrated. The epistemic importance is proposed to describe the contribution of epistemic state of basic event to top event. The FT of the aero-engines damaged by oil filter plug was analyzed and compared. The three importance measures of six basic events are computed, respectively. The ranking of importance of these events is attained. The results illustrate that the EN can quantify the imprecision and uncertainty and propagate this imprecision from the basic events to the top event. Moreover, the precise and imprecise probabilities are considered and compared.

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