A Discrete Stress-Strength Interference Model With Stress Dependent Strength

Hong-Zhong Huang, *Senior Member, IEEE*, and Zong-Wen An

***Abstract*—In structural reliability engineering, one often encounters situations where the strength of a structure is influenced by the stress, but the stress is irrelevant to the strength. This phenomenon can be called a unilateral dependency of strength on stress. To evaluate structural reliability in such cases, the stress on a structure is proposed to be a discrete random variable, and the stress dependent strength is represented by a discrete random variable that has different conditional probability mass functions under different stress amplitudes. Then a discrete stress-strength interference model with stress dependent strength is presented based on the universal generating function technique. Finally, the effectiveness of this model is demonstrated by an illustrative example.**

***Index Terms*—Discrete model, stress dependent strength, stress-strength interference, universal generating function.**

ACRONYM[1]

SDS	stress dependent strength
SSI	stress-strength interference
UGF	universal generating function
BEC	bivariate exponential conditionals
pmf.	probability mass function

NOTATION

$\Pr(e)$	probability of event e
R	reliability of a structure
X	discrete r.v.
$\mathbf{x}$	vector consisting of all possible values of X, $\mathbf{x} = (x_1, x_2, \ldots, x_k)$
$\mathbf{p}_x$	vector consisting of the probabilities corresponding to all elements of $\mathbf{x}$, $\mathbf{p}_x = (p_{x_1}, p_{x_2}, \ldots, p_{x_k})$, $p_{x_i} = \Pr(X = x_i)$
$u_X(z)$	UGF of X
Y	discrete r.v. that is dependent on X
$\mathbf{y}_i$	vector consisting of possible values of Y under the condition in which X has the value of x_i, $\mathbf{y}_i = (y_{1\|i}, y_{2\|i}, \ldots, y_{m_i\|i})$, $i = 1, 2, \ldots, k$
$\mathbf{q}_{y\|i}$	vector consisting of probabilities corresponding to all elements of $\mathbf{y}_i$, $\mathbf{q}_{y\|i} = (q_{1\|i}, q_{2\|i}, \ldots, q_{m_i\|i})$, $q_{j\|i} = \Pr(Y = y_{j\|i} \| X = x_i)$
$\mathbf{y}$	vector consisting of all possible values of Y
$\mathbf{p}_{y\|i}$	vector consisting of probabilities corresponding to all elements of $\mathbf{y}$
$\overline{u}_Y(z)$	UGF representing the conditional pmf of Y
$f(X, Y)$	function of X, and Y, in which Y is dependent of X
$\overline{\otimes}$	composition operator over UGF of unilaterally dependent r.v.
$\overline{u}_f(z)$	UGF of function $f(X, Y)$
S	discrete r.v. representing the stress on a structure
$\mathbf{s}$	vector consisting of the all possible values of S, $\mathbf{s} = (s_1, s_2, \ldots, s_k)$
$\mathbf{p}_s$	vector consisting of the probabilities corresponding to all elements of $\mathbf{s}$, $\mathbf{p}_s = (p_{s_1}, p_{s_2}, \ldots, p_{s_k})$, $p_{s_i} = \Pr(S = s_i)$
C	discrete r.v. representing the SDS of a structure
$\mathbf{c}_i$	vector consisting of possible values of C under the condition in which S has the value of s_i, $\mathbf{c}_i = (c_{1\|i}, c_{2\|i}, \ldots, c_{m_i\|i})$, $i = 1, 2, \ldots, k$
$\mathbf{q}_{c\|i}$	vector consisting of probabilities corresponding to all elements of $\mathbf{c}_i$, $\mathbf{q}_{c\|i} = (q_{1\|i}, q_{2\|i}, \ldots, q_{m_i\|i})$, $q_{j\|i} = \Pr(C = c_{j\|i} \| S = s_i)$
$\mathbf{c}$	vector consisting of all possible values of C
$\mathbf{p}_{c\|i}$	vector consisting of probabilities corresponding to all elements of $\mathbf{c}$
$\overline{u}_C(z)$	UGF representing the conditional pmf of C
$r(C, S)$	function of S, and C, in which C is dependent on S
$\overline{u}_r(z)$	UGF of function $r(C, S)$
$\alpha(r_i)$	binary-valued function with domain on the set of possible values of function $r(S, C)$, $\alpha(r_i) = \begin{cases} 1, & r_i > 0 \\ 0, & r_i \le 0 \end{cases}$

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H.-Z. Huang is with the School of Mechatronics Engineering, University of Electronic Science and Technology of China, Chengdu, Sichuan, China (e-mail: hzhuang@uestc.edu.cn).

Z.-W. An is with the School of Mechatronics Engineering, University of Electronic Science and Technology of China, Chengdu, Sichuan, China. He is also with the School of Mechatronics Engineering, Lanzhou University of Technology, Lanzhou, Gansu, China.

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[1]The singular and plural of an acronym are always spelled the same.

I. INTRODUCTION

The stress-strength interference (SSI) model has been widely used for reliability analysis of structures. In this model, the structural reliability is defined as the probability that strength is larger than stress, where stress implies all elements possibly producing structural failures, and strength implies the ability to resist structural failures. In further investigation concerning SSI models, there have been many attempts to compute or estimate reliability. Kotz *et al.* [1] gave a comprehensive survey of the SSI models presented in the literature. It should be mentioned that the majority of the models in [1] are based on an identical assumption that stress and strength are s-independent of each other. In real situations, this assumption does not always hold. For instance, a mechanical structure has different fatigue lives under different stress levels. And the fatigue life of a structure is essentially a kind of representation of the ability to resist structural failures. So the SSI model with stress dependent strength has attracted the attention of researchers lately.

Since the bivariate exponential conditionals (BEC) distribution was introduced by Arnold & Strauss [2], some applications have been found in evaluating the reliability of structures with stress dependent strength, and systems with dependent components. SenGupta [3] chose the BEC distribution as a probability model for accelerated life testing, and derived a reliability expression for structures with stress dependent strength. Nadarajah & Kotz [4] derived some explicit expressions for structural reliability when the joint distribution of stress and strength is bivariate exponential. Navarro *et al.* [5] studied the reliability properties of systems with exchangeable components, and exponential conditional distribution. Yu [6] presented a simple, fast algorithm for simulating the random variables from BEC distribution. Additionally, considering the system with dependent components, Eryilmaz [7], and Urkkan & Pham-Gia [8] established multivariate stress-strength models for complex systems with dependent components.

Note that the dependency between stress and strength represented by the BEC distribution is a bilateral dependency, in which both stress and strength are dependent upon each other. However, from a practical point of view, the stress on a structure is uniquely determined by the variation of external loads or environment. Stress is irrelevant to the strength of a structure. This means that only a unilateral dependency exists between stress and strength, in which the strength is dependent on the stress, but the stress is independent of strength.

Simultaneously, in some specific situations, the stress on a structure can be treated as a discrete random variable. For example, the stress pattern in a step-stress accelerated life test can be treated as a discrete random variable of which the possible values can be obtained from all stress levels, and the corresponding probabilities can be obtained from the acting times of each stress levels. Treating the stress on a structure as a discrete random variable, this paper will build a structural reliability model with unilateral dependency between stress and strength.

The rest of this paper is organized as follows. In Section II, a brief description of the universal generating function of unilaterally dependent discrete variables is given, which is employed to describe the characteristics of discrete stress and strength. In Section III, a generalized problem is formulated, from which the discrete SSI model with stress-dependent strength is established. For demonstrating the validity of the model, an illustrative example is provided in Section IV. Conclusions, and discussions are summarized in Section V.

II. UGF OF UNILATERAL DEPENDENT DISCRETE VARIABLES

The concept of UGF was introduced by Ushakov [9]. In a series of research by Levitin & Lisnianski [10]–[12], the UGF method has been applied to reliability analysis, and the optimization of multi-state systems.

Based on the basic principles of the UGF method, the UGF of a discrete r.v. X is defined as a polynomial function of variable z, $u_X(z)$, that relates the possible values of X to the corresponding probabilities.

$$u_X(z) = \sum_{i=1}^{k} p_{x_i} z^{x_i}. \tag{1}$$

where the variable X has k possible values.

Consider a function $f(X, Y)$ of discrete r.v. X, and Y, in which Y is dependent on X. Suppose that, when X has the value of $x_i \in \mathbf{x}, (i = 1, 2, \ldots, k)$, the conditional pmf of Y is characterized by the vectors $\mathbf{y}_i$, and $\mathbf{q}_{y|i}$. We can define the set of all possible values of Y as

$$\mathbf{y} = \bigcup_{i=1}^{k} \mathbf{y}_i = (y_1, y_2, \ldots, y_m), \tag{2}$$

and redefine the conditional pmf of Y when X takes the value of x_i as

$$\mathbf{p}_{y|i} = \left(p_{1|i}, p_{2|i}, \ldots, p_{m|i}\right), \tag{3}$$

where m is the number of all possible values of Y, and

$$p_{j|i} = \begin{cases} 0, & y_j \notin \mathbf{y}_i \\ q_{j|i}, & y_j \in \mathbf{y}_i \end{cases}, \qquad 1 \le j \le m. \tag{4}$$

Thus, the conditional pmf of Y is defined by the vectors $\mathbf{y}$, and $\mathbf{p}_{y|i}$. It can be represented in the form of the UGF with vector coefficients

$$\overline{u}_Y(z) = \sum_{j=1}^{m} \overline{p}_{j|i} z^{y_j}, \tag{5}$$

where $\overline{p}_{j|i}$ is a vector, $\overline{p}_{j|i} = (p_{j|1}, p_{j|2}, \ldots, p_{j|k})$.

Because each combination of the possible values of X and Y, (x_i, y_j), corresponds to a possible value of the function $f(X, Y)$, and the probability of the combination is $p_{x_i} p_{j|i}$, the UGF of the function $f(X, Y)$ can be obtained as

$$\begin{aligned}\overline{u}_f(z) &= \overline{\otimes}\left(u_X(z), \overline{u}_Y(z)\right) \\ &= \overline{\otimes}\left(\sum_{i=1}^{k} p_{x_i} z^{x_i}, \sum_{j=1}^{m} \overline{p}_{j|i} z^{y_j}\right) \\ &= \sum_{i=1}^{k} p_{x_i} \sum_{j=1}^{m} p_{j|i} z^{f(x_i, y_j)},\end{aligned} \tag{6}$$

TABLE I
THE UNILATERAL DEPENDENCY BETWEEN STRESS, AND STRENGTH

Stress S		Strength C			
s_1	p_{s_1}	$c_{1\mid 1}$	$c_{2\mid 1}$	...	$c_{m_1\mid 1}$
		$q_{1\mid 1}$	$q_{2\mid 1}$	...	$q_{m_1\mid 1}$
s_2	p_{s_2}	$c_{1\mid 2}$	$c_{2\mid 2}$	...	$c_{m_2\mid 2}$
		$q_{1\mid 2}$	$q_{2\mid 2}$	...	$q_{m_2\mid 2}$
...	...	...	...	...	...
		...	...	...	...
s_k	p_{s_k}	$c_{1\mid k}$	$c_{2\mid k}$	...	$c_{m_k\mid k}$
		$q_{1\mid k}$	$q_{2\mid k}$	...	$q_{m_k\mid k}$

where $\overline{\otimes}$ is a composition operator over UGF of unilaterally dependent r.v.

III. DISCRETE SSI MODEL WITH SDS

In this section, we will first formulate a generalized problem. Then the discrete SSI model with SDS will be constituted.

A. Formulation of the Problem

Suppose that the external loads with different amplitudes are randomly applied on a structure during the operation time. Accordingly, the stress of the structure will also randomly vary in magnitude, and occurrence. These random characteristics can be represented by a discrete r.v. S with a specific pmf. Without loss of generality, we suppose that the pmf of stress S is expressed by two vectors $\mathbf{s}$, and $\mathbf{p}_s$.

To describe the SDS denoted by C, we assume that, when stress S takes the value of s_i, the characteristics of strength can be described by a discrete r.v. $C_i (i = 1, 2, \ldots, k)$ (note that, a continuous r.v. is allowable, and it can be approximated by a discrete r.v. as presented in Section IV). The conditional pmf of C_i can be characterized by the vectors $\mathbf{c}_i$, and $\mathbf{q}_{c|i} (i = 1, 2, \ldots, k)$. Thus, we can obtain k conditional pmf of SDS C, which represent the unilateral dependency of the strength on the stress. We can summarize this unilateral dependency in Table I.

From the above discussions, the problem now can be expressed as calculating the reliability of the structure when the pmf of the stress, and conditional pmf of the SDS, are known. To solve this problem, we can use the UGF technique introduced in Section II.

B. Solution to the Problem

Suppose the stress on a structure, and the SDS of the structure are characterized by two pairs of vectors $(\mathbf{s}, \mathbf{p}_s)$, and $(\mathbf{c}_i, \mathbf{q}_{c|i})$, respectively. We can directly obtain the UGF of stress according to (1) as

$$u_S(z) = \sum_{i=1}^{k} p_{s_i} z^{s_i}. \tag{7}$$

According to (5), the conditional pmf of C can be represented in the form of the UGF with vector coefficients

$$\overline{u}_C(z) = \sum_{j=1}^{m} \overline{p}_{j|i} z^{c_j}. \tag{8}$$

We construct a function $r(S, C)$ of stress S, and strength C, in which the SDS C is dependent on the stress S.

$$r(S, C) = C - S. \tag{9}$$

Based on (6), the UGF of the function $r(C, S)$ can be obtained as

$$\begin{aligned} \overline{u}_r(z) &= \overline{\otimes}\left(u_S(z), \overline{u}_C(z)\right) \\ &= \overline{\otimes}\left(\sum_{i=1}^{k} p_{s_i} z^{s_i}, \sum_{j=1}^{m} \overline{p}_{j|i} z^{c_j}\right) \\ &= \sum_{i=1}^{k} p_{s_i} \sum_{j=1}^{m} p_{j|i} z^{r(s_i, c_j)}. \end{aligned} \tag{10}$$

It is not difficult to understand that the final form of the UGF of the function $r(C, S)$ is also a polynomial function. Therefore, (10) can be rewritten as

$$\overline{u}_r(z) = \sum_{i=1}^{K} P_i z^{r_i}, \tag{11}$$

where r_i, and $P_i (i = 1, 2, \ldots, K)$ are possible values of the function $r(C, S)$, and corresponding probabilities respectively.

As mentioned in Section I, the structural reliability is defined as the probability that strength is larger than stress. We can get the mathematical description of structural reliability as

$$R = \Pr(C > S) \tag{12}$$

Transforming (12), and substituting (9), the structural reliability can be further expressed as

$$\begin{aligned} R &= \Pr(C - S > 0) \\ &= \Pr\left(r(S, C) > 0\right). \end{aligned} \tag{13}$$

To obtain the probability in (13), the coefficients of polynomial $\overline{u}_r(z)$ represented by (11) can be summed for every term with $r_i > 0$. For the sake of depiction, we define a binary-valued function with domain on the set of possible values of function $r(C, S)$ as

$$\alpha(r_i) = \begin{cases} 1, & r_i > 0 \\ 0, & r_i \le 0 \end{cases}. \tag{14}$$

Then, (13) can be rewritten as

$$R = \sum_{i=1}^{K} P_i \alpha(r_i). \tag{15}$$

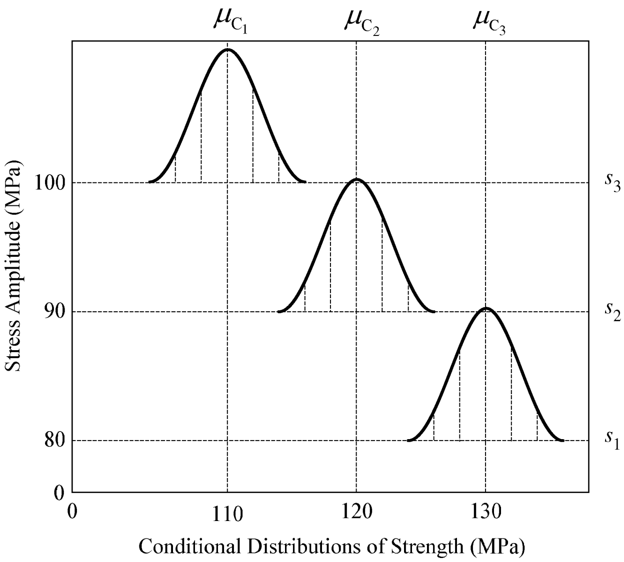

Fig. 1. The schematic illustration of stress, and strength.

For the comparison with the classical SSI model, (15) can be called a discrete SSI model with SDS.

IV. An Illustrative Example

Consider an example in which a structure, during the service time, is subjected to the variable stress S that is characterized by three kinds of stress amplitudes $(s_1, s_2, s_3) = (80, 90, 100)$ MPa, and the corresponding occurrence probabilities $(p_{s_1}, p_{s_2}, p_{s_3}) = (0.2, 0.6, 0.2)$. When the three kinds of stress act on the structure, the SDS can be represented by three standard normal r.v., C_1, C_2, and C_3 that have different mean values, $\mu_{C_1} = 130$ MPa, $\mu_{C_2} = 120$ MPa, and $\mu_{C_3} = 110$ MPa, and the identical standard deviations $\sigma_{C_1} = \sigma_{C_2} = \sigma_{C_3} = 10$ MPa. The stress-strength relation of the structure is depicted by Fig. 1.

According to (1), the UGF of stress S can be obtained as

$$u_S(z) = 0.2z^{80} + 0.6z^{90} + 0.2z^{100}. \tag{16}$$

To evaluate the structural reliability by using the SSI model suggested in Section III, we will translate the continuous r.v. representing the conditional distribution of SDS into discrete r.v. Firstly, we can determine an approximate range of SDS, which can be denoted by an interval. Then, the interval is divided into finite subintervals. The midpoint values of each subinterval are treated as possible values of SDS, and the area values of each subinterval are treated as the corresponding probabilities. In this example, the interval of SDS is approximately determined as $\langle \mu_{C_i} - 3\sigma_{C_i}, \mu_{C_i} + 3\sigma_{C_i} \rangle (i = 1, 2, 3)$, and it is divided into six subintervals (as shown in Fig. 1.). Accordingly, the SDS with a standard normal distribution can be translated into the discrete r.v. with the following vectors of possible values, and corresponding conditional probabilities.

$$\mathbf{c}_1 = (105, 115, 125, 135, 145, 155), \tag{17}$$

TABLE II
The Conditional Distributions of SDS Under Three Stress Amplitudes

Stress S		Strength C					
80	0.2	105	115	125	135	145	155
		0.02	0.14	0.34	0.34	0.14	0.02
90	0.6	95	105	115	125	135	145
		0.02	0.14	0.34	0.34	0.14	0.02
100	0.2	85	95	105	115	125	135
		0.02	0.14	0.34	0.34	0.14	0.02

TABLE III
All Possible Values, and Corresponding Conditional Probabilities of SDS

Stress S	Strength C							
	85	95	105	115	125	135	145	155
80	0	0	0.02	0.14	0.34	0.34	0.14	0.02
90	0	0.02	0.14	0.34	0.34	0.14	0.02	0
100	0.02	0.14	0.34	0.34	0.14	0.02	0	0

$$\mathbf{c}_2 = (95, 105, 115, 125, 135, 145), \tag{18}$$
$$\mathbf{c}_3 = (85, 95, 105, 115, 125, 135), \tag{19}$$
$$\mathbf{q}_{c|1} = \mathbf{q}_{c|2} = \mathbf{q}_{c3} = (0.02, 0.14, 0.34, 0.34, 0.14, 0.02). \tag{20}$$

The conditional distributions of SDS under three stress amplitudes can be also depicted by Table II.

According to (2), we can obtain the all possible values of SDS as

$$\mathbf{c} = \bigcup_{i=1}^{3} \mathbf{c}_i = (85, 95, 105, 115, 125, 135, 145, 155). \tag{21}$$

According to (3), and (4), we can obtain the conditional probabilities of SDS when stress amplitudes have the values of s_1, s_2, and s_3:

$$\mathbf{p}_{c|1} = (0, 0, 0.02, 0.14, 0.34, 0.34, 0.14, 0.02), \tag{22}$$
$$\mathbf{p}_{c|2} = (0, 0.02, 0.14, 0.34, 0.34, 0.14, 0.02, 0), \tag{23}$$
$$\mathbf{p}_{c|3} = (0.02, 0.14, 0.34, 0.34, 0.14, 0.02, 0, 0). \tag{24}$$

All possible values of SDS, and corresponding conditional probabilities, can also be represented by Table III.

According to (5), we can obtain the UGF of the SDS of the structure, $\overline{u}_C(z)$, in which each coefficient of the polynomial has the form

$$\begin{aligned}\overline{u}_C(z) = &(0, 0, 0.02)z^{85} + (0, 0.02, 0.14)z^{95} \\ &+ (0.02, 0.14, 0.34)z^{105} + (0.14, 0.34, 0.34)z^{115} \\ &+ (0.34, 0.34, 0.14)z^{125} + (0.34, 0.14, 0.02)z^{135} \\ &+ (0.14, 0.02, 0)z^{145} + (0.02, 0, 0)z^{155}.\end{aligned} \tag{25}$$

According to (6), the UGF of function $r(S, C)$ can be obtained as

$$\begin{aligned}\overline{u}_r(z) &= \overline{\otimes}\left(u_S(z), \overline{u}_C(z)\right)\\ &= (0.2z^{80} + 0.6z^{90} + 0.2z^{100})\\ &\quad \overline{\otimes}\big((0,0,0.02)z^{85} + (0,0.02,0.14)z^{95}\\ &\qquad + (0.02,0.14,0.34)z^{105} + (0.14,0.34,0.34)z^{115}\\ &\qquad + (0.34,0.34,0.14)z^{125} + (0.34,0.14,0.02)z^{135}\\ &\qquad + (0.14,0.02,0)z^{145} + (0.02,0,0)z^{155}\big)\\ &= 0.2(0z^{85-80} + 0z^{95-80} + 0.02z^{105-80}\\ &\qquad + 0.14z^{115-80} + 0.34z^{125-80} + 0.34z^{135-80}\\ &\qquad + 0.14z^{145-80} + 0.02z^{155-80})\\ &\qquad + 0.6(0z^{85-90} + 0.02z^{95-90} + 0.14z^{105-90}\\ &\qquad + 0.34z^{115-90} + 0.34z^{125-90} + 0.14z^{135-90}\\ &\qquad + 0.02z^{145-90} + 0z^{155-90})\\ &\qquad + 0.2(0.02z^{85-100} + 0.14z^{95-100} + 0.34z^{105-100}\\ &\qquad + 0.34z^{115-100} + 0.14z^{125-100}\\ &\qquad + 0.02z^{135-100} + 0z^{145-100} + 0z^{155-100})\\ &= 0.004z^{-15} + 0.028z^{-5} + 0.08z^{5}\\ &\qquad + 0.152z^{15} + 0.236z^{25} + 0.236z^{35}\\ &\qquad + 0.152z^{45} + 0.08z^{55} + 0.028z^{65} + 0.004z^{75}\end{aligned} \tag{26}$$

It can be seen that the above polynomial consists of ten terms. According to (15), the structural reliability can be obtained as

$$\begin{aligned}R &= \sum_{i=1}^{10} P_i \alpha(r_i)\\ &= 0.08 + 0.152 + 0.236 + 0.236\\ &\quad + 0.152 + 0.08 + 0.28 + 0.004\\ &= 0.9680\end{aligned} \tag{27}$$

It should be mentioned that, in this example, the conditional distributions of SDS are assumed to be normally distributed with different means, and identical standard deviations. We can calculate the structural reliability by employing the table of the standard normal distribution, and the total probability formula. The calculated result is equal to 0.9682. Thus, it can be seen that the relative error resulting from the discrete SSI model with SDS is equal to 0.021%.

V. Conclusions and Discussions

In this paper, we consider a unilateral dependency of strength on stress existing in some real situations, and present a discrete SSI model with SDS based on a UGF technique. In this model, the SDS of a structure is treated as a discrete r.v. that has different conditional pmf under different stress amplitudes. Calculation of the structural reliability is based on the UGF technique. An example illustrates that, by employing the discretization of a continuous r.v., this model can be also applied to the situations in which the SDS of a structure can be represented by a continuous r.v. with an arbitrary distribution.

Note that, when the discrete SSI model with SDS is applied to the case of continuous SDS, discretization of a continuous r.v will result in calculatioqqn errors. There is no way to avoid these errors completely, but we can control them. When a continuous r.v. is translated into a discrete r.v., reducing the length of the subinterval can improve the calculation accuracy. This topic has been discussed in detail by authors of [13].

Another limitation of the discrete SSI model with SDS is that the stress on a structure is assumed to be a discrete r.v. When the stress on a structure is regarded as a continuous r.v., the effectiveness of this model will be lost. In our future work, we will explore the unilateral dependency between continuous stress and strength.

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Hong-Zhong Huang is a Full Professor, and the Dean of the School of Mechanical, Electronic, and Industrial Engineering at the University of Electronic Science and Technology of China, Chengdu, Sichuan, China. He has held visiting appointments at several universities in Canada, USA, and elsewhere in Asia. He received a Ph.D. degree in reliability engineering from Shanghai Jiaotong University, China. He has published over 150 journal articles, and 5 books in the fields of reliability engineering, optimization design, fuzzy sets theory, and product development. He is a Regional Editor of *International Journal of Reliability and Applications*, an Editorial Board Member for *The International Journal of Reliability, Quality and Safety Engineering*, *International Journal of Reliability and Quality Performance*, *International Journal of Performability Engineering*, *Advances in Fuzzy Sets and Systems*, and *The Open Mechanical Engineering Journal*. He received the William A. J. Golomski Award from the Institute of Industrial Engineers in 2006. His current research interests include system reliability analysis, warranty, maintenance planning and optimization, and computational intelligence in product design. He is a senior member of the IEEE.

Zong-Wen An is an Associate Professor of the school of mechatronics engineering at Lanzhou University of Technology, Lanzhou, Gansu, China. He received the B.S., and M.S. degree in mechanical engineering from Lanzhou University of Technology in 1990, and 2004, respectively. Now he is pursuing the Ph.D. degree at the University of Electronic Science and Technology of China. His research interests include structural reliability analysis, and mechanical design theory and methodology.