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An approach to reliability evaluation of multiple V-belt drives considering the deviation of belt length

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Abstract: Multiple V-belt drives are applied widely to the mechanical engineering industry. To improve their performance and reduce their risk in the course of running, some reliability models of multiple V-belt drives have been presented in recent years. In these models the lengths of individual belts in a multiple V-belt drive were regarded as identical. However, in practice the length deviations among individual belts in a multiple V-belt drive do exist. In this paper, the effect of belt length deviation on the transmitted power of a multiple V-belt drive is analysed in detail. The operation risk of a multiple V-belt drive is defined as the probability that the actual transmitted power is less than the rated power that ensures the multiple V-belt drives to perform its intended task. Then an approach to risk evaluation of a multiple V-belt drive is presented by using the universal generating function technique and the reliability model of a multistate weighted k-out-of-n system. Finally, the effectiveness of this approach is demonstrated by an example.

Keywords: risk evaluation, multiple V-belt drive, deviation of belt length, multistate weighted k-out-of-n system, universal generating function

1 INTRODUCTION

Multiple V-belt drive (MVBD) systems can provide an efficient way of transmitting power to all accessories. Owing to the advantages of light weight and convenient operation, MVBD systems have been widely applied in automobile, machine tool, and other types of mechanical equipment. Consequently, it is necessary to study the reliability of MVBD systems so as to reduce their operation risk. In this regard, Zhang [**1**] presented reliability calculation formulas when fatigue failure of V-belts follows the logarithmic normal distribution and Weibull distribution. Zhao *et al.* [**2**] presented a design method of the MVBD using the stress-strength interference model. Wu *et al.* [**3**] proposed a method of reliability design of the MVBD after analysing the deficiency of conventional design criteria. Zhu and Xiao [**4**] constructed a fuzzy reliability model of fatigue strength of the MVBD according to fuzzy mathematics and reliability design theory.

However, aforementioned models or methods are based on an assumption that all belts in an MVBD are identical. Actually, geometrical deviations among individual belts do exist in a MVBD. These geometrical deviations are restricted to certain limits (tolerances) according to national and/or international standards. Considering belt geometrical deviations, Gerbert and Maré [**5**] presented a unified theory regarding the tension distribution in a MVBD. In this paper the effect of the length deviation of the belt on the transmitted power of the MVBD is considered, and an approach to risk evaluation of the MVBD is proposed using the universal generating function (UGF) technique and the reliability model of a multistate weighted k-out-of-n system.

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The rest of this paper is organized as follows. In sections 2 and 3 the UGF technique and the multistate weighted k-out-of-n system, which are necessary tools for risk evaluation of a MVBD, are introduced respectively. In section 4 an approach to risk evaluation of a MVBD is presented. To validate the suggested approach, an example is provided in section 5. The conclusions are summarized in section 6.

2 BRIEF DESCRIPTION OF THE UGF TECHNIQUE

The concept of the UGF was introduced by Ushakov [**6**]. In a series of research work by Lisnianski and Levitin [**7**, **8**], the UGF method has been applied to reliability analysis and optimization of multistate systems.

Suppose that a discrete random variable (r.v.) X has a probability mass function (p.m.f.) characterized by a pair of vectors ($\boldsymbol{x}$, $\boldsymbol{p}$), in which $\boldsymbol{x}$ consists of all possible values of X and $\boldsymbol{p}$ consists of the corresponding probabilities

$$\boldsymbol{x} = (x_1, x_2, \ldots, x_k)$$

$$\boldsymbol{p} = (p_1, p_2, \ldots, p_k)$$

$$p_v = \Pr(X = x_v), \quad v = 1, 2, \ldots, k$$

Based on the basic principle of the UGF method, the p.m.f. of discrete r.v. X can be represented by a polynomial function of variable z, $u_X(z)$, which relates the possible values of X to the corresponding probabilities

$$u_X(z) = p_1 z^{x_1} + p_2 z^{x_2} + \cdots + p_k z^{x_k} = \sum_{l=1}^{k} p_l z^{x_l} \tag{1}$$

Consider q independent discrete r.v. $X_1, X_2, \ldots, X_q$ and an arbitrary function $f(X_1, X_2, \ldots, X_q)$. Suppose that the number of possible values of each r.v. are k_1, $k_2, \ldots, k_q$ respectively. According to equation (1), the UGF of individual r.v. can be obtained. Then, by employing composition operator $\otimes$, the UGF of function $f(X_1, X_2, \ldots, X_q)$, $u_f(z)$, can be calculated by

$$u_f(z) = \otimes_f [u_{X_1}(z), u_{X_2}(z), \ldots, u_{X_q}(z)] \tag{2}$$

where composition operator $\otimes$ is defined as

$$\otimes_f \left(\sum_{l=1}^{k_1} p_{1l} z^{x_{1l}}, \sum_{l=1}^{k_2} p_{2l} z^{x_{2l}}, \ldots, \sum_{l=1}^{k_q} p_{ql} z^{x_{ql}} \right) = \sum_{l=1}^{k_1} \sum_{l=1}^{k_2} \cdots \sum_{l=1}^{k_q} \left[\prod_{v=1}^{q} p_{vl} z^{f(x_{1l},\, x_{2l},\, \ldots,\, x_{ql})} \right] \tag{3}$$

3 MULTISTATE WEIGHTED k-OUT-OF-n SYSTEM AND ITS RELIABILITY

3.1 Definition of a multistate weighted k-out-of-n system

If a system consisting of n independent components can perform its task if and only if at least k of its components are in working condition, this type of system is called a k-out-of-n system. Some physical structures of the k-out-of-n system in engineering applications can be found. For example, the engine system of an airplane can be regarded as a 2-out-of-4 system, in which the airplane can survive if no more than two of its four engines are damaged.

In a further investigation concerning reliability of k-out-of-n systems, Wu and Chen [**9**] proposed a model of the binary weighted k-out-of-n system. In this model each component has a weight representing its utility to the system, and the system consisting of n components can perform its task if and only if the total weight of all components is at least k, a prespecified value.

In the reliability analysis regarding the binary k-out-of-n system, it is assumed that each component, as well as the entire system, is in one of two possible states, i.e. working or failure. This assumption is one of the bases of binary reliability theory. However, in a multistate context, a component or the entire system may be in multiple states. When a component is in different states, it may have different contributions to the system. When a component completely fails, its contribution to the system is zero. From the view of multistate systems, Li and Zuo [**10**] proposed a multistate weighted k-out-of-n model. It should be mentioned that in the domain of a multistate system, a multistate weighted k-out-of-n system is actually a multistate parallel system according to system terminology. The formal definition of a multistate weighted k-out-of-n system is given below by Li and Zuo [**10**].

Definition

In a system with n components, each component may be in $m+1$ possible states, 0, 1, 2, ..., m. Component i ($1 \leqslant i \leqslant n$), when in state j ($0 \leqslant j \leqslant m$), has a utility w_{ij} to the system. The system also has multiple states, 0, 1, ..., M, which are determined by different combinations of all possible states of the components. The system is in state J ($0 \leqslant J \leqslant M$) or above if the total utility of all components is greater than or equal to a prespecified value k_J, where k_J is the minimum total utility required to ensure that the system is in state J or above.

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3.2 Reliability of a multistate weighted *k*-out-of-*n* system

Based on the definition above, the reliability of a multistate weighted k-out-of-n system can be defined as the probability that the total utility of all components is larger than or equal to k_J. To calculate the probability, the UGF technique introduced in section 2 will be employed.

In a multistate weighted k-out-of-n system, for each component i, its utility is uniquely determined by its state. Suppose that all possible utilities of component i and the corresponding probabilities can be represented by a pair of vectors ($\boldsymbol{w}_i$, $\boldsymbol{p}_i$), where

$$\boldsymbol{w}_i = (w_{i0}, w_{i1}, \ldots, w_{im})$$

$$\boldsymbol{p}_i = (p_{i0}, p_{i1}, \ldots, p_{im})$$

In fact, the pair of vectors ($\boldsymbol{w}_i$, $\boldsymbol{p}_i$) reflects the probability distribution of the utility value of component i, and it can be naturally regarded as the p.m.f. of a discrete r.v. denoted by W_i. According to equation (1), the UGF of r.v. W_i, $u_i(z)$, can be obtained as

$$u_i(z) = p_{i0}z^{w_{i0}} + p_{i1}z^{w_{i1}} + \cdots + p_{im}z^{w_{im}} = \sum_{j=0}^{m} p_{ij}z^{w_{ij}} \tag{4}$$

Thus, the UGF of all components, $u_1(z)$, $u_2(z)$, ..., $u_n(z)$, can be obtained.

Based on the definition of the multistate weighted k-out-of-n system, the total utility of the system is the sum of utilities of all individual components. Therefore the total utility of the system, denoted by W_s, can be expressed as

$$W_s = W_1 + W_2 + \cdots + W_n \tag{5}$$

W_s can be regarded as a function of discrete r.v. W_i ($i = 1, 2, \ldots, n$). According to equations (2) and (3), the UGF of function W_s, $u_s(z)$, can be obtained as

$$\begin{aligned} u_s(z) &= \otimes_+[u_1(z), u_2(z), \ldots, u_n(z)] \\ &= \otimes_+\left(\sum_{j=0}^{m} p_{1j}z^{w_{1j}}, \sum_{j=0}^{m} p_{2j}z^{w_{2j}}, \ldots, \sum_{j=0}^{m} p_{nj}z^{w_{nj}}\right) \\ &= \sum_{j=0}^{m}\sum_{j=0}^{m}\cdots\sum_{j=0}^{m}\left[\prod_{i=1}^{n} p_{ij}z^{(w_{1j}+w_{2j}+\ldots+w_{nj})}\right] \end{aligned} \tag{6}$$

It is not difficult to understand that the final form of equation (6) is also a polynomial function. Therefore equation (6) can be rewritten as

$$u_s(z) = \sum_{J=0}^{M} p_{sJ}z^{w_{sJ}} \tag{7}$$

where w_{sJ} and p_{sJ} are possible values of W_s and the corresponding probabilities respectively. It should be noticed that, theoretically, the number of possible states of the system is equal to m^n, i.e. $M = m^n$. However, in the real operation, the general case is $M < m^n$ owing to the collection of like terms. This case can be seen in section 5.

Remembering the definition of system reliability at the beginning of this subsection and using the notation $R_s(k_J, n)$ to denote the reliability of the system with n components and a prespecified value k_J, the system reliability can be expressed as

$$R_s(k_J, n) = \Pr(W_s \geqslant k_J) \tag{8}$$

To obtain the probability in equation (8), the coefficients of polynomial $u_s(z)$ represented by equation (7) can be summed for every term with $w_{sJ} \geqslant k_J$. To describe this better, a binary-valued function $\beta(w_{sJ})$ with the domain on the set of possible values of W_s is defined as

$$\beta(w_{sJ}) = \begin{cases} 1, & w_{sJ} \geqslant k_J \\ 0, & w_{sJ} < k_J \end{cases} \tag{9}$$

Then, equation (8) can be rewritten as

$$R_s(k_J, n) = \sum_{J=0}^{M} p_{sJ}\beta(w_{sJ}) \tag{10}$$

Equation (10) can be regarded as the reliability model of a multistate weighted k-out-of-n system.

4 RISK EVALUATION OF AN MVBD

In engineering, the design of an MVBD is based on the calculation of the working capacity of an individual belt. The working capacity of a belt is generally denoted by its rated power. In the design procedure, the required initial tension of the individual belt must be determined owing to the working capacity of the individual belt depends partially on the initial tension. Furthermore, when the centre distance of the MVBD is fixed, the initial tension of each belt is determined by its actual elongation and the actual elongation of each belt is related to its actual length. Therefore, deviation of the belt length will affect the working capacity of the MVBD. In this section the relation between length deviation and transmitted power of the individual belt will be analysed in detail, after which an approach to operation risk evaluation of a MVBD will be presented. In order to reduce the complexity of the problem, some assumptions are first given.

4.1 Assumptions

Theoretically the dimension errors of both the belt section and the groove section also result in variation of belt elongation in a MVBD. The changes of

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environment temperature also induce variation of the belt length. However, these factors have less influence on the initial tension in comparison with the deviation of the belt length. Simultaneously, in the practical application of a MVBD, the centre distance can be adjusted within a certain range in order to obtain an appropriate initial tension. Once the adjustment is finished, the centre distances of all belts in a MVBD are regarded as identical and fixed.

Further analyses in this paper are based on the following assumptions.

1. The effect of dimension errors of the belt section and the groove section on initial tension is so trivial that it can be ignored.
2. The belt elongations caused by changes in the working environment can be ignored.
3. The centre distances of individual belts in a MVBD are identical and fixed.
4. The geometrical sizes except the belt length and the mechanical properties of all belts in a MVBD are identical.

4.2 Relation between the length deviation and the transmitted power of a belt

Regarding a belt as a linear elastic body and employing Hooke's law, its elongation ΔL_0 when it produces an initial tension F_0 can be obtained as

$$\Delta L_0 = F_0 \frac{L_\mathrm{p}}{EA} \tag{11}$$

where L_p is pitch length of the belt (nominal size of belt length), E is the elastic modulus of the belt, and A is the cross-section area of the belt. Let the allowable tolerance of the belt length be t, which can be assigned bilaterally and symmetrically based on L_p. Then the maximum and minimum of the belt lengths in a MVBD can be obtained as

$$\begin{aligned} L_{\max} &= L_p + t/2 \\ L_{\min} &= L_p - t/2 \end{aligned} \tag{12}$$

Similarly, when the centre distance of a MVBD is fixed, the maximum and minimum of belt elongations in a MVBD can also be obtained as

$$\begin{aligned} \Delta L_{\max} &= \Delta L_0 + t/2 \\ \Delta L_{\min} &= \Delta L_0 - t/2 \end{aligned} \tag{13}$$

In a MVBD all possible elongations of individual belts are in the interval $<\Delta L_{\min}, \Delta L_{\max}>$. From the interval a point is randomly selected to represent the actual elongation of the belt, which is denoted ΔL_{ij}. According to equation (11), the corresponding initial tension F_{ij} can be obtained as

$$F_{ij} = EA \frac{\Delta L_{ij}}{L_{ij}} \tag{14}$$

where L_{ij} is the actual length of the belt, which is in interval $<L_{\min}, L_{\max}>$.

Figure 1 illustrates the schematic structure and partial parameters of a MVBD. According to reference [**11**], if the small pulley is assumed to be a driver, the relationship between initial tension F_{ij} and actual driving torque T_{ij} on the driver can be obtained as

$$F_{ij} = \frac{T_{ij}}{d_\mathrm{p}} \frac{\exp(f\alpha) + 1}{\exp(f\alpha) - 1} \tag{15}$$

where f is the coefficient of friction between the belt and groove, α is the contact angle of the driver, and d_p is the pitch diameter of the driver.

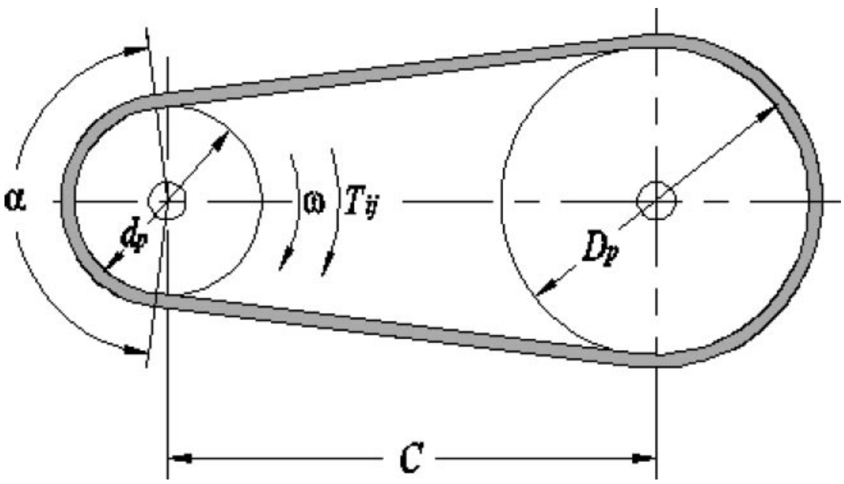

Fig. 1 A schematic view of a V-belt driver

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Let ω be the angular velocity of the driver; the transmitted power H_{ij} of the driver as well as that of the belt can be obtained as

$$H_{ij} = T_{ij}\,\omega \tag{16}$$

Transform equation (15) and substitute it into equation (16). Then equation (16) can be expressed as

$$H_{ij} = F_{ij}\omega d_{\mathrm{p}}\frac{\exp(f\alpha) - 1}{\exp(f\alpha) + 1} \tag{17}$$

Substitute equation (14) into equation (17). Then equation (17) can be expressed as

$$H_{ij} = \frac{\Delta L_{ij}}{L_{ij}}EA\omega d_{\mathrm{p}}\frac{\exp(f\alpha) - 1}{\exp(f\alpha) + 1} \tag{18}$$

Define a coefficient K as

$$K = EA\omega d_{\mathrm{p}}\frac{\exp(f\alpha) - 1}{\exp(f\alpha) + 1} \tag{19}$$

where K has the dimension of power and can be treated as a constant in a MVBD. Substitute equation (19) into equation (18). Then equation (18) can be expressed as

$$H_{ij} = K\frac{\Delta L_{ij}}{L_{ij}} \tag{20}$$

Equation (20) shows the relation between actual elongation and transmitted power of a belt in a MVBD.

4.3 Operation risk evaluation of a MVBD

Consider a MVBD with n belts as a system in which each individual belt can be regarded as a component of the system. Different elongations of a belt denote different states of a component. When belt i is in state j, its contribution to the MVBD is represented by its transmitted power H_{ij}. The MVBD can perform its intended task if total transmitted power of all belts is greater than or equal to a prespecified value H_J, where H_J denotes the rated power that ensures the MVBD satisfies the design requirements. Therefore a MVBD can be regarded as a multistate weighted k-out-of-n system and the operation risk of the MVBD can be defined as the probability that the total transmitted power is less than the rated power.

It should be noted that in a MVBD system, both a component and system cannot have state 0 because of installed tension. This means that in a MVBD system all belts are in a certain tense state but the values of tension are different among them. This is a slight modification to the model defined in section 3, which can represent the actual state of a MVBD more reasonably.

In section 4.2 the interval $<\Delta L_{\min}, \Delta L_{\max}>$ has been obtained, which represents all possible elongations of an arbitrary belt i in a MVBD. From this interval m points (including two boundary points of this interval) can be selected uniformly, which represent m different elongations of the belt as well as m different states of the component. Suppose the probabilities that a component is in each state are identical. The probabilities that belt i is in each possible state can be obtained as

$$p_{ij} = 1/m, \quad (j = 1, 2, \ldots, m) \tag{21}$$

For an arbitrary belt i in a MVBD, H_{ij} and p_{ij} can be obtained based on equations (20) and (21) respectively. Then, according to equation (4), the UGF of transmitted power of all belts in a MVBD can be obtained, which has an identical form of

$$u_i(z) = p_{i1}z^{H_{i1}} + p_{i2}z^{H_{i2}} + \cdots + p_{im}z^{H_{im}}, \quad (i = 1, 2, \ldots, n) \tag{22}$$

According to equations (6) and (7), the total transmitted power of all belts can be obtained as

$$u_{\mathrm{s}}(z) = \sum_{J=1}^{M} p_{\mathrm{s}J}z^{H_{\mathrm{s}J}} \tag{23}$$

where $H_{\mathrm{s}J}$ and $p_{\mathrm{s}J}$ are possible values of the total transmitted power of the MVBD and the corresponding probabilities.

According to equation (10), the operation risk of the MVBD with a given rated power value H_J, denoted by $r_{\mathrm{s}}(H_J, n)$, can be obtained as

$$\begin{aligned} r_{\mathrm{s}}(H_J, n) &= 1 - R_{\mathrm{s}}(H_J, n) \\ &= 1 - \sum_{J=1}^{M} p_{\mathrm{s}J}\beta(H_{\mathrm{s}J}) \end{aligned} \tag{24}$$

5 AN ILLUSTRATIVE EXAMPLE

In this example, provided by reference [**12**], an 11 kW motor with the speed of 1470 r/min drives a rotary pump, which operates 24 h per day. The speed of the pump shaft and the centre distance are 400 r/min and about 1500 mm respectively. The parameters of the MVBD are given in Table 1. Based on the

Table 1 Parameters of the MVBD

Description	Notation	Value
Rated power	H_j	14.3 kW
Belt section	None	B
Pitch diameter of the small pulley	d_{p}	140 mm
Pitch diameter of the large pulley	D_{p}	500 mm
Centre distance	C	1487 mm
Belt pitch length	L_{p}	4000 mm
Contact angle of the small pulley	α	2.90 rad
Number of belts	n	4
Required initial tension	F_0	286.9 N

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approach suggested in section 4, the operation risk of this MVBD can be evaluated.

It is given that $E = 150\,\text{MPa}$, $A = 143\,\text{mm}^2$, $L_p = 4000\,\text{mm}$, $F_0 = 286.9\,\text{N}$, $t = 12\,\text{mm}$ (from China National Standards: GB/T 11544-1997), $d_p = 140\,\text{mm}$, $f = 0.5132$, $\alpha = 2.90\,\text{rad}$, and $\omega = 153.86\,\text{rad/s}$. According to equation (11), ΔL_0 can be obtained as

$$\Delta L_0 = 53.5\,\text{mm}$$

Because t is equal to 12 mm, according to equation (13) $\Delta L_{\min}$ and $\Delta L_{\max}$ can be obtained as

$$\Delta L_{\min} = 47.5\,\text{mm}$$

$$\Delta L_{\max} = 59.5\,\text{mm}$$

Select five points ($m = 5$) from the interval $< 47.5, 59.5 >$ mm. The possible elongations of each belt can be obtained as

$$\Delta L_{i1} = 47.5\,\text{mm}$$

$$\Delta L_{i2} = 50.5\,\text{mm}$$

$$\Delta L_{i3} = 53.5\,\text{mm}$$

$$\Delta L_{i4} = 56.5\,\text{mm}$$

$$\Delta L_{i5} = 59.5\,\text{mm}$$

According to equation (19), the value of coefficient K is obtained as

$$K = 291.8\,\text{kW}$$

According to equation (20), the transmitted powers corresponding to every elongation value of each belt can be obtained respectively as

$$\begin{aligned} H_{i1} &= K\frac{\Delta L_{i1}}{L_{i1}} \\ &= K\frac{\Delta L_{i1}}{L_p + t/2} \\ &= 3.5\,\text{kW} \end{aligned}$$

$$H_{i2} = 3.7\,\text{kW}$$

$$H_{i3} = 3.9\,\text{kW}$$

$$H_{i4} = 4.1\,\text{kW}$$

$$H_{i5} = 4.3\,\text{kW}$$

According to equation (21), p_{ij} can be obtained as

$$p_{ij} = 0.2 \quad (i = 1, 2, 3, 4;\ j = 1, 2, 3, 4, 5)$$

According to equation (22), the UGF of four belts can be found, which have an identical form

$$\begin{aligned} u_i(z) &= 0.2z^{3.5} + 0.2z^{3.7} + 0.2z^{3.9} + 0.2z^{4.1} \\ &\quad + 0.2z^{4.3} \\ &(i = 1, 2, 3, 4) \end{aligned}$$

According to equation (23), the UGF of the actual transmitted power of the MVBD is obtained as

$$\begin{aligned} u_s(z) &= \otimes_{+}[u_1(z), u_2(z), u_3(z), u_4(z)] \\ &= \prod_{i=1}^{4} u_i(z) \\ &= (0.2z^{3.5} + 0.2z^{3.7} + 0.2z^{3.9} + 0.2z^{4.1} + 0.2z^{4.3})^4 \\ &= 0.0016z^{14.0} + 0.0064z^{14.2} + 0.016z^{14.4} \\ &\quad + 0.032z^{14.6} + 0.056z^{14.8} + 0.0832z^{15.0} \\ &\quad + 0.1088z^{15.2} + 0.128z^{15.4} + 0.136z^{15.6} \\ &\quad + 0.128z^{15.8} + 0.1088z^{16.0} + 0.0832z^{16.2} \\ &\quad + 0.056z^{16.4} + 0.032z^{16.6} + 0.016z^{16.8} \\ &\quad + 0.0064z^{17.0} + 0.0016z^{17.2} \end{aligned}$$

It can be seen that there are 17 terms in this polynomial, i.e. $M = 17$ which is the case of $M < m^n = 5^4$ mentioned in section 3.2.

The value of rated power H_J of this MVBD is 14.3 kW. According to equation (24) the operation risk of this MVBD can be obtained as

$$\begin{aligned} r_s(H_J, n) &= 1 - R_s(14.3, 4) \\ &= 1 - \sum_{j=1}^{17} p_{sJ}\beta(P_{sJ}) \\ &= 0.008 \end{aligned}$$

In a similar way, the operation risk of this MVBD with different values of H_J can be evaluated. For example, when rated power H_J is equal to 15.0 kW, the operation risk of this MVBD is equal to 0.112.

6 CONCLUSIONS

In this paper, the variation of transmitted power caused by the length deviations of belts in a MVBD is analysed. The operation risk of a MVBD is defined as the probability that the actual transmitted power is less than the rated power. An approach to operation risk evaluation of a MVBD is presented by using the UGF technique and the model of a multi-state weighted k-out-of-n system. The effectiveness of the suggested approach is demonstrated by an example.

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APPENDIX A

Notation

A	cross-section area of belt
C	centre distance of an MVBD
d_p	pitch diameter of the small pulley
D_p	pitch diameter of the large pulley
E	elastic modulus of the belt
f	coefficient of friction between the belt and groove
$f(X_1, X_2, \ldots, X_q)$	function of r.v. $X_1, X_2, \ldots, X_q$
F_0	required initial tension of the belt
F_{ij}	actual initial tension of belt i
H_{ij}	actual transmitted power of belt i
H_J	rated power of MVBD
i	ith component of system/ith belt of MVBD, $i = 1, 2, \ldots, n$
j	jth state of component/jth state of belt, $j = 0, 1, \ldots, m$
k_J	total utilities ensuring system in state J or above
J	Jth state of system/Jth state of MVBD, $J = 0, 1, \ldots, M$
K	coefficient with a dimension of power
L_{ij}	actual length of belt i
L_{max}	maximum of belt length
L_{min}	minimum of belt length
L_p	pitch length of belt
n	number of components in system-/number of belts in MVBD
$\boldsymbol{p}$	vector consisting of probabilities corresponding to all values of X
$\boldsymbol{p}_i$	vector consisting of probabilities corresponding to all values of W_i
p_{ij}	probability of component i in state j/probability of belt i in state j
$r_s(H_J, n)$	risk of MVBD with n belts and a rated power H_J
$R_s(k_J, n)$	reliability of system with n components and a prespecified value k_J
t	allowable tolerance of belt length
T_{ij}	actual driving torque of belt i
$u_f(z)$	UGF of function $f(X_1, X_2, \ldots, X_q)$
$u_i(z)$	UGF of W_i/UGF of transmitted power of belt i in MVBD
$u_s(z)$	UGF of W_s/UGF of total transmitted power of MVBD
$u_X(z)$	UGF of X
$\boldsymbol{w}_i$	vector consisting of all possible values of W_i
w_{ij}	utility of component i when in state j
W_i	discrete r.v. denoting utility of component i
W_s	discrete r.v. denoting the total utilities of the system
$\boldsymbol{x}$	vector consisting of all possible values of X
X	discrete r.v.
$X_1, X_2, \ldots, X_q$	q independent discrete r.v.
α	contact angle of the small pulley
$\beta(w_{sJ})$	binary-valued function with domain on the set of possible values of W_s
ΔL_0	belt elongation when it produces an initial tension of F_0
ΔL_{ij}	actual elongation of belt i

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ΔL_{max}	maximum of belt elongation
ΔL_{min}	minimum of belt elongation
ω	angular velocity of driver (small pulley)
$\otimes$	composition operator in the UGF technique

Acronyms (for singular and plural forms)

r.v.	random variable
p.m.f.	probability mass function
UGF	universal generating function
MVBD	multiple V-belt drive

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