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ZASTOSOWANIE METODY DEKOMPOZYCJI HIERARCHICZNEJ DO ALOKACJI NIEZAWODNOŚCI W DUŻYCH SYSTEMACH

A HIERARCHICAL DECOMPOSITION APPROACH FOR LARGE SYSTEM RELIABILITY ALLOCATION

Niezawodność stała się w ostatnich latach ważkim problemem, zwłaszcza w odniesieniu do dużych systemów składających się z wielu podsystemów, modułów i komponentów. Dążenie do osiągania niezawodności już na etapie projektu sprawiło, że coraz więcej uwagi zwraca się na alokację niezawodności, metodę, która pozwala na dobrze wyważony podział docelowej niezawodności systemu pomiędzy jego podsystemy i komponenty. Jednakże poszukiwanie optymalnego programu alokacji niezawodności dla systemu o dużej liczbie podsystemów i części składowych nie jest sprawą prostą i problem ten należy do klasy problemów trudnych. Przeprowadzono wiele prac badających przydatność wydajnych obliczeniowo metod, np., algorytmu dokładnego, algorytmu heurystycznego czy algorytmu meta-heurystycznego, itp., do optymalizacji alokacji niezawodności systemu złożonego. I chociaż zaproponowane w dotychczasowych badaniach metody sprawdzają się w przypadku systemów składających się z umiarkowanej liczby elementów składowych, to wciąż jednak ciąży na nich "przekleństwo wymiarowości," które nie pozwala na ich łączenie w przypadku systemów składających się z dziesiątek/setek podsystemów i części składowych jakie znajdują zastosowanie w inżynierii przemysłowej. Aby zminimalizować ten niedostatek, zaproponowano strategię dekompozycji, w której problem alokacji niezawodności dla systemu o dużej liczbie komponentów jest rozkładany na zespół mniejszych, skoordynowanych podproblemów, które dają się rozwiązać w sposób obliczeniowo wydajny za pomocą tradycyjnego algorytmu optymalizacyjnego. W niniejszej pracy zastosowano metodę kaskadowania celów, jako wydajną metodę dekompozycji hierarchicznej, której użyto do rozkładu problemu alokacji niezawodności dużego systemu na zespół hierarchicznie uporządkowanych problemów optymalizacyjnych zgodnie z konfiguracją systemu. Wydajność i efektywność proponowanej metody ilustruje przykład numeryczny oraz studia porównawcze.

***Słowa kluczowe**: hierarchiczna struktura systemu, optymalna alokacja niezawodności, projektowanie systemów złożonych, kaskadowanie celów, dekompozycja systemu.*

Reliability has become a great concern in recent years, especially for large system consisting of a large number of subsystems, modules and components. To achieve the reliability goal in design stage, reliability allocation, a method to apportion the system target reliability amongst subsystems and components in a well-balanced way, has since received increasing attention. However, seeking the optimal reliability allocation scheme for a system with bunch of subsystems and components is not straightforward, and it is known as an NP-hard problem. An abundance of work has been carried out to investigate the computational efficient methods, e.g. exact algorithm, heuristic algorithm and meta-heuristic algorithm etc., to handle the optimization of reliability allocation for the complex system. Even though the proposed methods in past research work well for system consisting of a moderate set of components, they will still suffer "curse of dimensionality" and be impossible to converge if the system consisting of tens/hundreds of subsystems and components which maybe exist in industrial engineering. To mitigate the deficiency, a decomposition strategy is proposed, in which the reliability allocation problem for the system with a large number of components is decomposed into a set of smaller, coordinated sub-problems which can be solved via traditional optimization algorithm in an computational efficient manner. Target cascading method, as an efficient hierarchical decomposition method, is employed in this paper to decompose the large system reliability allocation problem into a set of hierarchical optimization problems in according with the system configuration. To illustrate the efficiency and effectiveness of the proposed method, a numerical example is presented, as well as some comparative studies.

***Keywords**: hierarchical system structure, optimal reliability allocation, large system design, target cascading, system decomposition.*

1. Introduction

Reliability based design of large complicated systems, such as aircraft and automobiles, usually involves complicated nonlinear programming optimization problems. Sometimes, it turns out to be difficult or impossible to solve using general mathematical programming approaches. Many decomposition methods have been used in optimal design of large complicated systems [2,3,5]. These methods are nonhierarchical in design and few were used for solving reliability optimization problems. Wang [8] and Li [6] proposed a decomposition-coordination method (DCM), which transforms an all-at-once optimum allocation problem into many small-scale optimization problems in a multi-level nested optimization architecture. DCM is very sensitive to step size, which indicates that it is not very robust. In addition, DCM can not provide a general allocation framework. Zhang [9] proposed the collaborative allocation (CA) based on collaborative optimization. CA is a nested optimization process with a general non-hierarchic problem structure. In CA the auxiliary constraints are equality constraints, and the convergence has not been demonstrated yet.

According to the experience gained from studies reported in [1,4,7], it is found that target cascading (TC) has a few features which are applicable to optimum allocation. Firstly, TC is designed for early product development, is particularly suitable for problems with feed forward coupling, and has a unidirectional hierarchical communication structure, which matches the features of reliability optimization problems. Secondly, TC provides a general optimization framework. Thirdly, the hierarchic multilevel optimization of TC is similar to allocation of reliability requirements. And lastly, coordination for linking variables and responses of different object levels can easily be associated with nested coordination for design requirements allocation, and also the convergence has been proven.

In this study, a new method called target cascading reliability allocation (TCRA), is proposed to solve large complicated reliability allocation problems. Examples of reliability allocation are used to describe the implementation procedure of TCRA.

The paper is structured as follows: Section 2 briefly introduces the whole structure of reliability allocation. Section 3 provides an introduction of target cascading. Section 4 develops a target cascading reliability allocation model. Section 5 presents an example to illustrate the proposed method. Finally, conclusions are provided in Section 6.

2. Model of Reliability Allocation of Large Systems

The reliability allocation problem is to minimize cost, weight or size under system reliability requirements. Mostly we minimize the cost, equation (1) is the dynamic programming model of such a reliability allocation problem.

$$\begin{aligned} &\text{mim } C_s = \sum_{i=1}^{N} C_i(u_i) \\ &\text{s.t. } R_s[R_1(u_1), R_2(u_2), \cdots, R_N(u_N)] \geq R_{S0} \\ &\qquad u_i \in U_i \end{aligned} \tag{1}$$

where C_S is the system total cost, C_i is the cost of the ith subsystem, R_S is a function of subsystem reliabilities R_i (i=1,2, …,N), N is the number of subsystems, u_i denotes the decision vector of subsystem i, U_i is the allowable range of u_i.

3. Principle and Mathematical Model of TC

3.1. Principle of TC

A large complicated system can be decomposed into three levels, namely, the system level, the subsystem level, and the component level. This simple three level structure is shown in Figure 1.

The principles of TC are illustrated in Figure 2. In this work, performance responses from a subproblem j at level i are represented by a vector r_{ij}. The superscript T denotes the target values passed from a higher level. It can be seen that the design objective of each element in TC is composed of two parts: (1) to minimize the deviation of subproblem performances and linking variables from assigned targets, and (2) to minimize the deviation of children element performances and linking variables from targets identified in that subproblem. Therefore, the framework of TC represents a collaborative design effort such that the ultimate goal of each subproblem is to help meeting the system-level targets. In TC, sibling elements do not communicate directly with each other but are coordinated via their parent elements for design consistency.

3.2. Mathematical model of TC

The TC optimization of element j at level i (O_{ij} in Figure 3) with n_{ij} children is formulated in equation (2), based on the information flow shown in Figure 2. The vector $\mathbf{r}_{ij}$ represents the element's responses. The optimization variables include local design variables $\mathbf{x}_{ij}$, linking variables $\mathbf{y}_{ij}$, targets for children responses $\mathbf{r}_{(i+1)k}$, k=1,...,n_{ij}, targets for children linking variables $\mathbf{y}_{(i+1)k}$, and tolerance optimization variables ε_{ij}^{r} and ε_{ij}^{y} to coordinate children responses and linking variables for design consistency.

$$\begin{aligned} &\text{Given } \mathbf{r}_{ij}^{U}, \mathbf{y}_{ij}^{U}, \mathbf{r}_{(i+1)k}^{L}, \mathbf{y}_{(i+1)k}^{L}, \ k = 1,\dots,n_{ij} \\ &\text{find } \ \mathbf{r}_{(i+1)k}, \mathbf{x}_{ij}, \mathbf{y}_{ij}, \mathbf{y}_{(i+1)k}, \varepsilon_{ij}^{r}, \varepsilon_{ij}^{y}, \ \ k = 1,\dots,n_{ij} \\ &\text{to minimize } \left\| \mathbf{r}_{ij} - \mathbf{r}_{ij}^{U} \right\| + \left\| \mathbf{y}_{ij} - \mathbf{y}_{ij}^{U} \right\| + \varepsilon_{ij}^{r} + \varepsilon_{ij}^{y} \\ &\text{subject to } \ \sum_{k=1}^{n_{ij}} \left\| \mathbf{r}_{(i+1)k} - \mathbf{r}_{(i+1)k}^{L} \right\| \leq \varepsilon_{ij}^{r} \\ &\qquad\qquad \sum_{k=1}^{n_{ij}} \left\| \mathbf{y}_{(i+1)k} - \mathbf{y}_{(i+1)k}^{L} \right\| \leq \varepsilon_{ij}^{y} \\ &\qquad\qquad \mathbf{g}_{ij}(\mathbf{r}_{ij}, \mathbf{x}_{ij}, \mathbf{y}_{ij}) \leq 0 \\ &\text{where } \quad \mathbf{r}_{ij} = \mathbf{f}_{ij}(\mathbf{r}_{(i+1)1}, \dots, \mathbf{r}_{(i+1)n_{ij}}, \mathbf{x}_{ij}, \mathbf{y}_{ij}) \end{aligned} \tag{2}$$

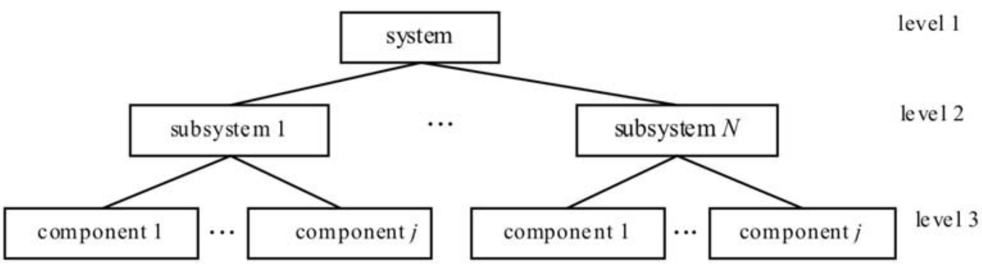

Fig. 1. A three -level system

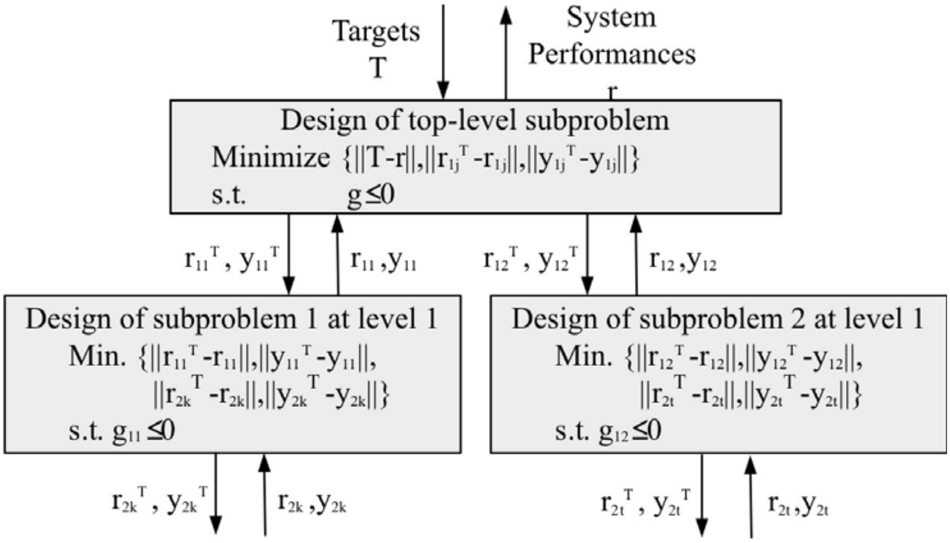

Fig. 2. The principles of TC

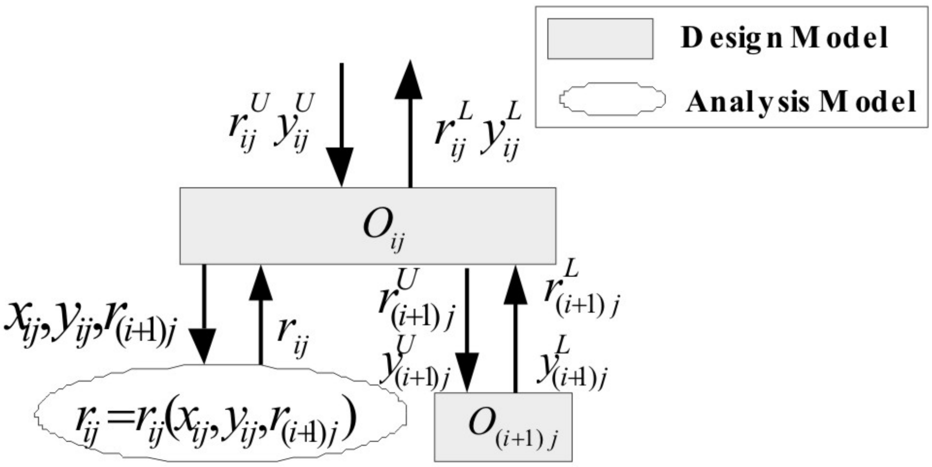

Fig. 3. Information Flow of Subsystem Oij in TC

In equation (2), the superscripts U indicate targets assigned by the parent element, while superscripts L indicate values passed from children elements. The targets for responses and linking variables of element O_{ij} are $\mathbf{r}_{ij}^{\mathrm{U}}$ and $\mathbf{y}_{ij}^{\mathrm{U}}$, respectively. The actual achievable values, $\mathbf{r}_{(i+1)k}^{\mathrm{L}}$ and $y_{(i+1)k}^{\mathrm{L}}$, are passed up to O_{ij} from its children. Solving the problem in equation (2), element O_{ij} finds the achievable values of its responses and linking variables that are the closest to $\mathbf{r}_{ij}^{\mathrm{U}}$ and $\mathbf{y}_{ij}^{\mathrm{U}}$, respectively. Then, O_{ij} passes them back to its parent element as $\mathbf{r}_{ij}^{\mathrm{L}}$ and $\mathbf{y}_{ij}^{\mathrm{L}}$ respectively. It also determines the optimal values for its children responses and linking variables with the least inconsistency from $\mathbf{r}_{(i+1)k}^{\mathrm{L}}$ and $\mathbf{y}_{(i+1)k}^{\mathrm{L}}$. These optimal values are passed down as targets, $\mathbf{r}_{(i+1)k}^{\mathrm{U}}$ and $\mathbf{y}_{(i+1)k}^{\mathrm{U}}$.

4. Mathematical Model of TCRA

The mathematical model of TCRA is established on the system level, subsystem level and component level respectively as shown in equations (3) to (5).

The system level programming P0

$$
\begin{aligned}
&\text{Find } [R_i, C_i]\ i = 1, 2, \cdots, N \\
&\min \left\| C_S - C_S^U \right\| + \left\| R_S - R_S^U \right\| \\
&\text{s.t. } \ R_S = f(R_1, R_2, \cdots R_N) \\
&\qquad C_S = \sum_{i=1}^{N} C_i,\ R_S \ge R_{S0} \\
&\qquad \left\| C_i - C_i^L \right\| \le \varepsilon_i,\ \ \left\| R_i - R_i^L \right\| \le \varepsilon_i \\
&\qquad \underline{R}_i \le R_i \le \overline{R}_i,
\end{aligned}
\tag{3}
$$

The subsystem level programming P11

$$
\begin{aligned}
&\text{Find } R_{ij}, C_{ij} \\
&\min\ f = \left\| R_i - R_i^U \right\| + \left\| C_i - C_i^U \right\| \\
&\text{s.t. } \left\| R_{ij} - R_{ij}^L \right\| + \left\| C_{ij} - C_{ij}^L \right\| \\
&\qquad C_i = \sum_j C_{ij} \\
&\qquad R_i = f(R_{i1}, R_{i2}, \cdots, R_{ij}) \\
&\qquad C_{ij} = f(R_{ij}) \\
&\qquad \underline{R}_{ij} \le R_{ij} \le \overline{R}_{ij}
\end{aligned}
\tag{4}
$$

The component level programming P21

$$
\begin{aligned}
&\text{Find } x_{ijm} \\
&\min\ \left\| R_{ij} - R_{ij}^U \right\| + \left\| C_{ij} - C_{ij}^U \right\| \\
&\text{s.t. } R_{ij} = f(x_{ij1}, x_{ij2}, x_{ijm}) \\
&\qquad C_{ij} = f(x_{ij1}, x_{ij2}, x_{ijm}) \\
&\qquad x_{ijm} \in X_{ijm}
\end{aligned}
\tag{5}
$$

5. Application of TC in Reliability Allocation

The reliability allocation problem in equation (6) is used to demonstrate TCRA for two-level optimal allocation. Through Figure 4 we can say that the system is composed of five subsystems and each subsystem encompasses two components.

$$
\begin{aligned}
&\text{Find } \mathbf{R} = [R_{11}\ R_{12}\ \cdots\ R_{51}\ R_{52}] \\
&\min\ C_s = \sum_{i=1}^{5} C_i \\
&\text{s.t. } R_S \ge 0.999 \\
&\qquad R_S = R_5 + R_1(1-R_5)(R_2R_3 + R_4 - R_2R_3R_4) \\
&\qquad 0.5 \le R_{ij} \le 0.98,\ i = 1,2; j = 1,2 \\
&\qquad 0.2 \le R_{ij} \le 0.99,\ i = 3,4,5; j = 1,2 \\
&\qquad R_i = R_{i1}R_{i2} \ge 0.5,\ i = 1,2 \\
&\qquad 0.5 \le R_i \le 0.998,\ i = 3,4,5 \\
&\qquad R_i = 1-(1-R_{i1})(1-R_{i2}), i = 3,4,5 \\
&\qquad C_{i1}\left(R_{i1}\right) = R_{i1}^2/3, C_{i2}\left(R_{i2}\right) = R_{i2}^2/2, i = 1,2 \\
&\qquad C_{i1}\left(R_{i1}\right) = [\ln(1-R_{i1})]^2/100, i = 3,4,5 \\
&\qquad C_{i2}\left(R_{i2}\right) = [\ln(1-R_{i2})]^2/100, i = 3,4,5 \\
&\qquad C_i = \sum_{i=1}^{2} C_{ij}\ ,\ i = 1,\cdots,5; j = 1,2
\end{aligned}
\tag{6}
$$

where R is the reliability requirement, and C is the cost. Subscripts 's', 'i' and 'ij' indicate the corresponding values of the system, subsystem i and component j in subsystem i, respectively.

According to TCRA, the partitioning structure as shown in Figure 5, the system optimization model as shown in equation (7) and the subsystem optimizations as shown in equations (8) and (9) are established. The subsystem 1 and subsystem 2 optimization models are the same as equation (8), while equation (9) shows that for subsystem 3 to 5. The system optimization takes the duty of allocating reliability requirements for subsystems, the subsystem optimizations feed back the subsystem optimum allocation to the system. Auxiliary variables C_i are also transmitted to subsystems in addition to reliability requirements to calculate the total cost. The system iteration process is showed in Figure 6.

$$
\begin{aligned}
&\text{Find } [\mathbf{R}\ \mathbf{C}] = [R_1\ R_2\ \cdots\ R_5\ C_1\ C_2\ \cdots\ C_5] \\
&\min\ \left\| C_s - C_s^U \right\| + \left\| R_s - R_s^U \right\| + \varepsilon_{ci} + \varepsilon_{Ri} \\
&\text{s.t. } R_S = R_5 + R_1(1-R_5)(R_2R_3 + R_4 - R_2R_3R_4) \\
&\qquad C_s = \sum_{i=1}^{5} C_i, R_S \ge 0.999 \\
&\qquad R_i \ge 0.5,\ i = 1,2, 0.5 \le R_i \le 0.998,\ i = 3,4,5 \\
&\qquad \left\| C_i - C_i^L \right\| \le \varepsilon_{ci},\ \left\| R_i - R_i^L \right\| \le \varepsilon_{Ri}
\end{aligned}
\tag{7}
$$

$$
\begin{aligned}
&\text{Find } [\mathbf{R}\ \mathbf{C}] = [R_{11}\ R_{12}\ \ R_{21}\ R_{22}\ C_1\ C_2] \\
&\min\ \left\| C_i - C_i^U \right\| + \left\| R_i - R_i^U \right\| \\
&\text{s.t. } 0.5 \le R_{ij} \le 0.98, \\
&\qquad R_i = R_{i1}R_{i2} \ge 0.5, \\
&\qquad C_{i1}\left(R_{i1}\right) = R_{i1}^2/3, \\
&\qquad C_{i2}\left(R_{i2}\right) = R_{i2}^2/2, \\
&\qquad C_i = \sum_{j=1}^{2} C_{ij} \\
&\qquad i = 1,2; j = 1,2
\end{aligned}
\tag{8}
$$

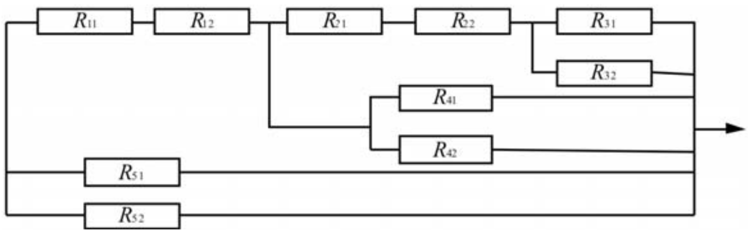

Fig. 4. System configuration

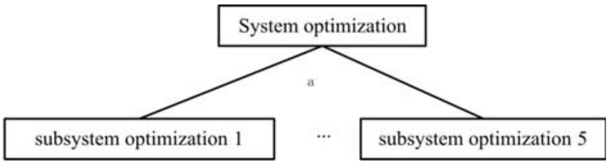

Fig. 5. The system partitioning structure

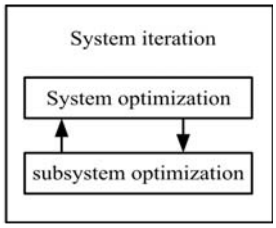

Fig. 6. The system iteration process

Tab.1. Two level reliability optimum allocation results using TCRA

		Subsystem (s_1)		Subsystem (s_2)		Subsystem (s_3)		Subsystem (s_4)		Subsystem (s_5)	
		S_{11}	S_{12}	S_{21}	S_{22}	S_{31}	S_{32}	S_{41}	S_{42}	S_{51}	S_{52}
DM	R_{ij}	0.8092	0.6607	0.7825	0.6389	0.3795	0.2518	0.7801	0.597	0.9794	0.9028
	C_{ij}	0.2183	0.2183	0.2041	0.2041	0.0019	0.0016	0.0222	0.0146	0.1499	0.0915
	R_i	0.5346		0.5017		0.5360		0.9114		0.998	
$R_{s=}$0.9990 C_s=1.1266											
DCM	R_{ij}	0.8472	0.6917	0.7826	0.6389	0.3372	0.3875	0.6159	0.4363	0.9795	0.9027
	C_{ij}	0.2392	0.2392	0.2042	0.2041	0.0066	0.0040	0.0092	0.0055	0.1510	0.0904
	R_i	0.5860		0.5000		0.7286		0.7835		0.9980	
R_s=0.9990 C_s=1.1533											
CA	R_{ij}	0.8102	0.6608	0.7830	0.6386	0.3757	0.2001	0.8465	0.2039	0.990	0.8000
	C_{ij}	0.2188	0.2183	0.2044	0.2039	0.0022	0.0008	0.0351	0.0009	0.2121	0.0432
	R_i	0.5354		0.5000		0.5006		0.8778		0.998	
R_s=0.9990 C_s=1.1397											
TCRA	R_{ij}	0.8151	0.6673	0.7822	0.6399	0.3557	0.2705	0.7688	0.6035	0.9778	0.9041
	C_{ij}	0.2215	0.2227	0.2039	0.2047	0.0019	0.0017	0.0214	0.0143	0.145	0.0916
	R_i	0.5439		0.5005		0.53		0.9083		0.9979	
R_s=0.9990 C_s=1.1287											

$$
\begin{aligned}
&\text{Find } [\mathbf{R}\ \mathbf{C}] = [R_{31}\ R_{32}\ R_{41}\ R_{42}\ R_{51}\ R_{52}\ C_3\ C_4\ C_5] \\
&\min\ \left\| C_i - C_i^U \right\| + \left\| R_i - R_i^U \right\| \\
&\text{s.t.}\ \ 0.5 \le R_i \le 0.998, \\
&\qquad 0.2 \le R_{ij} \le 0.99, \\
&\qquad R_i = 1 - (1 - R_{i1})(1 - R_{i2}), \\
&\qquad C_{i1}(R_{i1}) = [\ln(1 - R_{i1})]^2 / 100, \\
&\qquad C_{i2}(R_{i2}) = [\ln(1 - R_{i2})]^2 / 100, \\
&\qquad C_i = \sum_{j=1}^{2} C_{ij} \\
&\qquad i = 3,4,5;\ j = 1,2
\end{aligned}
\tag{9}
$$

DM, DCM, CA and TCRA are all adopted to solve the optimum allocation problem given in equation (6), and the results are listed in Table 1 for comparison, where S_{ij} (i=1,2,…,5,j=1,2) represents the component j in subsystem i.

Form Table 1, all the methods above can grant the system reliability, but the costs may not be the optimal, also we can find that the reliability of component are regarded as design variables, but in the physical structure, it is determined by some design parameter, such as material and environment, so the component level also a optimization level, the TCRA method can give more detail optimization process.

6. Conclusions

A target cascading method for reliability allocation is developed. TCRA is preliminarily validated and it still needs to be further studied. Through the present study, it is shown that:

1) Compared to DM, DCM and CA, the hierarchical structure of TC is closer to the reliability optimum allocation process.
2) TCRA can reflect the detailed relationship of system and subsystems, and give us some idea about the design of the lowest component level too.

3) Compared to DM, the dimensions of design variables are reduced through decomposition of optimization in CA and TCRA. Detailed analysis of subsystems may be performed inside its respective subsystem optimization.
4) Compared to DCM, the system level optimization is separated from subsystem optimizations in CA and TCRA. Subsystem optimization needs not to be performed in the process of the system level optimization. Accordingly the optimization is easier and the robustness is better.
5) Compared to CA, TCRA is a hierarchical multilevel optimization, and the auxiliary constraints are in the inequality form. The tolerances of the assigned targets are designed as optimization variables which improves the convergence.
6) As good hierarchical structure is critical to TCRA, further study is needed on its optimal partitioning and coordination of large complicated systems.

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This research was partially supported by the National High Technology Research and Development Program of China (863 Program) under contract number 2007AA04Z403 and the Open Project Program of the State Key Laboratory of Mechanical Transmission, Chongqing University, China under contract number 200802.

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