An Approach to Reliability Assessment Under Degradation and Shock Process

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***Abstract*—Product performance usually degrades with time. When shocks exist, the degradation could be more rapid. This research investigates the reliability analysis when typical degradation and shocks are involved. Three failure modes are considered: catastrophic (binary state) failure, degradation (continuous processes), and failure due to shocks (impulse processes). The overall reliability equation with three failure modes is derived. The effects of shocks on performance are classified into two types: a sudden increase in the failure rate after a shock, and a direct random change in the degradation after the occurrence of a shock. Two shock scenarios are considered. In the first scenario, shocks occur with a fixed time period; while in the second scenario, shocks occur with varying time periods. An engineering example is given to demonstrate the proposed methods.**

***Index Terms*—A fixed time period, degradation process, reliability analysis, shocks, varying time periods.**

ACRONYM

MSS	Multi-state System
ALT	Accelerated Life Testing
LS	Least Squares
MLE	Maximum Likelihood Estimation
BM	Bayesian Method
CDF	Cumulative Distribution Function
HPP	Homogeneous Poisson Process
TS	Test Specimens

NOTATION

t	time
$Y(t)$	general degradation path
K	number of shocks
A	damage caused by shocks
b	shock magnitude
$[0, A_s]$	small damage level
$[A_s, A_l]$	moderate damage level
$[A_l, +\infty)$	large damage level
λ	rate of HPP
$G(\bullet)$	cumulative distribution function for damage
p_s	probability that the damage falls into the region $[0, A_s]$
$p_{s,l}$	probability that the damage falls into the region $[A_s, A_l]$
p_l	probability that the damage falls into the region $[A_l, +\infty)$
hp_s	failure rate for the damage in the region $[0, A_s]$
$hp_{s,l}$	failure rate for the damage in the region $[A_s, A_l]$
q	number of shocks in the region $[A_s, A_l]$
$R(t)$	time-dependent reliability
$p_{0,l}$	probability that the damage falls into the region $[0, A_l]$
$hp_{0,l}$	failure rate that the damage falls into the region $[0, A_l]$
FM1	s-independent static failure mode
FM2	failure due to degradation
FM3	failure due to shocks
R_1	reliability associated with FM1
E_1	event of FM1
E_2	event of FM2
E_3	event of FM3
A_f	threshold of the damage of shocks
D_f	threshold of degradation
$\lambda(t, i)$	failure rate affected by the ith shock at time t
T	fixed period
α	constant changing failure rate after a shock
δY_i	random change in degradation Y caused by the ith shock

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I. INTRODUCTION

PRODUCT performance may deteriorate due to wear, fatigue, erosion, and other causes. Products may also suddenly fail due to excessive loading, shocks, and other stresses.

For the two kinds of failure, there are three major types of reliability analysis methods: binary-state methods, multi-state methods, and continuous state methods.

A binary-state method accounts for only two states: either perfect functioning, or complete failure. It is the earliest reliability analysis method, and has been widely applied to engineering problems. Because a number of intermediate states may exist between the perfect functioning and the complete failure states, the concept of a multi-state system (MSS) was introduced in the mid-1970s [1]–[4]. Consequently, MSS reliability theories have also been developed to analyse, model, and predict MSS performance.

The third type of reliability method, continuous state method, is suitable for performance degradation problems. Degradation is common in many components and systems, especially in mechanical and structural systems. Degradation is usually described by a continuous performance process in terms of time.

Obtaining enough failure data is time-consuming, and may be impossible for highly reliable systems. The accelerated life testing (ALT) methodology is commonly used because it is not only efficient to assess a product life, but also efficient to obtain the degradation data for highly reliable systems. Using degradation information becomes important for reliability prediction. Therefore, degradation data play an important role in evaluating system reliability. Reference [5] demonstrates how to use degradation data to estimate system reliability by using a degradation process model. To better describe the relationship between degradation and the implied lifetime distribution, two degradation models are investigated [6]. The first is the additive degradation model, where a degradation process is represented as the sum of a deterministic degradation path, and a random variation around the deterministic degradation path. The second is the multiplicative degradation model, where a degradation process is the product of a deterministic degradation path, and a random variation around the deterministic degradation path. In [7], statistical degradation data are analysed with three methods: random processes with a general distribution method, degradation path method, and linear regression method. Furthermore, a mixed model is built to assess the system reliability with degradation, and sudden failure data. The least squares (LS) method, maximum likelihood estimation (MLE), and Bayesian method (BM) are usually used to estimate parameters in both the degradation path method, and random processes with a general distribution method [8], [9]. In [10], the binary-state reliability method is extended to a continuous reliability method. Both degradation and sudden failure (catastrophe) are considered. The relation between degradation and catastrophe is studied by the state tree analysis, and the fault tree analysis. Statistical tools and regression techniques are used to build the mathematical model of degradation. However, the effect of shocking is not considered in [5]–[10].

Shocking is one of the major causes of system failures. Mallor [11] classifies the shock models into two categories based on the dependency of shocks. In the first method, the shock effect and its arrival time are assumed s-independent. In the second one, the shock effect and its arrival time are considered s-dependent. Furthermore, a general model is developed to describe the cumulative damage, extreme damage, and run damage. Under a generalized framework proposed by Bai *et al.* [12], a cumulative shock model, an extreme shock model, and a δ-shock model are created.

In many engineering applications, degradation and shocks occur at the same time. In Li and Pham's work [13], a reliability model with two degradation processes, and a random shock process, is investigated. If any of these processes exceeds a prefixed critical value, the system would break down. The two degradation processes are discretized into multiple states, and the MSS reliability theory is then applied. They also propose a maintenance model for systems with multiple competitive processes [14]. Li *et al.* [15] propose a reliability prediction method for s-independent and s-dependent degradation processes considering the effects of shocks. In [16], an approach to make an optimal replacement strategy for the system subject to shock and random threshold failure is proposed. Klutke and Yang [17] study the average availability of the systems with shocks, and graceful degradation. Some other competitive failure models by accounting for degradation and shock processes have also been studied recently [18], [19]. In their work, failure is assumed not self-announcing; the failure must be detected by inspection, and degradation is assumed to be caused by a stochastic environment. However, the relationship between a degradation process and a shocking process is not considered in their models.

It is obvious that shocks may affect a degradation process. Even though both the degradation process and shocks have been studied in previous literatures, their relationship has been rarely investigated. Shocks not only decrease the performance directly, but also speed up the degradation. The effect on the degradation due to shocks, especially those with serious damages, may not be neglected. Our objective in this paper is to conduct reliability analysis under both a degradation process and shocks with consideration of the effect of shocks on a degradation process. From the different perspectives of the effect, two methods are proposed. In the first method, the effect of shocks on degradation is considered as the sudden change in the failure rate; while in the second method, the effect of shocks is considered as the random change.

The organization of this paper is as follows. In Section II, degradation analysis and shocking process analysis are briefly reviewed. The three failure modes considered in this work are provided in Section III. In Section IV, the first method, i.e. reliability assessment with changing failure rate, is presented. And the second method, i.e. reliability assessment with random changes, is represented in Section V. An engineering example is given in Section VI. Conclusions and future research are provided in Section VII.

II. Degradation Analysis and Shocking Process Analysis

A. Degradation Analysis

The performance of many products, such as structures and machines, may deteriorate with time due to aging, fatigue, corrosion, and strength reduction. Degradation data about the performance are important for reliability analysis. They are usually collected from ALT, or operational systems.

The degradation path can be modeled by a stochastic process, where the additive degradation model or the multiplicative degradation model could be applied depending on the degradation characteristics [6]. Without loss of generality, we use $Y(t) = F_Y(t)$ to denote a general degradation path.

B. Shocking Process Analysis

In this subsection, we review several common shock models. Shocks may be produced internally within components or systems, or may come from the environment, or both. Most shocks are harmful. There are four typical shock models [11], [12]: the extreme shock model, the cumulative model, the run shock model, and the δ-shock model.

In the extreme shock model, a system is considered to fail as soon as the magnitude of any shock exceeds a threshold [20]. In the cumulative model, no shock is vital, and the shock magnitudes are cumulative; if the cumulative magnitudes cross over a critical level, the system will break down [21], [22]. The combination of the first and second models, the mixed shock model, has also been developed to account for both extreme shocks, and shocks whose consequences are cumulative [23].

The third model is the run shock model, which assumes that a system will be operational unless K consecutive shocks with critical magnitudes occur. This model is usually applicable for mechanical and electronic systems [24]. Mallor proposes a mixed model based on this model mixed with the cumulative shock model (the second model) to accommodate consecutive shocks whose consequences are also cumulative [25].

For the δ-shock model, a system will fail when the time lag between two successive shocks falls into a predefined time lag interval [26], [27].

In the extreme shock model and cumulative shock model, damage is considered proportional to the random shock magnitudes. The damage can be expressed as a function of the random shock magnitudes, namely $A = g(b)$, where A is the damage, and b is the shock magnitude.

For easy quantification, damage is divided into several levels. A damage level plays an important role in engineering practices because it may be an indicator for determining when maintenance should be made, and what kind of maintenance should be implemented. Too many levels, however, may bring a computational burden, or sometimes are not necessary. Hence, damage is usually divided into three levels [28]: small damage, moderate damage, and large damage. A small damage level is treated as harmless. A moderate damage level is considered as a harmful but not fatal impact. A large damage level is considered fatal, because when the damage reaches this level, a system will fail. Each damage level can be denoted by its respective region $[0, A_s]$, $[A_s, A_l]$, or $[A_l, +\infty)$.

Shock damage models have been studied by Finkelstein and Zarudnij [28]. In their work, a shock process is modeled by a HPP with rate λ, and the shock damage is assumed to follow s-independent Gaussian distributions. If the cumulative distribution function $G(A)$ of damage is known, the probability that the damage falls into regions $[0, A_s]$, $[A_s, A_l]$, and $[A_l, +\infty)$ can be computed respectively by

$$\begin{aligned} p_s &= G(A_s) \\ p_{s,l} &= G(A_l) - G(A_s) \\ p_l &= 1 - G(A_l). \end{aligned} \tag{1}$$

The corresponding failure rates are then given by [25]

$$\begin{aligned} hp_s &= \lambda p_s \\ hp_{s,l} &= \lambda p_{s,l} \\ hp_l &= \lambda p_l. \end{aligned} \tag{2}$$

If at least one shock from the process with failure rate hp_l occurs, or if more than q shocks from the region $[A_s, A_l]$ occur, the system will fail. Based on this assumption, the reliability at time t can be calculated by [28]

$$R(t) = \exp(-hp_l t)\exp(-hp_{s,l}t)\sum_{j=0}^{q}\frac{(hp_{s,l}t)^j}{j!}. \tag{3}$$

Shocks with large and moderate damage are considered in this work. Shocks with a fixed time period and varying time periods are also accounted for. Because shocks in the region $[0, A_s]$ have no consequence, we divided shocks into two regions: moderate damage region $[0, A_l]$, and large damage region $[A_l, +\infty)$. In the moderate damage region, damage is cumulative. And the probability that the damage falls into the region $[0, A_l]$ and $[A_l, +\infty)$ can be respectively calculated by

$$\begin{aligned} p_{0,l} &= G(A_l) \\ p_l &= 1 - G(A_l). \end{aligned} \tag{4}$$

Then the corresponding failure rates are

$$\begin{aligned} hp_{0,l} &= \lambda p_{0,l} \\ hp_l &= \lambda p_l. \end{aligned} \tag{5}$$

When shocks occur with a fixed time period, the reliability at time t can be expressed by

$$R(t) = (p_{0,l})^q \Pr\left\{\sum_{j=0}^{q} A_j \le A_f\right\} \quad (jT \le t < (j+1)T, \quad j = 1, 2, \ldots), \tag{6}$$

where A_j is the damage of the jth shock, and $A_0 = 0$; A_f is the threshold of the damage of shocks.

When shocks are assumed to follow a HPP, the reliability at time t can be expressed by

$$R(t) = \exp(-hp_l t)\sum_{j=0}^{q}\exp(-hp_{0,l}t) \times \frac{(hp_{0,l}t)^j}{j!}\Pr\left\{\sum_{k=0}^{j} A_k \le A_f\right\}, \tag{7}$$

where A_k is the damage of the kth shock.

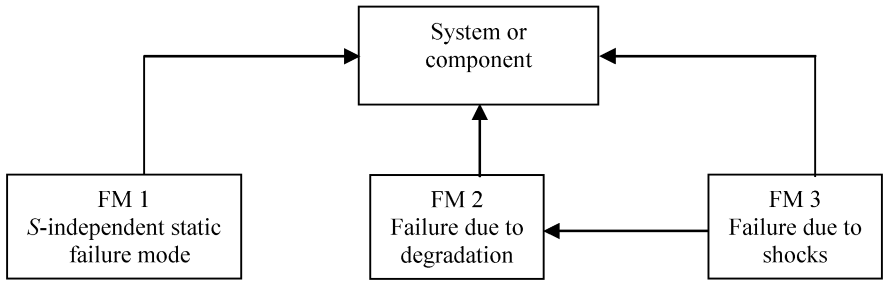

Fig. 1. Relationship between performance and three failure modes.

III. Three Failure Modes Considered in This Work

In this work, the performance of a system or component is assumed to degrade due to three failure modes, as shown in Fig. 1.

The three failure modes are s-independent static failure mode (FM1), failure due to degradation (FM2), and failure due to shocks (FM3). FM1 is statistically time independent, and is also s-independent of degradation and shocks. It may come from manufacture imprecision, damages during transportation, or storage. It may occur before the system or component is put into operation. R_1 is used to denote the reliability associated with this failure mode. FM2 is caused by the degradation during product servicing time. FM3 is resulted from shocks, which not only affect the performance of a system or component, but also speed up FM2.

Let E_1, E_2, and E_3 denote the corresponding events of FM1, FM2, and FM3, respectively. As discussed above, E_1 is s-independent of E_2 and E_3. Whichever failure mode occurs, the system or component is considered not functioning. Hence the overall reliability can be calculated by the competitive failure model

$$\begin{aligned} R &= \Pr\{E_1, E_2, E_3\} \\ &= \Pr\{E_1\} \cdot \Pr\{E_2, E_3\} \\ &= \Pr\{E_1\} \cdot \Pr\{E_2 | E_3\} \cdot \Pr\{E_3\}. \end{aligned} \tag{8}$$

As mentioned previously, the probability of the first event is $\Pr\{E_1\} = R_1$. For E_2, when the degradation $Y(t)$ is greater than a threshold D_f, FM2 occurs. The probability of $E_2|E_3$ is therefore computed by $\Pr\{Y(t) \le D_f | E_3\}$.

Then the reliability can be rewritten as

$$R = R_1 \cdot \Pr\{Y(t) \le D_f | E_3\} \cdot \Pr\{E_3\}. \tag{9}$$

From different perspectives about the effects of shocks on degradation, two reliability analysis methods are proposed. In the first method, the effects of shocks on degradation are considered as changes in the failure rate. In the second method, the effects of shocks on degradation are considered as random changes in the degradation process. In both methods, two shock models are used: shocks with a fixed time period, and shocks with varying periods. The first method is presented in Section IV, and the second method is presented in Section V.

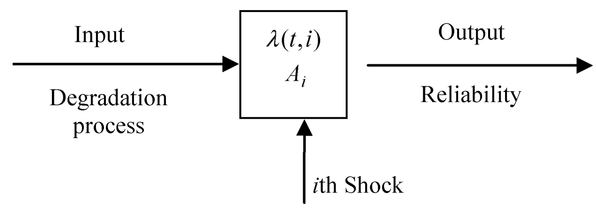

Fig. 2. Flowchart of reliability calculation process.

IV. Reliability Analysis With Increasing Failure Rates

In this Section, we consider the effect of shocks causing a sudden increase in the failure rate of a degradation process.

A flowchart (Fig. 2) is used to demonstrate the idea. In Fig. 2, the input denotes the degradation process, $\lambda(t, i)$ denotes the failure rate affected by the ith shock at time t, A_i denotes the random damage by the ith shock, and the output denotes the reliability at time t.

Shocks may occur internally within a system, or come from the operational environment. Some internal shocks within rotating machinery occur in a fixed time period. On the other hand, the occurrence of external shocks from the environment usually is random, and in this case a HPP is often used to model the shocking process. Both shocks with a fixed time period (Section IV-A), and those with varying time periods (Section IV-B) are studied in this paper.

Reliability can be computed by an integral if the failure rate is known. For shocks with a fixed time period, the integral limits are constants; and it is relatively easy to calculate (see Section IV-A). For those shocks with varying time periods, however, the integral limits are random numbers; it is difficult, or even impossible, to obtain an analytical solution. We then propose a simulation method (see Section IV-B).

A. Shock With a Fixed Time Period

As shown in Fig. 3, shocks with a fixed time period affect a product performance P with associated degradation processes. For example, an engine is fixed in a factory, where the shocks with a fixed period due to the manufacturing defect affect the engine performance. We consider the effect of shocks with a fixed time period on degradation by two approaches. In the first approach, the effect of shocks is an increasing function with respect to the failure rate. This approach is discussed in the following paragraphs. In the other approach, the effect of shocks is

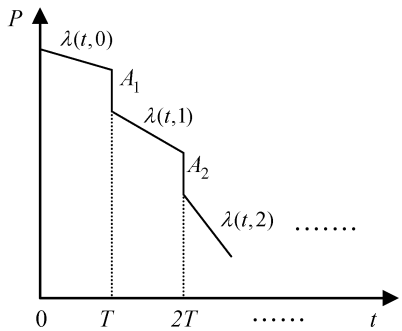

Fig. 3. Performance degrading process.

assumed to be the discrete random changes in the degradation. The latter approach will be discussed in Section V-A.

In Fig. 3, the vertical axis P denotes a performance, and the horizontal axis t denotes time. The fixed time period is T, and $iT(i=1,2,\cdots)$ are therefore the time instances when shocks occur. The shocks cause random damage $A_1, A_2,, \cdots$, in the performance P. $\lambda(t,0)$ is the initial failure rate without shocks while $\lambda(t,i)$ denotes the failure rate after the ith shock. For easy computation due to the additive property of the Gaussian distribution, $A_i(i=1,2,\cdots)$ are assumed to follow a s-independent Gaussian distribution in this paper. The proposed method can also be extended to the cases that the damage follows another distribution, such as the lognormal distribution, or the Weibull distribution. There may not be an analytical expression, but Monte Carlo simulation could assist the search of the solution for these extended cases.

We also assume the failure rate increases with a constant rate $\alpha(\alpha \geq 1)$ after each shock. The failure rates in terms of time can then be represented by

$$\lambda(t,i)=\alpha\lambda(t,i-1) \qquad (i=1,2,\ldots). \tag{10}$$

From the above equation, a general formulation is given by

$$\lambda(t,i)=\alpha^i\lambda(t,0) \qquad (i=1,2,\ldots). \tag{11}$$

The failure rate is one important measure for reliability which can be derived [29]. The initial failure rate, before the f

$$\lambda(t,0)=\frac{1}{\Pr\{Y(t)\leq D_f\}}\frac{d\left[1-\Pr\{Y(t)\leq D_f\}\right]}{dt}. \tag{12}$$

When $Y(t)$ is less than D_f, the component or system is considered operational. Therefore, in (12), $\Pr\{Y(t)\leq D_f\}$ represents reliability.

In most engineering problems, the mean and standard deviation of a degradation process increase in terms of time. The Gaussian process with increasing mean and standard deviation could be used to describe these engineering problems. Furthermore, when $Y(t)$ is a Gaussian process with increasing mean and standard deviation, $\Pr\{Y(t)\leq D_f\}$ is a monotone decreasing function with respect to time. Therefore, a Gaussian process with increasing mean and standard deviation could be considered as an approximate monotone process. Given the Gaussian distribution assumption, (12) becomes

$$\lambda(t,0)=\frac{\exp\left(-\frac{(D_f-\eta(t))^2}{\sigma^2(t)}\right)}{\sqrt{2\pi}\sigma(t)\left[1-0.5\left(1+erf\frac{D_f-\eta(t)}{\sqrt{2}\sigma(t)}\right)\right]}, \tag{13}$$

where $erf(x)=(2/\sqrt{\pi})\int_0^x e^{-t^2}dt$.

When only the degradation is considered without shocks, reliability is denoted by $R_0(t)$, and can be written as a function of the initial failure rate

$$R_0(t)=\Pr\{Y(t)<D_f\}=\exp\left\{-\int_0^t\lambda(\tau,0)d\tau\right\}. \tag{14}$$

In this paper, the mixed shock model in (6) is used. Based on (9), reliability can be expressed as follows.

When $0\leq t<T$,

$$\begin{aligned}R(t)&=\Pr\{E_1\}\Pr\{E_2|E_3\}\Pr\{E_3\}\\&=R_1\exp\left\{-\int_0^t\lambda(\tau,0)d\tau\right\},\end{aligned} \tag{15}$$

where $\Pr\{E_1\}=R_1$, $\Pr\{E_2|E_3\}=\exp\{-\int_0^t\lambda(\tau,0)d\tau\}$, and $\Pr\{E_3\}=1$.

When $iT\leq t<(i+1)T, i=1,2,\ldots,q$,

$$\begin{aligned}&\Pr\{E_2|E_3\}\\&=\Pr\{Y(t)\leq D_f|E_3\}\\&=\exp\left\{-\left(\sum_{j=1}^{i}\int_{(j-1)T}^{jT}\lambda(\tau,j-1)d\tau+\int_{iT}^{t}\lambda(\tau,i)d\tau\right)\right\}\\&=\exp\left\{-\left(\sum_{j=1}^{i}\int_{(j-1)T}^{jT}\alpha^{j-1}\lambda(\tau,0)d\tau+\int_{iT}^{t}\alpha^i\lambda(\tau,0)d\tau\right)\right\},\end{aligned} \tag{16}$$

$$\begin{aligned}&\Pr\{E_3\}\\&=(p_{0,l})^i\Pr\left\{\sum_{j=0}^{i}A_j\leq A_f\right\},\end{aligned} \tag{17}$$

$$\begin{aligned}&R(t)\\&=\Pr\{E_1\}\Pr\{E_2|E_3\}\Pr\{E_3\}\\&=R_1\exp\left\{-\left(\sum_{j=1}^{i}\int_{(j-1)T}^{jT}\alpha^{j-1}\lambda(\tau,0)d\tau+\int_{iT}^{t}\alpha^i\lambda(\tau,0)d\tau\right)\right\}\\&\quad\times(p_{0,l})^i\Pr\left\{\sum_{j=0}^{i}A_j\leq A_f\right\}.\end{aligned} \tag{18}$$

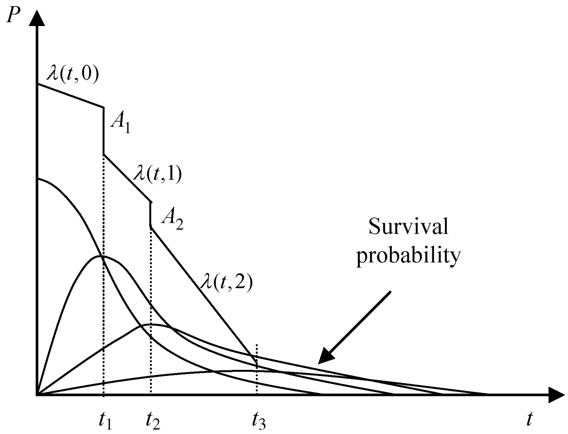

Fig. 4. Performance degrading process under random shocks.

Because of the relationship between reliability and failure rate, (18) can be rewritten as

$$R(t) = R_1 \left\{ \prod_{j=1}^{i} \left[R_0(jT)\right]^{\left(\alpha^{(j-1)} - \alpha^{j}\right)} \left[R_0(t)\right]^{\alpha^{i}} \right\} (p_{0,l})^{i} \times \Pr\left\{ \sum_{j=0}^{i} A_j \le A_f \right\}. \quad (19)$$

The number of periods can be determined by $i = \text{int}[t/T]$, where int stands for taking the integer part of t/T. When the product serviceable life t is given, i can then be determined. Because (19) is a recursive formulation, and $R_0(t)$ can be obtained by (14), it is straightforward to calculate $R(t)$.

B. Shocks With Poisson Process

In the above subsection, shocks with a fixed time period of occurrence are discussed. In this subsection, shocks with varying time periods of occurrence are considered. For example, an engine is put in a car, where the shocks due to the condition of the road affect the engine performance. The varying time periods of occurrence may follow a certain probability distribution. In this paper, we employ the most commonly used HPP to describe a shocking process. The performance process under the random shocks is depicted in Fig. 4.

In Fig. 4, the vertical axis P denotes the performance, and the horizontal axis t denotes time. The time periods are varying, and $t_i (i = 1, 2, \cdots)$ are the time instances when shocks occur. The shocks cause random damages, $A_1, A_2, \cdots$ on the performance metric P. $\lambda(t, 0)$ denotes the initial failure rate without shocks, while $\lambda(t, i)$ denotes the failure rate after the ith shock. In this paper, $A_i (i = 1, 2, \cdots)$ are assumed to follow a s-independent Gaussian distribution. However, the proposed method can also be extended to cases that $A_i (i = 1, 2, \cdots)$ do not follow the s-independent Gaussian distribution. The model (in Section IV-A) is a special case of the model herein.

Then reliability can be derived as follows.

When $0 \le t < t_1$,

$$R(t) = \Pr\{E_1\} \Pr\{E_2|E_3\} \Pr\{E_3\} = R_1 \exp\left\{ -\int_0^t \lambda(\tau, 0) d\tau \right\}. \quad (20)$$

When $t_i \le t < t_{i+1}$ $(i = 1, 2, \ldots, n)$,

$$\begin{aligned} \Pr\{E_2|E_3\} &= \Pr\{Y(t) \le D_f | E_3\} \\ &= \exp\left\{ -\left(\sum_{j=1}^{i} \int_{t_{(j-1)}}^{t_j} \lambda(\tau, j-1) d\tau + \int_{t_i}^{t} \lambda(\tau, i) d\tau \right) \right\} \\ &= \exp\left\{ -\left(\sum_{j=1}^{i} \int_{t_{(j-1)}}^{t_j} \alpha^{j-1} \lambda(\tau, 0) d\tau + \int_{t_i}^{t} \alpha^{i} \lambda(\tau, 0) d\tau \right) \right\}. \end{aligned} \quad (21)$$

From (7),

$$\Pr\{E_3\} = \exp(-hp_l t) \exp(-hp_{0,l} t) \times \frac{(hp_{0,l} t)^i}{i!} \Pr\left\{ \sum_{j=0}^{i} A_j \le A_f \right\}. \quad (22)$$

Because the time instances of occurrence follow a HPP, the survival probability under i shocks can be expressed by

$$\begin{aligned} P_i(t) &= \Pr\{E_1\} \Pr\{E_2|E_3\} \Pr\{E_3\} \\ &= R_1 \exp\left\{ -\left(\sum_{j=1}^{i} \int_{t_{(j-1)}}^{t_j} \alpha^{j-1} \lambda(\tau, 0) d\tau + \int_{t_i}^{t} \alpha^{i} \lambda(\tau, 0) d\tau \right) \right\} \\ &\quad \times \exp(-hp_l t) \exp(-hp_{0,l} t) \frac{(hp_{0,l} t)^i}{i!} Pr\left\{ \sum_{j=0}^{i} A_j \le A_f \right\} \\ &= R_1 \left\{ \prod_{j=1}^{i} \left[R_0(t_j)\right]^{\left(\alpha^{(j-1)} - \alpha^{j}\right)} \left[R_0(t)\right]^{\alpha^{i}} \right\} \exp(-hp_l t) \\ &\quad \times \exp(-hp_{0,l} t) \frac{(hp_{0,l} t)^i}{i!} \Pr\left\{ \sum_{j=1}^{i} A_j \le A_f \right\}. \end{aligned} \quad (23)$$

The survival probabilities are also plotted in Fig. 4. Then reliability, the sum of all the survival probabilities, is calculated by

$$\begin{aligned} R(t) &= \sum_{i=1}^{\infty} P_i \\ &= \sum_{i=1}^{\infty} R_1 \left\{ \prod_{j=1}^{i} \left[R_0(t_j)\right]^{\left(\alpha^{(j-1)} - \alpha^{j}\right)} \left[R_0(t)\right]^{\alpha^{i}} \right\} \exp(-hp_l t) \\ &\quad \times \exp(-hp_{0,l} t) \frac{(hp_{0,l} t)^i}{i!} \Pr\left\{ \sum_{j=0}^{i} A_j \le A_f \right\} \end{aligned} \quad (24)$$

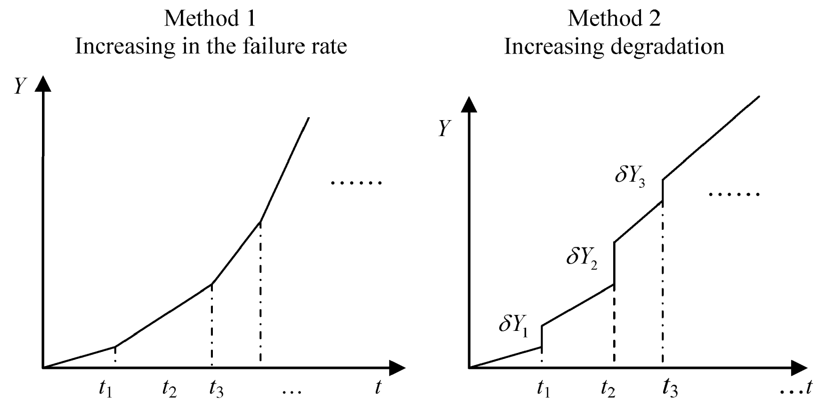

Fig. 5. Degradation process under random shocks.

Because the shocking process is assumed to follow a HPP, t_j is a Gamma distribution with parameters λ and j. Because no analytical solution is available, a simulation method is used to solve (23) and (24). A matrix is used to represent the samples of t_j, as shown below.

$$\begin{bmatrix} t_{11} & t_{12} & \dots & t_{1m} \\ t_{21} & t_{22} & \dots & t_{2m} \\ \dots & \dots & \dots & \dots \\ t_{j1} & t_{j2} & \dots & t_{jm} \end{bmatrix}$$

m samples of t_j (m columns in the matrix) are obtained from Monte Carlo simulation. Then m reliability functions can be obtained by

$$R(t;k) = \sum_{n=1}^{\infty} R_1 \left\{ \prod_{j=1}^{i} [R_0(t_{jk})]^{(\alpha^{(j-1)}-\alpha^j)} [R_0(t)]^{\alpha^i} \right\} \times \exp(-hp_l t)\exp(-hp_{0,l}t)\frac{(hp_{0,l}t)^i}{i!} \times \Pr\left\{ \sum_{j=0}^{i} A_j \le A_f \right\} \quad (k = 1,2,\dots,m). \tag{25}$$

Then the mean reliability is computed by

$$\overline{R}(t) = \frac{1}{m}\sum_{k=1}^{m} R(t;k). \tag{26}$$

And the standard deviation of the reliability is given by

$$\sigma_{R(t)} = \sqrt{\frac{1}{m-1}\sum_{k=1}^{m}\left[R(t;k) - \overline{R}(t)\right]^2}. \tag{27}$$

V. Reliability Analysis With Degrading Performances

In Section IV, the effect of shocks on degradation is considered as a sudden increase in the failure rate. A simulation method is used to calculate reliability when shocks occur with varying time periods. In this section, the effect of shocks is considered as random changes in the degradation process.

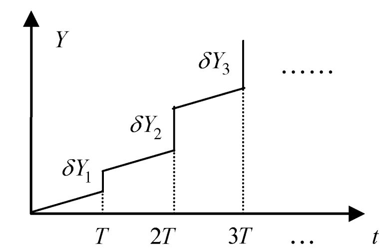

Fig. 6. Degradation process under shocks with a fixed time period.

The comparison between the two methods is depicted in Fig. 5, where Y is the degradation. $\delta Y_i (i = 1,2,\dots)$ is the random effect on degradation due to the ith shock, and is assumed to be a Gaussian distribution with $\delta Y_{\mathrm{i}} \sim N(\mu_1, \sigma_1^2)$. In the first method, the failure rate suddenly increases when a shock occurs. In the second method, a sudden random increase in degradation is assumed after a shock.

From Fig. 5, we know that shocks cause the degradation process to jump at time t_i for method 2. Then the degradation process is no longer continuous, and forms several states. Each shock can be considered as a state transmission signal. For more details, see [21].

A. Shocks With a Fixed Time Period

Fig. 6 describes the degradation process under shocks with a fixed time period.

In Fig. 6, the vertical axis Y is the degradation measure. The fixed time period is T, and $iT (i = 1,2,\cdots)$ are therefore the time instances when shocks occur. δY_i is the random change in Y caused by the ith shock.

The mixed shock model in (6) is used here. By accommodating three failure modes (see Fig. 1), (9), and (28) (shown at the bottom of the next page) is obtained to assess the reliability.

$$R(t) = \begin{cases} R_1 \Pr\{Y(t) \le D_f\} & (0 \le t < T) \\ R_1 \Pr\left\{Y(t) + \sum_{j=0}^{i}\delta Y_j \le D_f\right\}(p_{0,l})^i \Pr\left\{\sum_{j=0}^{i} A_j \le A_f\right\} & (iT \le t < (i+1)T, (i = 1,2,\dots,q)) \end{cases}. \tag{28}$$

$\delta Y_0 = 0$, and $\delta Y_i (i = 1, 2, \ldots, n)$ are s-independent, and are assumed to follow the Gaussian distribution with CDF

$$F(\delta y_i) = \frac{1}{\sqrt{2\pi}\sigma_1} \int_{-\infty}^{\delta y_i} e^{-\frac{(y-\mu_1)^2}{2\sigma_1^2}} dy. \quad (29)$$

Because $\delta Y = \sum_{i=1}^{q} \delta Y_i$ is a linear combination of random variables with a Gaussian distribution, δY also follows a Gaussian distribution with the CDF

$$F(\delta y) = \frac{1}{\sqrt{2\pi} q \sigma_1} \int_{-\infty}^{\delta y} e^{-\frac{(y-q\mu_1)^2}{2n\sigma_1^2}} dy. \quad (30)$$

Because $Y(t)$ is a stochastic process, and is assumed to be a Gaussian process in this paper, $Y(t) + \delta Y$ is also a stochastic process, and $Y(t) + \delta Y \sim N[q\mu_1 + \eta(t), q\sigma_1^2 + \sigma^2(t)]$.

Then $\Pr\{Y(t) + \sum_{j=0}^{i} \delta Y_j \le D_f\}$ in (28) can be computed by

$$\Pr\left\{Y(t) + \sum_{j=0}^{i} \delta Y_j \le D_f\right\} = \Pr\left\{Y(t) + \delta Y \le D_f\right\} = \Phi\left(\frac{D_f - \eta(t) - i\mu_1}{\sqrt{i\sigma_1^2 + \sigma^2(t)}}\right). \quad (31)$$

A_i are assumed to follow a s-independent Gaussian distribution with $A_i \sim N(\mu_2, \sigma_2^2)$. By substituting (31) into (28), reliability can be rewritten as (32), shown at the bottom of the page

B. Shocks With Poisson Process

In Section V-A, shocks occur with a fixed time period. In this subsection, shocks arrive with varying time periods, which follow a HPP. Given the shock model in (7), the survival probability under i shocks can be expressed by

$$P_i(t) = R_1 \Pr\left(Y(t) + \sum_{j=0}^{i} \delta Y_j \le D_f\right) \exp(-hp_l t) \times \exp(-hp_{0,l} t) \frac{(hp_{0,l} t)^i}{i!} \Pr\left\{\sum_{j=0}^{i} A_j \le A_f\right\}. \quad (33)$$

Then the reliability, the sum of all the survival probabilities, can be computed by

$$R(t) = \sum_{i=0}^{\infty} P_i(t) = R_1 \Pr\left(Y(t) + \sum_{j=0}^{i} \delta Y_j \le D_f\right) \exp(-hp_l t) \times \exp(-hp_{0,l} t) \frac{(hp_{0,l} t)^i}{i!} \Pr\left\{\sum_{j=0}^{i} A_j \le A_f\right\}. \quad (34)$$

Note from (34) that reliability is not related to time instances. Only the number of shocks is needed. Then an analytical result can be obtained, and is given by

$$R(t) = \sum_{i=0}^{\infty} R_1 \Phi\left(\frac{D_f - \eta(t) - i\mu_1}{\sqrt{i\sigma_1^2 + \sigma(t)}}\right) \exp(-hp_l t) \times \exp(-hp_{0,l} t) \frac{(hp_{0,l} t)^i}{i!} \Pr\left\{\sum_{j=0}^{i} A_j \le A_f\right\}. \quad (35)$$

VI. Case Study

The case study is based on the fatigue crack data in [30]. We demonstrate the proposed methods presented in Sections IV and V, with necessary modifications in the problem from [30]. Meanwhile, some information on the shocking process is provided to illustrate the proposed method. Even though the shocking process is hypothetical, the proposed methods are still useful with real data. Note that the effect of shocks on degradation reduces the reliability compared to that without consideration of any given information on the shock process.

A. Degradation and Shock Processes

In [30], the degradation path approach is used to analyse the fatigue crack growth data. The data were collected from 21 test specimens (TS). All specimens have an initial crack length of 0.90 cm. The degradation metric Y (crack length) in Table I are part of the data sets in [30]. The loading cycles in the original data are transformed into the loading time herein.

The formulation $\mu(t) = ae^{bt}$ for the mean, and $\sigma(t) = ce^{dt}$ for the standard deviation of the degradation process Y are assumed, where a, b, c, and d are undetermined coefficients. Then the LS method is employed to determine the coefficients a, b, c, and d. The results are

$$\hat{\mu}(t) = 0.8767e^{0.0398t}, \quad (36)$$
$$\hat{\sigma}(t) = 7.7602 \times 10^{-5} e^{0.4646t}. \quad (37)$$

From the data analysis, a Gaussian process is suitable to fit the data sets. Hence, Y is assumed to follow a Gaussian process, and the degradation path can be described by

$$Y(t) \sim N\left[\hat{\mu}(t), \hat{\sigma}(t)\right] = N(0.8767e^{0.0398t}, 7.7602 \times 10^{-5} e^{0.4646t}) \quad (38)$$

We then introduce shocks into the problem. As discussed previously, shocks are categorized into two types in terms of their arrival time: shocks with a fixed time period, and those with varying time periods.

For the shocks with a fixed time period, the time period $T = 2\ \mathrm{months}$ is used. Yet for the shocks with varying time periods, a HPP is used, and the occurrence rate is $\lambda = 0.5$.

$$R(t) = \begin{cases} R_1 \Phi\left(\frac{D_f - \eta(t)}{\sigma(t)}\right) & (0 \le t < T) \\ R_1 \Phi\left(\frac{D_f - \eta(t) - i\mu_1}{\sqrt{i\sigma_1^2 + \sigma^2(t)}}\right) (p_{0,l})^i \Phi\left(\frac{A_f - i\mu_2}{\sqrt{i}\sigma_2}\right) & (iT \le t < (i+1)T, (i = 1, 2, \ldots, q)) \end{cases}. \quad (32)$$

TABLE I
FATIGUE CRACK GROWTH DATA (CM)

TS	Loading time (month)												
	0	1	2	3	4	5	6	7	8	9	10	11	12
1	0.90	0.92	0.97	1.01	1.05	1.09	1.15	1.21	1.28	1.36	1.44	1.55	1.72
2	0.90	0.92	0.96	1.00	1.04	1.08	1.13	1.19	1.26	1.34	1.42	1.52	1.67
3	0.90	0.93	0.97	1.00	1.04	1.08	1.13	1.18	1.24	1.31	1.39	1.49	1.65
4	0.90	0.93	0.97	1.00	1.03	1.07	1.10	1.16	1.22	1.29	1.37	1.48	1.64
5	0.90	0.92	0.97	0.99	1.03	1.06	1.10	1.14	1.20	1.26	1.31	1.40	1.52
6	0.90	0.93	0.96	1.00	1.03	1.07	1.12	1.16	1.20	1.26	1.30	1.37	1.45
7	0.90	0.92	0.96	0.99	1.03	1.06	1.10	1.16	1.21	1.27	1.33	1.40	1.49
8	0.90	0.92	0.95	0.97	1.00	1.03	1.07	1.11	1.16	1.22	1.26	1.33	1.40
9	0.90	0.93	0.96	0.97	1.00	1.05	1.08	1.11	1.16	1.20	1.24	1.32	1.38
10	0.90	0.92	0.94	0.97	1.01	1.04	1.07	1.09	1.14	1.19	1.23	1.28	1.35
11	0.90	0.92	0.94	0.97	0.99	1.02	1.05	1.08	1.12	1.16	1.20	1.25	1.31
12	0.90	0.92	0.94	0.97	0.99	1.02	1.05	1.08	1.12	1.16	1.19	1.24	1.29
13	0.90	0.92	0.94	0.97	0.99	1.02	1.04	1.07	1.11	1.14	1.18	1.22	1.27

The damage on the performance due to the ith shock follows a Gaussian distribution with $A_i \sim N(2\,\text{units}, 0.5\,\text{units})$. The CDF of damage $G(A)$ is also assumed to follow a Gaussian distribution, and the threshold between moderate and large damage is defined as $A_l = 3.2\,\text{units}$. The probabilities of damage of one shock falling into the regions $[0, A_l]$, and $[A_l, \infty]$ are $p_{0,l} = 0.9552$, and $p_l = 0.0488$, respectively. Then the corresponding failure rates $hp_{0,l}$ and hp_l can be obtained through (5), and the results are $hp_{0,l} = 0.4776$, and $hp_l = 0.0224$. The probability associated with FM1 is $\Pr\{E_1\} = R_1 = 0.99$. The degradation, and shock thresholds are $D_f = 2\,\text{cm}$, and $A_f = 35\,\text{units}$, respectively. Next, we solve the problem in two scenarios: shocks resulting in an increasing failure rate, and shocks resulting in a direct change in the degradation process. If we did not consider the correlation between the degradation and shocks, we would use $R(t) = \Pr\{E_1\}\Pr\{E_2\}\Pr\{E_3\}$ to calculate reliability.

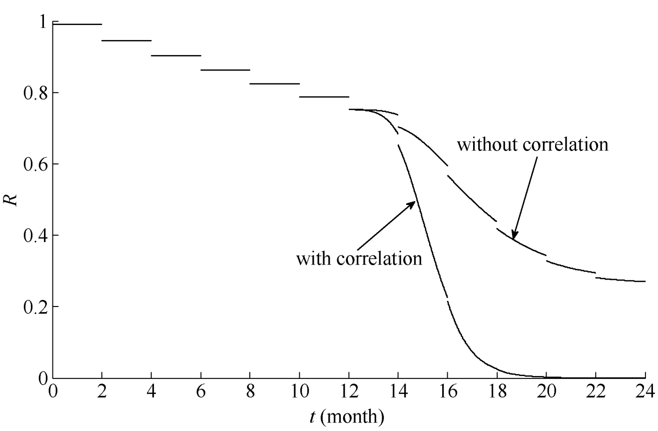

Fig. 7. Reliability curves under shocks with a fixed period.

B. Reliability Analysis With Increasing Failure Rate Due to Shocks

The theory of this method is presented in Section IV. The aforementioned two types of shocks are considered. The failure rate increases with a constant rate $\alpha = 1.1$.

1) Shocks with a fixed time period TWhen $0 \le t < T$, as shown in (15), reliability is computed by

$$R(t) = R_1 \Pr\{Y(t) \le D_f\} = 0.99\Phi\left(\frac{2 - 0.8767 \times e^{0.0398t}}{\sqrt{7.7602 \times 10^{-5} \times e^{0.4646t}}}\right). \quad (39)$$

When $iT \le t < (i+1)T$ $(1 \le i < \infty)$, according to (16), reliability is given by

$$R(t) = R_1 \left\{ \prod_{j=1}^{i} [R_0(jT)]^{(\alpha^{(j-1)} - \alpha^j)} [R_0(t)]^{\alpha^i} \right\} (p_{0,l})^i \times \left\{ \sum_{j=0}^{i} A_i \le A_f \right\} = 0.99 \left\{ \prod_{j=1}^{i} [R_0(2j)]^{(\alpha^{(j-1)} - \alpha^j)} [R_0(t)]^{\alpha^i} \right\} (0.9552)^i \times \Phi\left(\frac{35 - 2i}{\sqrt{0.5i}}\right). \quad (40)$$

The results are plotted in Fig. 7 (the lower curve). The curve shows how reliability changes in terms of time, considering the effect of shocks on degradation, e.g. $R(t) = \Pr\{E_1\}\Pr\{E_2|E_3\}\Pr\{E_3\}$. The reliability curves are not continuous because there is a sudden increase in the failure rate when a shock occurs.

When the correlation between the degradation and shocks is not taken into consideration, it means that the degradation is s-independent from the shocks. Because we did not account for the effects of shocks, the reliability estimate would be higher. The reliability curve without considering the correlation between the degradation and shocks is also plotted in Fig. 7 (the upper curve). The results indicate that the reliability estimation will be too optimistic without the consideration of the correlation between the degradation and shocks.

As shown in the figure, both reliability curves overlap before the 13th month. This overlap indicates that the effects of shocks are not significant before the 13th month. The curves began to separate at the 13th month, and the effects of shocks become a dominant factor of the reliability performance.

2) Shocks with a HPPWhen shocks occur with varying time periods, which are modeled by a HPP, a simulation method is used to calculate reliability as shown in Section IV-B. The number of simulations is 10,000. The reliability curves

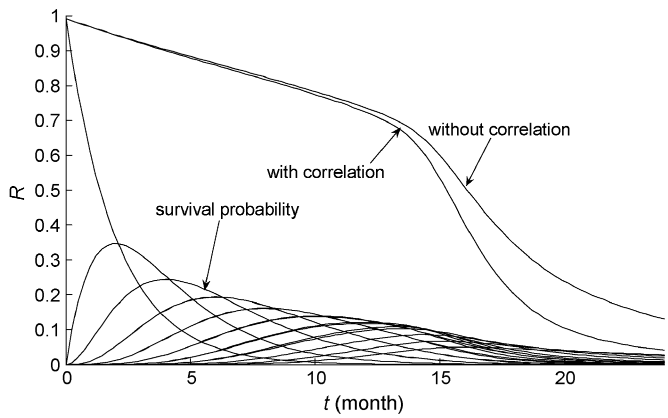

Fig. 8. Reliability curves under shocks with Poisson process.

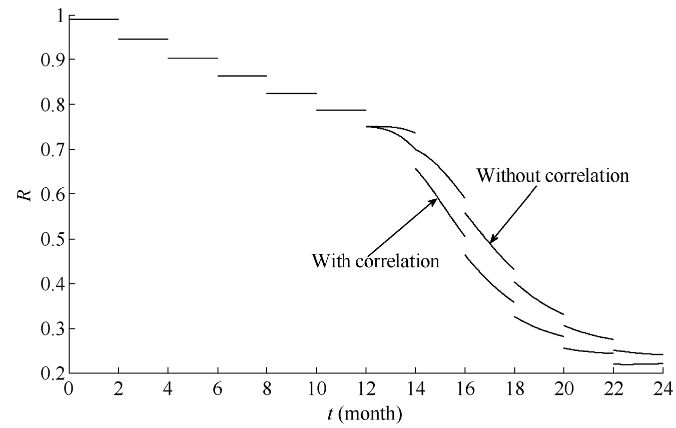

Fig. 9. Reliability curve under fixed time period shock.

are depicted in Fig. 8. The mean reliability curve (the lower curve) is plotted. The curves of the survival probability under i shocks are also plotted, which is a family of curves, and is determined by (23).

The reliability curve (the upper curve) without considering the correlation between the degradation and shocks is also plotted. Hence, the reliability would be lower when the correlation between the degradation and shocks is accounted for.

Because the effects of shocks are not significant before the 12th months, the two reliability curves nearly overlap before that. The curves separate after the 12th month, and the effects of shocks become larger.

C. Reliability Analysis With Random Change in Degradation

In Section VI-B, the effects of shocks on degradation are assumed to be changes in the failure rate. In this subsection, the effect is assumed to be a random change in degradation. Two types of shocks are also considered. The random change in degradation Y caused by the ith shock is a Gaussian distribution with $\delta Y_i \sim N(0.02\ \mathrm{cm}, 0.01\ \mathrm{cm})$.

1) Shocks with a fixed time period

When $0 \le t < T$, as shown in (32), reliability can be expressed as

$$R(t) = R_1 \Pr\{Y(t) \le D_f\} = 0.99\Phi\left(\frac{2-0.8767e^{0.0398t}}{\sqrt{7.7602\times10^{-5}e^{0.4646t}}}\right). \quad (41)$$

When $iT \le t < (i+1)T$ $(1 \le i < \infty)$, as shown in (32), reliability can be expressed by

$$R(t) = R_1 \Pr\left\{Y(t) + \sum_{j=1}^{i}\delta Y_j \le D_f\right\}(p_{0,l})^i \times \Pr\left\{\sum_{j=0}^{i} A_j \le A_f\right\} = 0.99\Phi\left(\frac{2-0.8767e^{0.0398t}-0.02i}{\sqrt{7.7602\times10^{-5}e^{0.4646t}+0.01i}}\right)(0.9552)^i \times \Phi\left(\frac{35-2i}{\sqrt{0.5i}}\right). \quad (42)$$

The reliability curve (the lower curve) under shocks with a fixed time period is plotted in Fig. 9. The curve shows how reliability changes in terms of time considering the effect of shocks on degradation, e.g. $R(t) = \Pr\{E_1\}\Pr\{E_2|E_3\}\Pr\{E_3\}$. The reliability curves are not continuous because there is a sudden increase in the degradation when a shock occurs.

Without considering the correlation between the degradation and shocks, the reliability (the upper curve shown in Fig. 9) is higher than the case with the correlation between the degradation and shocks.

As shown in the figure, the effects of shocks are not significant before the 12th month; they overlap before that month. However, the two curves begin to separate after the 12th month, and the effects of shocks become larger.

2) Shocks with Poisson process

The survival probability under i shocks, as shown in (33), can be expressed as

$$P_i = 0.99\Phi\left(\frac{D_f-0.8767e^{0.0398t}-0.02i}{\sqrt{7.7602\times10^{-5}e^{0.4361t}+0.01i}}\right) \times \exp(-0.0224t)\exp(-0.4776t)\frac{(0.4776t)^i}{i!}\Phi\left(\frac{35-2i}{\sqrt{0.5i}}\right). \quad (43)$$

The reliability, according to (34), can be expressed by

$$R(t) = \sum_{i=0}^{\infty} 0.99\Phi\left(\frac{D_f-0.8767e^{0.0398t}-0.02i}{\sqrt{7.7602\times10^{-5}e^{0.4361t}+0.01i}}\right) \times \exp(-0.0224t)\exp(-0.4776t)\frac{(0.4776t)^i}{i!}\Phi\left(\frac{35-2i}{\sqrt{0.5i}}\right). \quad (44)$$

The results are plotted in Fig. 10 (the lower curve). The curve shows how reliability changes with time with considering the effects of shocks on degradation, e.g. $R(t) = \Pr\{E_1\}\Pr\{E_2|E_3\}\Pr\{E_3\}$. Because the occurrences of shocks are random, the reliability curves are continuous.

As shown in Fig. 10, the reliability curve without considering the correlation is much higher than the actual reliability.

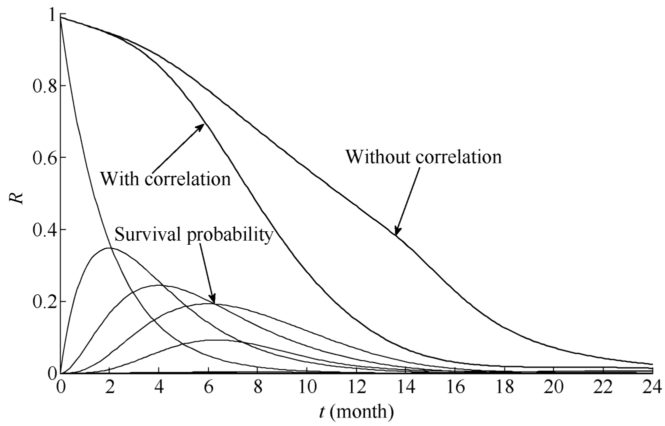

Fig. 10. Reliability curves under Poisson process shocks.

VII. Conclusion

The impact of shocks on product performance, especially on reliability performance, is critical. How to model such an impact is very complicated. This work is an attempt to analyse product reliability with performance degradation and shocks under two scenarios. In the first scenario, each shock is assumed to result in a sudden increase in the failure rate of the product. In the second scenario, each shock is assumed to result in a random increase in a degradation path. Shocks with a fixed time period, and shocks with varying time periods, are also considered. The four combinations of the two scenarios on the effects of shocks, and the two types of shocks, have been carefully studied.

To calculate product reliability with both degradation and shocks, three failure modes are considered, including the catastrophic (binary state) failure, degradation (continuous processes), and the failure due to shocks (impulse processes). The reliability for the four combinations with the three failure modes are derived, under stated assumptions. For instance, the change in performance and degradation is assumed to follow a s-independent Gaussian distribution. With suitable adjustments, the proposed methods are applicable to situations where those assumptions do not hold.

As shown in the example, shocks have a significant impact on the performance degradation, and on the product reliability. If the correlation between performance degradation and shocks is not considered, the predicted reliability would be higher than the actual case. For an accurate reliability prediction, it is necessary to consider the correlation between the performance degradation and shocks. The example indicates that, without the consideration of the correlation, the reliability estimation will be risky (higher than the accurate value).

Future research will seek the answers to three questions. 1) In what situations can the two scenarios be used? 2) How can the proposed methods be used for non-Poisson processes? 3) And when should maintenance be performed based on the estimated reliability?

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