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Reliability analysis on competitive failure processes under fuzzy degradation data

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ABSTRACT

Reliability is the ability of a system to perform its required functions under stated conditions for a specified period of time. Reliability analysis is an important tool to evaluate the performance of a system and make maintenance decision. In engineering practices, there are usually several processes to cause a system to failure. Generally, the failure processes can be categorized into degradation process and shock process. This research investigates the system reliability analysis when both degradation process and shock process are involved. Furthermore, the effect due to shock process on the degradation process is considered and degradation analysis is conducted under fuzzy degradation data. After that, a system reliability model on competitive failure processes under fuzzy degradation data is constructed. Since several states are formed when the effect due to shock process on degradation process is accounted for, multi-state system reliability theory is employed to evaluate the proposed model. This method could be used to assess reliability of many devices precisely in the real world, such as diesel engines. A practical engineering example is provided to illustrate the proposed model and method.

1. Introduction

System performance may degrade with time due to some factors from system themselves or environment, such as wear, erosion, shocks and so on. System reliability decreases correspondingly. System may also suddenly fail due to excessive loading, shocks or some other reasons. Basically, there are three major types of reliability analysis methods to deal with the two kinds of failures, including binary-state methods, multi-state methods and continuous state methods.

A binary-state method only considers two states: perfect functioning and full failure, which is the earliest reliability analysis method and has been widely applied in engineering practices. Since a number of intermediate states may exist between the perfect functioning and full failure, such as in power supply systems and communication systems, which have different performance levels and several failure modes with various effects on the entire system performance, multi-state system theory was introduced in the middle of 1970s [1–4].

Continuous state methods are suitable for the performance degradation problems. Degradation is common in many components and systems, especially in mechanical and structural systems. Performance degradation is usually a function of time.

Since it is very difficult to obtain the failure data in the real world, degradation data is very useful for the reliability analysis [5,6]. Accelerated testing method is an efficient approach to obtain the degradation data for the components and systems, especially high reliable components and systems [7]. Classical reliability analysis is based on the precise degradation data under the assumption that the collected data are crisp numbers. However, some collected data are imprecise and are represented as fuzzy numbers in the engineering practices [8].

Degradation is generally a stochastic process. There are two methods to express the stochastic process from the degradation data, including a degradation path curve method [9–11] and a random process with general distributions method [12–14]. The least-square method, maximum likelihood, and Bayesian method are usually used to estimate parameters in the both methods [7,15].

Shocking is an important source to cause systems to fail and an increasing attention has been focused on. Two methods to classify shocks model are proposed in [16]. In the first method, the shock effect and its arrival time are assumed independent. While in the second one, the shock effect and its arrival time are considered dependent. A general model is, furthermore, developed to describe the cumulative damage, extreme damage, and run damage [16]. Under a generalized framework proposed in [17], the cumulative shock model, extreme shock model and δ-shock model are created.

The binary-state method is extended to the continuous state method to assess system reliability under degradation in [12]. An integrated model of binary-state method and continuous method is created under the consideration that degradation and sudden failure may exist at the same time. Zuo et al. proposed a mixed model to assess system reliability with degradation and sudden failure data [13]. However, the effect of shock process was not considered in [12,13].

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Nomenclature

A_i	magnitude of the ith shock
B_i	time interval between the ith and $(i+1)$th shock
$C(\delta)$	the prefixed region
D_f	the prefixed critical degradation
$D(t; X(t))$	random degradation path
$G(c)$	the damage distribution
$g(t)$	the deviation of $X(t)$
k	the prefixed critical maximum number of middle shock
$N(t)$	Poison distribution
$r(t)$	failure rate without considering the effect of shocks
$X(t)$	a random process used to express the variation of degradation data
X_1	binary state failure mode $X_1 \in \{0, 1\}$
X_2	degradation process failure mode, $X_2 \in [0, M]$
X_3	failure mode caused by shock $X_3 \in \{0, 1, ..., M\}$
Y_i	a random variable used to express the effect on degradation process due to the ith shock
$\alpha(t)$	shape parameter in Weibull distribution
$\beta(t)$	scale parameter in Weibull distribution
$\eta(t)$	mean degradation path of $X(t)$
CDF	cumulative distribution function
HPP	homogenous poison process
IFR	increasing failure rate class
NHPP	non-homogenous poison process

In many engineering applications, degradation process and shock process occur at the same time. Li and Pham investigated a reliability assessment model with two degradation processes and a shock process. This is a competitive failure model, in which if any one of the three processes exceeds the critical value, the system is considered to fail. The two degradation processes are discretized into multiple states, and then multi-state system reliability method is employed [18]. Klutke and Yang [19] studied the average availability of the systems with shocks and graceful degradation. But in their models, the relation between the degradation process and shock process is not accounted for.

The organization of this paper is as follows. In Section 2, the independent competitive failure model (IDCFM) and dependent competitive failure model (DCFM) under degradation process and shock process is described. Shock process is analyzed in Section 3. In Section 4, fuzzy degradation data analysis is briefly introduced. Fault tree analysis (FTA) is used to evaluate the reliability of IDCFM and the multi-state system reliability method is implemented to evaluate the reliability of DCFM in Section 5. In Section 6, an engineering example is used to illustrate the proposed model. Some conclusions and further work are given in Section 7.

2. Model formulation

The failure of system is derived from three main failure modes: binary-state failure mode, degradation process failure mode, and failure mode due to shock process. Two types of models are proposed on the condition whether the shock process cause effect on the degradation process. The models are shown in Figs. 1 and 2, respectively.

In Figs. 1 and 2, X_1, X_2, X_3, Y_i, S denote binary state failure mode, degradation process failure mode, failure mode due to shock process, random effect by the ith shock on degradation process and system reliability, respectively. In Fig. 1, there is no relation among the three failure modes. However, in Fig. 2, shock process causes random effect on the degradation process.

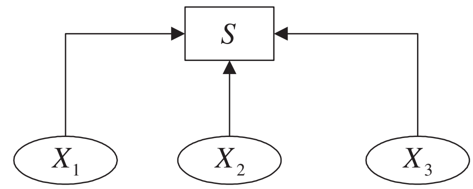

Fig. 1. Irrelative among failure modes.

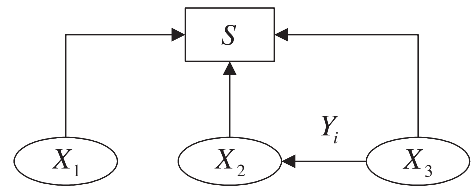

Fig. 2. Shock cause impact on degradation.

Models are studied under some conditions as follows:

(1) The observed data are imprecise;
(2) The life distribution class is subjected to IFR;
(3) Shock process follows statistically Poison distribution $N(t)$. $\{A_i, B_i\}_{i=1}^{\infty}$ can be used to describe the whole shock process, where A_i denotes the magnitude of the ith shock with normal distribution $A_i \sim N(\mu_2, \sigma_2{}^2)$ and B_i denotes the time interval between the ith and $(i+1)$th shock;
(4) The impact on the system due to shock process is independent;
(5) The impact on the degradation process due to shocks is independent identical distribution.

3. Shock process analysis

In engineering applications, many factors from devices themselves and random environment bring shocks to devices. Shock process caused by devices themselves usually has regular periods, especially for rotating devices, while shock process caused by the factors of random environment is usually considered to follow Poisson process.

Four principal shock models have been studied in [17] and [18]: extreme shock model where systems fails as soon as the magnitude of any shock exceeds the prefixed critical level; cumulative shock model where systems will break down when the cumulative magnitude exceeds; run shock model where system will work until k consecutive shocks with critical magnitude happen, and δ-shock model where the system will fail when the time lag between two successive shocks falls into $C(\delta)$. Different magnitude of shocks can lead to different damage to the system. Usually, a shock with magnitude below a prefixed value is considered harmless to the system, while a shock with magnitude above a prefixed value is considered fatal to the system. Another case that magnitude is between the low prefixed value and the high prefixed value is thought to cause middle damage to the system. Therefore, shocks are divided into three states in terms of the magnitude of the shocks. When the magnitude of shocks reaches the interval $[0, c_s]$, the shocks are harmless to the system; when the magnitude of shocks goes into the interval $[c_s, c_l]$, the shocks cause middle damage to the system and the damage is cumulative; and when the magnitude of any shock exceeds c_i, the system is down. The arrival times of shocks follow Poison distribution, and their magnitude and time interval can be described in Fig. 3.

As studied in [20], the probability of the damage corresponding to interval $[0, c_s]$, $[c_s, c_l]$ and $[c_s, \infty]$ are denoted as P_s, $P_{s,l}$, P_l for

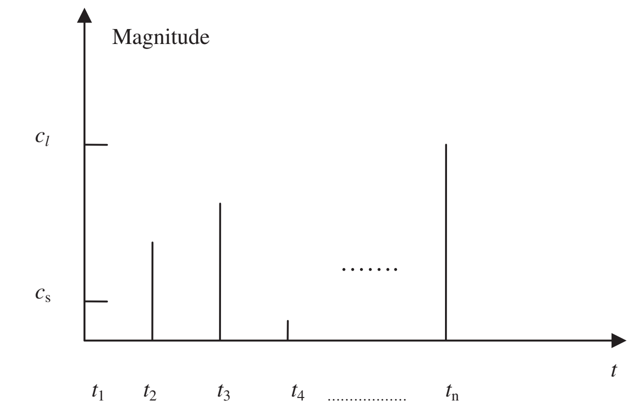

Fig. 3. The relation between magnitude and time interval t.

NHPP, respectively.

$$P_s(t) = \exp\{-\int_0^t (1-p_s)h(x)dx\}$$
$$P_{s,l}(t) = \exp\{-\int_0^t (1-p_{s,l})h(x)dx\} \quad (1)$$
$$P_l(t) = \exp\{-\int_0^t (1-p_l)h(x)dx\}$$

And corresponding failure rates can be obtained

$$hp_s = (1-p_s)h(x)$$
$$hp_{s,l} = (1-p_{s,l})h(x) \quad (2)$$
$$hp_l = (1-p_l)h(x)$$

where $p_s = G(c_s)$, $p_{s,l} = G(c_l) - G(c_s)$, $p_l = 1 - G(c_l)$.

For the HPP, as a special example for the NHPP, $h(x) = \lambda$.

If at least one shock from the process with failure rate hp_l occurs or if more than k shocks from the process with failure rate $hp_{s,l}$ occur, the system will fail. Then the system reliability is provided

$$P_{s,k}(t) = \exp(-h_{p_l}t)\exp(-h_{p_{s,l}}t)\sum_{i=0}^{k}\frac{(h_{p_{s,l}}t)^i}{i!} \quad (3)$$

4. Fuzzy degradation analysis

In the traditional degradation analysis, the observed data are treated as crisp numbers without fuzziness. Furthermore, coefficients used to fit the degradation data are treated as crisp number without fuzziness. In fact, in the practical engineering, many observation data are fuzzy. So far, fuzzy regression method has been studied by many authors [21–23]. Huang et al. [8] proposed a Bayesian reliability assessment method of gear lifetime under fuzzy environment. In this section, a new method to make degradation analysis is proposed. The proposed method can deal with the problem both shown in Table 1 and the observed data at different time t_i for different devices. And then connect all the observed data obtained from the same device to form curve family shown in Fig. 4. In succession, samples of observed data from different devices at the same time t_i. A table like Table 1 can also be obtained.

Some subjective factors, such as the warp of the observer, cause the observed data to be imprecise. In order to characterize the real-world conditions, every degradation data is considered as a fuzzy number with symmetric triangular membership function (l_{ji}, β_{ji}), as shown in Fig. 5, where l_{ji} is the center and β_{ji} is the spread width. The mean degradation value at time t_i can be expressed as

Table 1
Degradation data.

	t_1	t_2	...	t_n
V_1	$\tilde{y}_{11}$	$\tilde{y}_{12}$	...	$\tilde{y}_{1n}$
V_2	$\tilde{y}_{21}$	$\tilde{y}_{22}$	...	$\tilde{y}_{2n}$
...	...	...	...	...
V_n	$\tilde{y}_{n1}$	$\tilde{y}_{n2}$	...	$\tilde{y}_{nn}$

$$\tilde{\tilde{y}}_i = \frac{1}{n}\sum_{j=1}^{n}\tilde{y}_{ji} \quad (4)$$

Hence $\tilde{\tilde{y}}_i$ is also a fuzzy number with symmetric triangular membership function (l_i, β_i), as shown in Fig. 5, where $l_i = \frac{1}{n}\sum_{j=1}^{n} l_{ji}$ is the center and $\beta_i = \frac{1}{n}\sum_{j=1}^{n}\beta_{ji}$ is the spread width.

We can know that the spread width of $\tilde{y}_{ji}$ and $\tilde{\tilde{y}}_i$ is the same and the center of $\tilde{\tilde{y}}_i$ is the mean value of the degradation data at time t_i, if the spread width of each $\tilde{y}_{ji}$ is the same. For each device, the function of degradation *vs.* time is assumed to be expressed as

$$\eta(t) = ae^{bt} \quad (5)$$

By implementing the natural log operation on both the sides of the regression model and provided that $y = \log(\eta(t))$ and $m = \log(a)$, the formulation can be rewritten as

$$y = m + bt \quad (6)$$

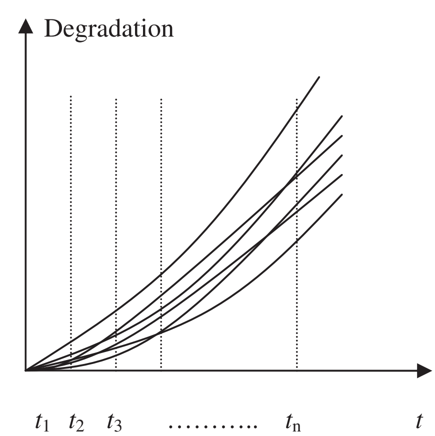

Fig. 4. Degradation path curve.

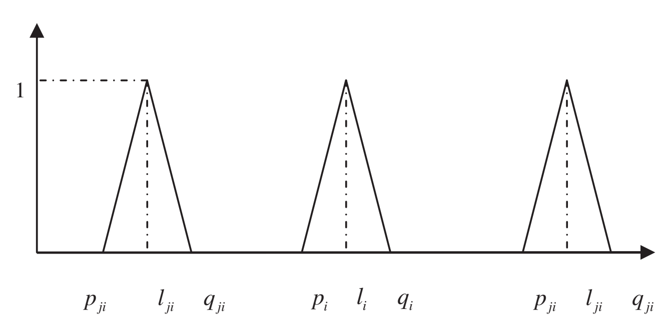

Fig. 5. Membership function of $\tilde{\tilde{y}}_i$ and $\tilde{y}_{ji}$.

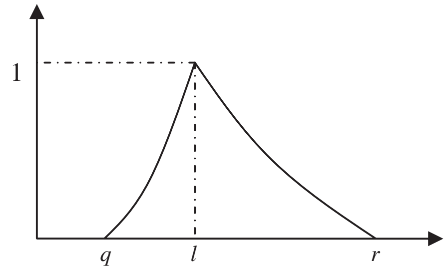

Fig. 6. The membership function of $\delta(t_i)$.

Fuzzy linear regression is employed to resolve data fitness. The basic model assumes a fuzzy linear function as

$$\tilde{Y} = \tilde{A}_0 X_0 + \tilde{A}_1 X_1 + \ldots + \tilde{A}_N X_N \tag{7}$$

where X_0, X_1,..., X_N are independent variables and $\tilde{A}_0, \tilde{A}_1 \ldots \tilde{A}_N$ are the fuzzy coefficients presented in the form of symmetric triangular fuzzy numbers denoted by $\tilde{A}_j = (\alpha_j, c_j)$,where α_j is the center and c_j is the spread width. Here, the fuzzy linear function can be written as

$$\tilde{y} = \tilde{m} + \tilde{b}t \tag{8}$$

where t is the independent variable and $\tilde{m}$, $\tilde{b}$ are fuzzy coefficients with the symmetric triangular fuzzy number denoted by $\tilde{m} = (\alpha_0, c_0)$ and $\tilde{b} = (\alpha_1, c_1)$, respectively.

Provided that the degree of fitting the estimated fuzzy regression model $\tilde{y}_i = \tilde{m} + \tilde{b}t$ to the mean observed data $\tilde{\bar{y}}_i$ has at least h degree. According to the analysis above, the linear programming problem is obtained [21].

$$\begin{aligned} &\min \sum_{i=1}^{n} (c_0 + c_1 t_i) \\ &\text{s.t.} \alpha_0 + \alpha_1 t_i + (1-h)(c_0 + c_1 t_i) \ge l_i + (1-h)\beta_i \\ &\alpha_0 + \alpha_1 t_i - (1-h)(c_0 + c_1 t_i) \le l_i - (1-h)\beta_i \\ &c_0 > 0, c_1 > 0 \\ &h > 0 \end{aligned}$$

By resolving the linear programming problem, $\alpha_0, \alpha_1, c_0, c_1$ can be obtained. Then the fuzzy number $\tilde{m}$ and $\tilde{b}$ can also be expressed.

From the view of statistical point, the deviation at time t_i can be given by

$$\delta(t_i) = \frac{1}{n} \sum_{j=1}^{n} (\tilde{y}_{ji} - \tilde{\bar{y}}_i)^2 \tag{9}$$

$\delta(t_\text{i})$ is fuzzy numbers with the similar membership function shown in Fig. 6.

This membership function of $\delta(t_\text{i})$ is neither symmetrical nor triangular, because there is multiplication operation in the process of solving $\delta(t_\text{i})$. According to the distribution of $\delta(t_\text{i})$, different type of function $\delta(t)$ is used to fit $\delta(t_\text{i})$ at time $t_i (i = 1, 2, ..., n)$. But intrinsically linear function and linear function are usually used. In [24], two types of degradation models, additive degradation model and multiplication degradation model, are analyzed. In the two models, X represents the random variance around a mean degradation level and is the same at any time t_i. In fact, the random variance around the mean degradation is not a fixed random variable, since it can change with time. So the degradation curve can be expressed as:

$$D(t, X(t), \Theta) = \eta(t, \Theta) + X(t) \tag{10}$$

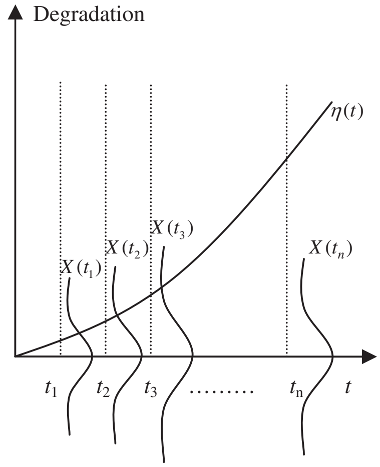

Fig. 7. The decomposed degradation path.

When all observations are operated in the same conditions, the Eq. (10) can be rewritten as

$$D(t, X(t)) = X(t) \tag{11}$$

The degradation data are rooted in different systems under nearly the same environment. The observed data at time t_i approximately follow normal distribution. And the mean value and deviation can be expressed as

$$\begin{aligned} \mu(t) &= E(X(t)) = \eta(t) \\ \delta(t) &= D(X(t)) = \delta(t) \end{aligned} \tag{12}$$

When the observed data at time t_i approximately follow Weibull distribution, shape parameter $\alpha(t)$ and scale parameter $\beta(t)$ can be obtained by resolving the following equations.

$$\begin{aligned} \mu &= E(X(t)) = \alpha(t)\Gamma(1 + \frac{1}{\beta(t)}) = \eta(t) \\ \delta &= D(X(t)) = \alpha^2(t)[\Gamma(1 + \frac{2}{\beta(t)}) - \Gamma^2(1 + \frac{1}{\beta(t)})] = \delta(t) \end{aligned} \tag{13}$$

When the observation data at time t_i follows normal distribution, the degradation path can be decomposed as shown in Fig. 7.

Where $\eta(t)$ is the mean degradation path, $X(t_\text{i})$ is the random variable at time t_i.

5. Reliability analysis

Reliability analysis, especially in the important large systems, plays an important role in the reliability engineering. It helps engineers to evaluate the current system performance and make maintenance decisions to reduce the probability of failure. Independent and dependent competitive failure model reliability analysis methods are studied as follows.

5.1. Independent competitive failure model reliability analysis

In independent competitive failure model, there is no interaction between the three failure modes. Furthermore, the three failure modes form the parallel connection. If any failure mode among them reaches the prefixed value, the system will fail. Based on analysis, fault tree analysis (FTA) is used to transact the reliability analysis problem. The configuration of FTA in term of this model can be described in Fig. 8.

The failure of the system is derived from three aspects: binary state failure mode X_1, degradation process failure mode X_2, and the

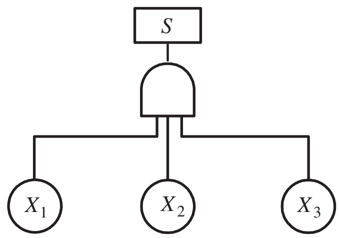

Fig. 8. The FTA of independent competitive failure model.

failure mode due to shocks X_3. The system will fail, when the degradation process exceeds the prefixed value D_f, or the failure of binary state occurs or the damage caused by shocks exceeds the prefixed value. Based on analysis abovementioned, the system reliability can be represented as

$$R = Pr\left\{X_1 = 1, X_2 \le D_f, Tp \in [t, t + \Delta t] | Tp > t\right\} = Pr\left\{X_1 = 1\right\} Pr \times \left\{X_2 \le D_f\right\} \exp(-hp_1 t)\exp(-hp_{s,1} t)\sum_{0}^{k}\frac{(hp_{s,1}t)^i}{i!} \quad (14)$$

where T_p denotes the random time to cause the system to fail due to the shocks.

First, the calculation of $Pr\{X_2 \le D_f\}$ can be realized as follows.

$$Pr\left\{X_2 \le D_f\right\} = p\left\{X(t) \le D_f\right\} \quad (15)$$

For different systems under nearly the same environment, the measured data usually follows normal distribution. Then

$$Pr\left\{X_2 \le D_f\right\} = \Phi\left(\frac{D_f - \eta(t)}{g(t)}\right) \quad (16)$$

Given a fixed value to k, the value of $\exp(-hp_l t)\exp(-hp_{s,l})\sum_{0}^{k}\frac{(hp_{s,l}t)^i}{i!}$ can be easily obtained.

5.2. Dependent competitive failure model reliability analysis

Dependent competitive failure model is usually met in the practical engineering, such as engine, and crane. The failure of systems is mainly caused by three failure modes: degradation process failure mode, binary state failure mode and the failure mode due to shock process. The shock process not only causes the decrease of system life directly but also accelerates the speed of degradation process indirectly. Because the interval between the consecutive shocks is not a real variable but a random variable, it is very difficult to resolve the varying failure rate problem with the random time interval with analytical methods. In this section, the effect due to shock process on degradation process is considered as a random variable instead of resolving the varying failure rate problem in the random time interval as shown in Fig. 9. Actually, the effect due to shock process gets along with the whole degradation process. A random degradation variable is provided as soon as a shock occurs to make the effect forward. Furthermore, only shocks with middle damage effects the continuous degradation process. The dependent competitive failure model can be seen in Fig. 2. Based on the analysis aforementioned, the system reliability of the dependent competitive failure model can be represented as

$$R = Pr\left\{X_1 = 1, X_2 + \sum_{i=0}^{n} Y_i \le D_f, T_p \in [t, t + \Delta t] | T_p > t\right\}$$
$$= Pr\left\{X_1 = 1\right\} Pr\left\{X_2 + \sum_{i=0}^{n} Y_i \le D_f\right\} \exp(-hp_1 t)\exp(-hp_{s,1}) \times \sum_{0}^{k}\frac{(hp_{s,1}t)^i}{i!} \quad (n = 1, 2, \ldots k) \quad (17)$$

The system maybe fail in term of the failure of the degradation process including original degradation and effect due to shock process, before the failure due to shock process and binary state components. So we should calculate the probability at the cause of every shock. Each shock is treated as the state transmission signal to the system. Then multi-state system reliability theory is employed to resolve this problem. The state transmission chart considering

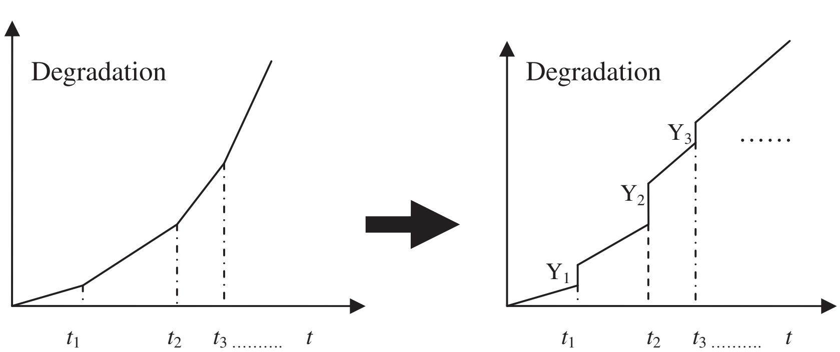

Fig. 9. Degradation process under the effect of shocks.

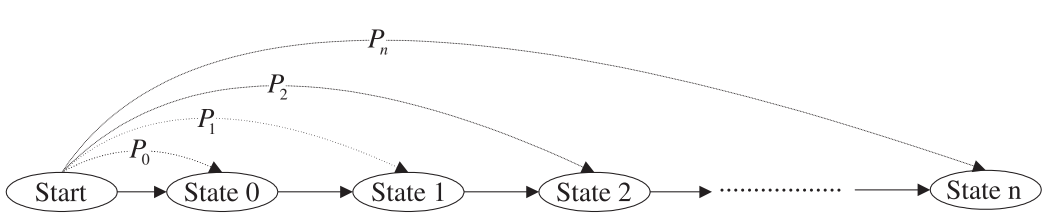

Fig. 10. State transmission chart.

the relation between degradation process and shock process can be described in Fig. 10.

The ith shock causes system to divert from state i to $(i+1)$ $(i=1, 2,\ldots,n)$. The actual transition path is from state i to state $i+1$ without jump as the solid line in Fig. 10. In the process of calculation, transition path considered is from state 1 to state j $(j=2, 3,\ldots,n)$, described as the dotted line in Fig. 10.

Probability in state j $(j=2, 3,\ldots,n)$ can be expressed as follows respectively.

$$P_1 = Pr\left\{X_1 = 1, N(t) = 1, \eta(t) + X(t) + Y_1 \le D_f, T_p \in [t, t+\Delta t] | T_p > t\right\}$$
$$= Pr\left\{X_1 = 1\right\} Pr\{N(t) = 1\} Pr\left\{\eta(t) + X(t) + Y_1 \le D_f\right\} \exp(-hp_1 t)\exp(-hp_{s,1})\sum_{0}^{1}\frac{(hp_{s,1}t)^i}{i!} \tag{18}$$

$$P_2 = Pr\left\{N(t) = 2, \eta(t) + X(t) + \sum_{i=1}^{2} Y_i \le D_f, X_1 = 1, T_p \in [t, t+\Delta t] | T_p > t\right\}$$
$$= Pr\left\{X_1 = 1\right\} Pr\{N(t) = 2\} Pr\left\{\eta(t) + X(t) + \sum_{i=1}^{2} Y_i < D_f\right\} \exp(-hp_1 t)\exp(-hp_{s,1})\sum_{i=1}^{2}\frac{(hp_{s,1}t)^i}{i!} \tag{19}$$

$$P_n = Pr\left\{N(t) = n, \eta(t) + X(t) + \sum_{i=1}^{n} Y_i \le D_f, X_1 = 1, T_p \in [t, t+\Delta t] | T_p > t\right\}$$
$$= Pr\left\{X_1 = 1\right\} Pr\{N(t) = n\} Pr\left\{\eta(t) + X(t) + \sum_{i=1}^{n} Y_i \le D_f\right\} \exp(-hp_1 t)\exp(-hp_{s,1})\sum_{i=1}^{n}\frac{(hp_{s,1}t)^i}{i!} \tag{20}$$

The system reliability of the system can be represented as

$$R = \sum_{i=1}^{\infty} p_i \tag{21}$$

where $n \le k$

To obtain the system reliability, probability in every state must be known. Y_i $(i=1, 2, \ldots n)$ is independent and follows normal distribution with CDF

$$F(y_i) = \frac{1}{\sqrt{2\pi}\sigma_1}\int_{-\infty}^{y_i} e^{-(t-\mu_1)^2/2\sigma_1^2} d \tag{22}$$

According to the characteristic of normal distribution, $Y = \sum_{i=1}^{n} Y_i$ still follows normal distribution with CDF

$$F(y) = \frac{1}{\sqrt{2\pi n}\sigma_1}\int_{-\infty}^{y} e^{-(y_i - n\mu_1)^2/2n\sigma_1^2} dt \tag{23}$$

The degradation under the ith shock can be written as

$$D(t) = X(t) + \sum_{i=1}^{n} Y_i \tag{24}$$

with CDF

$$F_D(t) = Pr\left\{X(t) + \sum_{i=1}^{n} Y_i < D_f\right\}$$
$$= Pr\left\{X(t) + Y < D_f\right\} \tag{25}$$

Because $X(t)$ is a random process following normal distribution and Y is a random variable following normal distribution, so $X(t)+Y$ is a random process following normal distribution

$$X(t) + Y \sim N(n\mu_1 + \eta(t), n{\sigma_1}^2 + g(t))$$

Then Eq. (25) can be rewritten as

$$F_D(t) = Pr\{X(t) + \sum_{i=1}^{n} Y_i < D_f\}$$
$$= Pr\{X(t) + Y < D_f\}$$
$$D_f - \eta(t) - n\mu_1$$
$$= \phi(\frac{D_f - \eta(t) - n\mu_1}{\sqrt{n\sigma^2 + g(t)}} \tag{24'}$$

6. Case study

6.1. Degradation analysis

Lu and Meeker used path curve approach to analyze the fatigue crack data sets including degradation data and failure data [11]. The data sets were collected from 21 test specimens. All specimens had an initial crack length of 0.90 in. The data in Table 2 is only a part of data sets in [11], which only includes degradation data.

Some subjective factors, such as the warp of the observer, cause the observed data to be imprecise. In order to characterize the real-world conditions, the observed data could be treated as fuzzy numbers and the value in Table 2 is considered as the center of the fuzzy numbers. The fuzziness is mainly derived from the approximate evaluation to the mantissa of observed data and given that they have the same spread width 0.01. Then the observed data can be expressed as $\tilde{y}_{ji} = (y_{ji}, 0.01)$ $(j=1, \ldots, 13; i=0, \ldots, 12)$. So the mean observed data at time t_i can also be represented as $\tilde{\bar{y}}_i = (\bar{y}_i, 0.01)$. According to the distribution of y_i $(i=0,\ldots,12)$ the formulation $\eta(t)=ae^{bt}$ is used to describe the path curve of the mean observed data at each time t_i. After implementing the natural log operation on both sides of the formulation, the equation $\ln(\eta(t))=\ln(a)+bt$ is derived. Provided that $y=\ln(\eta(t))$ and $m=\ln(a)$, the relation between y and t can be seen in Table 3.

When the numbers in Table 3 are crisp numbers without fuzziness, Least Square is used to fit the number in Table 3 and $m=-0.1316$, $b=0.0398$ is obtained.

In order to make the fuzziness of the coefficients $\tilde{m}$ and $\tilde{b}$ minimum, the linear program should be resolved

$$\min \sum_{i=0}^{12}(c_0 + c_1 t_i)$$
$$s.t.\ \alpha_0 + \alpha_1 t_i + (1-h)(c_0 + c_1 t_i) \ge l_i + (1-h)\beta_i$$
$$\alpha_0 + \alpha_1 t_i - (1-h)(c_0 + c_1 t_i) \le l_i - (1-h)\beta_i$$
$$i = 0, 1, \ldots, 12$$
$$h > 0, c_0 > 0, c_1 > 0$$

Table 2
Fatigue crack growth data (in inch).

TS	Loading cycle (10^4 cycles)												
	0	1	2	3	4	5	6	7	8	9	10	11	12
1	0.90	0.92	0.97	1.01	1.05	1.09	1.15	1.21	1.28	1.36	1.44	1.55	1.72
2	0.90	0.92	0.96	1.00	1.04	1.08	1.13	1.19	1.26	1.34	1.42	1.52	1.67
3	0.90	0.93	0.97	1.00	1.04	1.08	1.13	1.18	1.24	1.31	1.39	1.49	1.65
4	0.90	0.93	0.97	1.00	1.03	1.07	1.10	1.16	1.22	1.29	1.37	1.48	1.64
5	0.90	0.92	0.97	0.99	1.03	1.06	1.10	1.14	1.20	1.26	1.31	1.40	1.52
6	0.90	0.93	0.96	1.00	1.03	1.07	1.12	1.16	1.20	1.26	1.30	1.37	1.45
7	0.90	0.92	0.96	0.99	1.03	1.06	1.10	1.16	1.21	1.27	1.33	1.40	1.49
8	0.90	0.92	0.95	0.97	1.00	1.03	1.07	1.11	1.16	1.22	1.26	1.33	1.40
9	0.90	0.93	0.96	0.97	1.00	1.05	1.08	1.11	1.16	1.20	1.24	1.32	1.38
10	0.90	0.92	0.94	0.97	1.01	1.04	1.07	1.09	1.14	1.19	1.23	1.28	1.35
11	0.90	0.92	0.94	0.97	0.99	1.02	1.05	1.08	1.12	1.16	1.20	1.25	1.31
12	0.90	0.92	0.94	0.97	0.99	1.02	1.05	1.08	1.12	1.16	1.19	1.24	1.29
13	0.90	0.92	0.94	0.97	0.99	1.02	1.04	1.07	1.11	1.14	1.18	1.22	1.27
Avg.	0.90	0.9231	0.9562	0.9854	1.0177	1.0531	1.0915	1.1338	1.1862	1.2431	1.2969	1.3731	1.4723

Table 3
The relation between *y* and *t*.

t	0	1	2	3	4	5	6	7	8	9	10	11	12
y	−0.1054	−0.0800	−0.0448	−0.0147	0.0175	0.0517	0.0876	0.1256	0.1708	0.2176	0.2600	0.3171	0.3868

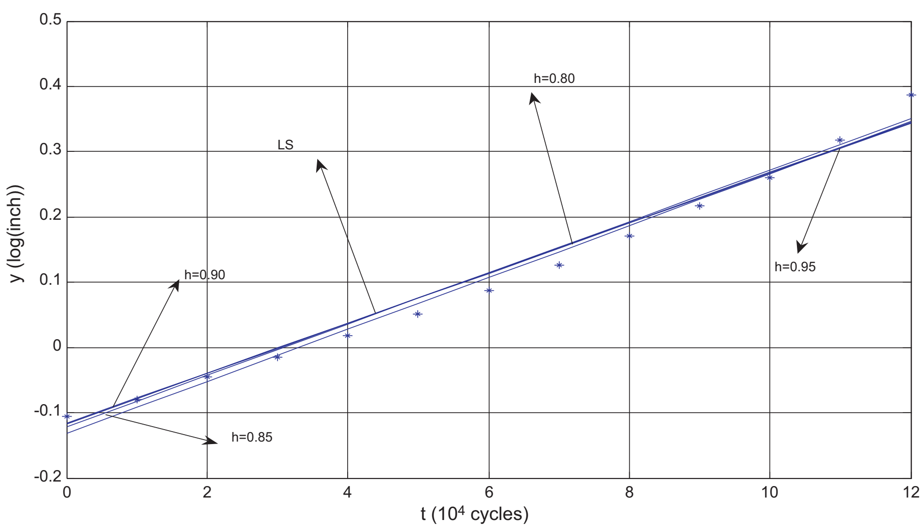

Fig. 11. Fitness cure paths of the mean of different *h*.

Table 4
Center and spread width of $\tilde{m}$ and $\tilde{b}$.

	$h = 0.80$	$h = 0.85$	$h = 0.90$	$h = 0.95$
α_0	−0.1152	−0.1213	−0.1165	−0.1166
α_1	0.0382	0.0393	0.0384	0.0385
c_0	0.0600	0.1167	0.1120	0.2340
c_1	0.0139	0.0121	0.0254	0.0510
f_{val}	1.8622	2.4572	3.5685	7.0200

Substituting the data in Table 3 into the linear program above, α_0, α_1, c_0 and c_1 can be obtained. Different α_0, α_1, c_0 and c_1 are shown in Table 4 in terms of different values of *h* and the fitness cure path is shown in Fig. 11.

According to the distribution of v_i ($i = 0,\ldots,12$) the formulation $g(t) = ce^{dt}$ is used to describe the path curve of the variance of observed data at each time t_i. After implementing the natural log operation on both sides of the formulation, the equation $\ln(g(t)) = \ln(c) + bt$ is derived. Provided that $f = \ln(g(t))$ and $n = \ln(c)$, the relation between *f* and *t* can be seen in Table 5. Because the variance in t_0 is equal to 0, the nature log at t_0 is meaningless. Then the variance in t_0 is ignored

When the numbers in Table 5 are crisp numbers without fuzziness, Least Square is used to fit the number in Table 5 and $n = -9.4709$, $b = 0.4646$ is obtained.

Because the variance should be big or equal to 0, $l_i - \beta_{Li}$ should be big or equal to 0. The conclusion can be reached that β_{Ri} should be biger than β_{Li}. In order to make the fuzziness of the coefficients

Table 5
The relation between *f* and *t*.

t	1	2	3	4	5	6	7	8	9	10	11	12
f	−8.5172	−8.5172	−8.5172	−7.6009	−7.4186	−6.7254	−6.1193	−5.7764	−5.2785	−4.8409	−4.3901	−3.7132

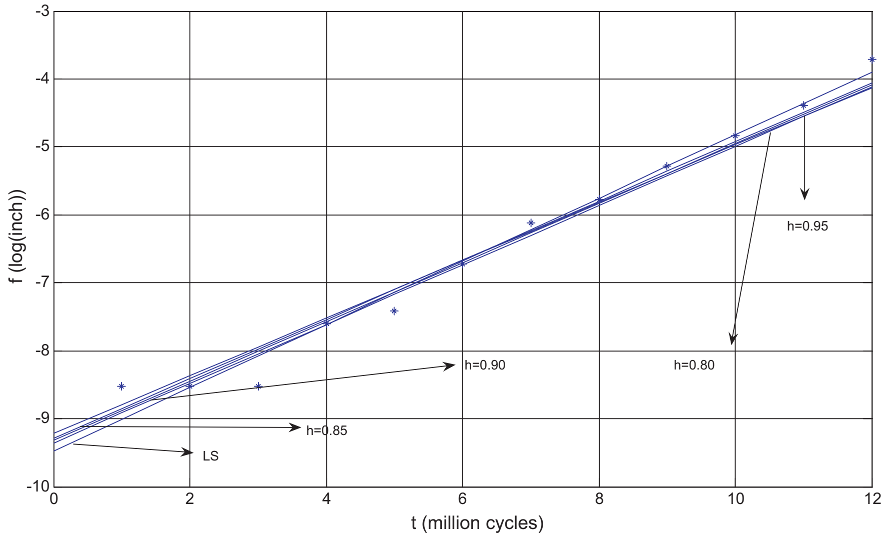

Fig. 12. Fitness cure paths of the variance of different h.

Table 6
Center and spread width of $\tilde{n}$ and $\tilde{d}$.

	$h=0.80$	$h=0.85$	$h=0.90$	$h=0.95$
α_3	−9.2156	−9.2854	−9.3205	−9.3556
α_4	0.4243	0.4361	0.4363	0.4365
c_3	2.6987	3.6001	5.0558	9.4231
c_4	0.0578	0	0	0
f_{val}	39.5929	46.8007	65.7255	122.5001

$\tilde{n}$ and $\tilde{d}$ minimum, the linear program should be resolved

$$\min \sum_{i=0}^{12} (c_3 + c_4 t_i)$$

$$s.t. \alpha_3 + \alpha_4 t_i + (1-h)(c_3 + c_4 t_i) \geq l_i + (1-h)\beta_R$$

$$\alpha_3 + \alpha_4 t_i - (1-h)(c_3 + c_4 t_i) \leq l_i - (1-h)\beta_L$$

$$i = 0, 1, \ldots, 12$$

$$h > 0, c_3 > 0, c_4 > 0$$

Substituting the data in Table 5 into the linear program above, α_3, α_4, c_3 and c_4 can be obtained. Different α_3, α_4, c_3 and c_4 are shown in Table 6 in terms of different value of h and the fitness curve path is shown in Fig. 12.

6.2. Independent competitive failure model

Consider fatigue crack analyzed above as degradation process failure mode to the system. Then $X(t) \sim N(\eta(t), g(t))$ where $\eta(t) = 0.891e^{0.0382t}$ and $g(t) = 9.2769 \times 10^{-5} e^{0.4361t}$. The parameter of Poisson distribution is denoted as $\lambda = 1$. Assumed that at least one shock from the process with failure rate hp_1 occurs or if more than 5 shocks from the process with failure rate $hp_{s,1}$ occur, the system will fail. The system reliability can be expressed as

$$R = Pr\{X_1 = 1, X_2 < D_f, Tp \in [t, t+\Delta t] | Tp > t\}$$

$$= Pr\left\{X_1 = 1\right\} Pr\{X(t) \leq D_f\} \exp(-hp_1 t) \exp(-hp_{s,1} t) \sum_{0}^{k} \frac{(hp_{s,1} t)^i}{i!}$$

$$= \Phi\left(\frac{D_f - 0.891e^{0.0382t}}{\sqrt{9.2769 \times 10^{-5} e^{0.4361t}}}\right) * 0.95 * \exp(-0.0498t) * \exp(-0.855t) * \sum_{0}^{5} \frac{(0.855t)^i}{i!}$$

The relation between reliability and time can be described as Fig. 13.

6.3. Dependent competitive failure model

Assume that the effect of every shock on the degradation process follows normal distribution with $Y_i \sim N(0.02, 0.1)$. Because the system will fail when more than 5 shocks from the process with failure rate $hp_{s,1}$ occur, the total states of the system are six. The

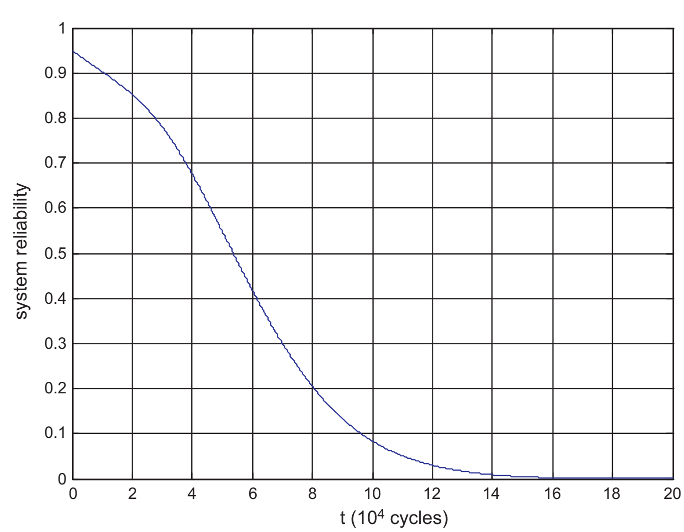

Fig. 13. The relation between reliability and time.

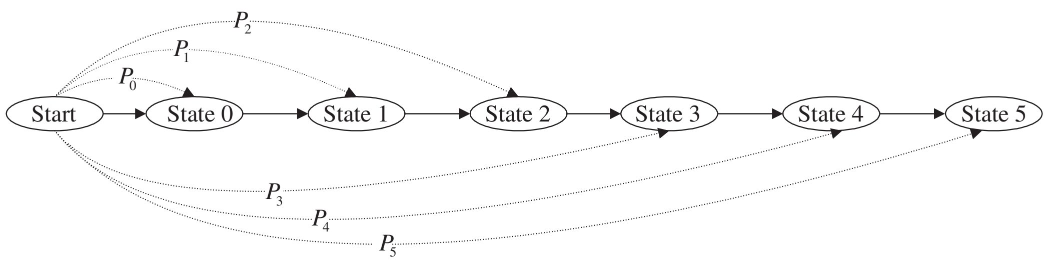

Fig. 14. State transmission chart.

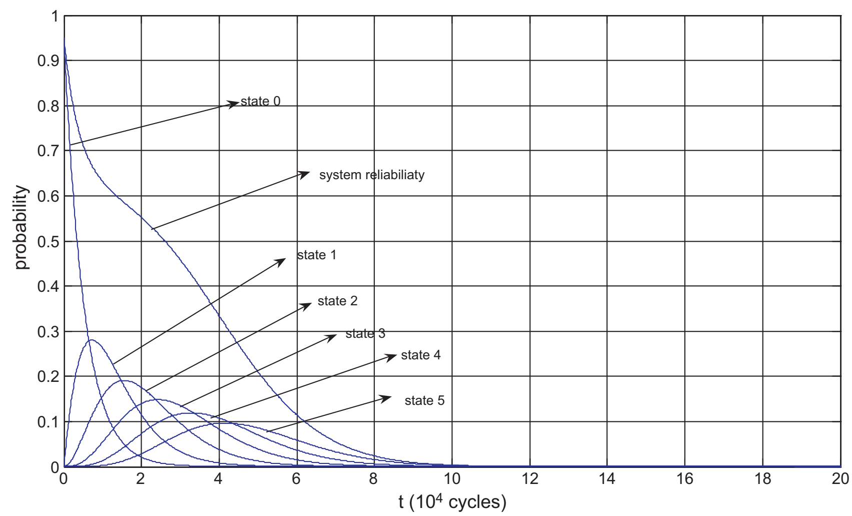

Fig. 15. Probability curve in every state and system reliability curve.

state transmission chart can be seen in Fig. 14.where

$$P_0 = e^{-t}\Phi\left(\frac{D_f - 0.891e^{0.0382t}}{\sqrt{9.2769\times 10^{-5}e^{0.4361t}}}\right) * 0.95 * \exp(-0.0498t) * \exp(-0.855t)$$

$$P_1 = te^{-t}\Phi\left(\frac{D_f - 0.891e^{0.0382t} - 0.02}{\sqrt{9.2769\times 10^{-5}e^{0.4361t} + 0.1}}\right) * 0.95 * \exp(-0.0498t) * \exp(-0.855t) \times \sum_{i=0}^{1}\frac{(0.855t)^i}{i!}$$

$$P_2 = \frac{1}{2}t^2e^{-t}\Phi\left(\frac{D_f - 0.891e^{0.0382t} - 0.04}{\sqrt{9.2769\times 10^{-5}e^{0.4361t} + 0.2}}\right) * 0.95 * \exp(-0.0498t) * \exp(-0.855t)\sum_{i=0}^{2}\frac{(0.855t)^i}{i!}$$

$$P_3 = \frac{1}{6}t^3e^{-t}\Phi\left(\frac{D_f - 0.891e^{0.0382t} - 0.06}{\sqrt{9.2769\times 10^{-5}e^{0.4361t} + 0.3}}\right) * 0.95 * \exp(-0.0498t) * \exp(-0.855t)\sum_{i=0}^{3}\frac{(0.855t)^i}{i!}$$

$$P_4 = \frac{1}{24}t^4e^{-t}\Phi\left(\frac{D_f - 0.891e^{0.0382t} - 0.08}{\sqrt{9.2769\times 10^{-5}e^{0.4361t} + 0.4}}\right) * 0.95 * \exp(-0.0498t) * \exp(-0.855t)\sum_{i=0}^{4}\frac{(0.855t)^i}{i!}$$

$$P_5 = \frac{1}{120}t^5e^{-t}\Phi\left(\frac{D_f - 0.891e^{0.0382t} - 0.10}{\sqrt{9.2769\times 10^{-5}e^{0.4361t} + 0.5}}\right) * 0.95 * \exp(-0.0498t) * \exp(-0.855t)\sum_{i=0}^{5}\frac{(0.855t)^i}{i!}$$

The probability curve in every state i ($i = 0$–5) and the system reliability can be shown in Fig. 15.

7. Conclusions

In the real world, the failure of many systems is mainly derived from three modes: binary state failure mode, degradation process failure mode and failure mode due to shock process. Degradation process failure has been focused on by many researchers and applied to the practical engineering. Degradation analysis plays an important role in the reliability evaluation, especially to the high reliable systems. However, in the traditional degradation analysis, the degradation data are considered as crisp numbers without fuzziness and the random variance around the mean degradation level is invariable with time stepping forward. In this paper, degradation data are treated as fuzzy number and random variance around the mean degradation level changes with time. We ignore some meaningless fuzzy numbers while some transformation is used, such as nature log, when the fuzzy variance is analyzed. Shock is another important factor to cause the system to fail. FTA is employed to evaluate the system reliability of independent competitive failure model. Because shocks not only bring a random direct effect to system, but also bring degradation a random indirect effect to cause the degradation not be expressed by a uniform function, multi-state system reliability theory is employed to evaluate the system reliability of dependent competitive model. The difference from traditional multi-state system theory is that the jump time from state i to $i+1$ is a random variable. This causes every state probability to be a function of t. This model will have wider application fields. For example, diesel engines have a degradation process by its effect, such as wear, erosion, crack and so on; meanwhile, shocks are caused by random environment not only on diesel engines but also on degradation process, especially in the case that the vibration is very significant in specific frequency ranges. DCFM is used to estimate the reliability of diesel engines and supply engineers a method to the design optimization and maintenance decision-making. Some theoretical research and application research are investigated as well.

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