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Reliability Analysis of Aircraft Servo-Actuation Systems Using Evidential Networks

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Abstract. Servo-actuation system is one of the key executing subsystems of the flight control system of an aircraft. With the development of fly-by-wire control systems, redundant servo-actuation systems have been extensively applied. A servo-actuation system has a long life and high reliability, which results in the lack of failure information. In the mean time, available data is insufficient and imprecise during its design stage. In this paper, evidential networks (EN) are used to handle imprecise probabilities. The formulae of marginal belief mass for series system, parallel system, series-parallel system and parallel-series system are represented respectively. The basic reliability model and the mission reliability model of a three-redundancy servo-actuation system in an aircraft flight control system are analyzed using EN approach, respectively. The EN manage and quantify the imprecision of the servo-actuation effectively, and propagate imprecision from the root nodes to the top nodes representing the system reliability.

Keywords. Servo-actuation system, Dempster–Shafer evidence theory, evidential networks, reliability analysis, imprecise probability.

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1 Introduction

A servo-actuation system is one of the key executing subsystems of an aircraft flight control system. The fault of servo-actuation system can result in an aborted mission and even an aircraft crash in a severe case. Consequently, its reliability directly affects the safety and security of the aircraft. Meanwhile, with the development of fly-by-wire control systems, redundant servo-actuation systems have been extensively applied. Therefore, reliability analysis of servo-actuation systems is essential for safety of aircraft. Quite often available data is insufficient and imprecise during product design stage. Since a servo-actuation system has a long lifetime and high reliability, a large amount of failure data is difficult to obtain. In fact, it is often difficult to accurately estimate the failure rates of individual components or subsystems. Consequently, imprecision and uncertainty needs to be taken into account [1]. A large body of literature makes great efforts on the investigation of the imprecision and uncertainty of system reliability analysis.

Carreras [1] encoded inherent uncertainty in the input data by modeling this data in terms of intervals. Appropriate interval arithmetic was used to propagate the data standard fault trees to generate output distributions reflecting the uncertainty in the input data. Sun [2] proposed a proper mathematical representation of uncertainties for reliability analysis of a practical engineering structural system. He introduced the concept of system reliability and its relationship to the reliability of its individual elements in an interval form. In terms of extension principle, interval arithmetic and possibility degree formula (PDF) for ranking interval numbers, basic properties of system reliability in interval form were investigated. The conclusion was that relationship between point reliability (point reliability used to describe a precise value of probability reliability is distinct from interval reliability) of some typical systems, and point reliability of their individual elements are maintained in their interval forms. The typical systems are series systems, parallel systems, series–parallel systems, parallel–series systems and r/n(G) systems. The proposed quasi-consistency establishes the foundations for interval reliability analysis of a complex engineering structural system.

Utkin [3] proposed new imprecise structural reliability models. These models were developed based on the imprecise Bayesian inference with imprecise Dirichlet, imprecise negative binomial, gamma-exponential and normal models. The models were applied to compute cautious structural reliability measures when the number of events of interest or observations was very small. The main feature of the models was that the prior ignorance was not modeled by a fixed single prior distribution, but by a class of priors which was defined by upper and lower probabilities that can converge as statistical data accumulate. Numerical examples

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are adopted to illustrate some features of the proposed approach. In real life situations, the reliability of an individual component may vary due to some realistic factors. And it is reasonable to treat this as a positive imprecise number which is represented by an interval.

Bhunia [4] formulated the reliability optimization problem as a problem of chance constraints based reliability stochastic optimization with interval valued reliability of components. Then, the chance constraints of the problem were converted into the equivalent deterministic form. The transformed problem had been formulated as an unconstrained integer programming problem with interval coefficients by Big-M penalty technique. To solve this problem, they developed a real coded genetic algorithm (GA) for integer variables with tournament selection, uniform crossover and one-neighborhood mutation. To study the stability of the developed GA with respect to the different GA parameters, sensitivity analyzes had been performed graphically.

Campi [5] addressed the problem of constructing reliable interval predictors directly from observed data. Different from standard predictor models, interval predictors return a prediction interval as opposed to a single prediction value. They showed that, in a stationary and independent observation framework, the reliability of the model was guaranteed a priori by an explicit and non-asymptotic formula, with no further assumptions on the structure of the unknown mechanism that generates the data. They attained a key result that the reliability of the model to its complexity was relational with the amount of available information.

When modeling variables with limited information using intervals with upper and lower bounds, the entire range of these bounds should be explored while estimating the system reliability. The computational cost involved in estimating reliability bounds increases tremendously because a single reliability analysis, which is a computationally expensive procedure, is needed for each configuration of the interval variables. To reduce the computational cost involved, Adduri [6] proposed a high quality function to approximate individual failure functions and the joint failure surface. The accuracy and efficiency of the proposed technique were demonstrated with numerical examples.

Based on the above review, the limitations of the conventional precise probability have been investigated. Aircraft servo-actuation systems typically are redundancy systems. Moreover, they have a long service time and high reliability resulting in the lack of the experiment information under the limitations of the finite product life cycle and expense. It is often difficult to accurately estimate the failure rates of individual components or subsystems. Evidential Networks (EN) are capable of dealing with random, epistemic and imprecise uncertainties in the reliability engineering. Moreover, the EN can model the propagation of uncertainty in reliability analysis of the system. In order to assessing the reliability performance, the servo-actuation systems of flight control systems are briefly introduced. EN approach is adopted to quantify and propagate the imprecision of failure rate of the three-redundancy servo-actuation system of the aircraft.

The rest of this paper is organized as follows. The servo-actuation system of the aircraft is shortly described in Section 2. In Section 3, the D-S evidence theory is briefly introduced. The EN of series system and parallel are presented in Section 4. In Section 5, the basic reliability model and the mission reliability model of the three-redundancy servo-actuation system are analyzed through the EN. The conclusion is given at the end of the paper.

2 Brief Description of Servo-actuation System of Aircraft

A servo-actuation system is an aggregate of the relational actuation systems of the pneumatic controls to control aircraft. The system consists of servo controllers, servo actuators and other components. A servo controller is the key component which controls the servo actuator and the interface device of flight control computer. The basic functional modules include: signal integrated circuits, preamplifier/correction, output circuits, feedback signal processing, gain setting circuit and so on. A servo controller is a part of flight control computer usually installed in the flight control computer.

A servo actuator is used to realize the signal transmission between the electrical and mechanical movement. And it is an executing component to drive the pneumatic surface of aircraft. Its basic function includes transformation, synthesis, equilibrium and amplification of the signal, mechanical transmission and power output, the acquisition of the mechanical motion feedback signal, the fault detection of the actuator, and the energy control of actuator. The electro-hydraulic servo actuator dominates the servo actuator in aviation.

Before the reliability of a system is analyzed, the reliability block diagram of the system should be constructed according to the design specification and system schematic diagram. The basic reliability model and the mission reliability model can be built up applying the reliability block diagram of system. For single channel servo actuation system, the basic reliability model and mission reliability model are the same. The mission reliability diagram and the basic reliability diagram are shown in Figure 1.

When the servo-actuation system is a redundant system, the basic reliability model and the mission reliability model are different. The reliability block diagram is respectively constructed according to the practical situation. Currently, the three-redundancy and the four-redundancy configurations are widely used.

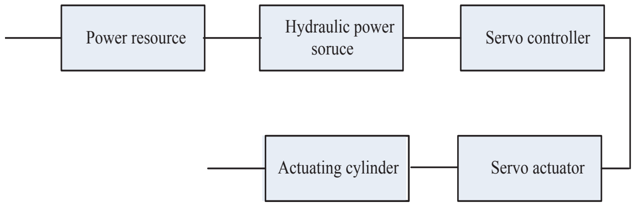

Figure 1. Mission reliability diagram and basic reliability diagram for single channel servo actuation system.

3 The Brief Introduction of D-S Evidence Theory

Evidence theory is expanded and proposed by Shafer [7] based on the innovative work of Dempster on interval belief assignment to the hypothesis [8], also called Dempster–Shafer evidence theory. This theory adopts the belief function and the plausibility function as the lower and upper limits of the belief interval to delineate the imprecision and uncertainty of the hypothesis. This can separate the ignorance from uncertainty in the hypothesis. Moreover, the D-S can distribute the basic belief to subsets of the discernment framework, which is more general than the probability theory that only allow to distribute the basic belief to the mutually exclusive singletons. Simultaneously, the combination rule of D-S evidence theory can aggregate multiple sources of evidence which may be imprecise and uncertain. Because of the flexibility of the basic axioms in evidence theory, no further assumptions are needed to quantify the uncertain information of system [9].

The D-S evidence theory starts with defining the frame of discernment (FD). The FD is a finite nonempty exhaustive set of mutually exclusive possibilities, denoted by Θ, which includes all the elementary proposition of the problem:

$$\Theta = \{q_1, q_2, \cdots, q_n\}. \tag{1}$$

The power set of Θ is all the possible subsets, denoted as 2^{Θ}. There are 2^n elements in the 2^{Θ}.

$$\begin{aligned} 2^{\Theta} = \{&\emptyset, \{q_1\}, \cdots, \{q_n\}, \{q_1, q_2\}, \{q_1, q_3\}, \cdots, \\ &\{q_1, q_n\}, \cdots, \{q_{n-1}, q_n\}, \{q_1, q_2, q_3\}, \cdots, \\ &\{q_1, q_2, \cdots, q_n\}\}, \end{aligned} \tag{2}$$

where $\emptyset$ is an empty set. Basic belief assignment is a primitive function of D-S evidence theory, which is denoted by $m(A)$. The function $m(A)$ is a mapping: $m(A) : 2^{\Theta} \to [0, 1]$, which satisfies the following conditions:

$$m(\emptyset) = 0, \tag{3}$$

$$\sum_{A \in 2^{\Theta}} m(A) = 1, \tag{4}$$

$$0 \le m(A) \le 1 \qquad A \in 2^{\Theta}, \tag{5}$$

$m(A)$ expresses the precise probability in which the evidence corresponds to m supports proposition A. $m(\emptyset) = 0$ means the closed world assumption (CWA). Relative to CWA, the open world assumption (OWA) is proposed by Smets and distributes the belief to $\emptyset m(\emptyset) \neq 0$. $\sum_{A \in 2^{\Theta}} m(A) = 1$ expresses the belief assignment function of the power set of Θ satisfies the normalization of the probability. $m(A)$ is not the probability in classical sense. The hypothesis A can be a set.

A belief function is a mapping: Bel: $2^{\Theta} \to [0, 1]$

$$\mathrm{Bel}(A) = \sum_{B \subseteq A} m(B) \tag{6}$$

and satisfies the following conditions:

$$\mathrm{Bel}(\emptyset) = 0 \tag{7}$$

$$\mathrm{Bel}(\Theta) = 1. \tag{8}$$

$\mathrm{Bel}(A)$ represents the total amount of probability that must be distributed among elements of A. It reflects the inevitability and signifies the total degree of belief of X and constitutes a lower limit function on the probability of A [10].

$\mathrm{Bel}(A)$ can be attained by BBA. Symmetrically, BBA can be attained by $\mathrm{Bel}(A)$ through the möbius transform [11]:

$$m(A) = \sum_{B \subseteq A} (-1)^{|A-B|} \mathrm{bel}(B), \tag{9}$$

where $A, B \in 2^{\Theta}$, $| \cdot |$ denotes the cardinality function.

A Plausibility function (Pl) is a mapping: Pl: $2^{\Theta} \to [0, 1]$. It is defined as the following

$$\mathrm{Pl}(A) = \sum_{A \cap B \neq \emptyset} m(B), \tag{10}$$

where $\overline{A}$ is the complement of a hypothesis A. $\mathrm{Pl}(A)$ measures the maximum amount of probability that can be distributed among the elements in A. It describes the total degree of belief related to A and constitutes an upper limit function on the probability of A [10].

Because the belief function and the plausibility function are derived from the basic belief assignment function, they can be transformed among each other [12].

$$\begin{aligned} \mathrm{Pl}(A) &= \sum_{A \cap B \neq \emptyset} m(B) \\ &= 1 - \sum_{A \cap B = \emptyset} m(B) = 1 - \sum_{B \subseteq \overline{A}} m(B) \\ &= 1 - \mathrm{Bel}(\bar{A}). \end{aligned} \tag{11}$$

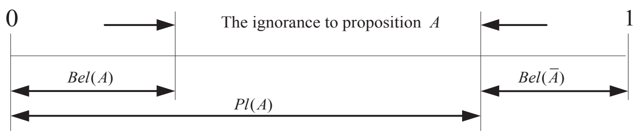

Figure 2. Relationship of Belief function and Plausibility function.

In the same way, the basic probability assignment function can be attained from the plausibility function by using the following formula:

$$m(A) = \sum_{B \subseteq A} (-1)^{|A|-|B|+1} \mathrm{Pl}(\overline{B}). \tag{12}$$

Moreover, the relationship of the belief function, Plausibility function and probability of hypothesis can be expressed

$$\mathrm{Bel}(A) \leq P(A) \leq \mathrm{Pl}(A), \tag{13}$$

where $P(A)$ describes the probability of the hypothesis A.

Based on the above functions, the posteriori confidence interval, $[\mathrm{Bel}(A), \mathrm{Pl}(A)]$, can be attained. This interval expresses the uncertainty of A. When the ignorance to proposition A is decreased, the length of interval diminishes. This difference provides a measurement of the imprecision and the uncertainty of the belief level in decision-marking [13].The relationship between $\mathrm{Bel}(A)$ and $\mathrm{Pl}(A)$ is illustrated in Figure 2.

Probability interval: $[\underline{\mathrm{P}(A)}, \overline{\mathrm{P}(A)}]$ is often considered as an effective metric for modeling uncertainty. Probability interval can be directly transformed into the posteriori confidence interval:

$$\underline{\mathrm{P}(A)} = \mathrm{Bel}(A) \tag{14}$$

$$\overline{\mathrm{P}(A)} = \mathrm{Pl}(A). \tag{15}$$

4 Evidential Networks

Evidential networks are proposed by Simon [14]. It is a directed acyclic graph (DAG). An evidential network is a DAG $G = ((N, A), M)$, where (N, A) represents the graph, "N" is a set of nodes, "A" is a set of arcs and "M" expresses the set of belief distributions that are distributed to each node [15]. When a node is a root node, its priori belief mass table is defined. When a node is not a root node, its belief mass distribution is defined by a conditional belief mass table quantifying the relation between the nodes and its parents [16]. The network propagates basic belief assignments as a priori belief mass on variables. A conditional belief function quantifies the dependency between a node and its parents And it allows for computing its mass distribution according to other variables [17].

In this paper, the system under study is homogeneous and no repair is considered. The reliability analysis is in the probist reliability model concerning systems or components with two states (functioning and malfunctioning). In evidential networks, the corresponding frame of discernment is described by the following:

$$\Theta = \{\{\mathrm{up}\}, \{\mathrm{down}\}\} 2^{\Theta} = \{\emptyset, \{\mathrm{up}\}, \{\mathrm{down}\}, \{\mathrm{up}, \mathrm{down}\}\}, \tag{16}$$

where {up} means the functioning of the components and systems, {down} means the malfunctioning of the components and systems.

If the basic belief assignment of {up, down} is equal to zero, the reliability analysis of system is a conventional probability reasoning method. If the basic belief assignment of {up, down} is not equal to zero, the probability theory cannot deal with this situation. This characterizes the ignorance on the real state of the components and systems, and the components and systems may be in the state {up} or {down}. Using Eqs. (6) and (10), the posteriori confidence interval of the occurring of event is attained. This describes the reliability uncertainty of the states of components and systems.

A system is a complex structure comprised of several interacting, interrelated and interdependent components. The structure must achieve certain specific functions. In system reliability analysis there are several typical models which must be considered. For example, Series configuration, Parallel configuration and so on.

4.1 Series System

A series system fails if any of the subsystems or components fails. A typical series system configuration and the corresponding to EN are shown in Figure 3. The detail conversion processing can be referred to [14] and [15].

The truth table defines the relationship between the state of system and component on the discernment framework of each parent's node. The conditional belief mass table describes the relationship between the belief masses on the discernment framework of each parent's node, and the belief masses of the discernment framework of child's nodes [14]. The truth table and the conditional belief mass table of the series system have been discussed in [14] and [15]. More detailed discussions can be found in [14–16].

To compute the marginal belief mass of top node of series system in EN, the inference thought of Simon [14] is adopted. The algorithm is expressed in Eq. (17) when series system has two components or subsystems. The algorithm updates the marginal belief mass on top node according to the evidence representing the knowledge of component and

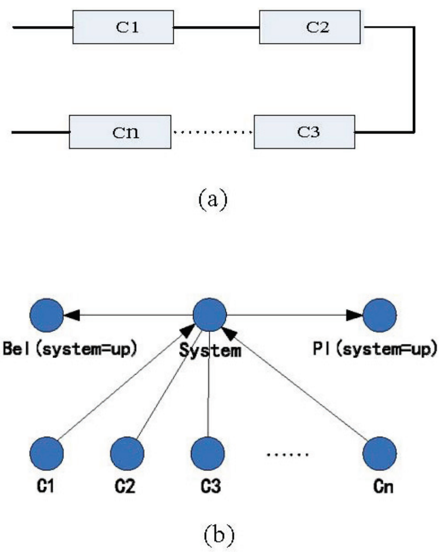

Figure 3. (a) A series system block diagram (b) Equivalent evidential network of the series system.

subsystem introduced into the EN in series system.

$$m_{ij}(\text{system} = C) = (m_i \odot m_j)(C)$$
$$= \begin{cases} m_i(C)\cdot m_j(C), \quad C = \{\text{up}\} \\ m_i(C) + m_j(C) - m_i(C)\cdot m_j(C), \quad C = \{\text{down}\} \\ m_i(X)\cdot m_j(C) + m_j(Y)\cdot m_i(C) + m_i(C)\cdot m_j(C), \\ \qquad X = \{\text{up}\}, Y = \{\text{down}\}, C = \{\text{up}, \text{down}\}, \end{cases} \tag{17}$$

where $m_i(\cdot)$ and $m_j(\cdot)$ express the belief masses of the parent nodes of component i and component j in series system. $m_{ij}(\cdot)$ expresses the marginal belief mass of top node of series system.

When there are more than two components or subsystems in series system, for example, there are n components or subsystems, the marginal belief mass of system or the top node of series system is generalized as shown in Eq. (18). It is both commutative and associative.

$$m = m_1 \odot m_2 \odot \cdots \odot m_n = (((m_1 \odot m_2) \odot \cdots) \odot m_n), \tag{18}$$

where, $m_i (i = 1, 2, \cdots, n)$ represents the belief mass of the parent nodes in series system. $m$is the belief mass of the child node.

4.2 Parallel System

A parallel system fails if and only if all the units in the system fail. A typical series system configuration corresponding to EN are shown in Figure 4. The detail conversion processing can be referred to paper [14].

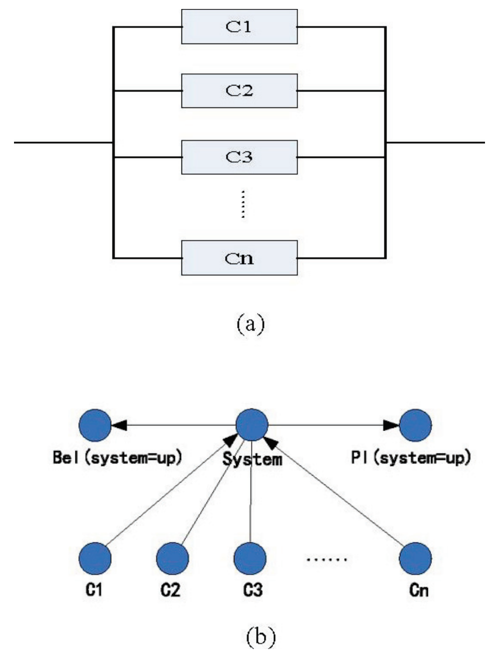

Figure 4. (a) A parallel system block diagram (b) Equivalent evidential network of the parallel system.

Similar to the series system, the truth table and the conditional belief mass table of the series system have been represented in [14] and [15]. More details can be found in [14]-[17]. The algorithm of computing the marginal belief mass of top node of parallel system is expressed in Eq. (19) when the parallel system have two components or subsystems in EN.

$$m_{ij}(\text{system} = C) = (m_i \otimes m_j)(C)$$
$$= \begin{cases} m_i(C) + m_j(C) - m_i(C)\cdot m_j(C), \quad C = \{\text{up}\} \\ m_i(C)\cdot m_j(C), \quad C = \{\text{down}\} \\ m_i(C) + m_j(C) - m_i(X)\cdot m_j(C) - m_j(X)\cdot m_i(C), \\ X = \{\text{up}\}, C = \{\text{up}, \text{down}\}. \end{cases} \tag{19}$$

Similar to the series system, when there are more than two components or subsystems in parallel system, the marginal belief mass of system or the top node of parallel system is shown as following

$$m = m_1 \otimes m_2 \otimes \cdots \otimes m_n = (((m_1 \otimes m_2) \otimes \cdots) \otimes m_n) \tag{20}$$

4.3 Series-parallel System

The series-parallel system represents a system consisting of multiple sub-systems connected in parallel redundancy. Meanwhile, there are multiple series components in every sub-system. A typical series-parallel system configuration corresponding to EN are shown in Figure 5. Note that m represents the number of the sub-system and n_i represents

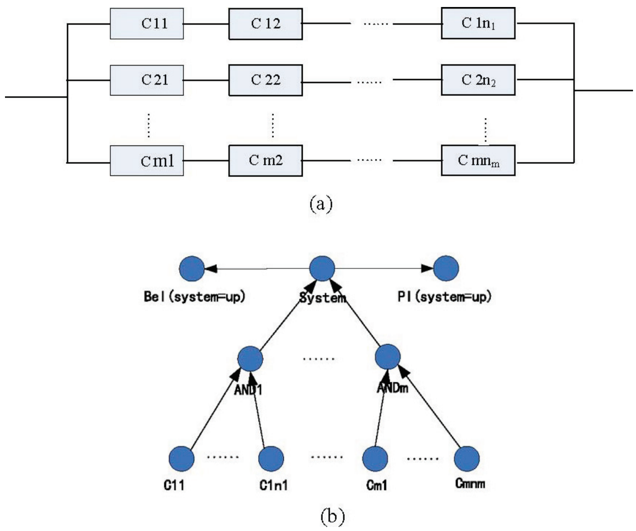

Figure 5. (a) A series-parallel system block diagram (b) Equivalent evidential network of the series-parallel system.

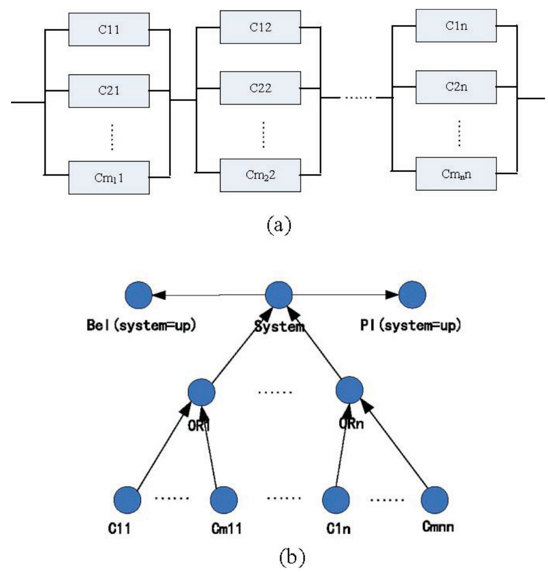

Figure 6. (a) A parallel-series system block diagram, (b) Equivalent evidential network of the parallel-series system.

the number of component. In the processing of the transforming EN, there are two steps which may be considered. First, each of sub-systems can be regarded as the series system. Second, the whole sub-systems can be regarded as the parallel system. The detailed processing of each step may be referred to the Section A and B. The transformed EN is showed in Figure 5.

The truth table and the conditional belief mass table can be attained by the corresponding table of series system and parallel system respectively. When there are m sub-systems and n_i components in ith sub-system, the marginal belief mass of system or the top node of series-parallel system is generalized as shown in Eq. (21). It is both commutative and associative:

$$\begin{aligned} m &= m_{C1} \otimes m_{C2} \otimes \cdots \otimes m_{Cm} \\ &= (m_{C11} \odot m_{C12} \odot \cdots \odot m_{C1n_1}) \otimes \\ &\quad (m_{C21} \odot m_{C22} \odot \cdots \odot m_{C2n_2}) \otimes \cdots \otimes \\ &\quad (m_{Cm1} \odot m_{Cm2} \odot \cdots \odot m_{Cmn_m}), \end{aligned} \tag{21}$$

where m_{Ci} $(i = 1, 2, \cdots, m)$ represents the belief mass of the child nodes in each of the sub-system. $m_{Cin_j}(i = 1, 2, \cdots, m; j = 1, 2, \cdots, \mathrm{n}_m)$ represents the belief mass of the n_j parent node in ith sub-system. m is the belief mass of the child node of whole system.

4.4 Parallel-series System

The parallel-series system represents a system which consists of multiple sub-systems connected in series. Meanwhile, there are multiple parallel components in every sub-system. A typical parallel-series system configuration and the corresponding to EN are shown in Figure 6. In Figure 6, n represents the number of the sub-system and m_i represents the number of component. In the processing of the transforming EN, there are two steps which may be considered. Firstly, each of sub-systems can be regarded as the parallel system. Secondly, the whole sub-systems can be regarded as the series system. The detailed processing of each step may be accorded to the Section A and B. The transformed EN is shown in Figure 6.

Similar to the series-parallel system, the truth table and the conditional belief mass table can be attained by the corresponding table of series system and parallel system respectively. The marginal belief mass of system or the top node of series-parallel system can be generalized as shown in Eq. (22).

$$\begin{aligned} m &= m_{C1} \odot m_{C2} \odot \cdots \odot m_{Cm} \\ &= (m_{C11} \otimes m_{C21} \otimes \cdots \otimes m_{Cm_11}) \odot \\ &\quad (m_{C12} \otimes m_{C22} \otimes \cdots \otimes m_{Cm_22}) \odot \cdots \odot \\ &\quad (m_{C1n} \otimes m_{C2n} \otimes \cdots \otimes m_{Cm_nn}), \end{aligned} \tag{22}$$

where m_{Ci} $(i = 1, 2, \ldots, m)$ represents the belief mass of the child nodes in each of the sub-system. m_{Cm_ij} $(i = 1, 2, \ldots, m; j = 1, 2, \ldots, \mathrm{n})$ represents the belief mass of the m_i parent node in jth sub-system. m is the belief mass of the child node of whole system.

5 Example Analysis

There is a three-redundancy servo-actuation system [18]. The basic reliability diagram of system is constructed in Figure 7. The corresponding mission reliability diagram of system is in Figure 8. In Figure 7, Ci $(i = 1, 2, \ldots, 11)$

	$C1$	$C2$	$C3$	$C4$	$C5$	$C6$
$\lambda_{Ci}(m^{-1})$	3.5×10^{-2}	3.5×10^{-2}	7×10^{-2}	7×10^{-2}	$[4.3 \times 10^{-2}, 6.6 \times 10^{-2}]$	5.5×10^{-2}

	$C7$	$C8$	$C9$	$C10$	$C11$
$\lambda_{Ci}(m^{-1})$	$[4.3 \times 10^{-2}, 6.6 \times 10^{-3}]$	5.5×10^{-2}	$[4.3 \times 10^{-2}, 6.6 \times 10^{-2}]$	5.5×10^{-2}	1.3×10^{-2}

Table 1. The failure rate of the component of the three-redundancy servo-actuation system.

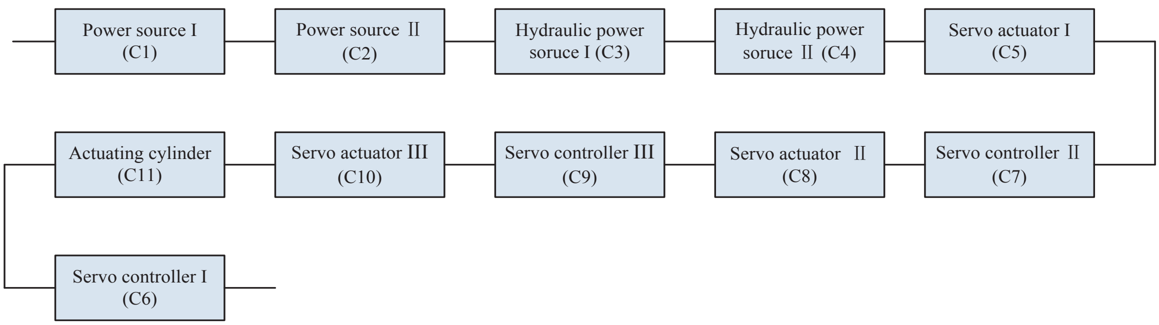

Figure 7. The basic reliability diagram of the three-redundancy servo-actuation system.

represents the corresponding component. Assuming that the life time of each component is exponentially distributed, the failure rates of the components are expressed in Table 1. The system is homogeneous and on repair is considered. Since the data of the servo controller to attain the precise probability is insufficient, the failure of the servo controller is adopted as an interval: $[1.3 \times 10^{-2}, 2.6 \times 10^{-2}]$ taking into account the expert opinion. A priori belief mass distribution defining event state is obtained by Eqs. (6), (9), (10) and (12):

$$\begin{aligned} m_{C5}(\text{up}) &= m_{C7}(\text{up}) = m_{C9}(\text{up}) = 9.974 \times 10^{-1} \\ m_{C5}(\text{down}) &= m_{C7}(\text{down}) = m_{C9}(\text{down}) \\ &= 4.3 \times 10^{-2} \\ m_{C5}(\text{up}, \text{down}) &= m_{C7}(\text{up}, \text{down}) = m_{C9}(\text{up}, \text{down}) \\ &= 2.3 \times 10^{-2}. \end{aligned}$$

For the basic reliability of the servo-actuation system, using Bayesialab [19], the consequence of EN analysis is shown in Figure 9. It is because all the components of system is regarded as series configuration in the basic reliability analysis.

The marginal belief mass of system can be attained by Eq. 18.

$$\begin{aligned} m &= m_{C1} \odot m_{C2} \odot \cdots \odot m_{C11} \\ &= (((m_1 \odot m_2) \odot \cdots) \odot m_{C11}). \end{aligned} \tag{23}$$

The priori belief mass distribution of the each node is shown in Figure 9 as well. The imprecision on the probabilities of servo controller is propagated through the network. The reliability of system is represented by the following interval:

$$[\underline{\text{p}}(\text{system} = \text{up}), \bar{\text{p}}(\text{system} = \text{up})] = [\text{Bel}(\text{system} = \text{up}), \text{Pl}(\text{system} = \text{up})] = [5.466 \times 10^{-1}, 5.880 \times 10^{-1}]. \tag{24}$$

This demonstrates that the EN method can delineate the propagation of the imprecise probability and the basic belief assignment of each node can be attained.

For the mission reliability of the servo-actuation system, the consequence of EN analysis is expressed in Figure 10 using Bayesialab [19]. Owing to imprecision of the servo controller in three-redundancy servo-actuation, the conventional probability cannot deal with the situation. The marginal belief mass of system can be attained by Eqs. (21) and (22)

$$m = (m_{C1} \otimes m_{C2}) \odot (m_{C3} \otimes m_{C4}) \odot ((m_{C5} \odot m_{C6}) \otimes (m_{C7} \odot m_{C8}) \otimes (m_{C9} \odot m_{C10})) \odot m_{C11}. \tag{25}$$

Using EN, the consequence is expressed in Figure 10. The priori belief mass distribution of component is expressed in Figure 10. The node basic belief mass assignment can be attained through EN. The imprecision of probability of servo controller propagated through the network. The reliability of system is attained as the interval:

$$[\underline{\text{p}}(\text{system} = \text{up}), \bar{\text{p}}(\text{system} = \text{up})] = [\text{Bel}(\text{system} = \text{up}), \text{Pl}(\text{system} = \text{up})] = [9.794 \times 10^{-1}, 9.801 \times 10^{-1}]. \tag{26}$$

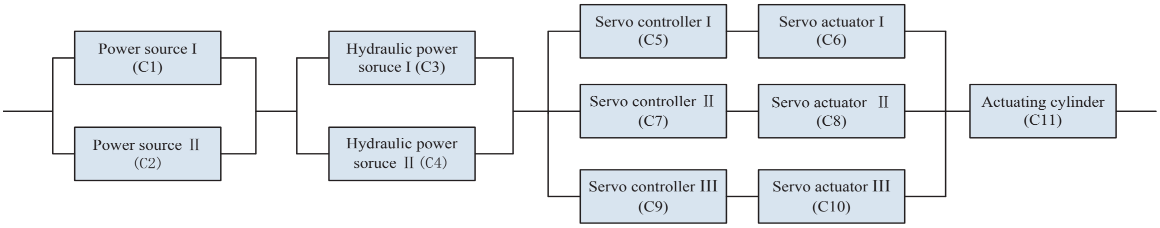

Figure 8. The mission reliability diagram of the three-redundancy servo-actuation system.

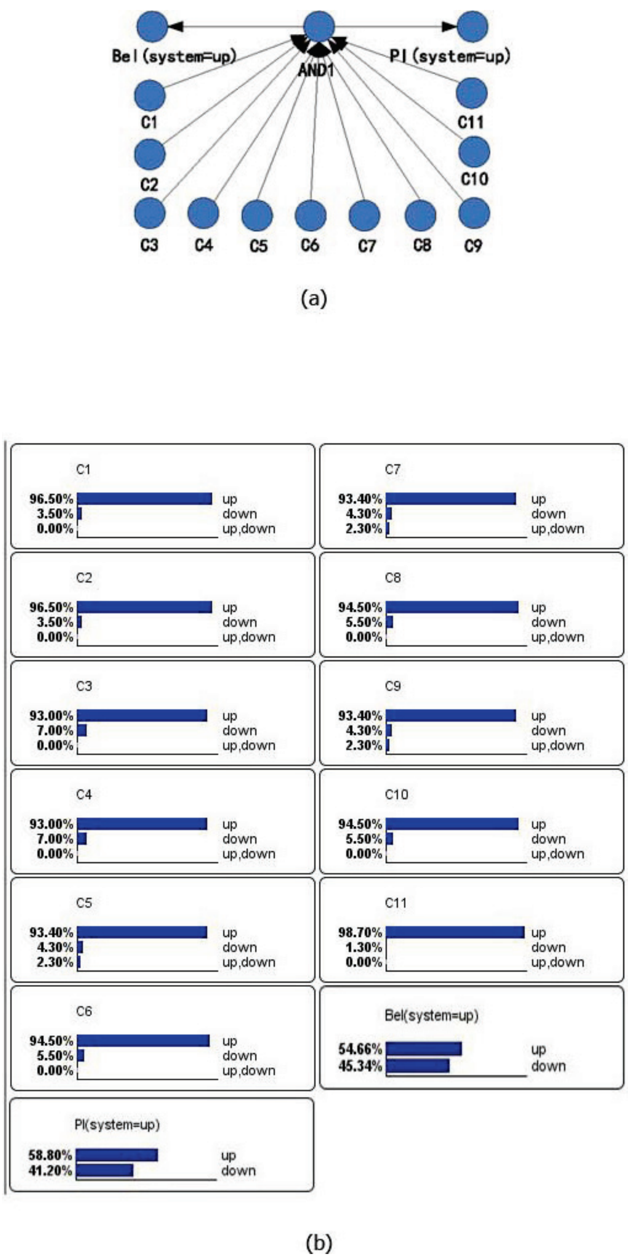

Figure 9. Evidential network to modeling the basic reliability model and basic belief assignments of each node.

Like the basic reliability analysis, this demonstrates that the evidential networks can delineate the propagation of the imprecise probability and the basic belief assignment of each node can be attained.

6 Conclusions and Discussions

Owing to the development of the fly-bywire control system, the redundant servo-actuation systems have been extensively applied. Therefore, the reliability analysis of a servo-actuation system is very important. Since failure data is insufficient and the reliability estimation is imprecise during the product design stage, and the long life and high reliability of a servo-actuation system leads to the lack of the experiment information, it is often difficult to estimate the failure rates of individual components or subsystems. Consequently, the variability and uncertainty need to be taken into account [12]. The EN approach is a powerful tool to handling random, epistemic and imprecise uncertainties in the reliability engineering. Moreover, the EN can model the propagation of uncertainty in reliability analysis of the systems. In this paper, the EN were adopted to quantify and propagate the imprecision of failure rate of the servo-actuation system. The servo actuation was briefly described. The transformation process of the series system and the parallel system was introduced respectively. The formulas of marginal belief mass of top node of two components in series and parallel systems in EN were expressed respectively. Meanwhile, the formula of marginal belief mass was represented when there are more than two components and subsystems. The EN of the basic reliability model and the mission reliability model of the three-redundancy servo-actuation system in aircraft control system were analyzed by considering the imprecision probability of the servo controller. The reliability interval of the respective system was attained through the EN.

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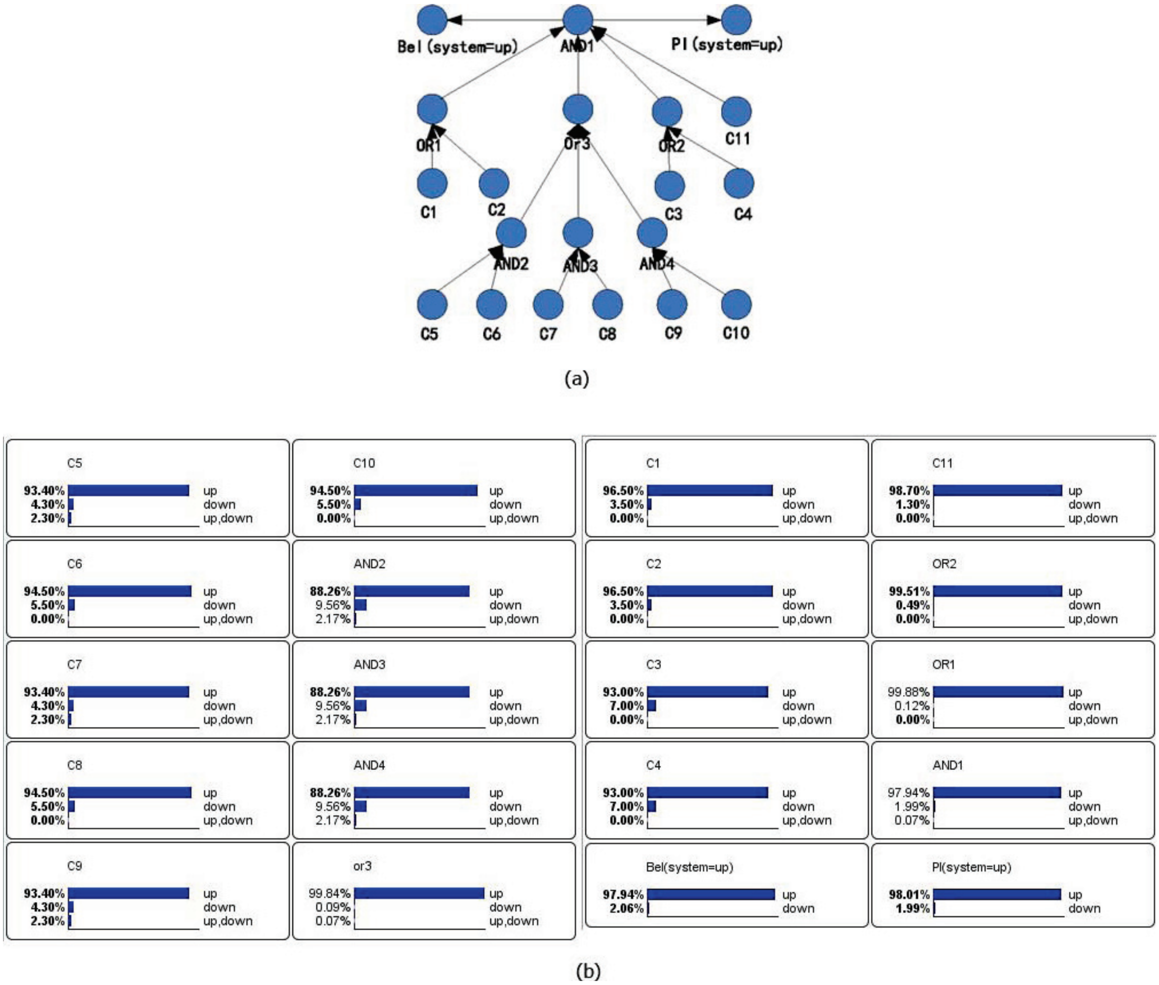

Figure 10. Evidential network to modeling the mission reliability model and basic belief assignments of each node.

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