Journal of Intelligent & Fuzzy Systems 16 (2005) 213–220
IOS Press

A coordination method for fuzzy multi-objective optimization of system reliability

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Abstract. A fuzzy directed graph is used to represent the dynamic relationships among the sub-objectives of a fuzzy multi-objective optimization problem. Using the comprehensive coordination function with exponential weights, we transform a fuzzy multi-objective optimization problem into a single-objective optimization problem. The optimal solution of the single-objective optimization problem can then be obtained with conventional single-objective optimization methods. The optimal solution represents the best trade-off among the possibly conflicting sub-objectives in the original optimization problem. An illustrative example is given to illustrate the effectiveness of the proposed approach.

Keywords: Fuzzy multi-objective optimization, system reliability, coordination method, coordination function, exponential weights

1. Introduction

Most engineering design problems involve optimization of several objectives subject to multiple inequality and equality constraints. These multiple design objectives often compete against one another and a simultaneous optimization of all these objectives is rarely achieved. Thus, in most cases, the design is accomplished by considering the best comprise among the competing objectives. Although the methods of finding a compromise solution to a multi-objective design problem are fairly well established in the literature of engineering optimization [1], the designer often encounters a problem in the development of a precise mathematical model of the system. Vagueness and impreciseness often arise due to poorly defined data, unclear system boundaries, unsatisfactory formulation of design objectives, and inability to evaluate the relative importance of the objectives. As the complexity of the system to be designed increases, more assumptions are made about its behavior and hence the ability of the designer to exactly model the system in precise mathematical terms is severely affected. To model the vague and imprecise nature of the design problem, one often uses the fuzzy sets theory [2].

The first application of fuzzy sets theory to the decision-making processes was presented by Bellman and Zadeh [3]. Their paper described the basic concepts and definitions associated with a decision-making process in a fuzzy environment. After their work, there come out a great number of articles dealing with the fuzzy optimization problems. The collection of papers on fuzzy optimization edited by Slowinski [4] and Delgado et al. [5] gives the main stream of this topic.

Formulating a fuzzy optimization problem entails developing membership functions for each constraint and each objective. These membership functions resemble normalization schemes. A variety of such functions are available in the literature (Rao) [6–8]. A rel-

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atively high value for a membership function of a constraint set indicates a near or definite membership in the set, i.e., a high likelihood of the constraint satisfaction. Therefore, the goal of a fuzzy optimization problem is to maximize all membership functions simultaneously. Most often, this is done using a formulation similar to the min-max formulation for multi-objective optimization [9]. Fuzzy objectives and constraints can be combined to provide an efficient algorithm for optimization. In terms of fuzzy optimization, both of the objective functions and constraint functions are treated as modified constraints. Consequently, fuzzy optimization lends itself to multi-objective optimization where additional objective functions are modeled as constraints [6, 10].

Zimmermann [10] initiated the application of fuzzy theory to optimization by solving theoretical, fuzzy, linear programming problems. Zimmermann [11] also proposed a max-min approach, which was used for solving fuzzy mathematical problems with fuzzy objectives and fuzzy constraints. Delgado et al. [12], Cadenas and Verdegay [13] and Verdegay [14] discussed fuzzy mathematical programming problems with fuzzy objective coefficients. In their approach the k-th objective λ-constraint approach was used for solving fuzzy multi-objective problems with fuzzy objective coefficients. Chanas [15] presented the possibility of the identification of a complete fuzzy decision in fuzzy linear programming by use of the parametric programming technique. This parametric approach can analytically describe the set of solutions incorporating the whole range of possible values of the fuzzy decision and provides some information on other alternatives close to the maximizing solution. Lai and Hwang [16] proposed an augmented max-min approach, which is essentially an extension of Zimmermann's approach in essence.

G.-Y. Wang and W.-Q. Wang [17] used a level-cuts approach to solve non-linear, structural problems with fuzzy constraints (and crisp objectives). Rao [6–8] used explicit, continuous membership functions for fuzzy constraints and fuzzy objectives to optimize mechanical systems and structure design; membership functions for the objective function and for the constraints are aggregated into a single, standard optimization problem. Rao's method of λ-formulation yields a unique compromise solution with maximum overall satisfaction for fuzzy optimum structural design. Furthermore, he introduced the α-cut approach which provided the results in a parametric form for multi-objective problems. Werner [18,19] proposed an interactive decision support system which aids in solving multiple objective programming problems subject to crisp and fuzzy constraints. One part of the system is an extension of a well-known fuzzy sets approach evaluating possible solutions by their degrees of membership to objectives and constraints.

Xu [20] also transforms problems with fuzzy constraints into standard optimization problems, though with a slightly different format; the final solution is then determined with a bound-constrained optimization approach. Despite the significant amount of work that has been completed with fuzzy optimization, there are few investigations on the use of fuzzy theory to determine feasible points of constrained problems.

Shih [21] presented a global criterion method by fuzzy logic to obtain solutions for multi-criteria crisp or fuzzy structural design, which is not only capable of acquiring the non-dominated solution, but also capable of achieving the highest degree of satisfactory design.

However, the conflicting degree among objectives and the designer's preferences are neglected to some extent. Loetamonphong et al. [22] studied the optimization problems that have multiple objective functions subject to a set of fuzzy relation equations. Huang [23] presented a fuzzy multi-objective optimization decision-making method, which can be used for the optimization decision-making on two or more objectives of system reliability.

A variety of other new techniques and applications for multi-objective optimization have been developed in recent years for overcoming the drawbacks of traditional methods. Representative work includes multi-objective collaborative optimization [24], interactive multi-objective optimization [25], physical programming [26], game theoretic approach [27], Taguchi routine based method [28], modified Dempster-Shafer theory [29], satisfaction metrics based method [30], genetic algorithms [31–33] and so on. They attempted to resolve the conflicting objectives so that the search for a compromise design over a feasible design space can be facilitated. A comprehensive survey of multi-objective optimization methods (traditional, evolutionary and interactive) is given in [34,35].

As discussed previously, conflicts in design objectives are unavoidable in any multi-objective optimization problem. How to model and resolve the conflicts is the ongoing and important topic. In this paper, we propose a coordination method for solving multi-objective optimization problems. In Section 2, we summarize the fuzzy multi-objective optimization problem. In Section 3, we develop the coordination method for solving multi-objective optimization problem with fuzzy sets theory. A case study is provided in Section 4 and conclusions are given in Section 5.

2. Model of fuzzy multi-objective optimization [6, 23]

Let the crisp multi-objective optimization problem be stated as follows:

Find X which minimizes $f(X)$

Subject to

$$g_m^{(l)} \leqslant g_m(X) \leqslant g_m^{(u)};\ m = 1, 2, \ldots, q \tag{1}$$

where X is the vector of n design variables, $f(X) = \{f_1(X), f_2(X), \ldots, f_p(X)\}^T$ is the vector of objective functions, $g_m(X)$ is the mth constraint function, and the superscripts (l) and (u) indicate the lower and upper bound values, respectively. This problem can also be expressed as

Find X which minimizes $f(X)$

Subject to

$$g_m(X) \in G_m;\ m = 1, 2, \ldots, q \tag{2}$$

where G_m denotes the allowable interval for the constraint function g_m; $G_m = [g_m^{(l)},\ g_m^{(u)}]$. When the constraints contain fuzzy information, the problem in Eq. (2) becomes

Find X which minimizes $f(X)$

Subject to

$$g_m(X) \in \tilde{G}_m;\ m = 1, 2, \ldots, q \tag{3}$$

where the tilde is used to denote that the operators or variables contain fuzzy information. Thus the constraint $g_m(X) \in \tilde{G}_m$ means that $g_m(X)$ is a member of the fuzzy set $\tilde{G}_m$ in the sense that the membership function value is greater than 0, i.e., $\mu_{\tilde{G}_m}(g_m) > 0$. To solve the fuzzy optimization problem in Eq. (3), we propose a coordination method in the following section.

3. A coordination method for fuzzy multi-objective optimization

3.1. Fuzzy directed graph for multi-objective optimization

In the literature of fuzzy systems several approaches to the idea of fuzzy graph have been introduced [36–40]. Reference [41] overviewed most of the literature and gave a general way of treating fuzzy graph problems. The concept of fuzzy directed graph was summarized in [36]. Because of the characteristics of a fuzzy directed graph, we find that it can also be used to represent the dynamic relationship of the entities in a fuzzy multi-objective optimization problem. As shown in Fig. 1, $\{f_1(X), f_2(X), \cdots, f_p(X), F(X)\}$ is a set consisting of p sub-objectives and one objective set function. The objective function measures the overall satisfaction of the p sub-objectives. We use $\mu_j(f_j)$, or μ_j for simplicity, to denote the degree of satisfaction of the jth sub-objective and call it the coordination function of the jth sub-objective $(j = 1, 2, \cdots, p)$. This coordination function is actually the membership function of $f_j(X)$. A parameter denoted by β_j is used in the coordination function μ_j $(j = 1, 2, \cdots, p)$. Correspondingly, we use $\mu(F)$ to denote the coordination function of the overall objective. In expressing the overall objective $F(X)$ as a function of the p sub-objectives, we use α_j to represent the relative importance of the jth sub-objective with the requirement of $\sum_{j=1}^{p} \alpha_j = 1$.

3.2. The coordination function of a sub-objective

A membership function is used to fuzzify a sub-objective into a coordination functions μ_j with an exponential weight as a parameter. This exponential weight denoted by β_j controls the pace at which the jth sub-objective changes between being satisfactory and non-satisfactory. If the jth sub-objective should be maximized, we formalize its coordination function using a monotonically increasing function given below:

$$\mu_j = \begin{cases} 0, & f_j \leqslant f_j^l \\ \left(\frac{f_j - f_j^l}{f_j^u - f_j^l}\right)^{\beta_j}, & f_j^l < f_j < f_j^u \\ 1, & f_j \geqslant f_j^u \end{cases} \tag{4}$$

If the jth sub-objective should be minimized, we formalize its coordination function using a monotonically decreasing function as follows:

$$\mu_j = \begin{cases} 1, & f_j \leqslant f_j^l \\ \left(\frac{f_j^u - f_j}{f_j^u - f_j^l}\right)^{\beta_j}, & f_j^l < f_j < f_j^u \\ 0, & f_j \geqslant f_j^u \end{cases} \tag{5}$$

In both Eqs (4) and (5), f_j^u and f_j^l are the upper and lower bounds of the jth sub-objective, respectively, and β_j is the exponential weight parameter. The upper and lower bounds are usually available from the description of the optimization problem while β_j has to be selected based on the following discussions.

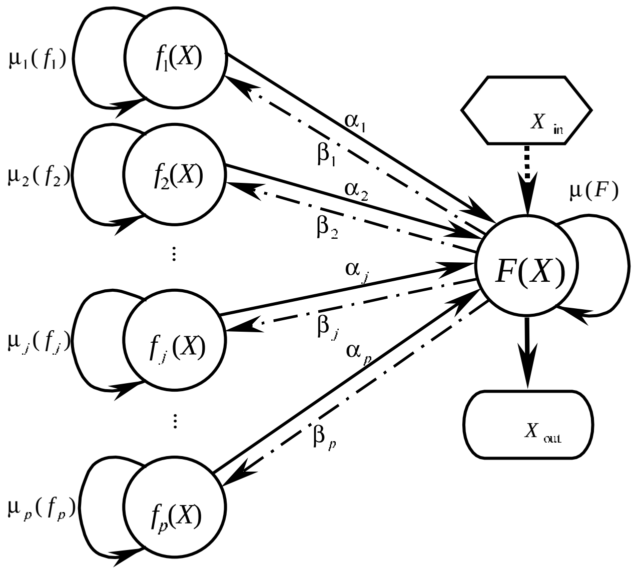


Fig. 1. Fuzzy directed graph for a multi-objective optimization problem.

The parameter β_j of the jth sub-objective used in Eqs (4) and (5) must be greater than 0. Its effects on the shape of the coordination function are illustrated in Fig. 2. When it is in the range between 0 and 1, we have the following observations.

When the value of the sub-objective function changes from being the least satisfactory to being the most satisfactory, the slope of the satisfaction decreases. Near the least satisfactory point, even if there is a little change in the sub-objective function value towards satisfaction, the coordination function will be improved greatly. Near the most satisfactory point, even if there is a great change in the sub-objective function value towards satisfaction, the value of the coordination function will change very little. Therefore, the smaller the β_j value is, the more easily for the jth sub-objective to become satisfactory. However, when β_j takes a value between 1 and ∞, the observations are just the opposite.

The exponential weights are usually determined by the designer's knowledge on the relative importance of the sub-objectives. The knowledge is usually fuzzy and can't lead to accurate β_j values. They can also be determined with a systematic approach called fuzzy comprehensive evaluation documented in [42–44].

3.3. Development of the coordination function of the overall objective

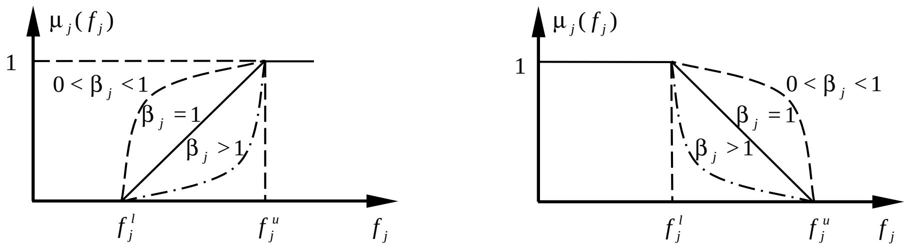
Once we have obtained the coordination functions for all sub-objectives as described in Section 3.2, we need to develop a method to find the coordination function of the overall objective function.

Given a solution X, we can find the coordination function values of all sub-objectives, that is, μ_j for $j = 1, 2, \ldots, p$. The degree of dissatisfaction of the jth sub-objective is measured by $1 - \mu_j$. The total degree of dissatisfaction of all p sub-objectives is equal to $\sum_{j=1}^{p} (1 - \mu_j)$. The relative degree of dissatisfaction of the jth sub-objective becomes $\frac{(1-\mu_j)}{\sum_{j=1}^{p} (1-\mu_j)}$. As we are in the process of finding a solution that would maximize a comprised satisfaction of all sub-objectives, we can use this relative degree of dissatisfaction as the weight in calculating the degree of satisfaction of the overall objective. In other words, we can calculate α_j using the following equation:

$$\alpha_j = \frac{(1 - \mu_j)}{\sum_{j=1}^{p} (1 - \mu_j)},\ j = 1, 2, \ldots, p \tag{6}$$

With α_j calculated with Eq. (6), we find the following expression of the coordination function of the overall objective:

$$\mu(F) = \sum_{j=1}^{p} \alpha_j \mu_j = \frac{\sum_{j=1}^{p} (1 - \mu_j)\, \mu_j}{\sum_{j=1}^{p} (1 - \mu_j)} \tag{7}$$



(a) Monotonically increasing function (b) Monotonically decreasing function

Fig. 2. Effects of β_j on the shape of the coordination function.

At any solution, if $\mu_j = 1$, that is, the jth sub-objective is totally satisfied, then this sub-objective has no effect on the comprehensive decision-making process any more. This sub-objective can then be removed from this comprehensive decision making process.

When the μ_j $(1 \leqslant j \leqslant p)$ values do not reach their maximum value 1 simultaneously, then there exists a point X^{**} and the kth sub-objective which satisfy the following equations:

$$\frac{\partial\mu(F(X^{**}))}{\partial(\mu_k(f_k(X^{**})))} = \left[\sum_{j=1}^{p}(1-\mu_j(f_j(X^{**})))(1-2\mu_k(f_k(X^{**})) + \mu_j(f_j(X^{**}))\right] \Bigg/ \left\{\left[\sum_{j=1}^{p}(1-\mu_j(f_j(X^{**})))\right]^2\right\} = 0 \tag{8}$$

$$\frac{\partial^2\mu(F(X^{**}))}{\partial(\mu_k(f_k(X^{**})))^2} = -4\cdot \frac{\sum\limits_{j=1,j\neq k}^{p}(1-\mu_j(f_j(X^{**})))\sum\limits_{l\geqslant j,l\neq k}^{p}(1-\mu_l(f_l(X^{**}))}{\left[\sum\limits_{j=1}^{p}(1-\mu_j(f_j(X^{**})))\right]^3} < 0 \tag{9}$$

Based on Eqs (8) and (9), once the values of the coordination function of all other sub-objectives are specified, there exists an optimal value of the coordination function of the kth sub-objective at which the coordination function of the overall objective attains its maximum. We take a multi-objective optimization problem with two sub-objectives as an example. When the value of one sub-objective function is specified (correspondingly its coordination function value is specified), there is an optimal value of the other sub-objective at which the overall objective function has the highest satisfaction value, as shown in Fig. 3.

4. Hypothetical case example

A system with five series sub-systems is considered. The relationship between the cost C_i and reliability R_i of the ith sub-system can be represented as

$$C_i(R_i) = a_i \ln(1/(1-R_i)) + b_i \quad (i = 1, 2, \ldots, 5) \tag{10}$$

where a_i and b_i are the reliability cost coefficient of the ith sub-system. For illustrative purposes, we use the following values of the parameters:

$$a_1 = 24,\ a_2 = 8,\ a_3 = 8.75,\ a_4 = 7.14,\ a_5 = 3.33$$

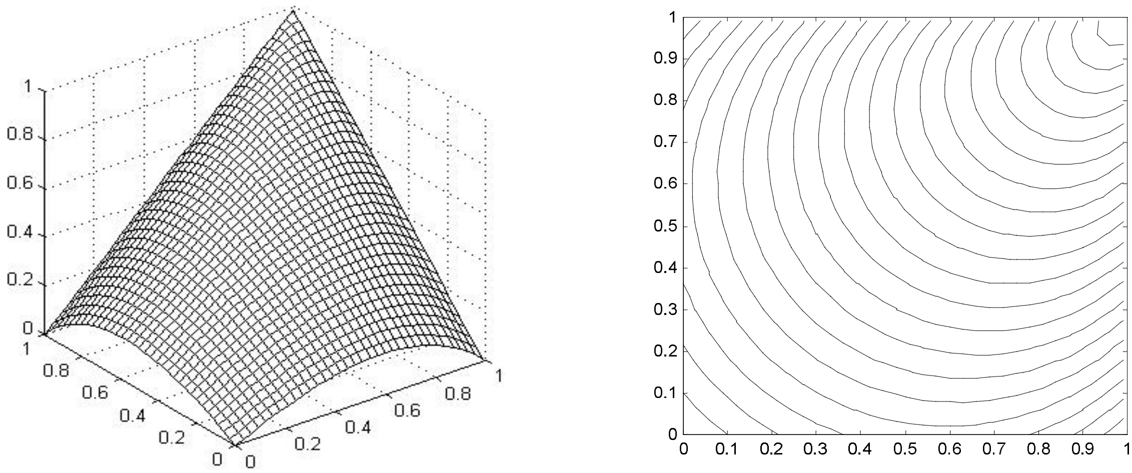
$$b_1 = 120,\ b_2 = 80,\ b_3 = 70,\ b_4 = 50,\ b_5 = 30$$

These parameters are in generic units. In addition, we are also given requirements that the cost of the system C_S should be between 500 and 600 and the system reliability R_S should be above 0.90. We need to determine the sub-systems' reliabilities $R = [R_1, R_2, \cdots, R_5]^T$ to maximize R_S and minimize C_S.

The multi-objective optimization model of the problem is expressed as

$$\text{find } R = [R_1, R_2, \cdots, R_5]^T \tag{11}$$

$$\text{to maximize } R_S\,(R) = \prod_{i=1}^{5} R_i \tag{12}$$



(a) mesh grid (b) contour line

Fig. 3. Illustrative diagram of coordination method.

and minimize

$$C_S = \sum_{i=1}^{5} \{a_i \ln[1/\,(1 - R_i)] + b_i\} \tag{13}$$

The coordination functions of the reliability and the cost of the system are expressed, respectively, as

$$\mu_1(R_S) = \begin{cases} 1, & R_S = 1 \\ ((R_S - 0.90)/0.1)^{\beta_R}, & 0.9 < R_S < 1 \\ 0, & R_S \leqslant 0.90 \end{cases} \tag{14}$$

$$\mu_2(C_S) = \begin{cases} 1, & C_S \leqslant 500 \\ ((600 - C_S)/100)^{\beta_C}, & 500 < C_S < 600 \\ 0, & C_S \geqslant 600 \end{cases} \tag{15}$$

The comprehensive coordination function considering both sub-objectives can be defined as

$$\mu(F(R, C)) = \alpha_1 \mu_1(R_S) + \alpha_2 \mu_2(C_S), \tag{16}$$

where

$$\alpha_1 = \frac{1 - \mu_1(R_S)}{2 - \mu_1(R_S) - \mu_2(C_S)}, \quad \alpha_2 = \frac{1 - \mu_2(R_S)}{2 - \mu_1(R_S) - \mu_2(C_S)}. \tag{17}$$

These expressions in Eq. (17) are obtained from Eq. (6).

The exponential weights β_R and β_C of the reliability and the cost of the system can be set at different values. The results obtained using an unconstrained optimization method [45] are shown in Table 1.

From the results given in Table 1, we can see the larger the exponential weight β_R of system reliability is, the larger the reliability R_S and the cost C_S will be, the larger the satisfaction degree in reliability will be, and the smaller the satisfaction degree in cost will be. When the exponential weight β_C of system cost increases, system reliability R_S decreases, and system cost C_S decreases. Then the satisfactory degree in reliability decreases, and the satisfactory degree in cost increases. When β_R and β_C are equal, the satisfactory degree in each sub-objective will be the lowest, the unsatisfactory degree in each sub-objective is the highest, and the comprehensive coordination ability of decision maker is the weakest. With the gradually increasing of the difference of the two exponential weights, the satisfactory degree of each sub-objective becomes larger, the total unsatisfactory degree becomes smaller, and the comprehensive coordination ability of decision maker becomes stronger, until the single-objective optimization is reached.

5. Conclusion

A coordination method for fuzzy multi-objective optimization is developed. It incorporates not only fuzzy factors in the decision-making process but also the designer's experience and knowledge. The optimization results of the numerical example illustrate that when

Table 1
Optimization results with different exponential weight values of reliability and cost

β_R	β_C	R_1	R_2	R_3	R_4	R_5	R_S	C_S	μ_1	μ_2	$\mu^{(F(R,C))}$
0.01	1	0.9499	0.9843	0.9829	0.9861	0.9934	0.9002	537.9	0.9416	0.6207	0.6635
0.1	1	0.9547	0.9844	0.9830	0.9861	0.9934	0.9050	540.5	0.7408	0.5955	0.6522
0.5	1	0.9655	0.9882	0.9871	0.9895	0.9951	0.9273	554.6	0.7231	0.4537	0.4860
0.8	1	0.9696	0.9897	0.9887	0.9908	0.9957	0.9360	561.3	0.4425	0.3871	0.4131
1	1	0.9715	0.9903	0.9894	0.9913	0.9959	0.9397	564.4	0.3973	0.3556	0.3758
1	0.8	0.9731	0.9909	0.9900	0.9918	0.9962	0.9432	567.5	0.4318	0.4067	0.4190
1	0.5	0.9764	0.9920	0.9913	0.9929	0.9967	0.9502	574.4	0.5020	0.7477	0.5039
1	0.1	0.9839	0.9946	0.9941	0.9952	0.9977	0.9659	594.1	0.6592	0.7531	0.6986
1	0.01	0.9849	0.9954	0.9950	0.9956	0.9980	0.9692	599.5	0.6923	0.9487	0.7290
1.5	1	0.9745	0.9914	0.9906	0.9923	0.9964	0.9463	570.4	0.3145	0.2957	0.3050
1	1.5	0.9690	0.9894	0.9885	0.9906	0.9956	0.9347	560.2	0.3466	0.2510	0.2956

the exponential weight of system reliability is equal to that of system cost, comprehensive coordination ability of decision maker is weakest. When the difference of the two exponential weights increases, the comprehensive coordination ability of decision maker is also strengthened until the single-objective optimization is reached.

The exponential weights of sub-objectives can be positive number, which is convenient to use in comparing and judging the complex relationship among sub-objectives. The range of an exponential weight should be determined based on specific problems. If the exponential weight is much larger than one, the slope of the coordination function will be very large near the most satisfactory point, and the ideal satisfactory solution will probably not be obtained. Therefore, the exponential weight is suggested to be less than one.

Acknowledgements

This research was partially supported by the National Excellent Doctoral Dissertation Special Foundation of China under the contract number 200232.

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