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ORIGINAL ARTICLE

Hong-Zhong Huang · Ping Wang · Ming J. Zuo
Weidong Wu · Chunsheng Liu

A fuzzy set based solution method for multiobjective optimal design problem of mechanical and structural systems using functional-link net

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Abstract The principle of solving multiobjective optimization problems with fuzzy sets theory is studied. Membership function is the key to introduce the fuzzy sets theory to multiobjective optimization. However, it is difficult to determine membership functions in engineering applications. On the basis of rapid quadratic optimization in the learning of weights, simplification in hardware as well as in computational procedures of functional-link net, discrete membership functions are used as sample training data. When the network converges, the continuous membership functions implemented with the network. Membership functions based on functional-link net have been used in multiobjective optimization. An example is given to illustrate the method.

Keywords Multiobjective optimization · Fuzzy sets · Membership function · Functional-link net · Neural network

H.-Z. Huang (✉)
School of Mechatronics Engineering,
University of Electronic Science and Technology of China,
Chengdu 610054, China
E-mail: hzhuang@uestc.edu.cn

W. Wu · C. Liu
School of Mechanical Engineering,
Heilongjiang Institute of Science and Technology,
Harbin 150027, China
E-mail: wu-weidong@163.com
E-mail: liu_chunsheng@163.com

P. Wang
School of Automation,
Chongqing University of Posts and Telecommunication,
Chongqing 400065, China
E-mail: wangping@cqupt.edu.cn

M. J. Zuo
Department of Mechanical Engineering, University of Alberta,
Edmonton, AB, Canada T6G 2G8
E-mail: ming.zuo@ualberta.ca

1 Introduction

More than 30 years have passed since the introduction of the concept of fuzzy optimization by Bellman and Zadeh [1]. Many new techniques in optimization such as fuzzy mathematical programming, fuzzy dynamic programming, and fuzzy multiobjective optimization have been developed which have enabled a higher qualitative level decision-making under uncertainty. Zimmermann [2, 3] reports some fundamental research results in this area. Luhandjula [4] provides a good state-of-the-art review of fuzzy optimization theory and applications. In the area of fuzzy multiobjective optimization, Rao [5] reports a formulation for fuzzy optimization of engineering systems involving multiple objectives. Dhingra and Rao [6] propose a cooperative fuzzy game theoretic approach to multiple objective design optimization. Rao and Chen [7] propose a novel hybrid method for solving general multiobjective optimization problems in which the objectives and the constraints are partly crisp and partly fuzzy. Huang [8] proposes a fuzzy multiobjective optimization decision-making method, which can be used for the optimization decision-making on two or more objectives of system reliability. Huang et al. [9] propose a coordination method for fuzzy multiobjective optimization of system reliability. Though hundreds of papers have been published in fuzzy optimization, many problems of theoretical as well as empirical types remain to be solved. It is expected that this field of research will remain fertile—in methodology as well as in applications—in the decades to come.

The latest development in the area of intelligent techniques is a combination of fuzzy sets theory with neural networks and genetic algorithms [10]. Pedrycz [11] gives an overview of the developments in the area of fuzzy neural networks. Sakawa and Sawada [12] employ neural computations to solve multiobjective 0–1 integer programming problems, an approach that is considered to be the most up-to-date method for solving an optimization problem with a special structure. Park and Pao

[13] use fuzzy rules to train a functional-link net in control path planning.

Development of the membership function is the key in using the fuzzy sets theory for solving multiobjective optimization problems. In engineering applications, it remains to be difficult to determine the membership functions of many fuzzy entities. In the present work, we utilize rapid quadratic optimization in the learning of weights, simplification in hardware as well as in computational procedures of functional-link net [14, 15] and use discrete membership functions as sample training data to train the neural network. The membership functions obtained from the functional-link net are then used in fuzzy multiobjective optimization.

The paper is organized as follows. The fuzzy model and computational procedure of multiobjective optimization is discussed in Sect. 2. In Sect. 3, the functional-link net architecture and learning algorithm is described. Our proposed method for multiobjective fuzzy optimization based on functional-link net is presented in Sect. 4 and an example is used to illustrate the effectiveness of the proposed method in Sect. 5. Conclusions are given in Sect. 6.

2 Fuzzy model and computational procedure of multiobjective optimization

Consider the following multiobjective optimization problem

$$\left.\begin{array}{l}\text{Find}\\ \mathbf{X}=[x_1,x_2,\ldots,x_n]^{\mathrm{T}}\\ \text{which minimizes}\\ F(\mathbf{X})=[F_1(\mathbf{X}),F_2(\mathbf{X}),\ldots,F_i(\mathbf{X}),\ldots,F_m(\mathbf{X})]^{\mathrm{T}}\\ \text{subject to}\\ g_j(\mathbf{X})\le 0; j=1,2,\ldots,p\\ h_k(\mathbf{X})=0; k=1,2,\ldots,q\end{array}\right\} \tag{1}$$

Because the m objectives of the problem may compete with one another, it is often impossible to minimize all these objectives simultaneously. As a result, a membership function is needed to indicate the degree of meeting the requirement of each objective. We propose the following computational procedure to find the optimal solution of the multiobjective optimization problem using fuzzy sets theory:

1. Find the minimum and maximum possible values of each individual objective function by solving the following optimization problems one by one:

$$\left.\begin{array}{l}Find\\ \mathbf{X}=[x_1,x_2,\ldots,x_n]^{\mathrm{T}}\\ \text{which minimizes}\\ F_i(\mathbf{X}); i=1,2,\ldots,m\\ \text{subject to}\\ g_j(\mathbf{X})\le 0; j=1,2,\ldots,p\\ h_k(\mathbf{X})=0; k=1,2,\ldots,q\end{array}\right\} \tag{2}$$

and

$$\left.\begin{array}{l}Find\\ \mathbf{X}=[x_1,x_2,\ldots,x_n]^{\mathrm{T}}\\ \text{which maximizes}\\ F_i(\mathbf{X}); i=1,2,\ldots,m\\ \text{subject to}\\ g_j(\mathbf{X})\le 0; j=1,2,\ldots,p\\ h_k(\mathbf{X})=0; k=1,2,\ldots,q\end{array}\right\} \tag{3}$$

Denote the minimum and maximum values of the ith objective function $F_i(\mathbf{X})$ by $F_i^{\min}$ and $F_i^{\max}$, respectively.

2. From the extreme values of the ith objective function $F_i(\mathbf{X})$, construct the membership function of this fuzzy objective function as follows:

$$\mu_{\tilde{F}_i}(\mathbf{X})=\left(\frac{F_i^{\max}-F_i(\mathbf{X})}{F_i^{\max}-F_i^{\min}}\right)^r \tag{4}$$

where r is a positive real number. Generally, we have [16]

$$r=1,\frac{1}{2},\frac{1}{3},\ldots \tag{5}$$

3. The measure of the optimization of all m objectives is expresses as

$$\tilde{D}=\bigcap_{i=1}^{m}\tilde{F}_i \tag{6}$$

The membership function of the fuzzy entity $\tilde{D}$ is given by

$$\mu_{\tilde{D}}(\mathbf{X})=\bigwedge_{i=1}^{m}\mu_{\tilde{F}_i}(\mathbf{X}) \tag{7}$$

where $\wedge$ denotes the "and" operation.

4. Find the optimal solution $\mathbf{X}$ of the following multiobjective optimization problem

$$\mu_{\tilde{D}}(\mathbf{X}^*)=\max\mu_{\tilde{D}}(\mathbf{X})=\max\bigwedge_{i=1}^{m}\mu_{\tilde{F}_i}(\mathbf{X}) \tag{8}$$

3 Functional-link net architecture and learning algorithm

Pao [14] provides a systematic description of a functional-link net. Klassen et al. [17] provide a further analysis of the characteristics of this type of neural network. Due to the strong nonlinear mapping and interpolation capability of the functional-link net, its learning rate is much larger than the multilayer perceptron. Therefore, it has been applied in pattern recognition, seismic trace editing, real-time optimal control, etc.

3.1 Topological characteristics

The topology of a functional-link net consists of a functional link and a single-layered flat network (Fig. 1).

The functional link alone acts on each input data point and expands the input pattern with the same function. This expansion is a kind of nonlinear transformation. The expansion functions are linearly independent. The expansion function expands each input pattern into the multidimensional vector space. In other words, the role of the functional link is to map the input pattern to a space of larger dimensions. Descriptions and classifications of the patterns are then conducted in this higher-dimensional space. It has been shown that with enough orthonormal basis functions for extending and eliminating redundant patterns in pre-treatment, the function expansion model can always find a flat network, i.e., a single-layered perceptron with no hidden layers [15].

Suppose that the input pattern is $x_s (s = 1, 2, ..., l)$. Three groups of expansion functions are given as follows:

1.

$$\phi_1(x_s) = x_s; \quad s = 1, 2, \ldots, l$$
$$\phi_2(x_s) = 1/x_s; \quad s = 1, 2, \ldots, l$$

2.

$$\phi_1(x_s) = x_s; \quad s = 1, 2, \ldots, l$$
$$\phi_2(x_s) = x_s x_{s+e}; \quad s = 1, 2, \ldots, l; \; e = 1, 2, \ldots, l - s$$

3.

$$\phi_1(x_s) = x_s; \quad s = 1, 2, \ldots, l$$
$$\phi_2(x_s) = \sin(x_s/x_{\max}); \quad s = 1, 2, \ldots, l$$
$$\phi_3(x_s) = \cos(x_s/x_{\max}); \quad s = 1, 2, \ldots, l$$

Generally speaking, any set of functions of linear independence can act as the group of expansion functions.

3.2 Learning scheme

Similar to a multilayer perceptron, the learning process of the functional-link net involves two phases: the feed-forward process and the error back-propagation process.

As shown in Fig. 1, the input pattern is $\mathbf{X} = [x_1, x_2, ..., x_l]^T$. There are k linearly independent expansion functions, $\phi_c(\cdot)(c = 1, 2, ..., k)$. The input data of l dimensions is expanded into an $l \times k$-dimensional vector with the expansion functions.

Let the input of the single-layered flat network be denoted by z_d ($d = 1,2, ..., l \times k$) and its output by y_v ($v = 1, 2, ..., h$). The relationship between the input and the output of the network is

$$z_d = \phi_c(x_s);$$
$$d = 1, 2, \ldots, l \times k; \; c = 1, 2, \ldots, k; \; s = 1, 2, \ldots, l \quad (9)$$

$$y_v = f\left(\sum_{d=1}^{l\times k} w_{dv} z_d - \theta_v\right); \quad v = 1, 2, \ldots, h \quad (10)$$

where w_{dv} is the connecting weight from the dth input neuron to the vth output neuron, θ_v is the threshold of the vth output neuron, and f is usually a sigmoid function given by

$$f(u) = 1/(1 + \exp(-\lambda u)) \quad (11)$$

where λ is the Sigmoidal gain. Once an output is obtained from the network, it is compared with the target output for this set of input. Any error that exists is used in the back-propagation process to modify the connection weights of the network.

Let $\hat{y}_v$ and y_v denote the desired (or target) output and the actual output of the vth output neuron, respectively. Then, the error function can be defined as

$$\varepsilon = \sum_{v=1}^{h} (\hat{y}_v - y_v)^2/2 \quad (12)$$

During the learning process, if $\varepsilon < \varepsilon_0$($\varepsilon_0$ is a small positive real number, e.g., 0.001), then the network converges and the learning is completed. If $\varepsilon \geq \varepsilon_0$, the error is then fed backwards through the network. The weights and the threshold updates are chosen to be as follows:

$$\left.\begin{array}{l} w_{dv}(t+1) = w_{dv}(t) + \alpha b_v z_d \\ \theta_v(t+1) = \theta_v(t) + \alpha b_v \\ b_v = y_v(1 - y_v)(\hat{y}_v - y_v) \end{array}\right\} \quad (13)$$

where t represents the iteration number and α is the learning rate which is a constant in the range of (0,1).

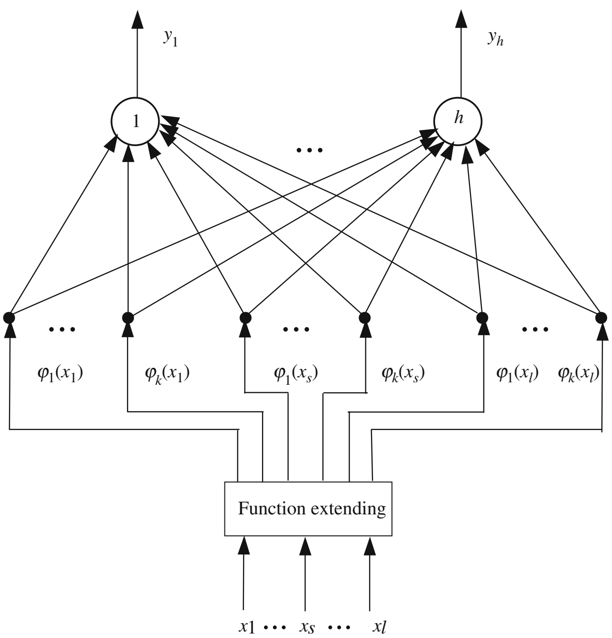

Fig. 1 Topology of functional-link net

4 A new method for multiobjective fuzzy optimization based on functional-link net

The key to solving a multiobjective optimization problem with fuzzy set theory lies in determining the proper membership functions. This is because different membership functions result in different optimal solutions. Explicit membership functions are usually used in solving multiobjective fuzzy optimization problems [5, 7–12, 18, 19]. However, it is difficult to determine proper explicit membership functions in engineering applications. In this paper, we propose a novel method for solving multiobjective optimization problems using the functional-link nets to map membership functions. With this approach, a multiobjective optimization problem is transformed into a single-objective optimization problem. The detailed procedure is as follows:

1. Choose network architecture and obtain sample training data.
2. Train the network and establish the network representation of membership functions.
3. Find an initial solution with $\mathbf{X}_0$ and ω_0.
4. Calculate $F_i(\mathbf{X}_0)$ for $i = 1, 2, ..., m$.
5. Use the trained network to determine the membership functions $\mu_{\tilde{F}_i}(\mathbf{X}_0)$ corresponding to $F_i(\mathbf{X}_0)$ ($i=1, 2, ..., m$) and calculate

$$\omega_i = \frac{\mu_{\tilde{F}_i}(\mathbf{X}_0)}{\eta_i}; \quad i = 1,\ 2,\ \ldots,\ m \tag{14}$$

$$\omega = \min(\omega_1, \omega_2, \ldots, \omega_m) \tag{15}$$

where η_i is a weight of ith objective.

6. If$(\omega - \omega_0) \leq \varepsilon_p$($\varepsilon_p$ is a certain given precision), then optimization process is complete and the optimal solution has been obtained. Otherwise go to the next step.
7. Suppose that ω_r corresponding to $F_r(\mathbf{X}_0)$ is the smallest. Then, let $F_r(\mathbf{X})$ be the objective function and solve the following single-objective optimization problem

$$\left.\begin{array}{l}\text{Find}\\ \mathbf{X} = [x_1, x_2 \ldots, x_n]^{\mathrm{T}}\\ \text{which minimizes}\\ F_r(\mathbf{X})\\ \text{subject to}\\ g_j(\mathbf{X}) \leq 0; j = 1, 2, \ldots, p\\ h_k(\mathbf{X}) = 0; k = 1, 2, \ldots, q\\ F_i(\mathbf{X}) \leq (1+\xi)F_i(\mathbf{X}_0); i = 1, 2, \ldots, m; i \neq r\end{array}\right\} \tag{16}$$

where ξ is a slack coefficient. After obtaining the optimal solution to this optimization problem, denoted by $\mathbf{X}_0$ again, go to Step 4.

5 Example

Consider a three bar truss design shown in Fig. 2. The material density is taken to be $\rho = 100$ kN/m^3. The allowable stress is set to be $[\sigma] = 200$ MPa. The cross-sectional area of each component has to be within the interval $[1 \times 10^{-5}$ m^2, 5×10^{-4} m$^2]$. In designing this truss, we aim to minimize the weight of the structure and minimize the vertical deflection of the loaded joint.

The two-objective optimization problem can be stated as follows:

$$\left.\begin{array}{l}\text{Find}\\ \mathbf{X} = [x_1, x_2]^{\mathrm{T}}\\ \text{which minimizes}\\ F_1(\mathbf{X}) = 2\sqrt{2}x_1 + x_2\\ F_2(\mathbf{X}) = 20/(x_1 + 2\sqrt{2}x_2)\\ \text{subject to}\\ 20(\sqrt{2}x_1 + x_2)/(\sqrt{2}x_1^2 + 2x_1x_2) - 20 \leq 0\\ 20\sqrt{2}x_1/(\sqrt{2}x_1^2 + 2x_1x_2) - 20 \leq 0\\ 20x_2/(\sqrt{2}x_1^2 + 2x_1x_2) - 15 \leq 0\\ 1 \times 10^{-5} \leq x_1 \leq 5 \times 10^{-4}\\ 1 \times 10^{-5} \leq x_2 \leq 5 \times 10^{-4}\end{array}\right\}$$

where x_1 and x_2 are the cross sectional areas of the bars, as indicated in Fig. 2.

We follow the proposed method outlined in Sect. 4 to solve this multiobjective optimization problem.

1. Choose a network architecture and train the network with training data. Since there are two objectives in the optimization problem, we need to train two functional-link nets. As shown in Fig. 3, let the value of the objective function be the input and the degree of satisfaction of the designer with the value of the objective function be the output. The data given in Table 1 is used to train the network.
2. Select an initial solution denoted by $\mathbf{X}_0 = [1.0, 1.0]^{\mathrm{T}}$.
3. Calculate the values $F_1(\mathbf{X}_0)$ and $F_2(\mathbf{X}_0)$ of the objective functions.
4. Use the trained network to find the membership function values $\mu_{\tilde{F}_1}(\mathbf{X}_0)$ and $\mu_{\tilde{F}_2}(\mathbf{X}_0)$ corresponding to $F_1(\mathbf{X}_0)$ and $F_2(\mathbf{X}_0)$. Then we calculate ω_1, ω_2, and $\omega = \min\{\omega_1, \omega_2\}$.
5. If$|\omega - \omega_0| \leq \varepsilon_p$ where $\omega_0 = 1.6$ and $\varepsilon_p = 0.01$, then output the optimal solution. Otherwise, go to the next step.

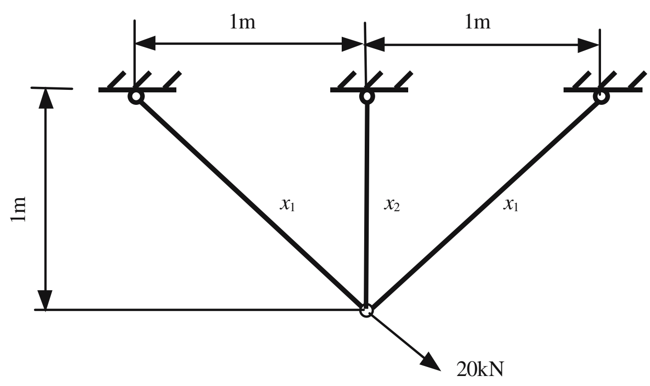

Fig. 2 Three-bar truss

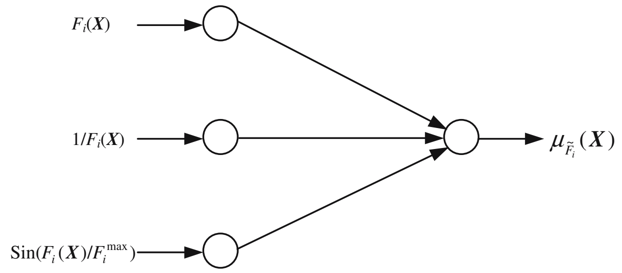

Fig. 3 Functional-link net mapping the membership degree of objective function

Table 1 Training data

Pattern number	Input $F_1(\mathbf{X})$	Output $\mu_{\tilde{F}_1}(\mathbf{X})$	Input $F_2(\mathbf{X})$	Output $\mu_{\tilde{F}_2}(\mathbf{X})$
1	2.2	1	1.4	1
2	4.1	0.9	3.7	0.9
3	4.9	0.8	4.7	0.8
4	5.6	0.7	5.6	0.7
5	6.3	0.6	6.5	0.6
6	7.0	0.5	7.3	0.5
7	7.7	0.4	8.1	0.4
8	8.5	0.3	9.1	0.3
9	9.5	0.2	10.4	0.2
10	10	0.1	12	0.1

6. If $\omega_1 \geq \omega_2$, then optimize the sub-problem H_1. If not, then optimize the sub-problem H_2.

H_1:

$$\left.\begin{array}{l}\text{find}\\ \mathbf{X}=[x_1,x_2]^T\\ \text{which minimizes}\\ F_2(\mathbf{X})=20/(x_1+2\sqrt{2}x_2)\\ \text{subject to}\\ 20\left(\sqrt{2}x_1+x_2\right)/\left(\sqrt{2}x_1^2+2x_1x_2\right)-20\leq 0\\ 20\sqrt{2}x_1/\left(\sqrt{2}x_1^2+2x_1x_2\right)-20\leq 0\\ 20x_2/\left(\sqrt{2}x_1^2+2x_1x_2\right)-15\leq 0\\ 1\times 10^{-5}\leq x_1\leq 5\times 10^{-4}\\ 1\times 10^{-5}\leq x_2\leq 5\times 10^{-4}\\ F_1(\mathbf{X})\leq(1+\xi)F_1(\mathbf{X}_0)\end{array}\right\}$$

H_2:

$$\left.\begin{array}{l}\text{find}\\ \mathbf{X}=[x_1,x_2]^T\\ \text{which minimizes}\\ F_1(\mathbf{X})=2\sqrt{2}x_1+x_2\\ \text{subject to}\\ 20\left(\sqrt{2}x_1+x_2\right)/\left(\sqrt{2}x_1^2+2x_1x_2\right)-20\leq 0\\ 20\sqrt{2}x_1/\left(\sqrt{2}x_1^2+2x_1x_2\right)-20\leq 0\\ 20x_2/\left(\sqrt{2}x_1^2+2x_1x_2\right)-15\leq 0\\ 1\times 10^{-5}\leq x_1\leq 5\times 10^{-4}\\ 1\times 10^{-5}\leq x_2\leq 5\times 10^{-4}\\ F_2(\mathbf{X})\leq(1+\xi)F_2(\mathbf{X}_0)\end{array}\right\}$$

After finding the optimal solution of the appropriate sub-problem, go to Step 3.

Following this procedure, we have obtained the following optimal solution:

$$\mathbf{X}_n^* = \left[1.32\times 10^{-4}, 1.01\times 10^{-4}\right]^{\mathrm{T}},\quad F_1\left(\mathbf{X}_n^*\right)=3.73$$
$$F_2\left(\mathbf{X}_n^*\right)=4.79.$$

6 Conclusions

The key to solving multiobjective optimization problems with fuzzy sets theory is to determine the membership functions. In this paper, we have proposed a method to use the functional-link nets to represent the membership functions of objectives. These membership functions are then used in solving the multiobjective optimization problem. Because a functional-link net enhances the representation of input data by extending input vector, the network architecture is simplified. Moreover, a single-layer functional-link net is competent to perform what the multi-layer perceptron can do.

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