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ORIGINAL PAPER

Multidisciplinary collaborative optimization using fuzzy satisfaction degree and fuzzy sufficiency degree model

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Abstract Collaborative optimization (CO) is a bi-level multidisciplinary design optimization (MDO) method for large-scale and distributed-analysis engineering design problems. Its architecture consists of optimization at both the system-level and autonomous discipline levels. The system-level optimization maintains the compatibility among coupled subsystems. In many engineering design applications, there are uncertainties associated with optimization models. These will cause the design objective and constraints, such as weight, price and volume, and their boundaries, to be fuzzy sets. In addition the multiple design objectives are generally not independent of each other, that makes the decision-making become complicated in the presence of conflicting objectives. The above factors considerably increase the modeling and computational difficulties in CO. To relieve the aforementioned difficulties, this paper proposes a new method that uses a fuzzy satisfaction degree model and a fuzzy sufficiency degree model in optimization at both the system level and the discipline level. In addition, two fuzzy multi-objective collaborative optimization strategies (Max–Min and α-cut method) are introduced. The former constructs the sufficiency degree for constraints and the satisfaction degree for design objectives in each discipline respectively, and adopts the Weighted Max–Min method to maximize an aggregation of them. The acceptable level is set up as the shared design variable between disciplines, and is maximized at the system level. In the second strategy, the decision-making space of the constraints is distributed in each discipline independently through the allocation of the levels of α. At the system level, the overall satisfaction degree for all disciplines is finally maximized. The illustrative mathematical example and engineering design problem are provided to demonstrate the feasibility of the proposed methods.

Keywords Multidisciplinary design optimization · Collaborative optimization · Fuzzy satisfaction degree · Fuzzy sufficiency degree

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1 Introduction

Large scale problems or complex systems usually involve multiple disciplines and coupling factors. When optimization is conducted serially, low efficiency and non-optimal solutions may be resulted in. Multidisciplinary design optimization (MDO) is a methodology based on the decomposition of the design problem of a system into problems of lower dimensions, which can be distributed to groups of engineers and experts in different disciplines. Collaborative optimization (CO) (Kroo et al. 1994) was developed to capture the multidisciplinary characteristics of engineering design. CO decomposes the design problem of a system into two levels, i.e. the system level and the disciplinary level in parallel. Each disciplinary subspace design is to satisfy its local constraints while minimizing the disciplinary objective discrepancy function. At the system level, target values are determined for the design variable used to keep consistency among disciplinary subspace designs while minimizing the overall system objective function. In the application of CO (Braun et al. 1995) employed it for the launch-vehicle design, and Sobieski and Kroo (1995) for the aircraft configuration.

Tappeta and Renaund (1997) presented Multi-objective collaborative optimization (MOCO), which was developed for comparative studies and tested three types of MOCO formulations for a modified aircraft sizing problem. CO has also been widely used in decision-making (Gu et al. 2002) and conceptual design (Balling and Rawlings 2000).

Collaborative optimization (CO) methods assume that all the design data are precisely known and the constraints delimit a well-defined set of feasible regions. However incompleteness and uncertainty of input information is often typical in many practical CO problems, especially in multi-objective CO. Such incompleteness and uncertainty is mainly caused by fuzzy performance criteria, fuzzy ideas of decision makers, as well as the presence of conflicting objectives (Balling and Wilkinson 1997; Huang et al. 2006a,b,c, 2005a; Huang and Li 2005; Huang et al. 2006d).

This paper proposes the use of a fuzzy satisfaction degree and a fuzzy sufficiency degree model in design models at the discipline level in order to construct the sufficiency degree for constraints and the satisfaction degree for objectives in each discipline. A typical CO architecture based on the model fuzzy satisfaction degree and the fuzzy sufficiency degree (FSSDCO) is developed. The two approaches developed are applied in two examples. The proposed strategies extend the work of Tappeta and Renaund (1997) to provide MOCO with the capability to handle fuzzy information. In the FSSDCO algorithm, every strategy follows the approach for CO suggested by Huang et al. (2004) and the formulation method of Tappeta and Renaund (1997). The first strategy utilizes the weighted Max–Min method [Fuzzy intersection by Bellman and Zadeh (1970)] to evaluate the sufficiency degree and the satisfaction degree, and employs the acceptable level as a shared design variable among coupled disciplines. This paper presents the FSSDCO based on an asymmetrical model using the α-cut method, and obtains a sequence of solutions using this model. A fuzzy CO problem is transformed into a series of satisfaction degree optimization problems in different sufficiency degree spaces, constructed by an optimum α-cut through the disciplinary analyses.

This paper is organized as follows. Section 2 discusses the fuzzy satisfaction degree and fuzzy sufficiency degree expression of decision-making problems as well as the process of mechanism type selection. Section 3 proposes the formulation of using the max-min method for the FSSDCO. Section 4 proposes the model based on the α-cut method for FSSDCO. An example is also used to illustrate the effectiveness of the proposed method. Finally, Section 5 is a summary of our conclusions.

2 Fuzzy satisfaction degree and fuzzy sufficiency degree

The conventional CO method assumes that all design variables are precisely known and the feasible region should be strictly satisfied. However, in applications some design variables or constraints often contain incomplete, imprecise, and vague (fuzzy) information, especially for a multi-objective problem (Balling and Wilkinson 1997; Huang et al. 2006b,c). Such imprecise information makes it difficult for CO to achieve the optimal solution. To avoid the aforementioned problems, the fuzzy satisfaction degree and fuzzy sufficiency degree models at the system level and discipline level have been proposed as an alternative to CO. Utilizing this style for modeling, the system-level objective becomes the acceptable levels or the system overall satisfaction degree, while disciplines adopt the inequalities of subspace satisfaction and the sufficiency degrees as their local constraints. In addition to the conventional analysis that aids the computation of local objectives or output variables coupled to other disciplines at the discipline level, the subspace analysis responsibility is extended to provide the sufficiency degree of local constraints and the satisfaction degree of design objectives, as illustrated in Fig. 1.

In Fig. 1 z^* is design variable at the system level, and it is the target which the discipline level will attempt to mach. d_i^* is the cumulative compatibility constraint returned from ith subspace optimization. $x_{\mathrm{ss}i}$ is the subspace design vector for ith discipline. y_{ij} is the coupling vector computed in discipline i and used in discipline j. $\mu_{\tilde{c}_{ij}}$ is the degree of sufficiency for the jth constraint function at ith discipline. $\mu_{\tilde{F}_i}$ is the degree of satisfaction for objective function at ith discipline.

This architecture of the FSSDCO is appealing because of its ability to distribute the model to decision-making or engineering design in different areas, with the capability to incorporate fuzzy or vague information. Firstly, the proposed particular mechanism of CO can efficiently solve an optimization problem with a distributed and parallel implementation. Secondly, the fuzzy satisfaction and sufficiency integrated approach is a significant enhancement of CO to deal with fuzzy information, which renders the optimum solutions to be acquired more easily and more acceptable.

2.1 Fuzzy satisfaction degree for objectives

In 1947, Nobel Prize winner Herbert A. Simon first introduced the satisfying criterion and proposed satisfying solutions in place of the traditional optimum ones under certain situations, which provided a new approach to solve an optimization problem (Simon 1996). Since then, the satisfaction degree theory has been extensively studied and applied. Takatsu presented the basic mathematical theory and characteristics of the latent satisfying decision criterion (Takatsu 1981). In engineering practice, Goodrich studied the theory of satisfying control, and applied it to some classic control problems (Goodrich 1990). Jin gives the satisfaction solution theory in neural computing (Jin 1992).

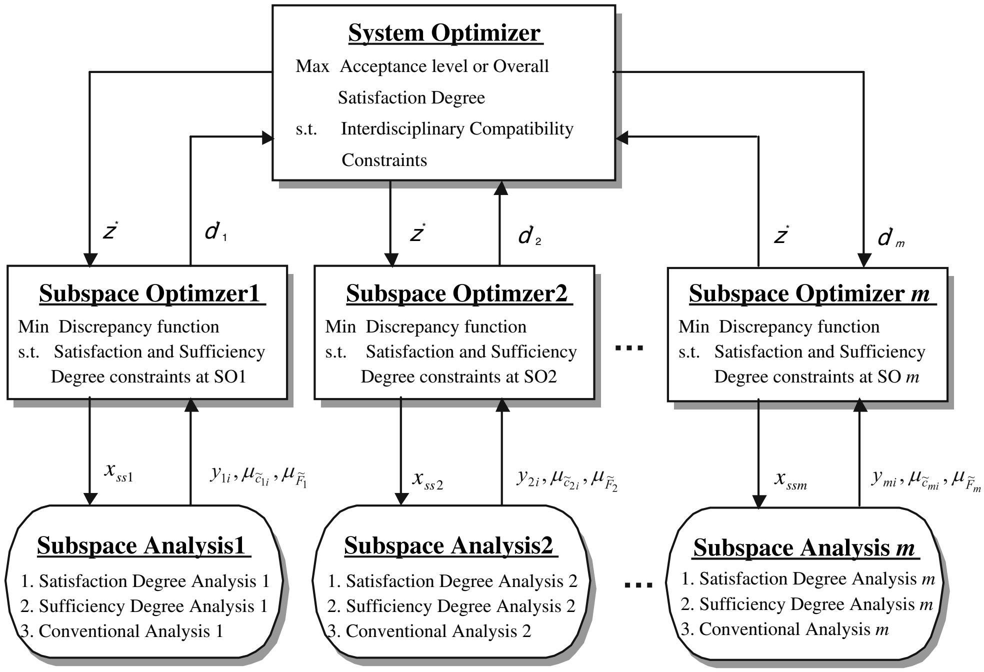

Fig. 1 The architecture of the FSSDCO

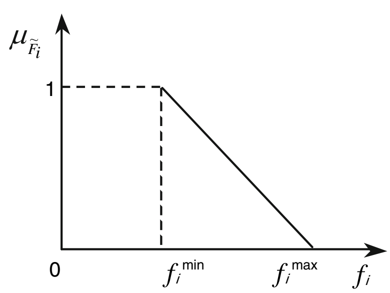

Fig. 2 Linear satisfaction degree function for ith objective

Definition 1 (Huang 1997; Wang 1992) The fuzzy satisfaction region of an objective is a fuzzy set which a decision maker determines according to the degree of satisfaction for different values of the objective. Suppose the fuzzy satisfaction region of the ith objective is denoted by $\tilde{F}_i$, then

$$\tilde{F}_i = \int \mu_{\tilde{F}_i}(f_i)/f_i \tag{1}$$

where $\mu_{\tilde{F}_i}(f_i)$ is the grade of membership of ith objective value in the fuzzy set $\tilde{F}_i$ and also called the satisfaction degree of the ith objective. It is denoted by α_i, namely,

$$\alpha_{\tilde{F}i} = \mu_{\tilde{F}_i}(f_i) \tag{2}$$

A proper function needs to be selected as the membership function of satisfaction degree of objectives according to their characteristics. For simplification a linear membership function of satisfaction degree of objectives is used, and the shape of $\tilde{F}_i$ is shown in Fig. 2.

The membership function $\mu_{\tilde{F}_i}(f_i)$ is defined as

$$\mu_{\tilde{F}_i}(f_i) = \begin{cases} 1, \ f_i \leqslant f_i^{\min} \\ \frac{f_i - f_i^{\max}}{f^{\min}{}_i - f_i^{\max}}, \ f_i^{\min} < f_i < f_i^{\max} \\ 0, \ f_i \geqslant f_i^{\max} \end{cases} \tag{3}$$

where $f_i^{\max}$, $f_i^{\min}$ are the upper and lower limits of the objective f_i, namely, the satisfaction interval of objectives for the decision maker. For a minimization problem, $f_i^{\min}$ is considered the most ideal value of the ith objective by the decision maker; that is to say if $f_i = f_i^{\max}$, then $\mu_{\tilde{F}_i}(f_i) = 0$, and at $f_i = f_i^{\min}$, $\mu_{\tilde{F}_i}(f_i) = 1$.

The satisfaction degree is an allowable interval of a membership function of an objective. It is difficult, especially for larger-scale design problems, to find the appropriate mathematical functions to express the satisfaction degree. However in many practical applications, acceptable extents may be determined before conducting design. For example if the engineering cost is selected as an objective function, the investor usually constrains the cost within an allowable limit. If the cost exceeds the upper limit, the design project cannot be accepted; that is, the satisfaction degree is 0. If the cost is less than the lower limit, the design project can be completely accepted; that is, the satisfaction degree is 1. Otherwise, the satisfaction degree is between 0 and 1.

2.2 Fuzzy sufficiency degree for constraints

In engineering optimization, constraints are limitations on design variables in terms of cost, geometry, and other

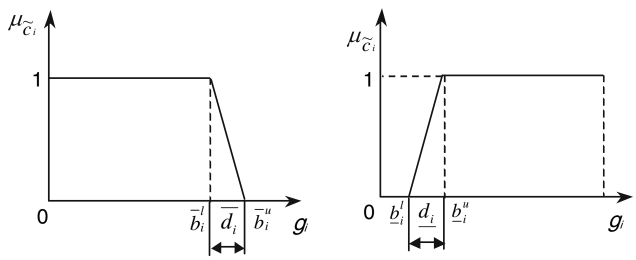

Fig. 3 Two linear sufficiency degree functions for constraints

conditions. These constraints likely contain fuzzy information. It is essential to formulate these constraints by fuzzy sets.

Definition 2 (Huang 1997; Wang 1992)
Similar to the definition for the satisfaction degree the fuzzy sufficiency region of constraints is also a fuzzy set, which represents the degree of a constraint being satisfied. Suppose that the fuzzy sufficiency region of the ith constraint is denoted by $\tilde{C}_i$, then

$$\tilde{C}_i = \int \mu_{\tilde{C}_i}(g_i)/g_i \tag{4}$$

The grade of membership of $\tilde{C}_i$ is called the sufficiency degree for the ith constraint function. It is denoted by

$$\alpha_{\tilde{C}i} = \mu_{\tilde{C}_i}(g_i) \tag{5}$$

For the membership functions of constraints, its shape may be selected according to constraint characteristics. For the inequality constraints, there are generally two forms, $g_i \leqslant b_i$ and $g_i \geqslant b_i$. The shapes of g_i for two types of expression are as shown in Fig. 3, and the function of sufficiency degree of constraints is defined in Eqs. (6) and (7).

$$\mu_{\tilde{c}i}(g_i) = \begin{cases} 1, 0 \leqslant g_i < \overline{b}_i^l \\ \frac{g_i - \overline{b}_i^u}{\overline{b}_i^l - \overline{b}_i^u}, \overline{b}_i^l \leqslant g_i < \overline{b}_i^u \\ 0, g_i \geqslant \overline{b}_i^u \end{cases} \tag{6}$$

$$\mu_{\tilde{c}i}(g_i) = \begin{cases} 0, g_i \leqslant \underline{b}_i^l \\ \frac{\underline{b}_i^l - g_i}{\underline{b}_i^l - \underline{b}_i^u}, \underline{b}_i^l < g_i \leqslant \underline{b}_i^u \\ 1, g_i > \underline{b}_i^u \end{cases} \tag{7}$$

$\overline{b}_{i,}^u \underline{b}_i^l$ are allowable upper and lower limits of the ith constraints respectively, and d_i is the length of the permissible deviation or tolerance which can be determined by decision makers.

2.3 The aggregation of fuzzy satisfaction degree and fuzzy sufficiency degree

According to the extension principle (Wang 1992; Zadeh 1975), fuzzy sets $\tilde{F}_i$ and $\tilde{C}_i$ are represented as $\tilde{\Phi}_F$ and $\tilde{\Phi}_C$ in the decision space. The grade of membership of decision vector $\mathbf{R}$ in $\tilde{\Phi}_F$ and $\tilde{\Phi}_C$ are respectively

$$\mu_{\tilde{\Phi}_F}(\mathbf{R}) = \bigcap_{i=1}^{n} \mu_{\tilde{F}_i}(f_i) = \bigcap_{i=1}^{n} \alpha_{\tilde{F}_i}(\mathbf{R}) \tag{8}$$

$$\mu_{\tilde{\Phi}_C}(\mathbf{R}) = \bigcap_{i=1}^{p} \mu_{\tilde{C}_i}(g_i) = \bigcap_{i=1}^{p} \alpha_{C_i}(\mathbf{R}) \tag{9}$$

The intersection of $\tilde{\Phi}_F$ and $\tilde{\Phi}_C$ is depicted by

$$\tilde{\Phi} = \tilde{\Phi}_F \cap \tilde{\Phi}_C \tag{10}$$

The fuzzy set $\tilde{\Phi}$ is called the acceptable region, and the grade of membership of decision vector $\mathbf{R}$ in $\tilde{\Phi}$ is called the acceptable degree denoted by

$$\begin{aligned}\alpha(\mathbf{R}) &= \mu_{\tilde{\Phi}}(\mathbf{R}) = T(\mu_{\tilde{\Phi}_F}(\mathbf{R}), \mu_{\tilde{\Phi}_C}(\mathbf{R})) \\ &= T(\alpha_{\tilde{F}_i}(\mathbf{R}), \quad \alpha_{\tilde{C}_i}(\mathbf{R}))\end{aligned} \tag{11}$$

Because different operators for the modulo arithmetic T in Eq. (11) create different results, here Zadeh's fuzzy operator (Zadeh 1975) is employed which is denoted by

$$\mu_{\tilde{\Phi}}(\mathbf{x}) = \left(\bigwedge_{i=1}^{n} \mu_{\tilde{F}_i}(\mathbf{x})\right) \wedge \left(\bigwedge_{i=1}^{p} \mu_{\tilde{C}_i}(\mathbf{x})\right) \tag{12}$$

3 The formulation of max–min method for FSSDCO

In 1970 Bellman and Zadeh provided the symmetric fuzzy mathematical programming model, named the Max–Min method, which can be described as follows:

Fuzzy objective set $\tilde{f}$
Fuzzy constraint set $\tilde{C}_j, \quad j = 1, 2, \ldots, J$
Find x^*, such that

$$\mu_{\tilde{D}}(x^*) = \max\left\{\mu_{\tilde{f}}(x) \wedge \left(\bigwedge_{j=1}^{J} \mu_{\tilde{C}_j}(x)\right)\right\} \tag{13}$$

The optimal decision is to select the best alternative from the satisfaction region in the fuzzy decision space which maximizes the membership function of the fuzzy decision. For simplification of the FSSDCO formulation, the acceptable level λ_i for ith discipline is formulated as the objective while the α-cut of level λ_i for the satisfaction degree and sufficiency degree at the ith discipline are adopted as the decision-making space for ith discipline. The model is given as follows:

$$\begin{aligned}&\text{Find } x \\ &\text{Max } \mu_{\tilde{\Phi}_i}(x) = \lambda_i \quad i = 1, 2, \ldots, m \\ &\text{s.t.} \quad \mu_{\tilde{C}_{ij}}(x) \geqslant \lambda_i \quad j = 1, 2, \ldots, p \\ &\qquad \mu_{\tilde{F}_{ik}}(x) \geqslant \lambda_i \quad k = 1, 2, \ldots, n \\ &\qquad 0 \leqslant \lambda_i \leqslant 1\end{aligned} \tag{14}$$

where n, p and m are the number of objectives, constraints at the ith discipline, and the sum of disciplines, respectively.

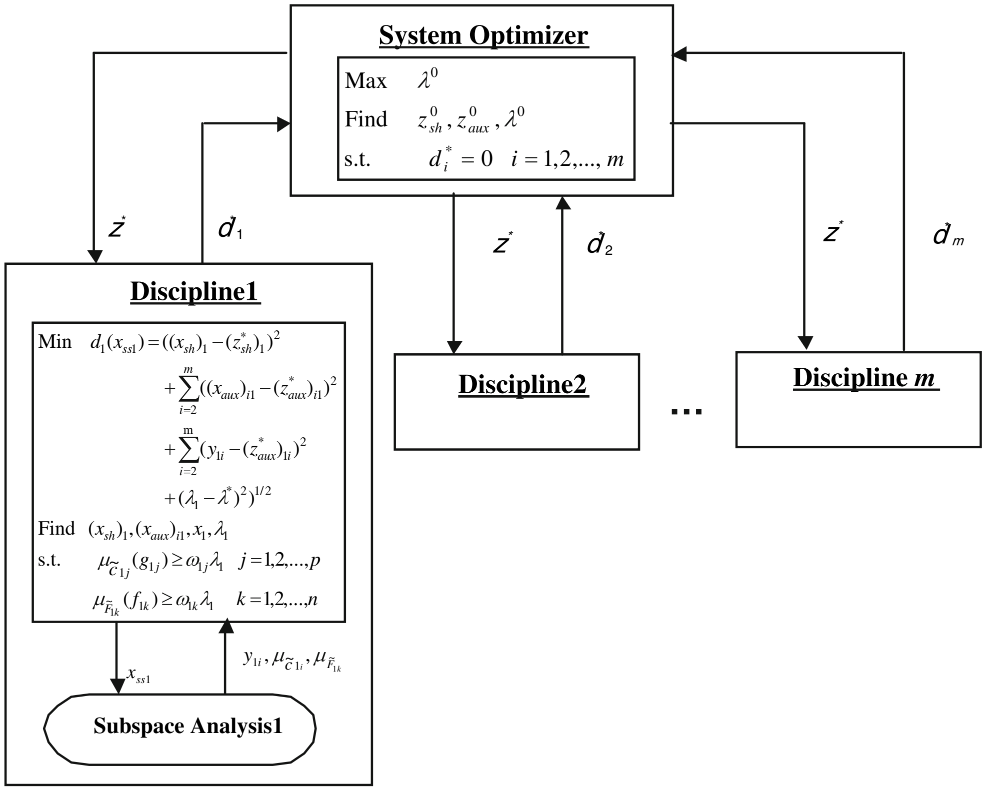

Fig. 4 The architecture for FSSDCO based on weighted Max–Min method

The Bellman–Zadeh's Max–Min operator reflects the conservative thought that the most dissatisfactory component is improved in the decision space determined by the satisfaction degree and sufficiency degree for the objectives and constraints. This results in a fact that although the most dissatisfactory component has been maximized, the variation of the remaining components within certain ranges does not directly influence the final result and some useful and relative information is likely to be lost. The weighted Max–Min method (Zuang et al. 1999) can effectively solve the aforementioned problem. Before practical optimization, using the relative importance of objectives and constraints, the decision maker may select suitable coefficients for the satisfaction and sufficiency degree. This method not only considers the fuzzy information for objectives and constraints, but also reflects the practical situation in which the decision maker pays unequal attention to the membership of objectives and constraints. The models at ith discipline are shown as follows:

$$
\begin{aligned}
&\text{Find } x \\
&\text{Max } \mu_{\tilde{\Phi}_i}(x) = \lambda_i && i = 1, 2, \ldots, m \\
&\text{s.t. } \mu_{\tilde{C}_{ij}}(x) \geqslant \omega_{ij}\lambda_i && j = 1, 2, \ldots, p \\
&\qquad \mu_{\tilde{F}_{ik}}(x) \geqslant \omega_{ik}\lambda_i && k = 1, 2, \ldots, n \\
&\qquad 0 \leqslant \lambda_i \leqslant 1 && 0 \leqslant \omega_i \leqslant 1
\end{aligned}
\tag{15}
$$

The architecture for FSSDCO based on the weighted Max–Min method is shown in Fig. 4. In this architecture, system level design variables include the shared variables z_{sh}^0 and the auxiliary variables z_{aux}^0. In addition the acceptable degree level λ^0 maximized by the system optimizer, is the target for all the disciplinary objectives for each discipline to maintain consistency. For the subspace analysis, each discipline's duty is extended to include three components, namely, conventional analysis for output variables, satisfaction degree analysis, and sufficiency degree analysis. The first analysis computes the coupling vector that is used in ith discipline, the second one provides the satisfaction degree of local objectives, and the third one provides the sufficiency degree of local constraints for the discipline optimizer. The acceptable degree level λ_i at ith discipline is used in the discrepancy function d_i which is minimized by the ith disciplinary optimizer. Also, it is used as a component of the compatibility constraints at the system level.

Note that FSSDCO separates the original problem into one system level and two discipline level problems, and that the system level coordinates the shared, auxiliary variables and level of satisfaction and sufficiency degree λ_i while maximizing λ^0. This enables two disciplines to search independently for the optimum solutions in the decision space when the satisfaction degree or the sufficiency degree level is greater

Table 1 Design vectors for the mathematical example

	Discipline 1	Discipline 2
System targets to be matched	$[z_1^*, z_3^*, z_4^*, z_5^*, \lambda^*]$	$[z_1^*, z_3^*, z_4^*, z_5^*, \lambda^*]$
Shared design vector	$[x_1, x_3, \lambda_1]$	$[x_1, x_3, \lambda_2]$
Auxiliary design vector	$[y_{21}]$	$[y_{12}]$
Local design vector	$[x_2]$	Empty
Subspace analysis	$[y_{12}, \mu_{\tilde{F}_1}, \mu_{\tilde{c}_{1i}}] = SA1[x_1, x_2, x_3, y_{12}] i = 1, 2, 3, 5$	$[y_{21}, \mu_{\tilde{F}_2}, \mu_{\tilde{c}_{2i}}] = SA2[x_1, x_3, y_{21}] i = 1, 3, 4$

Table 2 The possible lower and upper limits for constraints

g_i	$\underline{b}_i^u$	$\underline{b}_i^l$	$\overline{b}_i^u$	$\overline{b}_i^l$
x_1	4	2	–	–
x_2	3	1	–	–
x_3	–	–	5.5	3.5
y_{12}	10	8	–	–
y_{21}	–	–	11	9

than or equal to the weighted level $\omega_i \lambda_i$. Due to the minimization of the discrepancy function, λ^0 and λ_i can obtain mutual consistency to the utmost extent. Therefore as the system level design variable λ^0 reaches the maximum value, the disciplines may acquire the optimum solution simultaneously.

3.1 Mathematical example

This example is comprised of a coupled analysis to evaluate two objectives and the associated constraints. As shown in Eq. (16), the objective functions include two coupling variables y_{12} and y_{21}. Each calculates its value with the other as an input variable. Table 1 contains all the design vectors in CO standard notation for disciplines 1 and 2. This example focuses on the influence of fuzzy information. In view of the requirement of the degree of sufficiency for objectives, this paper presents the possible lower and upper limits for each of constraint as depicted in Table 2. With these data and adopting Eqs. (6, 7), the linear membership function for constraints can be constructed. Corresponding to the formulation for FSSDCO, the model is shown as Fig. 5.

$$
\begin{aligned}
&\text{Min } F_1(x) = (x_2+3)^2 - 9x_3^2 + (y_{12}+2)^2 \\
&\text{Min } F_2(x) = (y_{21}-10)^2 \\
&\text{find } x_1, x_2, x_3 \\
&\qquad s.t. \quad 4 \leqslant x_1 \leqslant 15 \\
&\qquad 3 \leqslant x_2 \leqslant 10 \\
&\qquad 0 \leqslant x_3 \leqslant 3.5 \\
&\qquad (y_{12}-3)/10 - 2 \leqslant 0 \\
&\qquad 5 - y_{12}/2 \leqslant 0 \\
&\qquad y_{21}/2 - 4.5 \leqslant 0 \\
&\qquad -y_{21} \leqslant 0
\end{aligned}
\tag{16}
$$

where

$$
\begin{aligned}
y_{12} &= x_1^2 + x_2 + x_3 - 0.2y_{21} \\
y_{21} &= x_1 + x_3 + \sqrt{x_4}
\end{aligned}
$$

The final result using the weighted Max-Min method for FSSDCO is listed in Table 3.

Note that when the weight coefficient is allocated with different values, the optimization by FSSDCO may obtain different results. If the weight is equivalent to each constraint and objective of the two disciplines (the decision-maker pays equal attention to constraint and objects), the framework of FSSDCO is attributed to the symmetric fuzzy optimization problem. The last row in Table 3 gives the corresponding results. Note also that from the first row to the third row, the larger the weight coefficient of the objective the smaller the sum of the objective function value. The data in Table 3 indicate that alteration of weight coefficients brings about the comprehensive influence to optimum solutions.

4 The formulation of α-cut method for the FSSDCO

In this section, we propose the model based on the α-cut method for FSSDCO and give an example to illustrate the effectiveness of the proposed method.

4.1 α-cut method using fuzzy satisfaction degree

In many engineering designs objectives and constraints commonly play different roles. That is, satisfying constraints is the precondition for achieving the optimum solutions of the objectives. For most of the engineering problems only the constraints include fuzzy information, while the design variables and objective functions are often deterministic. Therefore, the model using α-cuts method is described as follows:

$$
\begin{aligned}
&\text{Find } x \\
&\max\ \alpha_{\tilde{F}}(x) = T(\alpha_{\tilde{F}_1}(x), \alpha_{\tilde{F}_2}(x), \ldots, \alpha_{\tilde{F}_n}(x)) \\
&\text{s.t.} \quad \mu_{\tilde{c}_j}(x) \geqslant \lambda^* j = 1, 2, \ldots, p
\end{aligned}
\tag{17}
$$

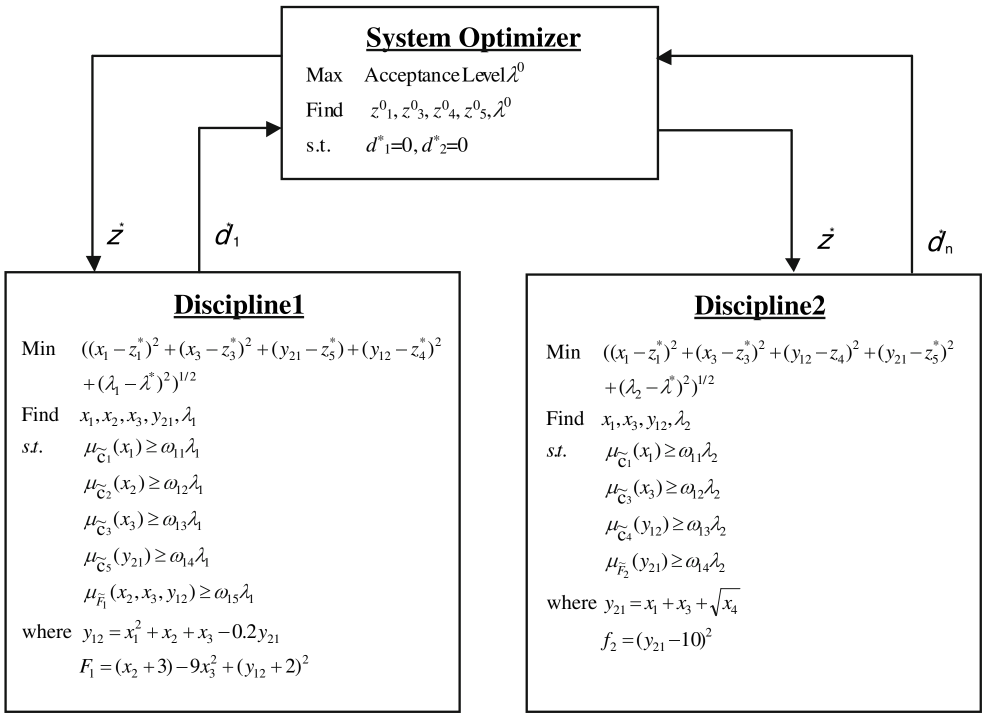

Fig. 5 The FSSDCO model for mathematical example

Table 3 The result of Max–Min method for FSSDCO using a sequence of weights

Discipline 1 ω_1	Discipline 2 ω_2	x_1	x_2	x_3	y_{12}	y_{21}	F_1+F_2	λ
[1.0,0.6,0.7,0.8,0.6]	[1.0,0.4,0.6,0.7]	2.8857	2.0333	3.7517	12.0892	10.1143	97.1732	0.7381
[0.8,0.6,0.7,0.8,0.6]	[0.8,0.4,0.6,0.7]	3.0331	2.2053	3.3599	12.7718	9.9669	143.7009	0.8610
[0.5,0.6,0.7,0.8,0.6]	[0.5,0.4,0.6,0.7]	3.2001	2.4002	2.4797	13.2566	9.3208	207.0485	1.0000
[0.9,0.2,0.5,0.4,0.7]	[0.8,0.2,0.3,0.7]	2.3713	1.9283	4.1972	9.8086	9.7004	5.2724	0.9283
[1.0,1.0,1.0,1.0,1.0]	[1.0,1.0,1.0,1.0]	3.2521	2.2521	2.7976	13.6764	9.7479	202.8609	0.6261

where $\alpha_{\tilde{F}}(x)$ is the overall satisfaction degree, $\alpha_{\tilde{F}_i}$ denotes the satisfaction degree of ith objective. $\mu_{\tilde{c}_j}$ is the membership function of the jth constraint, which constructs the feasible region for the overall satisfaction degree of α-cut $C_{j\lambda^*}$, i.e.

$$C_{j\lambda^*} = \left\{x|\mu_{\tilde{c}_j}(x) \geqslant \lambda^*\right\} \tag{18}$$

where $C_{j\lambda^*}$ is also called the level cut set λ^* which cuts the set for the sufficiency degree of the jth constraint function. The smaller the value of λ is, the greater the range of C_λ is. If λ is equal to 0, C_0 is the so-called support set. If λ is equal to 1, C_1 is the most rigorous acceptable value. Therefore from the point of view of engineering design, the selection of different values of λ can result in different designs. For a sequence of λ values there must exist an optimum λ^*, which is the most reliable, economical, and safe; meanwhile the corresponding C_{λ^*} is the optimum level cut set. For the sake of simplicity the overall satisfaction degree of the objective employs the Bellman–Zadeh's operator, and Eq. (17) is transformed into the following expression:

$$\begin{aligned} &\text{Find } x \\ &\max\ \mu_{\tilde{D}}(x) = \alpha_{\tilde{F}}(x) = \mathop{\wedge}_{i=1}^{n} \mu_{\tilde{F}_i}(x) \\ &\text{s.t.}\quad g_j(x) \leqslant \overline{b}_j^l + \overline{d}_j(1-\lambda) \qquad j = 1, 2, \ldots, J \\ &\qquad\ g_j(x) \geqslant \underline{b}_j^u - \underline{d}_j(1-\lambda) \qquad j = J+1, \ldots, p \\ &\qquad\ 0 \leqslant \lambda \leqslant 1 \end{aligned} \tag{19}$$

4.2 FSSDCO based on α-cut method

The formulation using the α-cut method is shown in Fig. 6. Note that depending upon the formulations, the level λ does not belong to the interdisciplinary variable set between different disciplines. In addition each discipline possesses the model for subspace optimum level analysis, returning the optimum level λ_i^* to form the allowable region for constraints

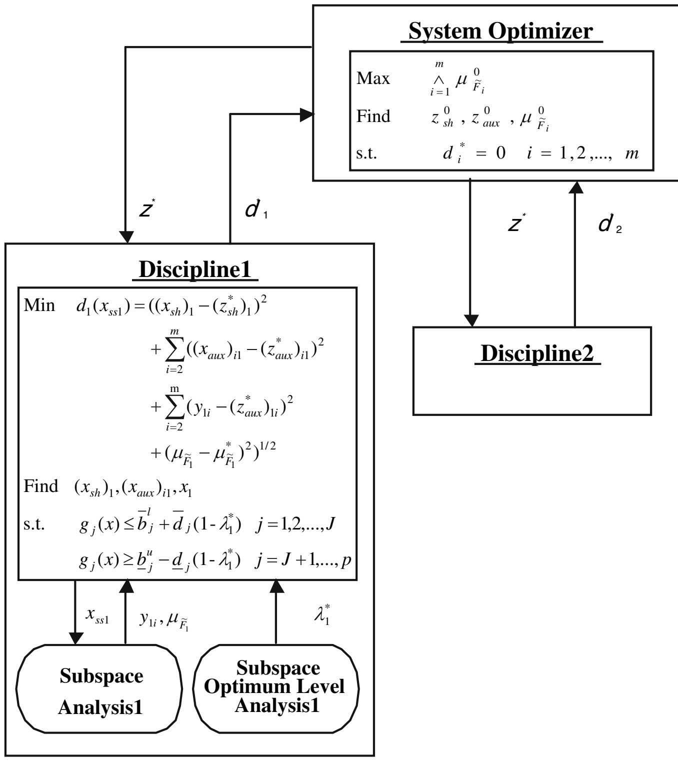

Fig. 6 The architecture for the FSSDCO using α-cuts method

at the ith discipline. The satisfaction degree $\mu_{\tilde{F}_1}$ used as the original objective function for the single level optimization problem becomes a part of the discrepancy function $d_i(x_{ssi})$. The system level optimizes design variables $\mu^0_{\tilde{F}_i}$, while satisfying the compatibility constraints d_i^* to keep $\mu_{\tilde{F}_i}$ and $\mu^0_{\tilde{F}_i}$ consistent. As the system objective, the aggregation of $\mu^0_{\tilde{F}_i}$ reaches the maximum value each discipline can obtain the optimum solution simultaneously.

Subspace optimum level analysis generally adopts two methods to achieve λ_i^*, i.e. mathematical programming and the fuzzy synthetical evaluation method (Huang 1996; Huang et al. 2005b). The former solves the mathematical programming problem to get λ_i^* according to the interrelationship between the maintenance and manufacture costs. In the fuzzy synthetical evaluation method, λ_i^* is evaluated by relevant engineers and experts based on influence factors, such as the capabilities of design and manufacturing, the characteristic of material, and the environment.

4.3 Gear reducer example using α-cut method in the FSSDCO

In this section, a gear reducer example is presented as shown in Fig. 7. The design objective is to minimize the overall volume. According to the CO method, the reducer example

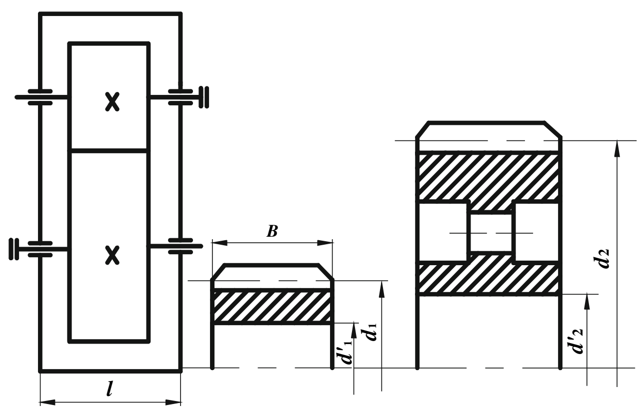

Fig. 7 A gear reducer

Table 4 Meanings of design variables in disciplines 1, 2

Item	Discipline 1	Discipline 2
Gear face width, B (cm)	x_1	x_1
The number of teeth of pinion	x_2	$y_{12}(ix_2)$
Module, m (cm)	x_3	x_3
Distance between bearings, l (cm)	x_4	x_4
Diameter of shaft 1, d'_1(cm)	x_5	Empty
Diameter of shaft 2, d'_2 (cm)	Empty	x_6

Table 5 Design vectors for gear reducer problem

	Discipline 1	Discipline 2
System targets to be matched	$[z_1^*, z_3^*, z_4^*, z_{\text{aux}}^*, \mu_{\tilde{F}_1}^*]$	$[z_1^*, z_3^*, z_4^*, z_{\text{aux}}^*, \mu_{\tilde{F}_2}^*]$
Shared design vector	$[x_1, x_3, x_4]$	$[x_1, x_3, x_4]$
Auxiliary design vector	Empty	$[y_{12}]$
Coupling vector	$[y_{12}]$	Empty
Local design vector	$[x_2, x_5]$	$[x_6]$
Subspace analysis	$[y_{12}, \mu_{\tilde{F}_1}] = SA1[x_1, x_2, x_3, x_4, x_5]$	$[\mu_{\tilde{F}_2}] = SA2[x_1, x_3, x_4, x_6, y_{12}]$

Table 6 Maximum and minimum values for objective function f_i

f_i	$f_i^{\max}$	$f_i^{\min}$
f_1	60965.9307	2772.50129
f_2	385411.7444	20476.9991

is decomposed into two disciplines, (1) drive pinion and shaft 1 and (2) driven pinion and shaft 2. The design variables for each discipline are as shown in Table 4.

In Table 4, i is the velocity ratio of the gear reducer, and y_{12} is coupling variable which is computed in discipline 1 and used in discipline 2. In Table 5, the design vectors for the gear reducer problem are listed. Note that, unlike the mathematical example, there is no inter-coupling vector between disciplines; that is, y_{12} is the only coupling vector. Note also that the subspace analysis assumes the responsibility of gaining the degree of satisfaction. The corresponding Max–Min interval for the original objective of disciplines is also shown in Table 6.

The subspace analysis makes use of data in Table 6 and Eq. (3) to construct the satisfaction degree as the objective function for each discipline. The nonlinear programming for different disciplines is as follows:

Discipline 1

$$
\begin{aligned}
&\text{Find} && x_{\text{SS}1} = [x_1, x_2, x_3, x_4, x_5] \\
&\text{Min} && d_1(x_{\text{SS}1}) = ((x_1 - z_1^*)^2 + (x_3 - z_3^*)^2 + (x_4 - z_4^*) \\
& && \quad + (y_{12} - z_{\text{aux}}^*)^2 + (\mu_{\tilde{F}_1} - \mu_{\tilde{F}_1}^*)^2)^{1/2} \\
&\text{s.t.} && 18 - (1 - \lambda_1) \leqslant x_2 \leqslant 37 + 3(1 - \lambda_1) \\
& && 0.9 - 0.1(1 - \lambda_1) \leqslant x_1/(x_2 x_3) \leqslant 1.3 + 0.1(1 - \lambda_1) \\
& && 0.3 - 0.1(1 - \lambda_1) \leqslant x_3 \leqslant 1.0 + 0.2(1 - \lambda_1) \\
& && x_2 x_3 \leqslant 30 \\
& && 10 - (1 - \lambda_1) \leqslant x_5 \leqslant 13 + 2(1 - \lambda_1) \\
& && 441630/(x_2 x_3 x_1^{0.5}) \leqslant 7695 + 855(1 - \lambda_1) \\
& && 70980/(x_1 x_2 x_3^2 (0.619 + 66.66 \times 10^{-4} x_2 \\
& && \quad - 85.4 \times 10^{-6} x_2^2)) \leqslant 2355 + 262(1 - \lambda_1) \\
& && 0.01233 x_4^3/(x_2 x_3 x_5^4) \leqslant 0.003 x_4 \\
& && 10(((29050 x_4/(x_2 x_3))^2 + (0.58 \times 27300)^2)^{0.5})/x_5^3 \\
& && \quad \leqslant 500 + 50(1 - \lambda_1) \\
&\text{where} && y_{12} = i x_2
\end{aligned}
\tag{20}
$$

Discipline 2

$$
\begin{aligned}
&\text{Find} && x_{\text{SS}2} = [x_1, x_3, x_4, x_6, y_{12}] \\
&\text{Min} && d_2(x_{\text{SS}2}) = ((x_1 - z_1^*)^2 + (x_3 - z_3^*)^2 + (x_4 - z_4^*) \\
& && \quad + (y_{12} - z_{\text{aux}}^*)^2 + (\mu_{\tilde{F}_2} - \mu_{\tilde{F}_2}^*)^2)^{1/2} \\
&\text{s.t.} && 18 - (1 - \lambda_2) \leqslant y_{12}/i \leqslant 37 + 3(1 - \lambda_2) \\
& && 0.9 - 0.1(1 - \lambda_2) \leqslant x_1/(y_{12} x_3/i) \\
& && \quad \leqslant 1.3 + 0.1(1 - \lambda_2) \\
& && 0.3 - 0.1(1 - \lambda_2) \leqslant x_3 \leqslant 1.0 + 0.2(1 - \lambda_2) \\
& && x_2 x_3 \leqslant 30i \\
& && 13 - (1 - \lambda_2) \leqslant x_6 \leqslant 18 + 2(1 - \lambda_2) \\
& && x_4 - x_1 - 0.5 x_6 \geqslant 4 \\
& && 70980/(x_1 y_{12} x_3^2/i(0.2824 + 17.7 \times 10^{-4} y_{12}/i \\
& && \quad - 39.4 \times 10^{-6} y_{12}^2/i^2)) \leqslant 1920 + 213(1 - \lambda_2) \\
& && 10(((29050 x_4/(y_{12} x_3/i))^2 \\
& && \quad + (0.58 \times 27300 \times 5)^2)^{0.5})/x_6^3 \leqslant 500 + 50(1 - \lambda_2)
\end{aligned}
\tag{21}
$$

In the actual deign, constraints in the disciplines are likely to include fuzzy information, which are listed in Table 7. $b_i^u b_i^l$ are the allowable upper and lower limits of the relevant variable, respectively.

For simplification and demonstration purposes this paper does not apply the optimum cut levels, but rather develops a sequence of cuts levels which are implemented in a comparison study. The FSSDCO method computes the optimum solutions at different cut levels, respectively. The results are shown in Table 8.

It can be seen from Table 8 that the fuzzy information in the CO influences the optimum solutions and objectives. Different cut levels for fuzzy constraint functions may result in different solutions. As indicated by the first four rows of Table 8, the smaller the cut level, the lower the overall objective F. Considering that CO supports distributed design environment, if cut levels for different disciplines take different values, an alteration of the allowable space of constraints can be obtained as well as the optimum solution in each discipline, even though some constraints are the same among disciplines.

The data in the last row of Table 8 is used as the optimum solution to this problem of the FSSDCO. According to

Table 7 The allowable intervals for fuzzy factors f_i

	$[\sigma_H]$(kg/cm^2)	$[\sigma_{F_1}]$(kg/cm^2)	$[\sigma_{F_2}]$(kg/cm^2)	$[\sigma_b]$(kg/cm^2)	$\phi_d x_1/(x_2x_3)$	x_2	x_3 (cm)	x_5 (cm)	x_6 (cm)
$\overline{b}^u$	8,550	2,617	2,133	550	1.4	40	1.2	15	20
$\overline{b}^l$	7,695	2,355	1,920	500	1.3	37	1.0	13	18
$\underline{b}^u$					0.9	18	0.3	10	13
$\underline{b}^l$					0.8	17	0.2	9	12

Table 8 The result using a sequence of cuts levels for FSSDCO

λ_1	λ_2	x_1 (cm)	x_2	x_3 (cm)	x_4 (cm)	x_5 (cm)	x_6 (cm)	F (cm^3)	$\mu_{\tilde{F}_1}$	$\mu_{\tilde{F}_2}$
0.4	0.4	12.6882	17.4010	0.8681	22.9935	9.6054	12.6106	33714.7518	0.9609	0.9776
0.6	0.6	13.0710	17.6004	0.8636	23.3710	9.6955	12.6000	34692.0792	0.9583	0.9753
0.8	0.8	13.4658	17.8000	0.8595	23.8658	9.8000	12.8000	36067.8237	0.9553	0.9720
1.0	1.0	14.1544	18.2010	0.8381	24.6544	10.0000	13.0000	37547.5487	0.9512	0.9686
0.2	0.4	12.5150	17.4013	0.8562	22.7150	9.5502	12.4000	32428.9626	0.9631	0.9807
0.4	0.6	12.8888	17.6007	0.8515	23.2538	9.6599	12.7299	33879.3971	0.9603	0.9772
0.4	0.8	13.0879	17.8004	0.8355	23.5125	9.7138	12.8492	34049.5825	0.9596	0.9768
0.6	0.8	13.2729	17.8001	0.8473	23.7047	9.7586	12.8637	35089.9619	0.9575	0.9743

Table 9 The rounded result for gear reducer design

λ_1	λ_2	x_1 (cm)	x_2	x_3 (cm)	x_4 (cm)	x_5 (cm)	x_6 (cm)	F (cm^3)	$\mu_{\tilde{F}_1}$	$\mu_{\tilde{F}_2}$
0.6	0.8	13.27	18	0.9	23.70	9.76	12.86	38430.66	0.9512	0.9662

the meaning of design variables, the final rounded optimum values are shown in Table 9.

5 Conclusions

This paper focuses on the use of the fuzzy satisfaction degree and the fuzzy sufficiency degree models for collaborative optimization. The illustrative mathematical example shows that the satisfaction and sufficiency degree theory based on the weighted Max–Min method is a rational and practical approach for decision making in multidisciplinary design optimization. In the gear reducer optimization example, the α-cut method is employed to construct allowable subspace in disciplines. The gear reducer problem also adopts the satisfaction degree to create the objective function for each discipline. The problem is effectively solved based on an asymmetrical model in MDO environments.

In addition, two strategies of the FSSDCO enhance the capability of CO containing fuzzy information. In engineering design practice, especially for the MOCO, there often exist conflicting objectives. This paper proposes using the satisfaction degree theory to solve MOCO problems. Moreover in MDO environments, the constraints generally are distributed to different disciplines. Therefore the α-cut method is applied to the FSSDCO, such that optimum cut levels can be obtained independently for each discipline to form various allowable spaces. The developed approaches in this paper can efficiently deal with CO problems in the fuzzy and distributed environment.

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