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The coupling relations between multi-quality characteristics (MQCs) and design parameters (DPs) are analysed. To improve the robustness of product quality, the coupled design is decoupled approximately by utilizing the non-linear relations between MQCs and DPs, and an approximate decoupling criterion is proposed. Based on the fundamental principle of fuzzy-robust design and axiomatic design theory, the methods for fuzzy-robust design are studied in the case of uncoupled design, decoupled design and coupled design. Three different models of fuzzy-robust optimization design with MQCs are established, and three different optimization strategies are proposed correspondingly. A back-propagation neural network is used as a substitute for non-linear stochastic functions in the established models to improve the computational efficiency. With the genetic algorithm combined with stochastic simulation and neural network, a hybrid intelligent optimization algorithm is developed for solving the established models. An example of fuzzy-robust design of a plastic part is presented.

Keywords: multi-quality characteristics; axiomatic design theory; fuzzy-robust optimization design; hybrid intelligent optimization algorithm

1. Introduction

Robust design under uncertainty has recently attracted a lot of attention. Uncertainties are usually modelled using probability theory. Taguchi's quality loss function and signal-to-noise ratio (S/N) are usually used as a measuring index of robustness (Chen *et al.* 2007, Joseph 2007, Kuo *et al.* 2008). However, recent researches show that the fuzzy factor is one of the influencing factors similar to random factors causing variations of quality characteristics of a product, and a fuzzy-robust design method was proposed (Guo 2002, 2004). This expands the study of uncertainty from probability in conventional robust design to fuzzy probability, and upgrades the robustness under stochastic uncertainty to the robustness under stochastic uncertainties and fuzzy uncertainties. The robust criteria used in fuzzy-robust design method, via fuzzy probability (Zadeh 1999), are entirely different from the robust criteria of conventional robust design. Recently, some scholars

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have discussed and improved the approach of fuzzy-robust design. Under the conditions of fuzzy objective and fuzzy design constraints, the percentile formulation design rules for fuzzy-robust design, which takes both the robustness of quality characteristic and the feasibility robustness of design constraints into account, were proposed by Liu *et al.* (2005) when the product quality possesses nominal-the-best or larger-the-better or smaller-the-better characteristics. Huang *et al.* (2005) developed a design method for high quality products using a fuzzy-robust design and bi-objective optimization model. The bi-objective functions are the tolerance of performance and the probability of high quality, and the bi-objective optimization model was solved using physical programming. Based on fuzzy Q-analysis, Chi and Teng (2008) proposed a fuzzy Taguchi method to solve the parameter design with multi-quality characteristics. Currently, fuzzy-robust design theory is still under development. Especially, the fuzzy-robust design with multi-quality characteristics (MQCs) is rarely discussed, and many issues need to be investigated. In this article, the optimization design principles and modelling methods of fuzzy-robust design with MQCs are discussed based on the principle of fuzzy-robust design (Guo 2002, 2004) and the design axioms (Suh 1990, 2001). A hybrid intelligent optimization algorithm is developed to solve the model of fuzzy-robust design with MQCs. As an example, the fuzzy-robust design optimization of a plastic part is presented.

2. The fundamentals of fuzzy-robust design

The quality characteristic index y of a product is a function of controllable-factor vector $\boldsymbol{X}_\mathbf{C}$, such as random design variables and their tolerances, and random noise-factor vector $\boldsymbol{Z}$. This function can be denoted as $y = y(\boldsymbol{X}_\mathbf{C}, \boldsymbol{Z})$. Suppose that the probability density function of y is $f(y)$. In Figure 1, y_0 is the expected value of quality index y, its tolerance is $\pm\Delta y$. y possesses nominal-the-best characteristic. Whether a design solution of quality characteristic y is a high-quality solution or not should be a fuzzy subset in the value region of y, and it can be expressed as a real fuzzy number denoted as $\tilde{A}$. Suppose that the fuzzy distributing function of y is $\mu_{\tilde{A}}(y)$ which expresses the subjecting degree of y to the expected design. Obviously, $\mu_{\tilde{A}}(y_0) = 1$. When $|y - y_0| \geq \Delta y$, $\mu_{\tilde{A}}(y) = 0$. When $|y - y_0| < \Delta y$, $\mu_{\tilde{A}}(y) \in (0, 1)$, and this indicates that the quality index y is a high-quality index to some extent or is close to the expected target value y_0 to some extent. The value of $\mu_{\tilde{A}}(y)$ can be defined according to the influence of the offset of y from the target value y_0 on the product performance. Using different $\mu_{\tilde{A}}(y)$, a designer can embody his/her desirability on the fuzzy distributing function of y. In Figure 1, a trapezoidal distributing function is defined as the membership function of $\tilde{A}$. When $|y - y_0| \geq \Delta y$, $\mu_{\tilde{A}}(y) = 0$. When $y \in [a, b]$, $\mu_{\tilde{A}}(y) = 1$, and this expresses that the design quality y can be regarded as satisfying. Therefore, $[a, b]$ can be considered as a high-quality interval of y. The $\mu_{\tilde{A}}(y)$ in the fuzzy transition region can be regarded as a linear function. The echelon distributing function of $\mu_{\tilde{A}}(y)$ is

$$\mu_{\tilde{A}}(y) = \begin{cases} 1.0; & (a \leq y \leq b) \\ \dfrac{y - y_0 + \Delta y}{\Delta y}; & (y_0 - \Delta y \leq y < a) \\ \dfrac{y_0 + \Delta y - y}{\Delta y}; & (b < y \leq y_0 + \Delta y) \\ 0; & (else) \end{cases}$$

As shown in Figure 1, the more the bell-shaped curve of $f(y)$ is embedded into the high-quality interval $[a, b]$, the larger the area enclosed between $f(y)$ and the y-axis falling into $[a, b]$ is, and

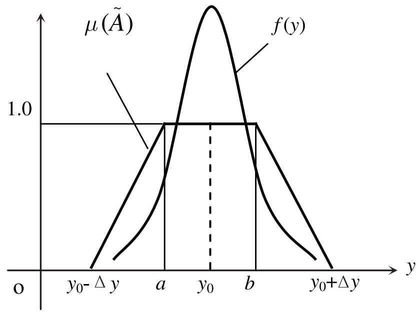

Figure 1. The membership function of $\tilde{A}$ and the distributing density function of y.

then the better the robustness of product quality is. Therefore, the robustness of y can be described by using the fuzzy probability of $\tilde{A}$, denoted as $P(\tilde{A})$. The basic criterion on fuzzy-robust design can be formulated as

$$P(\tilde{A}) = \int_{-\infty}^{+\infty} \mu_{\tilde{A}}(y) f(y) dy = \int_{y_0 - \Delta y}^{y_0 + \Delta y} \mu_{\tilde{A}}(y) f(y) dy \longrightarrow \max \tag{1}$$

or

$$P(\tilde{A}) \geq \beta. \tag{2}$$

Here, $P(\cdot)$ is an operator of probability calculation, and β is a scheduled level for controlling the robustness of a product. ($\beta \in [0, 1]$). '$\rightarrow$' expresses 'approach'. $P(\tilde{A})$ is the fuzzy probability of $\tilde{A}$, and it is called the fuzzy high-quality level. This robust design criterion based on fuzzy probability is called the fuzzy-robust design criterion (Guo 2002, 2004).

By using the fuzzy-robust design criterion, designers can felicitously deal with the fuzzy information contained in robust design about quality characteristics. Specifically, $P(\tilde{A})$ has a maximum when the mean of y is equal to the expected value y_0 and $P(\tilde{A})$ is a decreasing function of the standard deviation of y (Guo 2002, 2004). Therefore, Equation (1) has two effects on $f(y)$: on the one hand, it enables the mean value of y to approach the center of the high-quality interval $[a, b]$; on the other hand, it effectively reduces σ_y^2 which is the variance of y caused by controllable factors and random noise factors. Therefore, the controlling effect of the fuzzy-robust design criterion on the mean and variance of quality characteristic is similar to the effect of Taguchi's signal-to-noise ratio and quality loss function. For a quality index with a larger-the-better characteristic or a smaller-the-better characteristic, the criterion of fuzzy-robust design is similar to Equation (1). When modelling a fuzzy-robust optimization, some constraints of product performance and manufacturing cost should be taken into account. According to the design requirements of a product, Equation (1) is usually used as an objective to be optimized, and Equation (2) is usually used as a constraint of robustness.

3. Coupling and decoupling of multi-quality characteristics

$P(\tilde{A})$ is a measure of the information contained in design solution of a product. The larger the $P(\tilde{A})$ is, the less the information contained in a design solution is. Therefore, using Equation (1) as a criterion of fuzzy-robust design is consistent with the second design axiom (Suh 1990, 2001), namely the Information Axiom. By using Equation (1), the mean and variance of the quality characteristic y can be efficiently controlled, and then the expected fuzzy high-quality level $P(\tilde{A})$ can be achieved. However, Equation (1) is a basic criterion only suitable for the robust design with single quality characteristic index. When modelling the fuzzy-robust optimization with multi-quality characteristics, the coupling relationships between the functional requirements of a product must be taken into account. Every one of quality characteristics can be considered as a functional requirement (FR) of a product. Sometimes, the FR of a product is more than one in engineering design. Suppose the vector of multi-quality characteristics, namely FRs (functional requirements), is $\boldsymbol{Y} = [y_1, y_2, \ldots, y_m]^{\mathrm{T}}$; and the vector of design parameters (DPs), namely controllable factors that are defined by designer in physical domain, is $\boldsymbol{X} = [X_1, X_2, \ldots, X_m]^{\mathrm{T}}$. Then, Suh's design equation (Suh 1990, 2001) in the case of design with no redundancy of design parameters is

$$\boldsymbol{Y} = [\boldsymbol{A}] \cdot \boldsymbol{X}.$$

Here

$$[\boldsymbol{A}] = \begin{bmatrix} A_{11} & A_{12} & \ldots & A_{1m} \\ A_{21} & A_{22} & \ldots & A_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ A_{m1} & A_{m2} & \cdots & A_{mm} \end{bmatrix}$$

$[\boldsymbol{A}]$ is called a design matrix with elements $A_{ij} = \partial y_i / \partial X_j$, $(i, j = 1, 2, \ldots, m)$, and the noise factors $\boldsymbol{Z}_{\mathrm{k}}$ are impliedly contained in $\boldsymbol{A}_{ij}$. If $[\boldsymbol{A}]$ is a diagonal matrix, the design solution is defined as an uncoupled design; if $[\boldsymbol{A}]$ is a triangular matrix, the design solution is defined as a decoupled design; if $[\boldsymbol{A}]$ is a general matrix, the design solution is defined as a coupled design.

At the stage of conceptual design of a product, designers establish a mapping from the functional domain to the physical domain. They attempt to reduce or eliminate the coupling of the functional requirements through a reasonable design of the physical parts of a product. This design method of decoupling the functional requirements of a product is called 'hardware-based decoupling'. If the cost of 'hardware-based decoupling' is too large to be acceptable or 'hardware-based decoupling' cannot be achieved, the nonlinear functional relationships between MQCs and DPs can be used for the optimization of $[\boldsymbol{A}]$. Through optimization, the values of the off-diagonal elements of design matrix $[\boldsymbol{A}]$ are made to be close to zero or far less than the values of the diagonal elements of $[\boldsymbol{A}]$, and then a design solution with approximate non-coupling or decoupling can be achieved. This design method for reducing the coupling of product functional requirements is called 'numerical decoupling' or 'soft decoupling'. After testing and studying, a scalar quantity *CD* is proposed to measure the coupling degree between MQCs and DPs. According to design matrix $[\boldsymbol{A}]$, *CD* is defined as

$$CD = \left(\prod_{i=1}^{m} \frac{|A_{ii}|}{\sum_{j=1}^{m} |A_{ij}|} \right) \cdot \left(\frac{\sum_{i=1}^{m} A_{ii}^2}{\sum_{i=1}^{m} \sum_{j=i}^{m} |A_{ij} A_{ji}|} \right).$$

Here, $|\bullet|$ is an operator for calculating absolute value.

If *CD* is equal to 1.0, the design is uncoupled. As *CD* is close to 1.0, the values of the off-diagonal elements of design matrix $[\boldsymbol{A}]$ decrease, and an approximately uncoupled design can be achieved. The smaller the value of *CD* is, the stronger the coupling degree between MQCs and DPs is (or the weaker the independence of MQCs is). To reduce the variations of product quality propagated between MQCs and improve the robustness of a design solution, a criterion for 'numerical decoupling' is proposed as

$$CD \longrightarrow \max. \tag{3}$$

Here, $CD \leq 1.0$ and possesses a goal value of 1.0. *CD* is different from *R* (re-angularity) and *S* (semi-angularity) proposed by Suh (1990, 2001), and its advantage is that its calculation is simpler than the calculations of *R* and *S*.

4. Fuzzy-robust optimization design with multi-quality characteristics

According to Suh's first axiom (Suh 1990, 2001), namely the Independence Axiom, the functional requirements (FRs) of a product should be unattached. Therefore, the robust design of a product should begin at the stage of its conceptual design. At this stage, designers should make the design matrix [$\boldsymbol{A}$] a diagonal or triangular matrix; that is to say, designers should decouple the MQCs and DPs of a product in the conceptual design stage. Since the elements A_{ij} of [$\boldsymbol{A}$] are determined by the physical parts of a product, the elementary transformations of matrix are not suitable for [$\boldsymbol{A}$]. Sometimes, the decoupling between the MQCs and DPs may results in the increase of the manufacturing cost or structural complexity of a product, or the decoupling cannot be carried out because of the restriction of product structure. It is obvious that the Independence Axiom is too strict to be satisfied in some engineering designs. Therefore, after conceptual design of a product, the fuzzy-robust optimization of a product quality should be modelled according to the different expressions of the design matrix [$\boldsymbol{A}$].

4.1. *Fuzzy-robust optimization modelling in the case of uncoupled design*

For a product with m quality characteristics, the design equation in the case of uncoupled design is

$$\boldsymbol{Y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{bmatrix} = \begin{bmatrix} A_{11} & 0 & \dots & 0 \\ 0 & A_{22} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & A_{mm} \end{bmatrix} \cdot \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_m \end{bmatrix} = [\boldsymbol{A}] \cdot \boldsymbol{X}. \tag{4}$$

For the kth quality characteristic index ($k = 1, 2, \dots, m$, $y_k = A_{kk}X_k$), the criterion of fuzzy-robust design is

$$P(\tilde{A}_k) = \int_{-\infty}^{+\infty} \mu_{\tilde{A}_k}(y_k) f_k(y_k) dy_k = \int_{y_{0k}-\Delta y_k}^{y_{0_k}+\Delta y_k} \mu_{\tilde{y}_{0k}}(y_k) f_k(y_k) dy_k \longrightarrow \max. \tag{5}$$

Here, $P(\tilde{A}_k)$ is the fuzzy high-quality level of y_k. The definitions of the fuzzy membership function $\mu_{\tilde{A}_k}(y_k)$ of $\tilde{A}_k$ and the probability density function $f_k(y_k)$ of y_k are similar to those of single quality characteristic as shown in Section 2. Let Ω_k be the fuzzy-robust feasible domain of design parameters subjected to some constraints such as the performance, cost, as well as the geometric boundary of a product. Then, the optimization design for robustness can be modelled as

$$\begin{cases} \max & P(\tilde{A}_k) \\ \text{s.t.} & \boldsymbol{X}_k \in \Omega_k \quad (k = 1, 2, \dots, m) \end{cases} \tag{6}$$

Due to the non-coupling of the m quality characteristics y_k, Equation (6) expresses m unattached optimization design models aiming at the design robustness of y_k. These models can be solved by using any existing optimization algorithm, and the fuzzy-robust design parameters, namely DPs, can be obtained.

4.2. *Fuzzy-robust optimization modelling in the case of decoupled design*

For a product with m quality characteristics, the design equation in the case of decoupled design is

$$\boldsymbol{Y}=\begin{bmatrix} y_1\\ y_2\\ \vdots\\ y_m\end{bmatrix}=\begin{bmatrix} A_{11} & 0 & \dots & 0\\ A_{21} & A_{22} & \dots & 0\\ \vdots & \vdots & \ddots & \vdots\\ A_{m1} & A_{m2} & \cdots & A_{mm}\end{bmatrix}\cdot\begin{bmatrix} X_1\\ X_2\\ \vdots\\ X_m\end{bmatrix}=[\boldsymbol{A}]\cdot\boldsymbol{X}. \tag{7}$$

For the kth quality characteristic index $(k=1,2,\dots,m)$, $y_k=A_{k1}X_1+A_{k2}X_2+\cdots+A_{kk}X_k$. Because each quality index, except y_1, is not completely unattached, the fuzzy-robust optimization of this product should be modelled gradually from y_1 to y_m, according to the sequence of y_k. Hence, the fuzzy-robust design model of a decoupled design is a hierarchy optimization model. For example, the optimization design model, when $m=3$, is

$$\begin{cases}\max\ P(\tilde{A}_3)\\ \text{s.t.}\begin{cases}\boldsymbol{X}_3\in\Omega_3\\ (\boldsymbol{X}_2,\boldsymbol{X}_1)\in\begin{cases}\max\ P(\tilde{A}_2)\\ \text{s.t.}\begin{cases}\boldsymbol{X}_2\in\Omega_2\\ \boldsymbol{X}_1\in\begin{cases}\max\ P(\tilde{A}_1)\\ \text{s.t.}\ \boldsymbol{X}_1\in\Omega_1\end{cases}\end{cases}\end{cases}\end{cases}\end{cases} \tag{8}$$

This model can be solved by using a strategy of step-by-step optimization from y_1 to y_m. When solving the kth optimization sub-model $(k=2,\dots,m)$, the design parameters $(X_1, X_2,\dots, X_{k-1})$, determined in the former $(k-1)$ steps, should be considered as noise factors. When all sub-models are solved gradually, the optimal design parameters (DPs) are obtained. It is easy to see that the variations of the design parameters $(X_1, X_2,\dots, X_{k-1})$ solved in the former steps will directly propagate to y_k. Therefore, a decoupled design will be inferior to an uncoupled design in the quality robustness under the same manufacturing conditions and using circumstances.

4.3. *Fuzzy-robust optimization modelling in the case of coupled design*

For a product with m quality characteristics, the design equation in the case of coupled design is

$$\boldsymbol{Y}=\begin{bmatrix} y_1\\ y_2\\ \vdots\\ y_m\end{bmatrix}=\begin{bmatrix} A_{11} & A_{12} & \dots & A_{1m}\\ A_{21} & A_{22} & \dots & A_{2m}\\ \vdots & \vdots & \ddots & \vdots\\ A_{m1} & A_{m2} & \cdots & A_{mm}\end{bmatrix}\cdot\begin{bmatrix} X_1\\ X_2\\ \vdots\\ X_m\end{bmatrix}=[\boldsymbol{A}]\cdot\boldsymbol{X}. \tag{9}$$

Here, $[\boldsymbol{A}]$ is a general matrix.

For the kth quality characteristic index $(k=1,2,\dots,m)$, $y_k=A_{k1}X_1+A_{k2}X_2+\cdots+A_{kk}X_k+\cdots+A_{km}X_m$, the criterion of fuzzy-robust design is

$$P(\tilde{A}_k)\longrightarrow\max\quad(k=1,2,\dots,m). \tag{10}$$

Here, the definition and calculation of $P(\tilde{A}_k)$ are similar to Equation (5).

Since the quality characteristic index y_k is coupled with other quality characteristics, its probability density function $f_k(y_k)$ will be associated with all elements of the design parameter vector

$\boldsymbol{X}$. That is to say, m quality characteristics are not unattached. Hence, the fuzzy-robust design model is different from the model of uncoupled or decoupled design discussed previously. With Equation (3) and (10) synthesized, a multi-objective design optimization for the robustness of a coupled design can be modelled as

$$\begin{aligned} &\max\,[P(\tilde{A}_1),\, P(\tilde{A}_2), \ldots, P(\tilde{A}_m), CD]^{\mathrm{T}} \\ &\text{s.t. } [\boldsymbol{X}_1, \boldsymbol{X}_2, \ldots, \boldsymbol{X}_m]^{\mathrm{T}} \in \Omega \end{aligned} \tag{11}$$

Here, Ω is the fuzzy-robust feasible domain of design parameters subjected to constraints such as the performance, cost, as well as the geometric boundary of design parameters.

Equation (11) is a multi-objective optimization problem with complex correlations between objectives and design parameters. Generally, a compromise solution can be only obtained. In order to solve this problem, any existing method of multi-objective optimization, such as the method of goal programming (Igzino 1985, Tanino *et al.* 2003), the method of physical programming (Chen 2000), etc., can be used.

5. Hybrid intelligent optimization algorithm for the fuzzy-robust design with multi-quality characteristics

According to Equation (6), (8) and (11), it is obvious that those models are fuzzy-random optimization problems. In Section 4, three modelling methods for robustness of product quality have been presented. However, no numerical optimization algorithms were discussed. Because the product quality index y_k ($k = 1, 2, \ldots, m$) is a nonlinear function of design parameters and noise factors, it is difficult to formulate the probability density function $f_k(y_k)$ of y_k, and then the nonlinear functions such as fuzzy probability $P(\tilde{A}_k)$ will not be derived analytically. As a result, the statistic values such as fuzzy probability $P(\tilde{A}_k)$ cannot be calculated directly. Theoretically, it is possible to calculate these values by means of stochastic simulations, but sometimes the computational efficiency of simulations is too low to be acceptable. Hence, it is necessary to find more efficient substituting models for simulations. Thus, a back-propagation neural network (BPNN) (Zakarian *et al.* 1999) is used as a substituting model of non-linear functions such as fuzzy probability $P(\tilde{A}_k)$. Then, based on the genetic algorithm (GA) (Goldberg 1989, Guo 2006) combined with stochastic simulation and neural network, a hybrid intelligent optimization algorithm is developed for the fuzzy-robust design with multi-quality characteristics. The GA is used as the optimizer since it has better global convergence than gradient-based methods. The procedure of this algorithm is as follows.

Step 1: Make the conceptual design and structure design of a product to be optimized.
Step 2: Analyse the relations of MQCs and DPs, and establish the design equation $\boldsymbol{Y} = [\boldsymbol{A}] \cdot \boldsymbol{X}$.
Step 3: Distinguish the type of $[\boldsymbol{A}]$. If $[\boldsymbol{A}]$ is a diagonal matrix, establish the mathematical model of fuzzy-robust design according to Equation (6). If $[\boldsymbol{A}]$ is a triangular matrix, establish the mathematical model of fuzzy-robust design according to Equation (8). If $[\boldsymbol{A}]$ is a general matrix, establish the mathematical model of fuzzy-robust design according to Equation (11).
Step 4: According to the type of the established model, choose a strategy for solving the model.
Step 5: According to the probability density functions of design parameters and noise factors, calculate the statistic values of fuzzy probability $P(\tilde{A}_k)$ by means of stochastic simulations or fuzzy-stochastic simulations, and then establish a learning sample matrix and a testing sample matrix for BPNN.

Step 6: Design a BPNN as the substituting model for fuzzy probability $P(\tilde{A}_k)$. Determine the architecture of the BPNN, including the numbers of hidden layer, input node and output node, etc.
Step 7: Train the BPNN by using the learning sample matrix established.
Step 8: Check whether the simulation accuracy of the BPNN is satisfactory or not by using the testing sample matrix. If yes, then do next. If no, then go to Step 7.
Step 9: Use the checked BPNN as the substituting model for fuzzy probability $P(\tilde{A}_k)$ and solve the established mathematical model by using GA.
Step 10: Check whether the feasibility and the accuracy of the fuzzy-robust design solution obtained in previous step are satisfactory or not by using stochastic simulations or fuzzy-stochastic simulations. If yes, do next. If no, then increase the evolution generations of GA and go to Step 9.
Step 11: Output the design parameters of fuzzy-robust design optimization and terminate the optimization procedure.

When the developed optimization algorithm is used, three issues should be emphasized. Firstly, the BPNN should be well trained and have extensive ability to approximate the substituted models such as $P(\tilde{A}_k)$. Therefore, the learning samples should be representative and sufficient for training BPNN. Secondly, approximation errors exist consequentially because of substituting BPNN for stochastic simulations or fuzzy-stochastic simulations, and then the checkout of feasibility and the accuracy of the obtained solution, in Step 10, are necessary. Thirdly, the developed algorithm, from Step 1 to Step 11, is open to many algorithms for fuzzy-robust optimization. Any effective optimization algorithm, such as ant colony algorithm, can be use as a substitute for the GA in Step 9.

6. Example

6.1. *The formulation of multi-objective optimization for fuzzy-robust design*

For the purpose of comparison, a plastic part which was discussed by Chen (1999) is taken as an example of fuzzy-robust design. The part has two quality characteristics, the first is the ability to resist impact pressure, denoted as g_1 (MPa), and the second is the fluidity of plastic, denoted as g_2(g/10 minutes). The unit of fluidity (g/10 minutes) expresses the gram mass injected in 10 minutes by a special plastic injector for experiment. Through experiments (Chen 1999), it is validated that g_1 and g_2 are the functions of the rotating speed of injector's screw n(r/min) and the technical temperature t(°C). It is expected to find out the optimal parameters n and t subjected to $g_1 \geq 21.3$ (MPa) and $g_2 \geq 52.4$ (g/10 minutes). After factorial experiment (Chen 1999), two response surface models have been established as follows:

$$g_1(x) = 21.531 + 0.612x_1 + 0.626x_2 - 1.142x_1^2 - 1.492x_2^2 + 0.049x_1x_2 \tag{12}$$

$$g_2(x) = 52.030 + 0.021x_1 + 1.329x_2 - 0.500x_1^2 - 0.300x_2^2 + 0.098x_1x_2 \tag{13}$$

where, $x_1 = n - 270/28$; $x_2 = t - 250/14$; $242(r/\min) \leq n \leq 298(r/\min)$; $236(°\mathrm{C}) \leq t \leq 264(°\mathrm{C})$.

The parameters n and t are random variables distributed normally, and have deviations Δn and Δt respectively ($\Delta n = \pm 0.05 \cdot n$, $\Delta t = \pm 0.05 \cdot t$). Since n and t can be adjusted independently, n and t are statistically independent. Obviously, g_1 and g_2 have a larger-the-better characteristic. According to the theories of fuzzy-robust design (Guo 2002, 2004), two fuzzy subsets $\tilde{A}_1$ and $\tilde{A}_2$ are defined, and their membership functions are graphically defined as shown in Figure 2.

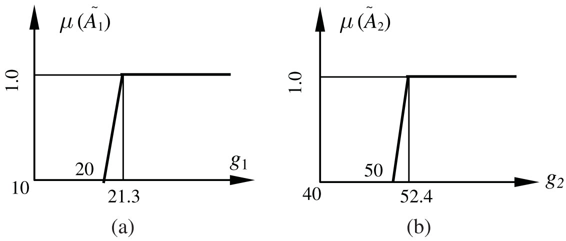

Figure 2. The membership functions of $\tilde{A}_1$ and $\tilde{A}_2$.

In Figure 2(a), the satisfying value of g_1 is 21.3 (MPa) and its acceptable lower limit is 21 (MPa). In Figure 2(b), the satisfying value of g_2 is 52.4 (g/10 minutes) and its acceptable lower limit is 52 (g/10 minutes).

The fuzzy high-quality levels of quality indexes g_1 and g_2 can be calculated by the following equations:

$$P(\tilde{A}_1) = \int_{20}^{+\infty} \mu(\tilde{A}_1) f_1(g_1) dg_1$$

$$P(\tilde{A}_2) = \int_{50}^{+\infty} \mu(\tilde{A}_2) f_2(g_2) dg_2$$

Here, $f_1(g_1)$ and $f_2(g_2)$ are the probability density functions of g_1 and g_2 respectively, and their mathematical expressions are not known. The goals of $P(\tilde{A}_1)$ and $P(\tilde{A}_2)$ are all 1.0.

Because $g_1(x)$ and $g_2(x)$ are coupled by design variables x_1 and x_2, the decoupling of g_1 and g_2 cannot be achieved. Therefore, the formulation of fuzzy-robust design should be modelled as a multi-objective optimization problem according to the method presented in Section 4.3. The compromise decision support problem (DSP) method is an effective method to deal with multi-objective optimization proposed by Bras and Mistree (1993). Now, this method is used to formulate this multi-objective problem as shown in Figure 3. In this figure, two different objectives under the keyword 'minimize' are defined and will be optimized. The first objective is the deviation function $\boldsymbol{D}_1$, and the second objective is the deviation function $\boldsymbol{D}_2$. d_i^- and d_i^+ are the deviation variables of the goals ($i = 1, 2, 3$), which are the measures for the achievement of the goals.

For the deviation function $\boldsymbol{D}_1$, the multi-objective scenario for fuzzy-robust design is modelled using pre-emptive formulation of the deviation variables. Here, $r_i (i = 1, 2, 3)$ are the priority levels which is determined according to the importance of the quality characteristics. Since the ability to resist impact pressure (g_1) determines the mechanical performance of the plastic part, g_1 is the principal quality characteristic to be optimized. The fluidity of plastic (g_2) is considered as a subsidiary quality characteristic since it does not determine the mechanical property of the plastic part and is only an influencing factor on the injecting performance of plastic. Moreover, the 'numerical decoupling' of g_1 and g_2 is wished to be achieved approximately for reducing the coupling of functional requirements. Therefore, the priority of r_1 is higher than that of r_2 and the priority of r_2 is higher than that of r_3. For the deviation function D_2, the multi-objective scenario for fuzzy-robust design is modelled by using the weighted summation of the deviation variables and the three deviation variables are considered to be comparably important. Here, $w_i (i = 1, 2, 3)$ are the weighted coefficients and let be equivalent.

6.2. *Numerical results and discussion of results*

The hybrid intelligent optimization algorithm developed in Section 5 is used to solve the optimization problem. Since n and t are normal variables, x_1 and x_2 are also normal random

Given

(a) $\Delta n = \pm 0.05n$; $\Delta t = \pm 0.05t$

(b) $x_1 = \dfrac{n-270}{28}$; $x_2 = \dfrac{t-250}{14}$

Find

x_1, x_2

Satisfy

(a) $P(\tilde{A}_1) - 1 + d_1^- - d_1^+ = 0$; $(d_1^+ = 0)$

(b) $P(\tilde{A}_2) - 1 + d_2^- - d_2^+ = 0$; $(d_2^+ = 0)$

(c) $CD - 1 + d_3^- - d_3^+ = 0$; $(d_3^+ = 0)$

(d) $x_1, x_2 \in [-1,1]$

Minimize

$\boldsymbol{D}_1 = [r_1(d_1^-),\ r_2(d_3^-),\ r_3(d_2^-)]$

or

$\boldsymbol{D}_2 = w_1 d_1^- + w_2 d_3^- + w_3 d_2^-$

Figure 3. The compromise DSP formulation of the design example.

variables. According to the method for determining the parameters of BPNN discussed by Funahsshi (1989), a BPNN is designed with two hidden layers, two input nodes and two output nodes. This neural network is used as the substitute for $P(\tilde{A}_1)$ and $P(\tilde{A}_2)$. The sample matrix $[\boldsymbol{X}]_S$ is created by repeatedly using stochastic simulations of x_1 and x_2. By means of repeated stochastic simulations of $P(\tilde{A}_1)$ and $P(\tilde{A}_2)$, the sample matrix $[\boldsymbol{P}]_S$ is established. By using $[\boldsymbol{X}]_S$ and $[\boldsymbol{P}]_S$ as the learning samples, the designed BPNN is trained. After checking the simulation accuracy of the trained BPNN, the trained BPNN is placed into the optimization problem shown as in Figure 3 to substitute for $P(\tilde{A}_1)$ and $P(\tilde{A}_2)$. When the deviation function $\boldsymbol{D}_1$ is used as an objective, the compromise DSP can be solved by using the method of lexicographic goal programming (Igzino 1985, Tanino *et al.* 2003). According to the priority levels, the lexicographic minimum of the deviation function $\boldsymbol{D}_1$ is achieved by using the hybrid GA (Guo 2006) which deals with constraints by using the penalty function method, and the first solution of fuzzy-robust design is obtained. When the deviation function $\boldsymbol{D}_2$ is used as an objective, the compromise DSP is solved directly by using the hybrid GA (Guo 2006), and the second solution of fuzzy-robust design is also obtained accordingly. Two solutions of fuzzy-robust design are shown in Table 1.

In Table 1, the former solution, which was determined experientially and was used previously in manufacture, and the first solution of robust design was presented by Chen (1999). The second solution of robust design was presented by Zhang *et al.* (2006). The nominal values of the ability to resist impact pressure (g_1) and the fluidity of colophony (g_2) are directly calculated by using Equation (12) or (13), but their means and variances are obtained through 10^6 stochastic simulations. The mean and variance of the second solution of robust design (Zhang *et al.* 2006) are different from the values in Table 1 since those (Zhang *et al.* 2006) are approximately calculated using the first-order Taylor expansion by considering the standard deviation of n as $\Delta n/3$ and the standard deviation of t as $\Delta t/3$.

For the first solution of fuzzy-robust design as shown in Table 1, the variance of the ability to resist impact pressure (g_1), which is the principal quality characteristic to be optimized, has been reduced significantly, and the mean of g_1 is larger than 21.3 (MPa); but for the old solutions, the means of g_1 are less than 21.3 (MPa) and are not satisfactory. Therefore, the robustness of

Table 1. Comparison between the fuzzy-robust solutions and the old solutions of discussed example.

		Old solutions				
		Former solution	First solution of robust design	Second solution of robust design	First solution of fuzzy-robust design	Second solution of fuzzy-robust design
Design variables	n/(r/min)	250	277	274.3128	271.1382	268.5116
	t/(°C)	240	257	256.2634	252.8502	253.5163
Ability to resist impact pressure (MPa)	Nominal value	19.5878	21.3767	21.4201	21.5459	21.4663
	Mean	19.4319	21.2048	21.2543	21.3819	21.300
	Variance	0.5997	0.1798	0.1479	0.0551	0.0746
Fluidity of plastic (g/10 minutes)	Nominal value	50.7076	52.6057	52.5627	52.2890	52.3410
	Mean	50.6644	52.5654	52.5185	52.2497	52.3000
	Variance	0.2443	0.1302	0.1295	0.13301	0.127
CD		0.2883	0.0364	0.0364	0.6062	0.5430

principal quality characteristic (g_1) has been obviously improved. For the quality characteristic g_2, its robustness has not be improved distinctly since the priority level r_3 about g_2 in Figure 3 is the lowest, and its mean is less than 52.4 (g/10 minutes), but it is larger than its acceptable lower limit. Because g_2 is only a subsidiary quality characteristic and does not determine the mechanical property of the plastic part, this new solution about g_2 can be considered acceptable. Especially, the design matrix of the first solution of robust design $[\boldsymbol{A}]_{\text{old}}$, as an example of old solutions, and the new design matrix of the first fuzzy-robust design solution $[\boldsymbol{A}]_{\text{new}}$ are

$$[\boldsymbol{A}]_{\text{old}} = \begin{bmatrix} 0.0655 & -1.2177 \\ -0.1800 & 1.0535 \end{bmatrix}$$

$$[\boldsymbol{A}]_{\text{new}} = \begin{bmatrix} 0.5291 & -0.3435 \\ 3.014 \times 10^{-4} & 1.2108 \end{bmatrix} \approx \begin{bmatrix} 0.5291 & -0.3435 \\ 0 & 1.2108 \end{bmatrix}$$

Obviously, the absolute values of diagonal elements of $[\boldsymbol{A}]_{\text{new}}$ are larger than those of $[\boldsymbol{A}]_{\text{old}}$, and the absolute values of off-diagonal elements of $[\boldsymbol{A}]_{\text{new}}$ are less than those of $[\boldsymbol{A}]_{\text{old}}$. At the same time, the coupling measurement *CD* of new solution is larger than *CD* of old solutions, which means that the coupling degree of two quality characteristics has been reduced effectively and the independence between g_1 and g_2 is maximized. By using fuzzy-robust design optimization, $[\boldsymbol{A}]_{\text{new}}$ has been made approximately be a diagonal matrix and the 'numerical decoupling' of g_1 and g_2 is achieved approximately. The advantage of the 'numerical decoupling' is that the desired quality characteristics can be easily achieved by adjusting the values of n and t respectively. Therefore, the first fuzzy-robust solution matches with the design axioms documented by Suh and its robustness is better than that of old solutions.

For the second solution of fuzzy-robust design as shown in Table 1, the variances of g_1 and g_2 are smaller than the variances of old solutions; the mean of g_1, which is the principal quality characteristic, is larger than 21.3 (MPa). Therefore, the robustness of g_1 and g_2 have been effectively improved. Since $\boldsymbol{D}_2$, the objective of this solution, is modelled using the weighted summation of the deviation variables, the *CD* of this solution is different from the *CD* of the first solution of fuzzy-robust design, which shows that the 'numerical decoupling' of g_1 and g_2 is not achieved entirely; however, since the *CD* of this solution is larger than the *CD* of old solutions, the independence between g_1 and g_2 has been effectively enhanced.

According to the above discussion, it can be seen that the design robustness and the independence between g_1 and g_2 of the first or the second solution of fuzzy-robust design are superior to that of old solutions as shown in Table 1. Hence, every one of the two new solutions of fuzzy-robust design can be used in the manufacture of the plastic part discussed. Furthermore, since the BPNN is used as a substitute for $P(\tilde{A}_1)$ and $P(\tilde{A}_2)$, the computational efficiency is improved. In this example, the CPU time to calculate the $P(\tilde{A}_1)$ or $P(\tilde{A}_2)$ by using BPNN is only one fifth of that of 10^5 stochastic simulations used.

7. Closure

In engineering design, the study of fuzzy-robust design with multi-quality characteristics is important, and the fuzzy-robust design of single quality characteristic is just its special example. The modelling methods, discussed in the case of uncoupled, decoupled and coupled design, are three typical cases. Those methods accord with the fundamental principles of fuzzy-robust design and the axiomatic design theory (Suh 1990, 2001). For the fuzzy-robust design of multi-quality characteristics with redundancy of design parameters, its fuzzy-robust design model can be transformed into one of the three typical cases discussed previously by predefining some design parameters. The presented example of the fuzzy-robust design with multi-quality characteristics shows that the proposed modelling methods and optimization algorithm are effective and practical. In summary, in our opinion, the methodology proposed in this article appears to have several advantages. Firstly, by using the fuzzy-robust design criterion as shown in Section 2, designers can felicitously deal with the fuzzy information about design robustness of quality characteristics when the mean and variance of quality characteristics are controlled effectively. Secondly, by using the modelling methods proposed in Section 4, the independence between multi-quality characteristics can be enhanced while the variances of quality characteristics decrease, and then the design robustness can be distinctly improved. Thirdly, the computational efficiency of the fuzzy-robust design optimization with multi-quality characteristics can be improved by using the hybrid intelligent optimization algorithm developed in Section 5, and the ability for the fuzzy-robust design method to solve complex engineering problems is improved.

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