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RESEARCH PAPER

Sequential optimization and reliability assessment for multidisciplinary design optimization under aleatory and epistemic uncertainties

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Abstract In engineering design, to achieve high reliability and safety in complex and coupled systems (e.g., Multidisciplinary Systems), Reliability Based Multidisciplinary Design Optimization (RBMDO) has been received increasing attention. If there are sufficient data of uncertainties to construct the probability distribution of each input variable, the RBMDO can efficiently deal with the problem. However there are both Aleatory Uncertainty (AU) and Epistemic Uncertainty (EU) in most Multidisciplinary Systems (MS). In this situation, the results of the RBMDO will be unreliable or risky because there are insufficient data to precisely construct the probability distribution about EU due to time, money, etc. This paper proposes formulations of Mixed Variables (random and fuzzy variables) Multidisciplinary Design Optimization (MVMDO) and a method of MVMDO within the framework of Sequential Optimization and Reliability Assessment (MVMDO-SORA). The MVMDO overcomes difficulties caused by insufficient information for uncertainty. The proposed method enables designers to solve MDO problems in the presence of both AU and EU. Besides, the proposed method can efficiently reduce the computational demand. Examples are used to demonstrate the proposed formulations and the efficiency of MVMDO-SORA.

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Nomenclature

$\mathbf{d}_s$	sharing deterministic design variables for all disciplines
$\mathbf{d}_i$	local deterministic design variables of the ith discipline
$\mathbf{X}_s$	sharing random and fuzzy variables, which are input variables to all disciplines
$\mathbf{X}_i$	local input variables to the ith discipline, composed by random and fuzzy variables
$\mathbf{X}$	a vector of input variables, composed by $\mathbf{X}_i$ $i = 1, \cdots,$ nd
$\mathbf{Y}$	a vector of linking variables with the maximal grade points $\mathbf{Y}^M$
$\mathbf{Y}_{\bullet i}$	a vector of linking variables, input of discipline i
y_{ij}	linking variable, the output of discipline i and input to discipline j
$\mathbf{P}$	a vector of all random and fuzzy parameters
$\mathbf{P}_i$	random and fuzzy parameters of discipline i
$g^{(i)}$	common deterministic constraints in discipline i
nd	total number of disciplines

$\Pi(G > 0)$ possibility of failure with the failure mode defined as $G > 0$

$\Pi(G^{(i)} > 0) \leq \alpha_t$ probability/possibility constraints in discipline i

$\mathbf{X}_s^{*,(i),(k-1)}$ Most Probable/Possible Point (MPPP) corresponding to $\mathbf{X}_s$ in the ith discipline in the $(k-1)$th cycle

$\mathbf{X}_i^{*,(i),(k-1)}$ MPPP of $\mathbf{X}_i$ in the ith discipline of the $(k-1)$th cycle

$\mathbf{P}_i^{*,(i),(k-1)}$ MPPP of $\mathbf{P}_i$ in the ith discipline obtained in the $(k-1)$th cycle

$\mathbf{Y}^*$ a vector of linking variables at the MPPPs, corresponding to probability/possibility constraint

$\mathbf{Y}_{\bullet i}^{*,(i)}$ a vector of linking variables at the MPPPs, corresponding to probability/possibility constraint in discipline i

$G_{\Pi}^{(i)}$ shifted deterministic constraint functions corresponding to probability/possibility constraints of discipline i

α_t allowable target possibility of failure and β_t is equal to $-\mathbf{\Phi}^{-1}(\alpha_t)$

$\mathbf{U}_s^{(i)}$ standard normal random variables in U-space, corresponding to random variables of $\mathbf{X}_s$ in the ith discipline

$\mathbf{V}_s^{(i)}$ standard fuzzy variables in V-space, corresponding to fuzzy variables of $\mathbf{X}_s$

$\mathbf{U}_i^{(i)}$ standard normal random variables in U-space, corresponding to random variables of $\mathbf{X}_i$ in the ith discipline

$\mathbf{V}_i^{(i)}$ standard fuzzy variables in V-space, corresponding to fuzzy variables of $\mathbf{X}_i$

$\mathbf{U}^{(i)}$ all standard normal random variables of all disciplines

$\mathbf{V}^{(i)}$ all standard fuzzy variables of all disciplines

$\mathbf{U}_{\mathbf{P}}^{(i)}$ a vector of standard normal random parameters in U-space, corresponding to all random parameters of $\mathbf{P}$

$\mathbf{V}_{\mathbf{P}}^{(i)}$ a vector of standard fuzzy parameters in V-space, corresponding to all fuzzy parameters of $\mathbf{P}$

$\mathbf{U}_{\mathbf{P}i}^{(i)}, \mathbf{V}_{\mathbf{P}i}^{(i)}$ standard normal random parameters, standard fuzzy parameters of discipline i

$\mathbf{Y}_{\bullet i}^{(i)}$ a vector of linking variables, the input of the ith discipline

1 Introduction

In the last two decades, the consideration of the effect of uncertainty has been one of the focus areas in engineering design. With the goal to achieve high reliability and safety in complex and coupled systems (e.g., multidisciplinary systems) design, the Reliability Based Multidisciplinary Design Optimization (RBMDO) has gained increasing attention (Sues et al. 1995; Sues and Cesare 2000; Koch et al. 2000; Padmanabhan and Batill 2002a, b; Du and Chen 2000, 2005; Du et al. 2008). To replace the computationally expensive simulation models, response surface models created at the system level are used in the reliability analysis for MDO under uncertainty (Sues et al. 1995). Within the RBMDO framework presented by Sues and Cesare (2000), the reliability analysis is decoupled from the optimization loop. Reliabilities are initially computed before the first execution of the optimization loop and updated after accomplishing the optimization loop during which approximations of reliability constraints are employed. In Koch et al. (2000), a multi-stage parallel implementation of probabilistic design optimization is utilized with the aim to integrate existing reliability analysis method into MDO framework. To search the Most Probable Point (MPP), concurrent subsystem optimization techniques were used in Padmanabhan and Batill (2002a, b) and Du and Chen (2000) and collaborative reliability analysis method was proposed in Du and Chen (2005) and Du et al. (2008). In Du et al. (2008), a Sequential Optimization and Reliability Assessment (SORA) method for the RBMDO was proposed. The main idea of the SORA method is to decouple the reliability analysis from the design optimization. By using the Most Probable Point (MPP) obtained from the previous iteration, constraints in the deterministic optimization are modified to make sure the MPP of current iteration falling into the deterministic feasible region. After solving the deterministic optimization, a new design solution is obtained. The feasibilities of probabilistic constraints will then be checked by reliability analysis at the new design point. In many cases, the whole process of solution will be convergent in few cycles (Du et al. 2008; Du and Chen 2004).

However in the design of most multidisciplinary systems, both aleatory and epistemic uncertainties exist. The aleatory uncertainty (stochastic uncertainty, irreducible uncertainty, inherent uncertainty, variability) can be modeled with the probability theory, and variables with aleatory uncertainty can be treated as random variables. The epistemic uncertainty (reducible uncertainty, subjective uncertainty), which is caused by lack of knowledge, can be modeled by possibility theory (Agarwal et al. 2004; Oberkampf et al. 2000; Youn et al. 2005; Du and Choi 2008). In single disciplinary design, Reliability Based Design Optimization (RBDO) is commonly utilized. When there are insufficient data to construct the precise statistical distribution of input with uncertainty (especially epistemic uncertainty) due to time, money, etc, the results of the probabilistic method will be unreliable or risky in this case. Since improper modeling of uncertainty could cause greater degree of statistical uncertainty than those of the physical uncertainty (Youn et al. 2005): 92% reliability with imprecise statistical data turns out to be 77 reliability with precise statistical information when the performance function is a linear model; system nonlinearity and smaller amount of data will increase the error associated with the reliability level. With the possibility theory, fuzzy variables are utilized to represent epistemic uncertainties (uncertainties with insufficient data) (Du and Choi 2008). It has been pointed out that when little information is available for input data, the possibility based method is better as it provides a more conservative design than the probabilistic design that is consistent with the limited available information (Youn et al. 2005). To deal with both types of uncertainties in single system design, Mixed Variables Design Optimization (MVDO) is proposed in Du and Choi (2008). This method is applied in design where only one discipline is involved. However, for MS, both formulations and solutions become much more complex. Few works have been done in MDO design when both types of uncertainties are associated with design inputs. In this paper, a formulation of MVMDO is proposed and a MVMDO-SORA method is developed to solve MDO problems involving both AU and EU.

This paper is organized as follows. In Section 2, Multidisciplinary Design Optimization (MDO) and Mixed Variables Design Optimization (MVDO) are briefly reviewed. The formulation of the MVMDO, the strategy, procedure, and formulation of the MVMDO-SORA are introduced and explained in detail in Section 3. In Section 4, examples are utilized to demonstrate the proposed formula and the efficiency of the MVMDO-SORA, followed by the conclusions in Section 5.

2 Briefly review of multidisciplinary design optimization (MDO) and mixed variables design optimization (MVDO)

2.1 Multidisciplinary design optimization (MDO)

The formulation of MDO is given as:

$$\begin{aligned} \min \quad & f\left(\mathbf{X}'', \mathbf{Y}''\right) \\ s.t. \quad & g^{(i)}\left(\mathbf{X}''_s, \mathbf{X}''_i, \mathbf{Y}''_{\bullet i}\right) \leq 0 \\ & h^{(i)}\left(\mathbf{X}''_s, \mathbf{X}''_i, \mathbf{Y}''_{\bullet i}\right) = 0 \\ & \mathbf{Y}''_{i\bullet} = \mathbf{Y}''_{i\bullet}\left(\mathbf{X}''_s, \mathbf{X}''_i, \mathbf{Y}''_{\bullet i}\right) \\ & i = 1, 2, \cdots, \mathrm{nd} \end{aligned} \tag{1}$$

where $\mathbf{X}'' = \left(\mathbf{X}''_s, \mathbf{X}''_1, \mathbf{X}''_2, \cdots, \mathbf{X}''_{\mathrm{nd}}\right)^T$ is a vector of design variables, $\mathbf{X}''_s$ represents a vector of sharing variables, and $\mathbf{X}''_i$ are local input variables to discipline i. $Y'' = (\mathbf{Y}''_{1\bullet}, \mathbf{Y}''_{2\bullet}, \cdots, \mathbf{Y}''_{(\mathrm{nd})\bullet})^T$ stands for a vector of linking variables, $Y''_{i\bullet} = (y''_{ij};\ \ j \neq i,\ \ j = 1, 2, \cdots, \mathrm{nd})$ refers to a vector of outputs obtained from the ith discipline, and y''_{ij} denotes the output of discipline i and the input to discipline j. $\mathbf{Y}''_{\bullet i}$ is a vector of input linking variables to the ith discipline. $f(\cdot)$ is the objective function, $g^{(i)}(\cdot)$ are inequality constraint functions and $h^{(i)}(\cdot)$ are equality constraint functions in discipline i. nd is the total number of disciplines.

A MDO problem with three disciplines is illustrated in Fig. 1. During the optimization, evaluations of the cost function and constraints require the multidisciplinary analysis as shown by the dashed box because each function has the common component $\mathbf{Y}''$ which maintains the consistencies among multiple disciplines. For example, when evaluating y''_{12} or y''_{13} in discipline 1, y''_{31} is an input. y''_{31} is a function of y''_{13}, but y''_{13} is also the result of discipline 1 (Du and Chen 2005).

2.2 Mixed variables design optimization (MVDO)

When design with both random and fuzzy variables, the formulation based on the Performance Measure Approach (PMA) (Du and Choi 2008) is:

$$\begin{aligned} \min_{(\mathbf{d}', X'^M)} \quad & f\left(\mathbf{d}', \mathbf{X}'^M, \mathbf{P}'^M\right) \\ s.t. \quad & G_{\Pi i}\left(\mathbf{d}', \mathbf{X}', \mathbf{P}'\right) \leq 0, i = 1, 2, \cdots, \mathrm{nc} \\ & \mathbf{d}'^L \leq \mathbf{d}' \leq \mathbf{d}'^U \\ & \mathbf{X}'^{M,L} \leq \mathbf{X}'^M \leq \mathbf{X}'^{M,U} \end{aligned} \tag{2}$$

where $G_{\Pi i}\left(\mathbf{d}', \mathbf{X}', \mathbf{P}'\right)$ stands for the value of the ith constraint function at the most probable/possible point (MPPP). $\mathbf{d}'$ is a vector of deterministic design variables. $\mathbf{X}' = (\mathbf{X}'_r, \mathbf{X}'_f) = [X'_i]^T \in R^{\mathrm{nr}+\mathrm{nf}}$ refers to a vector

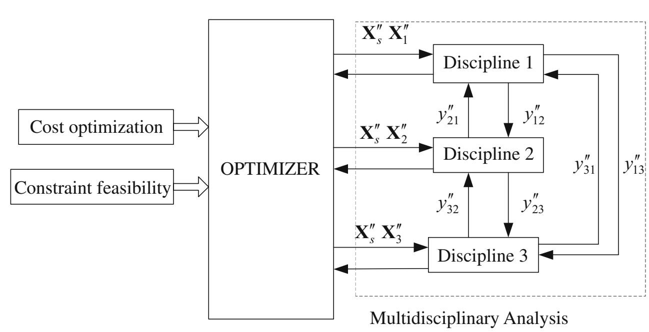


Fig. 1 Flowchart of MDO

of random and fuzzy variables, where each random variable X'_i has the probability density function $f_{X'_i}\left(x'_i\right)$ with the mean value $X'^M_i = E\left(X'_i\right)$, and each fuzzy variable X'_i has the membership function $\Pi_{X'_i}\left(x'_i\right)$ with the maximal grade point $X'^M_i = \{x' | \max\{\Pi_{X'_i}(x'_i)\}\}$, $\mathbf{X}'^M = [X'^M_i]^T \in R^{\mathrm{nr+nf}}$ is also a design vector. $\mathrm{P}' = (\mathbf{P}'_r, \mathbf{P}'_f) = \left[P'_i\right]^T \in R^{\mathrm{npr+npf}}$ stands for a vector of random and fuzzy parameters. nr, nf, nc, npr and npf are numbers of random variables, fuzzy variables, constraints, random parameters and fuzzy parameters, respectively.

To calculate the value of $G_{\Pi i}\left(\mathbf{d}', \mathbf{X}', \mathbf{P}'\right)$, all random variables and parameters need to be transformed into a set of independent standard normal random ones $\mathbf{U}$ using the Rosenblatt transformation (Du et al. 2008). All fuzzy variables and parameters need to be transformed into the standard fuzzy ones $\mathbf{V}$. The standard fuzzy variable has the isosceles triangular membership function as:

$$\Pi_{V_i}(v_i) = \begin{cases} v_i + 1 & -1 \le v_i \le 0 \\ 1 - v_i & 0 \le v_i \le 1 \end{cases} = 1 - |v_i|, \quad |v_i| \le 1 \tag{3}$$

This transformation can be written as:

$$V_i = \begin{cases} \Pi_{X_i}(X_i) - 1 & X_i \le X_i^M \\ 1 - \Pi_{X_i}(X_i) & X_i > X_i^M \end{cases} \tag{4}$$

where X_i^M is the maximal grade point of the membership function (Youn et al. 2005).

A single-loop optimization to find ($\mathbf{u}$, $\mathbf{v}$) can be formulated as

$$\begin{aligned} &\max_{(\mathbf{u},\mathbf{v})} G_i(\mathbf{u}, \mathbf{v}) \\ &s.t. \quad \|\mathbf{u}\|_2 \le \beta_t \\ &\qquad \|\mathbf{v}\|_\infty \le 1 - \alpha_t \end{aligned} \tag{5}$$

where α_t is the allowable possibility of failure. β_t is equal to $-\Phi^{-1}(\alpha_t)$ and Φ stands for the cumulated probability function of the standard normal distribution. The solutions are the MPPP ($\mathbf{u}^*$, $\mathbf{v}^*$) and performance measure at the MPPP $G_i(\mathbf{u}^*, \mathbf{v}^*)$ (Du and Choi 2008).

3 Mixed variables multidisciplinary design optimization in the framework of SORA (MVMDO-SORA)

The SORA method is originally developed for Reliability Based Design Optimization (RBDO) in Du and Chen (2004) and is introduced into MDO in Du et al. (2008). In this paper, adopting the basic ideas of SORA, SORA is further developed to solve MDO problems in which both aleatory uncertainty and epistemic uncertainty are associated with design variables and parameters. The method is called Mixed Variables Multidisciplinary Design Optimization within the framework of SORA (MVMDO-SORA). In this section, the MVMDO-SORA is presented in detail including associated strategy, procedure and formulations generated in MVMDO-SORA.

3.1 Strategy

To efficiently solve MVMDO problem, two key technologies are adopted.

1. Performance Measure Approach (PMA). In RBDO, PMA is more efficient than evaluating the actual probability directly. Some non-active probability constraints will dominate the whole computational process in directly evaluating their actual probability which results in low computational efficiency (Du et al. 2008). PMA is also effective for possibility based design optimization (Youn et al. 2005; Du and Choi 2008). Meanwhile utilizing the PMA, the probability or possibility requirement of failure is initially set to be an acceptable value and treated as constraint. The maximal value of probability or possibility constraint function (performance measure at most probable or possible point) is calculated. The probability or possibility requirement is achieved when the value is not larger than zero.
2. Sequential Optimization and Reliability Assessment (SORA). In this paper, with the basic idea of SORA, the solution process of MDO under aleatory and epistemic uncertainties is decoupled into sequential cycles of deterministic MDO and probability/possibility analysis. In each cycle, the probability/possibility analysis follows the solution of deterministic MDO. After solving the deterministic MDO, the maximal grade point of each fuzzy design variable and the mean value of each random variable are obtained. Then probability/possibility analysis is applied to analyze the feasibility of each probability/possibility constraint at the current design point. Based on this, in each iteration cycle, the optimization problem and the probability/possibility analysis are not nested but sequential. So the efficiency will be improved obviously.

3.2 Formulation of MVMDO

The formulation of Mixed Variables (random and fuzzy variables) Multidisciplinary Design Optimization (MVMDO) is given as:

$$\begin{aligned}
&\min_{(\mathbf{d}_s,\mathbf{d},\mathbf{X}_s^M,\mathbf{X}^M)} f\left(\mathbf{d}_s,\mathbf{d},\mathbf{X}_s^M,\mathbf{X}^M,\mathbf{P}^M\right)\\
&s.t.\ \Pi\left(G^{(i)}\left(\mathbf{d}_s,\mathbf{d}_i,\mathbf{X}_s,\mathbf{X}_i,\mathbf{P}_i,\mathbf{Y}_{\bullet i}\right)>0\right)\le\alpha_t\\
&g^{(i)}\left(\mathbf{d}_s,\mathbf{d}_i,\mathbf{X}_s^M,\mathbf{X}_i^M,\mathbf{P}_i^M,\mathbf{Y}_{\bullet i}^M\right)\le 0\\
&\mathbf{d}_s^L\le\mathbf{d}_s\le\mathbf{d}_s^U,\quad \mathbf{d}^L\le\mathbf{d}\le\mathbf{d}^U\\
&\mathbf{X}_s^{M,L}\le\mathbf{X}_s^M\le\mathbf{X}_s^{M,U},\mathbf{X}^{M,L}\le\mathbf{X}^M\le\mathbf{X}^{M,U}\\
&i=1,2,\cdots,\mathrm{nd}
\end{aligned}\tag{6}$$

where the superscript M denotes the mean value of a random variable and parameter or the maximal grade point of a fuzzy variable and parameter. $\mathbf{d}_s$ are sharing deterministic design variables for all disciplines. $\mathbf{d}_i$ denote local deterministic design variables of the ith discipline. $\mathbf{X}_s$ refer to sharing random and fuzzy variables, which are input variables to all disciplines. $\mathbf{X}_i$ composed of random and fuzzy variables is a vector of local input variables to the ith discipline. $\mathbf{X}$ is composed of $\mathbf{X}_i$, $i=1,2,\cdots,\mathrm{nd}$. $\mathbf{Y}$ is a vector of linking variables and $\mathbf{Y}^M$ are maximal grade points of $\mathbf{Y}$. $\mathbf{Y}_{\bullet i}$, a vector of linking variables, is the input of discipline i. y_{ij} is the output of discipline i and the input to discipline j. $\mathbf{P}$ represents a vector of all random and fuzzy parameters and $\mathbf{P}_i$ denotes a vector of random and fuzzy parameters in discipline i. $g^{(i)}$ denote common deterministic constraints in discipline i. nd is the total number of disciplines. $\Pi(G>0)$ denotes the possibility of failure with the failure mode defined as $G>0$. $\Pi\left(G^{(i)}>0\right)\le\alpha_t$ are probability/possibility constraints in discipline i.

3.3 Procedure

The procedure of the MVMDO-SORA includes the following steps:

Step 1: Set initial values for design variables $\mathbf{d}_s^{(0)}$, $\mathbf{d}^{(0)}$, $\mathbf{X}_s^{M,(0)}$, $\mathbf{X}^{M,(0)}$; $k=1$.

Step 2: Solve the deterministic MDO problem. The purpose of solving the deterministic MDO is to get the mean value or maximal grade point of each variable $\left(\mathbf{X}_s^{M,(k)},\mathbf{X}^{M,(k)}\right)$, as well as the value of each deterministic variable $\left(\mathbf{d}_s^{(k)},\mathbf{d}^{(k)}\right)$.

In the first cycle, the design problem is treated as a deterministic MDO problem without considering uncertainties. Because there is no information about MPPPs, the values of MPPPs are set to be equal to $\mathbf{X}_s^{M,(0)}$, $\mathbf{X}^{M,(0)}$ and $\mathbf{P}^M$. Variables in deterministic constraints are mean values of random variables, maximal grade points of fuzzy variables and deterministic variables.

From the second cycle, constraints in the deterministic MDO are modified with the MPPPs information from the previous cycle when requirements of probability/possibility constraints are not all satisfied.

Step 3: Probability/possibility analysis. First, all random variables and parameters are transformed into the independent standard normal random ones $\mathbf{U}$ using the Rosenblatt transformation. All fuzzy variables and parameters are transformed into the standard fuzzy ones $\mathbf{V}$ using (4) based on the value $\mathbf{X}_s^{M,(k)}$ and $\mathbf{X}^{M,(k)}$. Then the MPPP can be obtained after probability/possibility analysis. In the formulations of the probability/possibility analysis, consistencies between disciplines are treated as extra constraints as in Du et al. (2008).

Step 4: Check convergence. If requirements of probability/possibility constraints are all satisfied and the value of objective function is stable: $\left(G^{(i)}\le 0, i=1\sim\mathrm{nd}; |f(k)-f(k-1)|\le\varepsilon\right)$ where ε is an arbitrary small positive constant, stop the process of solution; otherwise set $k=k+1$ and go to Step 2 with the MPPPs obtained in Step 3.

If requirement of probability/possibility constraint $\Pi(G^{(i)}>0)\le\alpha_t$ is not satisfied in Cycle $k-1$ (the performance measure at its MPPP satisfies $G^{(i)}>0$), then the MPPP $\left(\mathbf{X}_s^{*,(i),(k-1)},\ \mathbf{X}^{*,(i),(k-1)},\ \mathbf{P}^{*,(i),(k-1)}\right)$ obtained from probability/possibility analysis in Cycle $k-1$ will be used to modify the constraint in the kth deterministic MDO. To make sure the feasibility of probability/possibility constraint, the MPPP should fall into the deterministic feasible region. Let $\mathbf{S}$ be the shift vector.

The shift is based on the idea of SORA in Du and Chen (2004) as:

$$\begin{aligned}
&\mathbf{S}_s^{(i)}=\mathbf{X}_s^{M,(k-1)}-\mathbf{X}_s^{*,(i),(k-1)}\\
&\mathbf{S}_j^{(i)}=\mathbf{X}_j^{M,(k-1)}-\mathbf{X}_j^{*,(i),(k-1)}\\
&i,j=1,2,\cdots,\mathrm{nd}
\end{aligned}\tag{7}$$

where $\mathbf{X}_s^{*,(i),(k-1)}$ is the MPPP corresponding to $\mathbf{X}_s$ in discipline i of the $(k-1)$th cycle; $\mathbf{X}_j^{*,(i),(k-1)}$ denotes the MPPP of $\mathbf{X}_j$ in discipline i of the $(k-1)$th cycle. $S_s^{(i)}$ and $S_j^{(i)}$ are shift vectors of $\mathbf{X}_s$ and $\mathbf{X}_j$ in discipline i, respectively.

Values of the MPPPs of parameters directly substitute those of $\mathbf{P}$ in $G^{(i)}$. The deterministic constraint in the kth deterministic MDO is modified as: $G_\Pi^{(i)}\left(\mathbf{d}_s, \mathbf{d}_i, \mathbf{X}_s^M - \mathbf{S}_s^{(i)}, \mathbf{X}_i^M - \mathbf{S}_i^{(i)}, \mathbf{P}_i^{*,(i),(k-1)}, \mathbf{Y}_{\bullet i}^{*,(i)}\right) \leq 0.$

3.4 Deterministic MDO of the kth cycle

The deterministic MDO of the kth Cycle is given as:

$$\begin{aligned}
&\min_{(\mathbf{d}_s,\mathbf{d},\mathbf{X}_s^M,\mathbf{X}^M,\mathbf{Y}^M,\mathbf{Y}^*)} f\left(\mathbf{d}_s, \mathbf{d}, \mathbf{X}_s^M, \mathbf{X}^M, \mathbf{P}^M, \mathbf{Y}^M\right)\\
&s.t.\ G_\Pi^{(i)}\left(\mathbf{d}_s, \mathbf{d}_i, \mathbf{X}_s^M - \mathbf{S}_s^{(i)}, \mathbf{X}_i^M - \mathbf{S}_i^{(i)}, \mathbf{P}_i^{*,(i),(k-1)}, \mathbf{Y}_{\bullet i}^{*,(i)}\right) \leq 0\\
&g^{(i)}\left(\mathbf{d}_s, \mathbf{d}_i, \mathbf{X}_s^M, \mathbf{X}_i^M, \mathbf{P}_i^M, \mathbf{Y}_{\bullet i}^M\right) \leq 0\\
&i = 1, 2, \cdots, \mathrm{nd}\\
&y_{ij}^M = y_{ij}\left(\mathbf{d}_s, \mathbf{d}_i, \mathbf{X}_s^M, \mathbf{X}_i^M, \mathbf{P}_i^M, \mathbf{Y}_{\bullet i}^M\right) i,\ j = 1, 2, \cdots, \mathrm{nd},\ i \neq j\\
&y_{jm}^{*,(i)} = y_{jm}^{*,(i)}\left(\mathbf{d}_s, \mathbf{d}_j, \mathbf{X}_s^M - \mathbf{S}_s^{(i)}, \mathbf{X}_j^M - \mathbf{S}_j^{(i)}, \mathbf{P}_j^{*,(i),(k-1)}, \mathbf{Y}_{\bullet j}^{*,(i)}\right)\\
&i,\ j, m = 1, 2, \cdots, \mathrm{nd},\ j \neq m\\
&\mathbf{d}_s^L \leq \mathbf{d}_s \leq \mathbf{d}_s^U,\ \mathbf{d}^L \leq \mathbf{d} \leq \mathbf{d}^U\\
&\mathbf{X}_s^{M,L} \leq \mathbf{X}_s^M \leq \mathbf{X}_s^{M,U}, \mathbf{X}^{M,L} \leq \mathbf{X}^M \leq \mathbf{X}^{M,U} \qquad (8)
\end{aligned}$$

where $\mathbf{S}_s^{(i)}, \mathbf{S}_i^{(i)}, \mathbf{S}_j^{(i)}$ are shift vectors of $\mathbf{X}_s$, $\mathbf{X}_i$ and $\mathbf{X}_j$ in discipline i, respectively. $\mathbf{Y}^*$ represents a vector of linking variables at the MPPPs, corresponding to probability/possibility constraints. $\mathbf{Y}_{\bullet i}^{*,(i)}$ corresponds to the probability/possibility constraint in discipline i. $\mathbf{P}_i^{*,(i),(k-1)}$ refers to the MPPP of $\mathbf{P}_i$ in the ith discipline obtained in the $(k-1)$th cycle. $G_\Pi^{(i)}$ stand for shifted deterministic constraint functions corresponding to probability/possibility constraints in discipline i.

The equality constraints for achieving consistencies among disciplines are modified with $\mathbf{X}_s^{*,(i),(k-1)}$, $\mathbf{X}_j^{*,(i),(k-1)}$, $\mathbf{P}_j^{*,(i),(k-1)}$ as $y_{jm}^{*,(i)} = y_{jm}^{*,(i)}\big(\mathbf{d}_s, \mathbf{d}_j, \mathbf{X}_s^M - \mathbf{S}_s^{(i)}$, $\mathbf{X}_j^M - S_j^{(i)},\ \mathbf{P}_j^{*,(i),(k-1)}, \mathbf{Y}_{\bullet j}^{*,(i)}\big)$, $i,\ j, m = 1, 2, \cdots, \mathrm{nd},\ j \neq m$. $\mathbf{S}_s^{(i)}, \mathbf{S}_j^{(i)}$ are shift vectors obtained from (7) respectively corresponding to X_s and X_j in discipline i. $\mathbf{X}_j^{*,(i),(k-1)}$ and $\mathbf{P}_j^{*,(i),(k-1)}$ are MPPPs obtained in the $(k-1)$th cycle relevant to $\mathbf{X}_j$ and $\mathbf{P}_j$ in discipline i.

3.5 Formula of probability/possibility analysis under the environment of MDO

The formulation of probability/possibility analysis in the environment of MDO is:

$$\begin{aligned}
&\max_{\substack{\left(\mathbf{U}_s^{(i)}, \mathbf{V}_s^{(i)}, \mathbf{U}^{(i)},\right.\\ \left.\mathbf{V}^{(i)}, \mathbf{U}_{\mathbf{P}}^{(i)}, \mathbf{V}_{\mathbf{P}}^{(i)}, \mathbf{Y}^{(i)}\right)}} G^{(i)}\left(\mathbf{d}_s, \mathbf{d}_i, \mathbf{U}_s^{(i)}, \mathbf{V}_s^{(i)}, \mathbf{U}_i^{(i)}, \mathbf{V}_i^{(i)}, \mathbf{U}_{\mathbf{P}i}^{(i)}, \mathbf{V}_{\mathbf{P}i}^{(i)}, \mathbf{Y}_{\bullet i}^{(i)}\right)\\
&s.t.\ \left\|\left(\mathbf{U}_s^{(i)}, \mathbf{U}^{(i)}, \mathbf{U}_{\mathbf{P}}^{(i)}\right)\right\|_2 \leq \beta_t\\
&\left\|\left(\mathbf{V}_s^{(i)}, \mathbf{V}^{(i)}, \mathbf{V}_{\mathbf{P}}^{(i)}\right)\right\|_\infty \leq 1 - \alpha_t \qquad (9)\\
&y_{jm}^{(i)} = y_{jm}^{(i)}\left(\mathbf{d}_s, \mathbf{d}_j, \mathbf{U}_s^{(i)}, \mathbf{V}_s^{(i)}, \mathbf{U}_j^{(i)}, \mathbf{V}_j^{(i)}, \mathbf{U}_{\mathbf{P}j}^{(i)}, \mathbf{V}_{\mathbf{P}j}^{(i)}, \mathbf{Y}_{\bullet j}^{(i)}\right)\\
&i = 1, 2, \cdots, \mathrm{nd};\ j = 1, 2, \cdots, \mathrm{nd};\\
&m = 1, 2, \cdots, \mathrm{nd};\ \ j \neq m
\end{aligned}$$

where α_t is the allowable target possibility of failure. β_t is equal to $-\Phi^{-1}(\alpha_t)$. $\mathbf{U}_s^{(i)}$ and $\mathbf{U}_i^{(i)}$ refer to the standard normal random variables in U-space respectively corresponding to random variables of $\mathbf{X}_s$ and $\mathbf{X}_i$ in discipline i. $\mathbf{V}_s^{(i)}$ and $\mathbf{V}_i^{(i)}$ denote standard fuzzy variables in V-space respectively corresponding to fuzzy variables of $\mathbf{X}_s$ and $\mathbf{X}_i$ in discipline i. $\mathbf{U}^{(i)}$ and $\mathbf{V}^{(i)}$ include all standard normal random variables and standard fuzzy variables of all disciplines, respectively. $\mathbf{U}_{\mathbf{P}}^{(i)}$ is a vector of standard normal random parameters in U-space corresponding to all random parameters of $\mathbf{P}$. $\mathbf{V}_{\mathbf{P}}^{(i)}$ denotes a vector of standard fuzzy parameters in V-space corresponding to all fuzzy parameters of $\mathbf{P}$. $\mathbf{U}_{\mathbf{P}i}^{(i)}$ and $\mathbf{V}_{\mathbf{P}i}^{(i)}$ are standard normal random and standard fuzzy parameters of discipline i, respectively. $\mathbf{Y}_{\bullet i}^{(i)}$, a vector of linking variables, is the input of the ith discipline.

It is implied that the probability/possibility constraint function includes all design inputs because of the existences of linking variables. In (9), the first and second constraints include all design inputs associated with uncertainties.

Solutions of probability/possibility analysis are MPPP $\left(\mathbf{U}_s^{*,(i)}, \mathbf{V}_s^{*,(i)}, \mathbf{U}^{*,(i)}, \mathbf{V}^{*,(i)}, \mathbf{U}_{\mathbf{P}}^{*,(i)}, \mathbf{V}_{\mathbf{P}}^{*,(i)}\right)$, linking variables $y_{jm}^{*,(i)}$ at MPPP and performance measure at MPPP. The MPPP in X-space can be obtained using the inverse Rosenblatt transformation and the inverse (4). The MPPPs are used to construct the deterministic MDO formulation for the next cycle if requirements of probability/possibility constraints are not all satisfied.

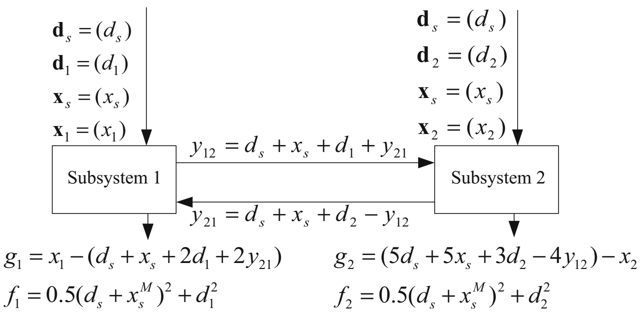


Fig. 2 Mathematical problem

4 Numerical examples

In this section, a mathematical problem is firstly used to demonstrate the proposed formulation and method in detail. The method proposed in this paper and the RBMDO-SORA is compared based on the obtained results. The efficiency of the proposed method is demonstrated through results. An engineering design example is subsequently provided.

4.1 A mathematical example for MVMDO

Modified from Du et al. (2008), a mathematical example is given as:

$$\min_{(d_s,d_1,d_2)} \; f\left(\mathbf{d},\mathbf{x}^M\right) = \left(d_s + x_s^M\right)^2 + d_1^2 + d_2^2$$

$$s.t. \quad \Pi\left\{G_1\left(\mathbf{d},\mathbf{x}\right) = x_1 - d_s - x_s - d_1 - d_2 > 0\right\} \le \alpha_t$$

$$\Pi\left\{G_2\left(\mathbf{d},\mathbf{x}\right) = d_s + x_s - 2d_1 + d_2 - x_2 > 0\right\} \le \alpha_t$$

$$0 \le d_s, d_1, d_2 \le 5 \tag{10}$$

where $x_s \sim N(0{,}0.3)$, $x_1 \sim N(5{,}0.5)$. $N(\mu, \sigma)$ stands for a normal distribution with mean value (μ) and standard deviation (σ). The triangular membership function of x_2 is (0.7,1,1.3). In ($x^M - dt$, x^M, $x^M + dt$), x^M is the maximal grade point of membership function of x; the value dt is the deviation of each side from the maximal grade point. The problem is decomposed into two subsystems as in Fig. 2 in the same way as that in Du et al. (2008).

It should be noted in this problem, x_s, x_1 and x_2 are design parameters but not design variables because the mean values and maximal grade point are all fixed.

The formulation of the deterministic MDO is given by:

$$\min_{\substack{(d_s, d_1, d_2),\, (y_{12}^M, y_{21}^M) \\ \left(y_{12}^{*,(1)}, y_{21}^{*,(1)}\right), \left(y_{12}^{*,(2)}, y_{21}^{*,(2)}\right)}} f = (f_1 + f_2) = \left(d_s + x_s^M\right)^2 + d_1^2 + d_2^2$$

$$s.t. \; G_1 = x_1^{*,(1)} - \left(d_s + x_s^{*,(1)} + 2d_1 + 2y_{21}^{*,(1)}\right) \le 0$$

$$y_{12}^{*,(1)} = d_s + x_s^{*,(1)} + d_1 + y_{21}^{*,(1)}$$

$$y_{21}^{*,(1)} = d_s + x_s^{*,(1)} + d_2 - y_{12}^{*,(1)}$$

$$G_2 = \left(5d_s + 5x_s^{*,(2)} + 3d_2 - 4y_{12}^{*,(2)}\right) - x_2^{*,(2)} \le 0$$

$$y_{12}^{*,(2)} = d_s + x_s^{*,(2)} + d_1 + y_{21}^{*,(2)}$$

$$y_{21}^{*,(2)} = d_s + x_s^{*,(2)} + d_2 - y_{12}^{*,(2)}$$

$$0 \le d_s, d_1, d_2 \le 5 \tag{11}$$

The optimal point (d_s, d_1, d_2) is then used in the probability/possibility analysis. At first, all random parameters are transformed into independent standard normal random parameters and all fuzzy parameters into standard fuzzy parameters.

The formulation for searching the MPPP of G_1 is given as:

$$\max_{\substack{\left(u_s^{(1)}, u_1^{(1)}, v_2^{(1)}\right) \\ \left(y_{12}^{(1)}, y_{21}^{(1)}\right)}} G_1 = \left(x_1^M + u_1^{(1)}\sigma_1\right) - \left[d_s + \left(x_s^M + u_s^{(1)}\sigma_s\right) + 2d_1 + 2y_{21}^{(1)}\right]$$

$$s.t. \; \left\|\left(u_s^{(1)}, u_1^{(1)}\right)\right\|_2 \le \beta_t$$

$$\left\|v_2^{(1)}\right\|_\infty \le 1 - \alpha_t \tag{12}$$

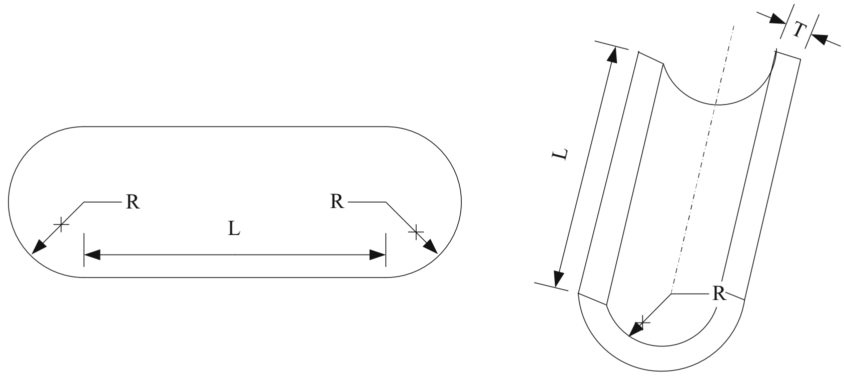
$$y_{12}^{(1)} = d_s + \left(x_s^M + u_s^{(1)}\sigma_s\right) + d_1 + y_{21}^{(1)}$$

$$y_{21}^{(1)} = d_s + \left(x_s^M + u_s^{(1)}\sigma_s\right) + d_2 - y_{12}^{(1)}$$

Table 1 Results of RBMDO and MVMDO

	Design variables			Objective			Number		
	d_s	d_1	d_2	f	G_1	G_2	n_1	n_2	k
RBMDO ($\beta = 3$)	2.2497	2.2498	2.2498	15.1843	0	−0.0513	451	635	
MVMDO ($\beta_t = 3$, $\alpha_t = 0.0013$)	2.2165	2.3163	2.2165	15.1909	1.4211×10^{-13}	0	522	522	3
($\beta_t = 4$, $\alpha_t = 0$)	2.3608	2.6108	2.3608	17.9629	1.3802×10^{-12}	0	522	522	3
($\beta_t = 2$, $\alpha_t = 0.0228$)	2.0554	2.0554	2.0554	12.6740	1.7764×10^{-15}	−0.1068	529	529	3

Fig. 3 Pressure vessel



The solution MPPP $\left(u_s^{*,(1)}, u_1^{*,(1)}, v_2^{*,(1)}\right)$ to G_1 is then transformed into the MPPP $\left(x_s^{*,(1)}, x_1^{*,(1)}, x_2^{*,(1)}\right)$ in the X-space. The formulation for searching the MPPP of G_2 $\left(x_s^{*,(2)}, x_1^{*,(2)}, x_2^{*,(2)}\right)$ can be derived in the same way. The MPPPs will be used to reconstruct the deterministic constraints of MDO for the next cycle if requirements of probability/possibility constraints are not all satisfied.

The judgment of convergence is $G_i \leq 0$, $i=1\sim 2$ and $|f(k) - f(k-1)| \leq 0.0001$. The results of RBMDO with $x_s \sim N(0,0.3)$, $x_1 \sim N(5,0.5)$ and $x_2 \sim N(1,0.1)$ solved by SORA in Du et al. (2008) and MVMDO are listed in Table 1. The results with different α_t and β_t are also listed in the same table.

The starting point is (0,0,0). The second row lists the results of RBMDO from Du et al. (2008). n_1 is the number of disciplinary analyses for subsystem 1 and n_2 is for subsystem 2. When the target possibility of failure is set the same as the target probability of failure $(1 - \Phi(3) = 0.0013 = \alpha_t)$, MVMDO obtains a more conservative design. But the number of disciplinary analyses is in the same order of magnitude as that in RBMDO. MVMDO-SORA efficiently solves the MVMDO problem in three cycles. Based on the values of objective function f in the fifth column, the less the possibility of failure is, the more conservative the design will be. The case of $\beta_t = 4$ and $\alpha_t = 0$ $(1 - \Phi(4) = 0 = \alpha_t)$ proposes the most conservative design.

Table 2 Nomenclature of the pressure vessel design problem

W	Weight of the pressure vessel
V	Volume, in.^{3}
R	Radius, in.
T	Thickness, in.
L	Length, in.
P	Pressure internal the cylinder, Klb
S_t	Allowable tensile strength of the cylinder material, Klb
σ_{circ}	Circumference stress

4.2 Design of a pressure vessel

The second example is the design of a pressure vessel showed in Fig. 3. This example is derived from Lewis and Mistree (1997), in which this problem is solved in a multi-player formulation based on game theory. The nomenclature of this example is showed in Table 2. The design variables are radius (R), length (L) and thickness (T). There are two design parameters: the internal pressure (P) and the allowable tensile strength of the material (S_t). The objective is to maximize the internal volume while minimizing the weight. In this paper, this problem is modified to an MVMDO problem.

The pressure vessel is designed by two design groups, and the coupled variables are thickness (T), length (L) and radius (R). The multidisciplinary systems and notations are given in Fig. 4.

Because of insufficient information to construct probability distributions of design variables, T, R and

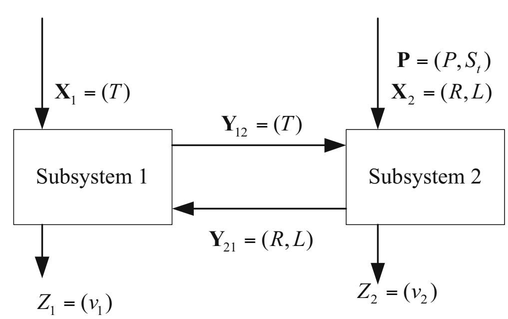


Fig. 4 MDO problem of pressure vessel

Table 3 Membership functions of design variables and distributions of design parameters

	Maximal grade	Deviation dt	Membership function	Low boundary of maximal grade	Up boundary of maximal grade	Mean value	Standard deviation	Distribution
Variables								
R		0.03	Triangular	0.1	36			
T		0.03	Triangular	0.5	6.0			
L		0.03	Triangular	0.1	140			
Parameters								
P						3.89	0.389	Normal
S_t						40	4	Normal

L are all fuzzy variables. But P and S_t are random parameters. The membership functions of design variables and probability distributions of parameters are shown in Table 3.

Sharing design variables $\mathbf{d}_s = \phi$.

Sharing random or fuzzy variables ϕ.

Subsystem 1:

Fuzzy variable $\mathbf{X}_1 = \{T\}$.

Input linking variables $\mathbf{Y}_{21} = \{y_{21,1}, y_{21,2}\} = \{R,L\}$.

Output linking variables $\mathbf{Y}_{12} = \{y_{12}\} = \{T\}$.

Output $Z_1 = \{v_1\}$. $v_1 = \frac{4}{3}\pi\left(T^M + y_{21,1}^M\right)^3 + \pi\left(T^M + y_{21,1}^M\right)^2 y_{21,2}^M - \left[\frac{4}{3}\pi\left(y_{21,1}^M\right)^3 + \pi\left(y_{21,1}^M\right)^2 y_{21,2}^M\right]$. In this subsystem the objective is to minimize the weight, which is equal to minimize the relevant volume.

The constraints in Subsystem 1 are as follows:

The probability/possibility constraints are given by:

$$\Pi\left(G_{11} = 5T - y_{21,1} > 0\right) \leq \alpha_t$$

$$\Pi\left(G_{12} = T + y_{21,1} - 40 > 0\right) \leq \alpha_t.$$

Subsystem 2:

Fuzzy variables $\mathbf{X}_2 = \{R,L\}$.

Input random parameters $\mathbf{P} = \{P,S_t\}$.

Input linking variables $\mathbf{Y}_{12} = \{y_{12}\} = \{T\}$.

Output linking variables $\mathbf{Y}_{21} = \{y_{21,1}, y_{21,2}\} = \{R,L\}$.

Output $Z_2 = \{v_2\}$. $v_2 = \frac{4}{3}\pi\left(R^M\right)^3 + \pi\left(R^M\right)^2 L^M$. In this subsystem, the objective is to maximize internal volume.

The constraints in Subsystem 2 are as follows:

The probability/possibility constraints are

$$\Pi\left(G_{21} = \frac{PR}{y_{12}} - S_t > 0\right) \leq \alpha_t$$

$$\Pi\left(G_{22} = L + 2R + 2y_{12} - 150 > 0\right) \leq \alpha_t.$$

The whole objective, v, is to minimize $v_1 - v_2$.

For comparing the methods of MVMDO and RBMDO, the results of MVMDO and RBMDO with $T \sim N(\mu_T, 0.01)$, $R \sim N(\mu_R, 0.01)$, $L \sim N(\mu_L, 0.01)$, $P \sim N(3.89, 0.389)$ and $S_t \sim N(40, 4)$ are listed in Tables 4 and 5.

Design solutions of MVMDO and RBMDO are shown in Table 4, including the optimal designs and values of objective function. The possibility requirement in MVMDO ($\alpha_t = 0.0013$) is equal to that in the RBMDO ($\beta = 3$). However, from the optimal values of objective, MVMDO provides a more conservative design. Because design variables, for which there are no sufficient data to construct their probability distributions, are characterized as fuzzy variables; this results in the difference in the mathematical formulations for the uncertainty analysis. Meanwhile as the value of possibility of failure decreases, the more conservative the design solution is, and vice versa. MVMDO-SORA works as efficiently as RBMDO-SORA. The whole design optimization process converges after a few cycles.

Table 4 Optimal design of MVMDO and RBMDO

	μ_T	μ_R	μ_L	T^M	R^M	L^M	v	v_1	v_2	n_1	n_2	k
MVMDO ($\alpha_t = 0.0013$)				5.2750	34.6650	69.9700	-2.5973×10^5	1.7890×10^5	4.3863×10^5	3528	3528	6
($\alpha_t = 0$)				6.0000	33.3983	71.0534	-2.0743×10^5	1.9761×10^5	4.0504×10^5	3718	3718	6
($\alpha_t = 0.0049$)				4.9827	34.9576	69.9701	-2.7759×10^5	1.6998×10^5	4.4757×10^5	3548	3548	6
RBMDO ($\beta = 3$)	5.2475	34.7100	69.9949				-2.6187×10^5	1.7822×10^5	4.4009×10^5	3428	3428	5

Table 5 Values of each probability/possibility constraint at MPPP and probability constraint at MPP

	G_{11}	G_{12}	G_{21}	G_{22}
MVMDO ($\alpha_t = 0.0013$)	-8.1101	0	-8.9956×10^{-10}	0
($\alpha_t = 0$)	-3.2183	-0.5417	-8.1439×10^{-11}	0
($\alpha_t = 0.0049$)	-9.8650	-7.1054×10^{-15}	-7.0472×10^{-9}	-2.8422×10^{-14}
RBMDO ($\beta = 3$)	-8.3194	1.8666×10^{-11}	-2.7477×10^{-11}	8.6914×10^{-11}

Table 5 lists the values of probability/possibility constraints at each relevant MPPP and values of probability constraints at each corresponding MPP. In MVMDO with different possibility of failure, all values are not larger than zero. This indicates the probability/possibility constraints are all satisfied. Some values are equal to zero, this means that corresponding constraints are active and the requirements are just satisfied.

5 Conclusions

When there are sufficient data to describe uncertainties associated with variables, RBMDO performs well to find an optimal solution. However, in engineer design, there often exist both aleatory and epistemic uncertainties. In such situations, MVMDO is more recommended than RBMDO. MVMDO formulation and MVMDO under the framework of SORA (MVMDO-SORA) are developed in this research. This proposed method can efficiently solve MDO problems in which both aleatory and epistemic uncertainties exist in design inputs; the method can also reduce the computational demand.

The MVMDO problem can be solved efficiently with sequential deterministic MDO and probability/possibility analysis. In each cycle, probability/possibility analysis follows the deterministic MDO. After solving the deterministic MDO, the maximal grade point of each fuzzy design variable and mean value of each random variable can be obtained. Then probability/possibility analysis is applied to analyze the feasibility of each probability/possibility constraint at the current new design point. To improve feasibilities of constraints which violate probability/possibility requirements, constraints in deterministic MDO are modified with the MPPPs obtained in probability/possibility analysis of previous cycle. Most importantly, in each iteration cycle, the solution is sequential but not nested. So the efficiency is improved and the process can converge in a few cycles.

As demonstrated in the two examples, when the possibility of failure in MVMDO is set to be the same as the probability of failure in RBMDO, MVMDO offers a more conservative design. The reason is that in MVMDO, some design inputs with limited data are treated as fuzzy variables. Meanwhile in RBMDO, there are sufficient data to precisely construct the probability distributions of design inputs. This results in differences in the formulation of uncertainty analysis. On the condition of limited data, the MVMDO obtains conservative results comparing with those of RBMDO. When the value of possibility of failure decreases, the more conservative the design will be. In both examples, MVMDO-SORA solves MVMDO problem efficiently, and the whole process converges in a few cycles.

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