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Collaborative optimization with inverse reliability for multidisciplinary systems uncertainty analysis

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This article introduces a method which combines the collaborative optimization framework and the inverse reliability strategy to assess the uncertainty encountered in the multidisciplinary design process. This method conducts the sub-system analysis and optimization concurrently and then improves the process of searching for the most probable point (MPP). It reduces the load of the system-level optimizer significantly. This advantage is specifically more prominent for large-scale engineering system design. Meanwhile, because the disciplinary analyses are treated as the equality constraints in the disciplinary optimization, the computation load can be further reduced. Examples are used to illustrate the accuracy and efficiency of the proposed method.

Keywords: reliability analysis; multidisciplinary design optimization (MDO); collaborative optimization; uncertainty; inverse reliability analysis

Abbreviations and acronyms

CO	Collaborative optimization
MPP	Most probable point
MDO	Multidisciplinary design optimization
MDF	Multidisciplinary feasible method
IDF	Individual discipline feasible method
CSSO	Concurrent subspace optimization
BLISS	Bi-level integrated system synthesis
AIO	All-in-one
FORM	First order reliability method
SORM	Second order reliability method
PMA	Performance measure approach
RIA	Reliability index approach

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1. Introduction

In modern complex engineering design, multidisciplinary design optimization (MDO) (Balling and Sobieski 1995) has been developed to optimize large-scale and coupled systems, where 'multi-disciplinary' implies that a system involves multiple interacting disciplines. Numerous approaches have been proposed for analysing such MDO problems, such as multidisciplinary feasible method (MDF), individual discipline feasible method (IDF) (Balling and Wilkison 1997), collaborative optimization (CO) (Kroo *et al.* 1994), concurrent subspace optimization (CSSO) (Sobieszczanski-Sobieski 1989) and bi-level integrated system synthesis (BLISS) (Sobieszczanski-Sobieski *et al.* 1998), where CO uses separate optimization routines for each sub-system to satisfy interdisciplinary compatibility, while a system-level optimizer coordinates the tradeoffs among sub-systems (McAllister and Simpson 2003).

However, to improve the overall performance of an engineering multidisciplinary system especially under uncertainty, the effect of uncertainty must be taken into account. In recent developments, some preliminary results of multidisciplinary design under uncertainty are reported (Mavris *et al.* 1999, Koch *et al.* 1999, Padmanabhan and Batill 2000, Du and Chen 2000b, 2002). In these works, the mean and variance of system performance are evaluated through uncertainty analysis and then utilized to obtain optimal solutions based on robustness considerations. A framework for reliability-based MDO was proposed in Sues and Cesare (2000). In their work, the reliability analysis is decoupled from the optimization. Reliabilities are computed initially before the first execution of the optimization loop, and then updated after the optimization loop is executed. Figure 1 shows this approach. However, inside each optimization loop, approximate forms of probabilistic constraints are introduced in the optimization formulation. To integrate the existing reliability analysis techniques into the MDO framework more tightly, a multi-stage, parallel implementation strategy of probabilistic design optimization was proposed by Koch *et al.* (2000). Du and Chen (2005) presented a collaborative reliability analysis that particularly is implemented in the IDF method for MDO. In this method, the procedure of the traditional most probable point (MPP) based reliability analysis method (Du and Chen 2000a, 2005) is combined with the

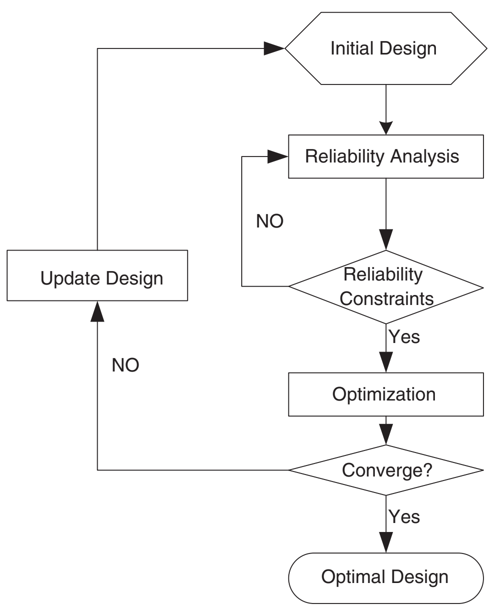

Figure 1. Double loop of RBO.

collaborative disciplinary analysis to automatically satisfy the interdisciplinary consistency in reliability analysis.

The objective of this article is to incorporate a novel collaborative reliability analysis in the multidisciplinary CO framework. An inverse reliability analysis strategy (Der Kiureghian *et al.* 1994, Li and Foschi 1998, Tu *et al.* 1999) is adopted in the reliability analysis that uses percentile performance for assessing probabilistic constraints. This article is organized as follows: The background of reliability analysis is presented in Section 2. Deterministic MDO using the CO method is introduced in Section 3. In Section 4, the proposed method, the collaborative reliability analysis in the framework of multidisciplinary system using the inverse reliability analysis strategy is discussed. Examples for the proposed method are given in Section 5, followed by the conclusions in Section 6.

2. Reliability analysis

The fundamental task of reliability analysis is to find a solution to a multidimensional integral representing the reliability, which is expressed by

$$R = \text{Prob}\{g(\mathbf{X}) \leq 0\} = \int_{g(\mathbf{X})\leq 0} f_x(\mathbf{X})\mathrm{d}x. \tag{1}$$

In the above model, $\mathbf{X} = \{x_1, x_2, \cdots, x_n\}$ is a vector of random design variables. $f_x(\mathbf{X})$ is the joint probability density function (PDF) of $\boldsymbol{X}$. $g(\mathbf{X})$ is a limit-state function, and the safe mode is defined as $g(\mathbf{X}) \leq 0$. $R = \text{Prob}\{g(\mathbf{X}) \leq 0\}$ stands for the probability of safety of a limit-state function.

Often it is impossible to obtain an analytical solution to the probability integration in Equation (1); therefore approximation methods, such as the first order reliability method (FORM) (Hasofer and Lind 1974) and the second order reliability method (SORM) (Breitung 1984), are employed to calculate the probability integration. The common key point of these two methods is the use of the most probable point (MPP). The reliability analysis based on the MPP using the FORM and SORM is introduced as follows.

Firstly, the original random variables $\mathbf{X} = \{x_1, x_2, \cdots, x_n\}$ (in x-space, the original design space) are transformed into a set of normalized random variables $\mathbf{U} = \{u_1, u_2, \cdots, u_n\}$ (in u-space, the standard normalized design space). Each element of $\mathbf{U}$ follows a standard normal distribution. Secondly, the limit-state function $g(\mathbf{U})$ is approximated by a linear form (in the FORM) or a quadratic form (in the SORM) at the MPP. After the two-step simplification and approximation, the probability integration in Equation (1) can be solved analytically. To reduce the accuracy loss, the expansion point is selected at the MPP which has the highest contribution to the probability integration. The MPP can be located by solving the following optimization problem that maximizes the joint PDF of random variables on the hyper surface of the integration region $g(\mathbf{U}) = 0$ in u-space (Du *et al.* 2004).

$$\begin{aligned} \min \quad & \|\mathbf{u}\| \\ \text{s.t.} \quad & \mathrm{g}(\mathbf{u}) = 0 \end{aligned}, \tag{2}$$

where $\|\cdot\|$ stands for the norm of a vector.

After the MPP $\mathbf{u}^*$ is identified, the reliability can then be simply expressed in the FORM as

$$R = \Phi(\beta), \tag{3}$$

where $\beta = \|\mathbf{u}^*\|$ is the shortest distance from the surface $g(\mathbf{U}) = 0$ to the origin in u-space, and is called the 'reliability index'. The reliability solution using the SORM can be found in Breitung's work (Breitung 1984).

However, to use Equations (1) and (2), the reliability $\text{Prob}\{g(\mathbf{X}) \leq 0\}$ for each limit-state function $g(\mathbf{X})$ needs to be evaluated. In presence of multiple constraints, some constraints may never be active and consequently their reliabilities are extremely high (approaching 1.0). Although these constraints are the least critical, the evaluation of reliability will unfortunately dominate the computational effort in probabilistic optimization. The solution to this problem is to perform the reliability assessment only up to the necessary level. Hence, using the percentile performance measure (inverse reliability) will be more efficient than directly evaluating the reliability. The percentile performance is shown as:

$$g^R \leq 0, \tag{4}$$

where g^R is the R-percentile performance of $g(\mathbf{X})$, namely,

$$\text{Prob}\{g(\mathbf{x}) \leq g^R\} = R. \tag{5}$$

If the FORM is used, the R-percentile performance can be obtained by solving the following model:

$$\begin{aligned} \max \quad & \text{g}(\mathbf{u}) \\ \text{s.t.} \quad & \|\mathbf{u}\| = \beta \end{aligned}. \tag{6}$$

Then g^R is the function value at the solution $\mathbf{u}^*_{MPP}$ (Du *et al.* 2004).

3. Collaborative optimization (CO)

For simplicity, a 3-sub-system example (Du and Chen 2005) shown in Figure 2 is used to present the CO method.

In this system, $\mathbf{x}_s$ are the system input variables which are the input for all disciplines, also called sharing variables. $\mathbf{x}_i (i = 1, 2 \text{ and } 3)$ are the input variables of disciplines i, $\mathbf{y} = (y_{21}, y_{31}, y_{12}, y_{32}, y_{13}, y_{23})$ are state variables (linking variables), and y_{ij} is the output of sub-system i which is taken as input to sub-system j. $\mathbf{z}_i$ are outputs of disciplines i.

The model of MDO is presented as follows,

$$\begin{aligned} \min \quad & f(\mathbf{x}, \mathbf{y}) \\ \text{s.t.} \quad & g_i(\mathbf{x}_s, \mathbf{x}_i, \mathbf{y}_{.i}) \geq 0 \\ & h_i(\mathbf{x}_s, \mathbf{x}_i, \mathbf{y}_{.i}) = 0 \\ & i = 1 \sim 3 \end{aligned}, \tag{7}$$

where $\mathbf{x} = (\mathbf{x}_s, \mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3)$ is a vector of design variables, f is the design objective function, g and h denote inequality constraints and equality constraints. $\mathbf{y}_{.i}$ is the input linking variables of

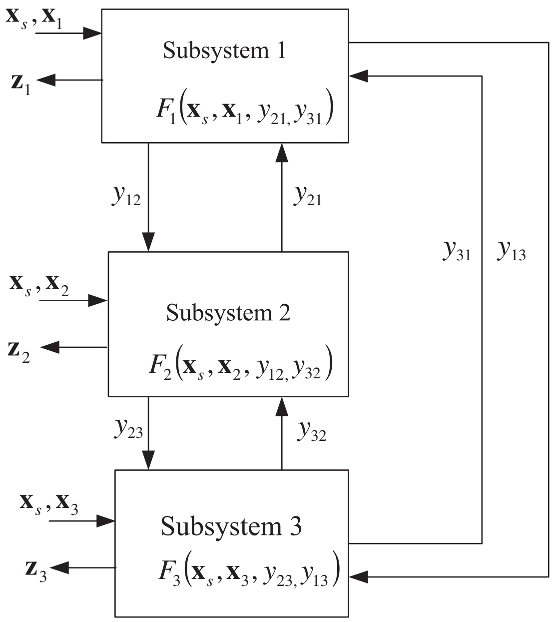

Figure 2. A multidisciplinary system.

discipline i from other disciplines.

$$\begin{aligned} y_{12} &= F_{y12}\,(\mathbf{x}_s, \mathbf{x}_1, y_{21}, y_{31}) \\ y_{13} &= F_{y13}\,(\mathbf{x}_s, \mathbf{x}_1, y_{21}, y_{31}) \\ y_{21} &= F_{y21}\,(\mathbf{x}_s, \mathbf{x}_2, y_{12}, y_{32}) \\ y_{23} &= F_{y23}\,(\mathbf{x}_s, \mathbf{x}_2, y_{12}, y_{32}) \\ y_{31} &= F_{y31}\,(\mathbf{x}_s, \mathbf{x}_3, y_{13}, y_{23}) \\ y_{32} &= F_{y32}\,(\mathbf{x}_s, \mathbf{x}_3, y_{13}, y_{23}) \end{aligned}. \tag{8}$$

The CO method divides the original problem in Equation (7) into two levels: one system level and parallel sub-system (discipline) levels. The system level assigns the targets of design variables to all sub-systems. The objective of each sub-system is to minimize the gap between the design variables and the target values under the condition of satisfying its own constraints. After the optimization of sub-systems is accomplished, the objective functions are passed to the system as consistent constraints to resolve the inconsistency among the design variables of all sub-systems.

Compared with other methods for MDO, CO preferably solves computational complexity and structural complexity, since it utilizes sub-system optimizer to make the disciplinary decisions. One of the merits of CO is that it reduces the complexity of a system so that sub-system problems can be analysed and optimized synchronously. The framework is similar to the modern engineering design structure and is suitable for large-scale optimization problems.

The CO method is used to reformulate the model Equation (7) as follows,

System level:

$$\begin{aligned} &\min \quad f(\mathbf{x}, \mathbf{y}) \\ &\text{s.t.} \quad q_j = 0 \\ &\text{DV} = \{\mathbf{x}, \mathbf{y}\} \end{aligned}. \tag{9}$$

Sub-system level:

$$
\begin{aligned}
\min \quad & q_j = \|\mathbf{x}_s - \mathbf{x}_{s0}\|_2^2 + \|\mathbf{x}_j - \mathbf{x}_{j0}\|_2^2 + \sum_{l=1}^{m} (y_l - y_{l0})^2 \\
\text{s.t.} \quad & y_{j.} = F_{y_{j.}}(\mathbf{x}_s, \mathbf{x}_j, \mathbf{y}_{.j}) \\
& g_j \geq 0 \\
& h_j = 0 \\
& \mathrm{DV} = \{\mathbf{x}_s, \mathbf{x}_j, \mathbf{y}\}
\end{aligned}, \tag{10}
$$

where $\mathbf{x}_s$ denotes the sharing variable, $\mathbf{x}_j$ is a vector of local variables in disciplines j, and m is the number of linking variables. x_{s0}, x_{j0}, y_{l0} are the target values.

The calculation process is as follows:

(1) Firstly, system level assigns target values (x_{s0}, x_{i0}, y_{l0}) of design variables (x_s, x_i, y_l) to each sub-system.
(2) Sub-system optimizers find local optimal solutions to meet the targets assigned at the system level. However, the solution from each sub-system may not exactly match the target. So, there exists an inconsistency between the target and the response. The inconsistency should be overcome.
(3) System optimizer obtains the global optimal solution, and passes the solution to the sub-system as new target values. (System optimizer assigns new targets for the sub-system level with the consideration of sub-systems' capabilities based on the response passed from sub-systems).
(4) Steps (2) and (3) are repeated until convergence (Li 2003).

4. Reliability analysis under the framework of multidisciplinary systems using inverse reliability

With the existence of uncertainty, the deterministic MDO model (7) is reformulated as follows:

$$
\begin{aligned}
\min \quad & f(\mathbf{x}, \mathbf{y}) \\
\text{s.t.} \quad & \mathrm{P}\{g_i(\mathbf{x}_s, \mathbf{x}_i, \mathbf{y}_{.i}) \leq 0\} \geq R_i
\end{aligned}. \tag{11}
$$

The design feasibility under uncertainty is defined as the probability of the constraint satisfaction $g_i \leq 0$ being greater than or equal to the desired probability R_i. The probability of the constraint satisfaction is also called the reliability. The reliability assessment is a critical component that demands much more computational effort for MDO under uncertainty than deterministic MDO. The traditional method, also called as multidisciplinary feasible method (MDF) reliability analysis, integrates MPP method directly with multidisciplinary systems (Du and Chen 2005), and Figure 3 shows this method in detail. This approach needs large-scale disciplinary analyses in the system-level analysis for locating the MPP.

In this section, the method which combines inverse reliability analysis with CO for MDO problems is proposed. Figure 4 shows the flowchart of this proposed method with related numeric formulation as follows:

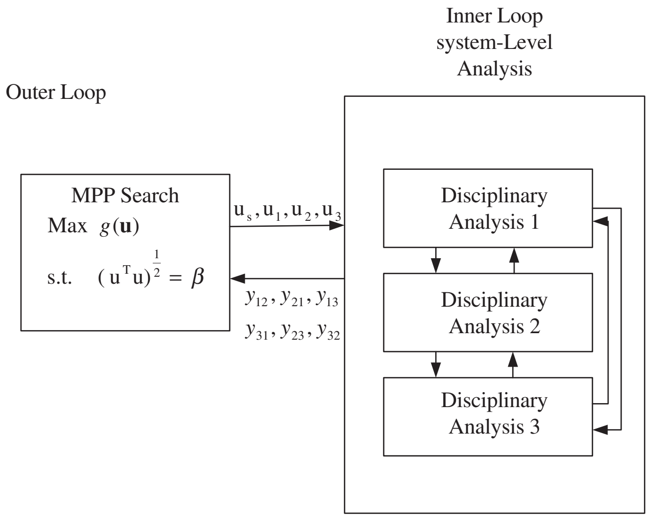

Figure 3. MPP search using the MDF method.

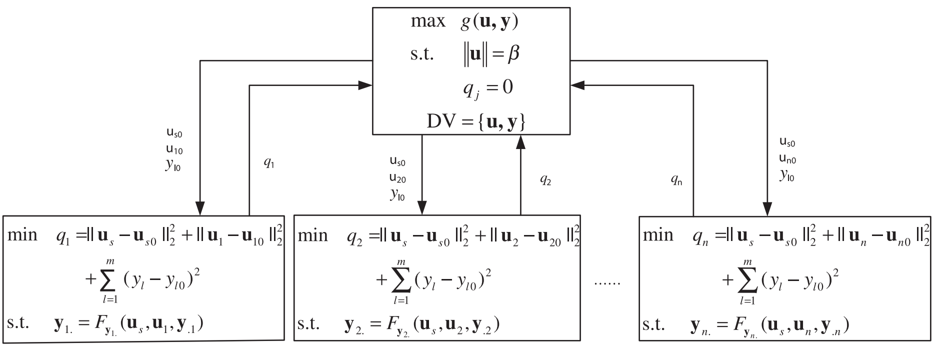

Figure 4. The model of the integration of CO and inverse reliability for MPP.

System level:

$$\begin{aligned} \max \quad & g(\mathbf{u}, \mathbf{y}) \\ \text{s.t.} \quad & \|\mathbf{u}\| = \beta \\ & q_j = 0 \\ & \mathrm{DV} = \{\mathbf{u}, \mathbf{y}\} \end{aligned} . \tag{12}$$

Sub-system level:

$$\begin{aligned} \min \quad & q_j = \|\mathbf{u}_s - \mathbf{u}_{s0}\|_2^2 + \|\mathbf{u}_j - \mathbf{u}_{j0}\|_2^2 + \sum_{l=1}^{m} (y_l - y_{l0})^2 \\ \text{s.t.} \quad & \mathbf{y}_{j.} = F_{\mathbf{y}_{j.}}(\mathbf{u}_s, \mathbf{u}_j, \mathbf{y}_{.j}) \end{aligned} . \tag{13}$$

In Equation (13), $\mathbf{u}$ is the design vector in u-space (standard normal space) corresponding to $\mathbf{x}$ vector in x-space (original design space). $\mathbf{u}_{s0}$, $\mathbf{u}_{j0}$, $\mathbf{y}_{l0}$ are the target values from the system level, other symbols are the same as those in the foregoing formulations. Corresponding to the given R,

$\beta = \phi^{-1}(R)$, where ϕ^{-1} is the inverse function of cumulative probabilistic function of a standard normal distribution.

In this method, optimization at the disciplinary level can be performed independently. This not only decomposes coupling among disciplines, but also can improve the process of searching the MPP. The load of the system-level optimizer will be reduced significantly, because of the optimization in disciplines. This advantage will especially be more prominent in large-scale engineering system design.

Using this proposed method, $\mathbf{u}^*_{MPP}$ will be obtained for each probability constraint, and then the percentile performance $g^R = g(\mathbf{u}^*_{MPP})$ can be calculated at the MPP. The deterministic optimization problems of Equations (12) and (13) can be solved by traditional optimization algorithms.

5. Example

In this section, two examples are utilized to demonstrate the proposed methods.

5.1. *Mathematical example*

The mathematical example is from the work of Du and Chen (2005). This example includes two disciplines and five random variables. Figure 5 illustrates the multidiscipline system of this example.

In discipline 1:

$$\mathbf{x}_s = \{x_1\}, \quad \mathbf{x}_1 = \{x_2, x_3\}, \quad \mathbf{z}_1 = \{z_1\}$$
$$y_{12} = x_1^2 + 2x_2 - x_3 + 2\sqrt{y_{21}}$$
$$z_1 = 5 - \left(x_1^2 + 2x_2 + x_3 + x_2 e^{-y_{21}}\right).$$

In discipline 2:

$$\mathbf{x}_s = \{x_1\}, \quad \mathbf{x}_2 = \{x_4, x_5\}, \quad \mathbf{z}_2 = \{z_2\}$$
$$y_{21} = x_1 x_4 + x_4^2 + x_5 + y_{12}$$
$$z_2 = \sqrt{x_1} + x_4 + x_5(0.4x_1).$$

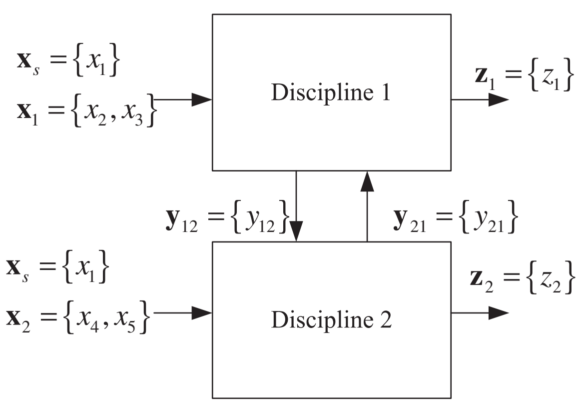

Figure 5. Multidiscipline system of the mathematical example.

When $x_1 - x_5 \sim N(1, 0.1)$, the proposed method is utilized to perform reliability analysis while treating z_1 as the limit-state function. The formulations are as follows.

System level:

$$\begin{aligned} \max \quad & z_1(\mathbf{u}, \mathbf{y}) = 5 - \left((1 + 0.1u_1)^2 + 2(1 + 0.1u_2) - (1 + 0.1u_3) + (1 + 0.1u_2)e^{-y_{21}}\right) \\ \text{s.t.} \quad & q_j = 0 \\ & \sqrt{u_1^2 + u_2^2 + u_3^2 + u_4^2 + u_5^2} = \beta \end{aligned}$$

.

Sub-system level:

Discipline 1:

$$\begin{aligned} \min \quad & q_1 = (u_1 - u_{10})^2 + (u_2 - u_{20})^2 + (u_3 - u_{30})^2 + (y_{12} - y_{120})^2 + (y_{21} - y_{210})^2 \\ \text{s.t.} \quad & y_{12} = (1 + 0.1u_1)^2 + 2(1 + 0.1u_2) - (1 + 0.1u_3) + 2\sqrt{y_{21}} \end{aligned}$$

Discipline 2:

$$\begin{aligned} \min \quad & q_2 = (u_1 - u_{10})^2 + (u_4 - u_{40})^2 + (u_5 - u_{50})^2 + (y_{12} - y_{120})^2 + (y_{21} - y_{210})^2 \\ \text{s.t.} \quad & y_{21} = (1 + 0.1u_1)(1 + 0.1u_4) + (1 + 0.1u_4)^2 + (1 + 0.1u_5) + y_{12} \end{aligned}$$

The solutions are $\mathbf{u}_{1MPP}^* = (u_1, u_2, u_3, u_4, u_5) = (-1.7774, -2.1616, -1.0809, 0.0001, 0.0000)$ and $z_1(\mathbf{u}_{MPP}^*) = 1.8643$. For comparisons, the results of MDF and the proposed method are listed in Table 1.

From Table 1, the solution of CO is very close to that of MDF, but the function evaluation number of CO is much less than that of MDF. This indicates that the proposed method is more efficient than MDF.

5.2. *Heart dipole*

The heart dipole is a well-known MDO example modified by NASA (Yuan 2005). The multidiscipline system is illustrated in Figure 6.

Table 1. Comparisons of MDF and the proposed method for mathematical example.

Method	$\mathbf{u}_{MPP}^* = (u_1, u_2, u_3, u_4, u_5)$	$z_1(\mathbf{u}_{MPP}^*)$	n
MDF	(−1.7772, −2.1617, −1.0810, 0.0005, −0.0019)	1.8643	305
CO	(−1.7774, −2.1616, −1.0809, 0.0001, 0.0000)	1.8643	169

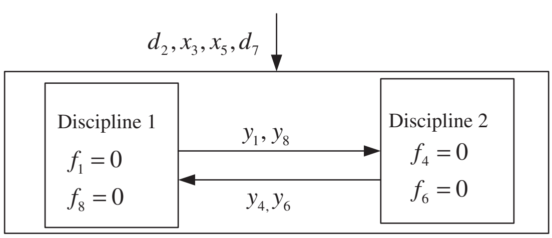

Figure 6. Multidiscipline system for heart dipole example.

Table 2. Reliability analysis results of MDF and the proposed method.

Method	$\mathbf{u}^*_{MPP} = (u_3, u_5)$	$z_5(\mathbf{u}^*_{MPP})$	n
MDF	(2.2574, 1.9759)	0	865
CO	(2.2573, 1.9759)	0	447

The RBMDO formulation of the heart dipole is:

$$\begin{aligned} \min \quad & f(d_2, d_7, x_3, x_5, y_{12}, y_{21}) = f_5 + f_6 + f_7 + f_8 \\ \text{s.t.} \quad & P\{f_i(d_2, d_7, x_3, x_5, y_{12}, y_{21}) \le 0\} \le R \quad i = 5, 7 \\ & f_6(d_2, d_7, x_3, x_5, y_{12}, y_{21}) \ge 0 \\ & f_8(d_2, d_7, x_3, x_5, y_{12}, y_{21}) \ge 0 \end{aligned}$$

where y_{12}, y_{21} can be obtained by solving the following equations

$$\begin{aligned} f_1 &= y_1 + d_2 - 0.63254 = 0 \\ f_4 &= d_7 y_1 + y_8 d_2 + x_5 x_3 + y_6 y_4 - 1.7345334 = 0 \\ f_6 &= x_3 x_5^2 - x_3 d_7^2 + 2y_1 x_5 d_7 + y_4 y_6^2 - y_4 y_8^2 - 2d_2 y_6 y_8 + 0.843453 = 0. \\ f_8 &= x_3 x_5^3 - 3x_3 x_5 d_7^2 + y_1 d_7^3 - 3y_1 d_7 x_5^2 + y_4 y_6^3 - 3y_4 y_6 y_8^2 \\ &\quad + d_2 y_8^3 - 3d_2 y_8 y_6^2 - 1.2342523 = 0 \end{aligned}$$

The proposed method is used while treating the function f_5 as the limit-state function. x_3, x_5 are normally distributed with the radio of mean value and standard variance equal to 10. The mean values of x_3, x_5 are $\mu_3 = 0.4674$, $\mu_5 = 1.1653$, respectively. The results of MDF and CO are listed in Table 2.

From Table 2, the MPP and R-percentile performance obtained by CO for reliability analysis are very close to that of MDF. But the function evaluation number of CO is much less than that of MDF. This also indicates that the proposed method is much more efficient than MDF reliability analysis method.

6. Conclusions

In the MDF reliability analysis method for multidisciplinary systems, two nested loops are involved. The outer loop searches for the MPP, and the inner loop is the system-level analysis. In the process of system-level analysis, a number of individual disciplinary analyses are performed. This results in a high computational cost which sometimes is unacceptable.

The proposed method in this article combines CO, which is an efficient MDO method, with inverse reliability analysis to assess the reliability. In this method, disciplinary analyses and optimization are performed concurrently. This strategy can improve the process for searching the MPP. It considers the individual disciplinary analyses as equality constraints of the disciplinary optimization, which reduces the computational load. The inverse reliability analysis strategy for analysing reliability is used but not the reliability index approach (RIA), because the former is more efficient and robust. Results of examples illustrate that the proposed method is much more efficient than the MDF reliability analysis method.

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