A New Method for Achieving Flexibility in Hierarchical Multilevel System Design

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A New Method for Achieving Flexibility in Hierarchical Multilevel System Design

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Abstract: Analytical target cascading (ATC) method has been widely applied to solve multilevel decomposed system design optimization problems. In the ATC method, concurrent design is achieved by target cascading. However, due to the complexity and the presence of uncertainty, it is a challenging task to set proper targets. In this article, instead of using point value targets, interval targets are analyzed and propagated through the multilevel system with the goal of reducing the effects of uncertainty while providing more flexibility to a design process. In the proposed method, the design of a hierarchical system at each level is taken as a single-objective optimization problem, by minimizing the degree of deviation between the target response interval and the achievable response interval. Not only the optimal design performance is considered in this method, but also the acceptable variation range of the performance is analyzed. When the present target for a lower level system and the achievable response from a lower level system are not point values, but rather intervals, their probability distributions are not available. Therefore, these variables are treated as interval variables. When the random and interval variables are present, the most probable point-based first-order reliability and the interval analysis methods are used to calculate the reliability bounds. The proposed method for flexibility under uncertainty provides more degree of freedom to the design of lower level systems, while also keeping the performance of the upper systems stable within a tolerable range. The accuracy of the proposed method is demonstrated *via* comparing results from both the proposed and traditional methods.

Key Words: flexibility, uncertainty, multilevel system, deviation degree, reliability bounds.

1. Introduction

Design of an engineering system is often a challenging task due to its complexity and the presence of uncertainty. It is also viewed as a decision-making process which involves target setting and target coordination. Analytical target cascading (ATC) is a decomposition methodology developed for hierarchical multilevel system optimization. In a hierarchical decomposition problem, the coordination procedure focuses on the minimization of the norm of the deviation between a given target set by the upper level model and the response of the lower level model.

The original target setting [1] and the target coordination [2,3] methods in multilevel systems are point-based design optimization. The optimization requires complete knowledge of design concepts and design models at each level [4]. The point-based target offers little flexibility since the targets have to be achieved exactly to avoid penalty. Instead of seeking a single-point solution for the design model at each level, it is desired that the target setting should possess flexibility *via* providing a range of solutions based on information passed along systems. Flexible design target provides designers more design freedom and can avoid rework by postponing immature commitments in the early stages of a design process.

Different measures of design flexibility have been proposed in literature. Suh [5] used the information content and proposed the notion of flexibility by considering both the achievable design performance and the target ranges. The probability density functions of the system and the target ranges were all assumed to be uniformly distributed. Simpson et al. [6] pointed out that developing ranged sets of the top-level system was a means to enhance openness and system flexibility. The design freedom and information certainty were used to measure system's flexibility. Chen and Yuan [7] proposed a 'preference function of performance levels' to evaluate the 'degree of desirability' of a varying performance to meet a range of design requirements. The worst-case scenario of constraints was used to guarantee that the whole range of design solutions satisfy the constraints. In Olewnik et al. [8], the flexibility of a system was represented by a range which was the line segment

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connecting two points in a performance space with a penalty function for deviation from that range, though the mathematical model of this approach was not provided. Moe et al. [9] proposed a prototype partitioning strategy considering the flexibility in requirements on cost, performance, and schedule. However, they did not develop a general model. Chen and Lewis [10] applied the robust design concept and game theory to obtain flexible top-level design specifications by treating targets as design variables. Their method assumed that the leader assumes rationality of the followers, and the unilateral dependency of the system on subsystem did not exist in hierarchical multilevel systems. Kalsi et al. [11] introduced a sequential approach using the concepts from comprehensive robust design to provide an additional option for handling uncertainty. In his method, unknown common variables needed by the system designer were modeled as noise variables with uniform probabilistic distributions, which varying within modified bounds that lie within the actual bounds of those common variables. Liu [12] proposed a method that provides the maximum design flexibility while incorporating the design heterogeneity. In their method, a design attribute space was decomposed into subregions first, and then, a flexibility measure was developed and used as the metric to obtain the most desired ranged set of targets.

There are two aspects that the aforementioned works have not covered. First, the flexible target set in a multilevel system is not studied. Second, these works only consider a flexibility range, but do not provide optimal performance. Many works have been done for the interval uncertainty analysis [13–26]. The hierarchical optimization and coordination is a multi-objective problem [27–30]. From [4], the convergence of ATC is achieved under proper weighting, which needs large amount of computation. In this article, our primary focus is threefold: First, a single-objective design optimization and coordination method is formulated. Second, the ranged targets are assigned to lower level systems instead of point-valued targets, and the formulation of ranged target coordination method is provided. Third, with the mixed random and interval variables, the maximum possible and the minimum possible of the probability of failure are computed. The design requirement is that these two bounds must be within the acceptable range of the probability of failure.

This article is organized as follows. Technical background and terminologies are introduced in Section 2. In Section 3, the approach for the design flexibility in a hierarchical multilevel system is put forth. The augmented ATC formulation for design flexibility is presented. Numerical examples are given in Section 4 to illustrate the effectiveness of the proposed method and followed by a conclusion in Section 5.

2. Technical Background

2.1 The Principle of ATC

ATC is a decomposition methodology developed for hierarchical multilevel system optimization. The design objective of each element in ATC is composed of two parts:

(1) To minimize the deviation of the current level subsystem performances and common variables from the assigned targets given by the upper system.
(2) To minimize the deviation of the lower level subsystem performances and common variables from response identified in lower system.

Therefore, the framework of ATC represents a collaborative design effort such that the ultimate goal of each subproblem is to meet the system-level targets.

In a multilevel hierarchical optimization problem, the formulations of the system and subsystem follow a top-down fashion as shown in Equations (1) and (2).

$$
\begin{aligned}
&\min : \left\| R_{sys} - \mathrm{T}^* \right\| + \sum_{i=1}^{n_{sub}} \left\| R_{subi} - R_{subi}^{sub} \right\|_2^2 \\
&\quad + \sum_{i=1}^{n_{sub}} \sum_{k=1}^{c_i} \left\| y_{subik} - y_{subik}^{sub} \right\|_2^2 \\
&\text{s.t. } R_{sys} = R_{sys}(\mathbf{x}_{sys}, \mathbf{R}_{sub}) \\
&g_{sysj}(\mathbf{R}_{sub}, \mathbf{x}_{sys}) \le 0, j = 1, 2, \ldots, m \\
&h_{sysj}(\mathbf{R}_{sub}, \mathbf{x}_{sys}) = 0, j= 1, 2, \ldots, l \\
&n_{sub} : \text{number of subsystems} \\
&c_i : \text{number of common varaibles in the } i\text{th subsystem}
\end{aligned} \tag{1}
$$

$$
\begin{aligned}
&\min : \left\| R_{subi} - R_{subi}^{sys} \right\|_2^2 + \sum_{k=1}^{c_i} \left\| y_{subik} - y_{subik}^{sys} \right\|_2^2 \\
&\text{s.t. } R_{subi} = R_{subi}(\mathbf{x}_{sub}, \mathbf{y}_{sub}) \\
&g_{subj}(R_{subi}, \mathbf{x}_{sub}, \mathbf{y}_{sub}) \le 0, j = 1, 2, \ldots, p \\
&h_{subj}(R_{subi}, \mathbf{x}_{sub}, \mathbf{y}_{sub}) = 0, j = 1, 2, \ldots, q
\end{aligned} \tag{2}
$$

where superscript ‘sub’, refer to the variable’s optimum in subsystem level, and it will be taken as design parameter in system level optimization design, and ‘sys’ refer to the variable’s optimum in system which will be given to the subsystem as target. The deviation terms is coordinated by L-2 norm.

2.2 Deviation Degree of Intervals

If the superscripts I denotes interval, the underscore and overscore denote the lower and the upper bounds of an interval respectively, for intervals $x^I = [\underline{x}, \overline{x}]$ and

$z^I = [\underline{z}, \overline{z}]$, the norm of $D(x^I, z^I) = \|x^I - z^I\| = \|\underline{x} - \underline{z}\|_2^2 + \|\overline{x} - \overline{z}\|_2^2$ refers to the degree of deviation of intervals x^I and z^I.

2.3 Uncertainty Analysis with the Mixed Interval Variables and Random Variables

The reliability constraints can be formulated as Equation (3).

$$P\{g_i(\mathbf{x}, \mathbf{p}) \le 0\} = \int_{g_i \le 0} f(\mathbf{X}) \mathrm{d}x \ge 1 - p_{fi}, i = 1, 2, \ldots, n \tag{3}$$

where $g_i(\mathbf{x}, \mathbf{p}) \le 0$ is a constraint function, $\mathbf{x}$ is the vector of both deterministic and random design variables, vector $\mathbf{p}$ denotes the deterministic and random parameters which are uncontrollable, p_{fi} the probability of failure, and n is the number of reliability constraints.

When both random and interval variables are present, a single probability of failure is not available, but its lower and upper bounds are needed. The bounds of probability failure p_f are defined by Equation (4).

$$\begin{aligned} p_f^{\min} &= P\left\{\min_{\mathbf{z}} g(\mathbf{x}, \mathbf{p}, \mathbf{z}) \le 0\right\} \\ p_f^{\max} &= P\left\{\max_{\mathbf{z}} g(\mathbf{x}, \mathbf{p}, \mathbf{z}) \le 0\right\}, \end{aligned} \tag{4}$$

where $\mathbf{z}$ is the vector of interval variables, and $\min_{\mathbf{z}} g(\mathbf{x}, \mathbf{p}, \mathbf{z})$ and $\max_{\mathbf{z}} g(\mathbf{x}, \mathbf{p}, \mathbf{z})$ are the minimum and maximum values of the limit state function g over the intervals of $\mathbf{z}$.

In the uncertainty analysis, the most probable point, which having the highest probability of failure value, is used to compute the maximal and minimal failure probability.

3. The Formulation for Achieving Design Flexibility in Multilevel System

3.1 Ranged Targets Coordination in Each Level

Instead of point-valued targets, ranged targets are assigned and propagated in each level. The coordination approach of ranged targets is formulated in Equation (5). In Equation (5), the degree of deviation between ranges is used to measure the uncertainty and achievable degree of the targets.

$$\begin{aligned} \min : &\left\|(R_{sub} - \Delta R_{sub}) - (R_{sub}^{sys} - \Delta R_{sub}^{sys})\right\|_2^2 \\ &+ \left\|(R_{sub} + \Delta R_{sub}) - (R_{sub}^{sys} + \Delta R_{sub}^{sys})\right\|_2^2 \end{aligned} \tag{5}$$

Equation (5) can be rewritten by the min (max) objective function as in Equation (6).

$$\min : \max\begin{pmatrix} \left\|(R_{subk} - \Delta R_{subk}) - (R_{subk}^{sys} - \Delta R_{subk}^{sys})\right\|_2^2 \\ \left\|(R_{subk} + \Delta R_{subk}) - (R_{subk}^{sys} + \Delta R_{subk}^{sys})\right\|_2^2 \end{pmatrix} \tag{6}$$

The intervals of common variables are calculated by the sensitivity of design variables with respect to common variables, as shown in Equation (7).

$$\Delta y_{subik} = \frac{\partial y_{subik}}{\partial R_{subi}} \Delta R_{subi} \tag{7}$$

where, $\frac{\partial y_{subik}}{\partial R_{subi}}$ is calculated by Equation (8).

$$\frac{\partial y_{subik}}{\partial R_{subi}} = -\frac{\partial h_{subi}/\partial R_{subi}}{\partial h_{subi}/\partial y_{ik}} \tag{8}$$

The intersection set of intervals of common variables in different subsystems is calculated and is assigned as interval targets to the subsystems, which is shown in Equation (9).

$$\begin{aligned} [y_{subik}^{sys} - \Delta y_{subik}^{sys}, y_{subik}^{sys} + \Delta y_{subik}^{sys}] &= \bigcap_{i=1}^{n_{sub}} y_{subik}^{subI} \\ y_{subik}^{subI} &= [y_{subik}^{sub} - \Delta y_{subik}^{sub}, y_{subik}^{sub} + \Delta y_{subik}^{sub}] \end{aligned} \tag{9}$$

The difference between general ATC and the proposed ranged target cascading (RTC) is presented in Table 1. The mathematical formulation is shown by Equations (10) and (11).

$$\begin{aligned} \min : &\left\| R_{sys} - \mathrm{T} \right\|_2^2 \\ \text{s.t. } & R_{sys} = R_{sys}(\mathbf{x}_{sys}, \mathbf{R}_{sub}) \\ & P\{g_{sysj} = g_{sysj}(\mathbf{x}_{sys}, \mathbf{R}_{sub} \pm \Delta\mathbf{R}_{sub}) \le 0\} \\ & \ge 1 - p_{fj}, j = 1, 2, \cdots, m \\ & h_{sysj} = h_{sysj}(\mathbf{x}_{sys}, \mathbf{R}_{sub}) \le 0, \ j = 1, 2, \cdots, l \\ & \mathbf{R}_{sub} - \Delta\mathbf{R}_{sub} \le \mathbf{R}_{sub}^{sub} - \Delta\mathbf{R}_{sub}^{sub} \\ & \mathbf{R}_{sub}^{sub} + \Delta\mathbf{R}_{sub}^{sub} \le \mathbf{R}_{sub} + \Delta\mathbf{R}_{sub} \\ & [y_{ik}^{sys} - \Delta y_{ik}^{sys}, y_{ik}^{sys} + \Delta y_{ik}^{sys}] = \bigcap_{i=1}^{n_{sub}} y_{ik}^{subI} \\ & y_{ik}^{subI} = [y_{ik}^{sub} - \Delta y_{ik}^{sub}, y_{ik}^{sub} + \Delta y_{ik}^{sub}] \end{aligned} \tag{10}$$

$$\begin{aligned} \min : &\max\begin{pmatrix} \left\|(R_{subi} - \Delta R_{subi}) - (R_{subi}^{sys} - \Delta R_{subi}^{sys})\right\|_2^2 \\ \left\|(R_{subi} + \Delta R_{subi}) - (R_{subi}^{sys} + \Delta R_{subi}^{sys})\right\|_2^2 \end{pmatrix} \\ \text{s.t. } & R_{subi} = R_{subi}(\mathbf{x}_{sub}, \mathbf{y}_{sub}) \\ & P\{g_{subj} = g_{subj}(\mathbf{x}_{sub}, \mathbf{y}_{sub} \pm \Delta\mathbf{y}_{sub}, R_{subi} \pm \Delta R_{subi}) \le 0\} \\ & \ge 1 - p_{fj}, \ j = 1, 2, \ldots, p \\ & h_{subj} = h_{subj}(\mathbf{x}_{sub}, \mathbf{y}_{sub}, R_{subi}) \le 0, j = 1, 2, \ldots, q \\ & \Delta y_{subik} = \frac{\partial y_{subik}}{\partial R_{subi}} \Delta R_{subi} \\ & \mathbf{y}_{sub}^{sys} - \Delta\mathbf{y}_{sub}^{sys} \le \mathbf{y}_{sub} - \Delta\mathbf{y}_{sub}, \mathbf{y}_{sub} + \Delta\mathbf{y}_{sub} \le \mathbf{y}_{sub}^{sys} \\ & \quad + \Delta\mathbf{y}_{sub}^{sys} \end{aligned} \tag{11}$$

In this article, the mathematical formulation of the target cascading is composed of levels of the system and

Table 1. The difference between general ATC and the proposed RTC.

	General ATC	Proposed RTC
Target setting	Point targets	Ranged target
Target cascading	Point solution	Ranged solution
System objective	$\Vert R_{sys} - \mathrm{T}\Vert_2^2 + \sum_{i=1}^{n_{sub}} (R_{subi} - R_{subi}^{sub})^2 + \sum_{i=1}^{n_{sub}} \sum_{k=1}^{c_i} (y_{subik} - y_{subik}^{sub})^2$	$\Vert R_{sys} - \mathrm{T}\Vert_2^2$
Subsystem objective	$(R_{subi} - R_{subi}^{sys})^2 + \sum_{k}^{c_i} (y_{subik} - y_{subik}^{sys})^2$	$\min : \max\begin{pmatrix} (R_{subi} - \Delta R_{subi} - (R_{subi}^{sys} - \Delta R_{subi}^{sys}))^2 \\ (R_{subi} + \Delta R_{subi} - (R_{subi}^{sys} + \Delta R_{subi}^{sys}))^2 \end{pmatrix}$
Constraints coordination	$(\mathbf{R}_{sub} - \mathbf{R}_{sub}^{\mathrm{sub}})^2 \le \varepsilon_R$ $(\mathbf{y}_{sub} - \mathbf{y}_{sub}^{\mathrm{sub}})^2 \le \varepsilon_y$	$\mathbf{R}_{sub} - \Delta\mathbf{R}_{sub} \le \mathbf{R}_{sub}^{sub} - \Delta\mathbf{R}_{sub}^{sub}$ $\mathbf{R}_{sub}^{sub} + \Delta\mathbf{R}_{sub}^{sub} \le \mathbf{R}_{sub} + \Delta\mathbf{R}_{sub}$ $[y_{ik}^{sys} - \Delta y_{ik}^{sys}, y_{ik}^{sys} + \Delta y_{ik}^{sys}] = \bigcap_{i=1}^{n_{sub}} y_{ik}^{subl}$ $y_{ik}^{subl} = [y_{ik}^{sub} - \Delta y_{ik}^{sub}, y_{ik}^{sub} + \Delta y_{ik}^{sub}]$

the subsystems, which correspond to the top and the lower levels of the modeling hierarchy, respectively. However, the target cascading formulation is not limited to two levels of modeling hierarchy. It can be further expanded to the system with multiple levels. The information flow of the proposed RTC for achieving flexibility is shown in Figure 1.

4. Case Study

4.1 Reliability Allocation and Design Optimization Problem

The all-in-once (AIO) model of a reliability allocation and design optimization problem is defined in Equation (12).

$$\begin{aligned} \min : f &= (\mu_{x_1} + \mu_{x_2} + \mu_{x_3}) + R_1^2 + R_2^2 \\ \text{s.t. } R_1 &= P\{g_1(\mathbf{x}) = 1 - x_1^2 x_2/20 \le 0\} \\ R_2 &= P\{g_2(\mathbf{x}) = 1 - (x_3^2 - 3x_2 + 5)/30 \le 0\} \\ g_3 &= 0.997 - R_1 R_2 \le 0 \\ &0 \le x_1 \le 10,\ 0 \le x_2 \le 10,\ 0 \le x_3 \le 10 \\ &R_1 \ge 0.9987,\ R_2 \ge 0.9987 \end{aligned} \tag{12}$$

where x_1, x_2, and x_3 are three random variables, μ_{x_1}, μ_{x_2}, and μ_{x_3} the mean values of the three random variables, and $P\{g_i(\mathbf{x}) \le 0\} \ge R_i^{\mathrm{T}} i = 1, 2, 3 \ldots$ the reliability constraints.

Based on the ATC method, the problem in Equation (12) can be formulated as a two-level reliability design optimization problem. In which, the system level is formulated in Equation (13). The subsystem level is composed of two subsystems design optimization problems which are given in Equations (14) and (15), respectively.

$$\begin{aligned} &\min : f = R_1^2 + R_2^2 \\ &\text{s.t. } g_3 = 0.997 - R_1 R_2 \le 0 \\ &\quad R_1 \ge 0.9985,\ R_2 \ge 0.9985 \end{aligned} \tag{13}$$

$$\begin{aligned} &\min : \mu_{x_1} + \mu_{x_2} + (R_1 - R_1^{sys})^2 + (x_2 - x_2^{sys})^2 \\ &\text{s.t. } R_1 = P\{g_1(\mathbf{x}) = 1 - x_1^2 x_2/20 \le 0\} \\ &\quad 0 \le x_1 \le 10, 0 \le x_2 \le 10, R_1 \ge 0.9985 \end{aligned} \tag{14}$$

$$\begin{aligned} &\min : \mu_{x_2} + \mu_{x_3} + (R_2 - R_2^{sys})^2 + (x_2 - x_2^{sys})^2 \\ &\text{s.t. } R_2 = P\{g_2(\mathbf{x}) = 1 - (x_2^2 - 3x_3 + 5)/30 \le 0\} \\ &\quad 0 \le x_3 \le 10, 0 \le x_2 \le 10, R_2 \ge 0.9985 \end{aligned} \tag{15}$$

Different from ATC formulations, the RTC model is formulated in Figure 2.

The comparisons of optimization results from the proposed method, the probabilistic all-in-one (PAIO) and probabilistic ATC (PATC) are presented in Tables 2 and 3.

In terms of both the reliability allocation and design optimization problem, the optimal range obtained by the proposed RTC method is the same as that by the interval PAIO method. In the target cascading process of the proposed method, the sensitivity of the system to each subsystem can also be calculated, which will be studied in future research. The sensitivity provides information for the reliability allocation and system reliability design. The ranged targets are propagated though the multilevel system, which provides more choices and degree of flexibility to designers.

$$
\begin{aligned}
\min:\ & \left\| R_{sys} - \mathrm{T} \right\|_2^2 \\
\text{s.t. }\ & R_{sys} = R_{sys}(\mathbf{x}_{sys}, \mathbf{R}_{sub}) \\
& P\left\{ g_{sysj} = g_{sysj}(\mathbf{x}_{sys}, \mathbf{R}_{sub} \pm \Delta\mathbf{R}_{sub}) \le 0 \right\} \ge 1 - p_{fj},\ j = 1, 2, \cdots, m \\
& h_{sysj} = h_{sysj}(\mathbf{x}_{sys}, \mathbf{R}_{sub}) \le 0,\ j = 1, 2, \cdots, l \\
& \mathbf{R}_{sub} - \Delta\mathbf{R}_{sub} \le \mathbf{R}_{sub}^{sub} - \Delta\mathbf{R}_{sub}^{sub} \\
& \mathbf{R}_{sub}^{sub} + \Delta\mathbf{R}_{sub}^{sub} \le \mathbf{R}_{sub} + \Delta\mathbf{R}_{sub} \\
& [y_{ik}^{sys} - \Delta y_{ik}^{sys},\ y_{ik}^{sys} + \Delta y_{ik}^{sys}] = \bigcap_{i=1}^{n_{sub}} y_{ik}^{subI} \\
& y_{ik}^{subI} = [y_{ik}^{sub} - \Delta y_{ik}^{sub},\ y_{ik}^{sub} + \Delta y_{ik}^{sub}]
\end{aligned}
$$

$$\downarrow (R_{subk}^{sys}, \Delta R_{subk}^{sys}, y_{subk}^{sys}, \Delta y_{subk}^{sys}) \qquad \uparrow (R_{subk}^{sub}, \Delta R_{subk}^{sub}, y_{subk}^{sub}, \Delta y_{subk}^{sub})$$

$$
\begin{aligned}
\min:\ & \max \begin{pmatrix} \left\| (R_{subi} - \Delta R_{subi}) - (R_{subi}^{sys} - \Delta R_{subi}^{sys}) \right\|_2^2 \\ \left\| (R_{subi} + \Delta R_{subi}) - (R_{subi}^{sys} + \Delta R_{subi}^{sys}) \right\|_2^2 \end{pmatrix} \\
\text{s.t. }\ & R_{subi} = R_{subi}(\mathbf{x}_{sub}, \mathbf{y}_{sub}) \\
& P\left\{ g_{subj} = g_{subj}(\mathbf{x}_{sub}, \mathbf{y}_{sub} \pm \Delta\mathbf{y}_{sub}, R_{subi} \pm \Delta R_{subi}) \le 0 \right\} \ge 1 - p_{fj},\ j = 1, 2, \cdots, p \\
& h_{subj} = h_{subj}(\mathbf{x}_{sub}, \mathbf{y}_{sub}, R_{subi}) \le 0,\ j = 1, 2, \cdots, q \\
& \Delta y_{subik} = \frac{\partial y_{subik}}{\partial R_{subi}} \Delta R_{subi} \\
& \mathbf{y}_{sub}^{sys} - \Delta\mathbf{y}_{sub}^{sys} \le \mathbf{y}_{sub} - \Delta\mathbf{y}_{sub},\ \mathbf{y}_{sub} + \Delta\mathbf{y}_{sub} \le \mathbf{y}_{sub}^{sys} + \Delta\mathbf{y}_{sub}^{sys}
\end{aligned}
$$

Figure 1. Information flow of RTC for achieving flexibility.

$$
\begin{aligned}
\min:\ & f = R_1^2 + R_2^2 \\
\text{s.t. }\ & g_3 = 0.997 - R_1 R_2 \le 0 \\
& R_1 \ge 0.9985,\ R_2 \ge 0.9985 \\
& R_1^{sub} - \Delta R_1^{sub} \le R_1 - \Delta R_1 \\
& R_1 + \Delta R_1 \le R_1^{sub} + \Delta R_1^{sub} \\
& R_2^{sub} - \Delta R_2^{sub} \le R_2 - \Delta R_2 \\
& R_2 + \Delta R_2 \le R_2^{sub} + \Delta R_2^{sub}
\end{aligned}
$$

$$
\begin{aligned}
\min:\ & \mu_{x_1} + \mu_{x_2} + ((R_1 - \Delta R_1) - (R_1^{sys} - \Delta R_1^{sys}))^2 \\
& + ((R_1 + \Delta R_1) - (R_1^{sys} + \Delta R_1^{sys}))^2 \\
\text{s.t. }\ & R_1 = P\left\{ g_1(\mathbf{x}) = 1 - x_1^2 x_2 / 20 \le 0 \right\} \\
& x_2^{sys} - \Delta x_2^{sys} \le x_2 - \Delta x_2 \\
& x_2 + \Delta x_2 \le x_2^{sys} + \Delta x_2^{sys} \\
& 0 \le x_1 \le 10,\ 0 \le x_2 \le 10,\ R_1 \ge 0.9985
\end{aligned}
$$

$$
\begin{aligned}
\min:\ & \mu_{x_2} + \mu_{x_3} + ((R_2 - \Delta R_2) - (R_2^{sys} - \Delta R_2^{sys}))^2 \\
& + ((R_2 + \Delta R_2) - (R_2^{sys} + \Delta R_2^{sys}))^2 \\
\text{s.t. }\ & R_2 = P\left\{ g_2(\mathbf{x}) = 1 - (x_3^2 - 3x_2 + 5) / 30 \le 0 \right\} \\
& x_2^{sys} - \Delta x_2^{sys} \le x_2 - \Delta x_2 \\
& x_2 + \Delta x_2 \le x_2^{sys} + \Delta x_2^{sys} \\
& 0 \le x_3 \le 10,\ 0 \le x_2 \le 10,\ R_2 \ge 0.9985
\end{aligned}
$$

Figure 2. The formulation of the proposed RTC method.

Table 2. The optimal results of reliability allocation.

	Interval PAIO	Point PATC	Proposed RTC
R_1	[0.997, 1]	0.9985	[0.997, 1]
R_2	[0.997, 1]	0.9985	[0.997, 1]
Objective function	[11.7978, 12.8279]	11.8178	[11.7978, 12.8279]

Table 3. The optimal results of design variables.

	Interval PAIO	Point PATC	Proposed RTC
x_1	[3.7583, 3.764]	3.7625	[3.7583, 3.764]
x_3	[5.5225, 5.5354]	5.5321	[5.5225, 5.5354]
Common variable	[1.52, 1.5285]	1.5262	[1.52, 1.5285]

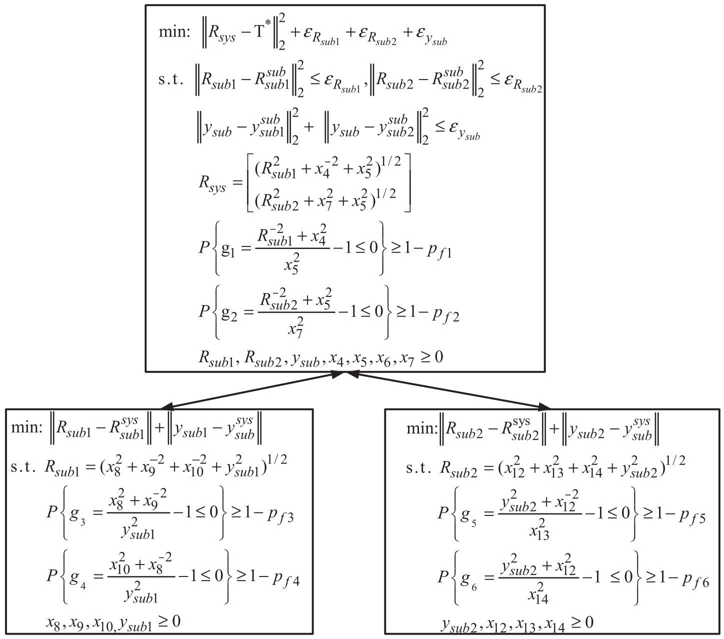

Figure 3. The formulation of original PATC.

4.2 A Geometric Programming Problem

Geometric programming problems with polynomials are known to have a unique global optimal solution. The AIO formulation of uncertainty analysis is provided in Equation (16), and the original PATC formulations and the proposed RTC formulation of the geometric programming problem are shown in Figures 3 and 4, respectively.

$$
\begin{aligned}
&\min f = x_1^2 + x_2^2 \\
&\quad \text{s.t. } P\{g_i(\mathbf{x}, \mathbf{p}) \le 0\} \ge 1 - p_{fi}, i = 1, 2, \ldots, 6 \\
&\quad g_1 = \frac{x_3^{-2} + x_4^2}{x_5^2} - 1, g_2 = \frac{x_6^{-2} + x_5^2}{x_7^2} - 1 \\
&\quad g_3 = \frac{x_8^2 + x_9^2}{x_{11}^2} - 1, g_4 = \frac{x_8^{-2} + x_{10}^2}{x_{11}^2} - 1 \\
&\quad g_5 = \frac{x_{12}^{-2} + x_{11}^2}{x_{13}^2} - 1, g_6 = \frac{x_{11}^2 + x_{12}^2}{x_{14}^2} - 1 \\
&\qquad x_3, x_4, ..., x_{14} \ge 0, i = 1, 2, \ldots, 6
\end{aligned}
\tag{16}
$$

where

$$
\begin{aligned}
&x_1 = \left(x_3^2 + x_4^{-2} + x_5^2\right)^{1/2}, x_2 = \left(x_5^2 + x_6^2 + x_7^2\right)^{1/2} \\
&x_3 = \left(x_8^2 + x_9^{-2} + x_{10}^{-2} + x_{11}^2\right)^{1/2}, \\
&x_6 = \left(x_{11}^2 + x_{12}^2 + x_{13}^2 + x_{14}^2\right)^{1/2}
\end{aligned}
$$

where p_{fi} is a range that within interval [0.001, 0.1] and x_8 the random variable, normally distributed with constant standard deviations $\sigma_{x_8} = 0.1$.

The proposed RTC optimization models for the three systems O_0, O_{11}, O_{12} are formulated in Equations (17)–(19).

O_0:

$$
\begin{aligned}
&\min : \|R_{sys} - T\|_2^2 \\
&\text{s.t. } R_{sys} = \begin{bmatrix} (R_{sub1}^2 + x_4^{-2} + x_5^2)^{1/2} \\ (R_{sub2}^2 + x_7^2 + x_5^2)^{1/2} \end{bmatrix} \\
&P\left\{ g_1 = \frac{(R_{sub1} \pm \Delta R_{sub1})^{-2} + x_4^2}{x_5^2} - 1 \le 0 \right\} \ge 1 - p_{f_1} \\
&P\left\{ g_2 = \frac{(R_{sub2} \pm \Delta R_{sub2})^{-2} + x_5^2}{x_7^2} - 1 \right\} \ge 1 - p_{f2} \\
&R_{sub1}^{sub} - \Delta R_{sub1}^{sub} \le R_{sub1} - \Delta R_{sub1} \\
&R_{sub1} + \Delta R_{sub1} \le R_{sub1}^{sub} + \Delta R_{sub1}^{sub} \\
&R_{sub2}^{sub} - \Delta R_{sub2}^{sub} \le R_{sub2} - \Delta R_{sub2} \\
&R_{sub2} + \Delta R_{sub2} \le R_{sub2}^{sub} + \Delta R_{sub2}^{sub} \\
&y_{sub} \pm \Delta y_{sub} = y_{sub1}^{sub} \pm y_{sub1}^{sub} \bigcap y_{sub2}^{sub} \pm y_{sub2}^{sub} \\
&R_{sub1}, R_{sub2}, y_{sub}, x_4, x_5, x_6, x_7 \ge 0
\end{aligned}
\tag{17}
$$

O_{11}:

$$
\begin{aligned}
&\min : \|(R_{sub1} \pm \Delta R_{sub1}) - (R_{sub1}^{sys} \pm \Delta R_{sub1}^{sys})\|_2^2 \\
&\quad \text{s.t. } R_{sub1} = (x_8^2 + x_9^{-2} + x_{10}^{-2} + y_{sub1}^2)^{1/2} \\
&\qquad P\left\{ g_3 = \frac{x_8^2 + x_9^{-2}}{(y_{sub1} \pm \Delta y_{sub1})^2} - 1 \le 0 \right\} \ge 1 - p_{f3} \\
&\qquad P\left\{ g_4 = \frac{x_{10}^2 + x_8^{-2}}{(y_{sub1} \pm \Delta y_{sub1})^2} - 1 \le 0 \right\} \ge 1 - p_{f4} \\
&\qquad y_{sub1}^{sys} - \Delta y_{sub1}^{sys} \le y_{sub1} - \Delta y_{sub1} \\
&\qquad y_{sub1} + \Delta y_{sub1} \le y_{sub1}^{sys} + \Delta y_{sub1}^{sys} \\
&\qquad x_8, x_9, x_{10}, y_{sub1} \ge 0
\end{aligned}
\tag{18}
$$

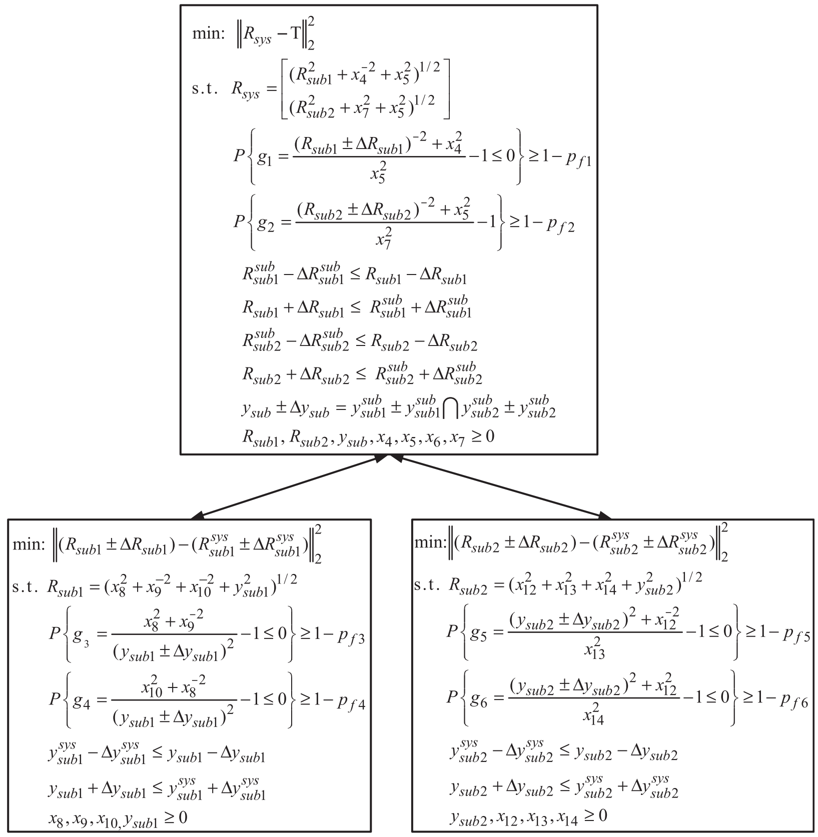

Figure 4. The formulation of proposed RTC.

O_{12}:

$$
\begin{aligned}
\min :& \|(R_{sub2} \pm \Delta R_{sub2}) - (R_{sub2}^{sys} \pm \Delta R_{sub2}^{sys})\|_2^2 \\
\text{s.t. }& R_{sub2} = (x_{12}^2 + x_{13}^2 + x_{14}^2 + y_{sub2}^2)^{1/2} \\
& P\left\{ g_5 = \frac{(y_{sub2} \pm \Delta y_{sub2})^2 + x_{12}^{-2}}{x_{13}^2} - 1 \le 0 \right\} \ge 1 - p_{f5} \\
& P\left\{ g_6 = \frac{(y_{sub2} \pm \Delta y_{sub2})^2 + x_{12}^2}{x_{14}^2} - 1 \le 0 \right\} \ge 1 - p_{f6} \\
& y_{sub2}^{sys} - \Delta y_{sub2}^{sys} \le y_{sub2} - \Delta y_{sub2} \\
& y_{sub2} + \Delta y_{sub2} \le y_{sub2}^{sys} + \Delta y_{sub2}^{sys} \\
& y_{sub2}, x_{12}, x_{13}, x_{14} \ge 0
\end{aligned} \tag{19}
$$

The solutions from PAIO and PATC are presented in Tables 4 and 5. The optimum solution for both random and interval variables are provided by the proposed RTC as shown in these tables.

From Table 5, it shows that the value of common variable can be obtained form interval [1.24, 1.64]. Based on the sensitivity of each subsystem to the common variable, system level can set different target of common variable to subsystem. Therefore, the interval solution provides more design freedom and flexibility to designers.

Table 4. Comparison of optimal design variables.

	Initial point	Interval PAIO	Point PATC	Proposed RTC
x_4	5.0	0.7598	0.7597	0.76,482
x_5	5.0	0.855	0.8659	0.85,677
x_7	5.0	0.913	0.9209	0.96,283
μx_8	5.0	1.09	1.2013	1.042
x_9	5.0	0.815	0.7912	0.99,423
x_{10}	5.0	0.845	0.7229	0.74,461
x_{12}	5.0	0.8409	0.8419	0.84,563
x_{13}	5.0	1.953	2.1080	1.8698
x_{14}	5.0	1.705	1.9344	1.67

Table 5. Comparison of optimal solutions.

	Interval PAIO	Point PATC	Proposed RTC
R_{sub1}	2.51±0.07	3.1019	2.33±0.12
R_{sub2}	3.165±0.175	3.5599	2.7631±0.15
Common variable	1.55±0.12	1.4830	1.44±0.2
Objective function	[18.9817, 21.996]	22.3038	[17.5197, 22.4872]

5. Conclusions

In this study, a new method to capture the effects of uncertainty and to improve design flexibility in a hierarchical multilevel system design process is proposed. Given the flexible interval, intervals of reliability are computed. The proposed method is capable of measuring how reliable a design is quantitatively under the required reliability target. Minimization of the interval deviation between the target and the achievable performance ranges, not only provides the optimum design value, but also gives the acceptable variation range for subsystem. This provides the flexibility that helps to resolve conflicts between the top-level system and lower level subsystems. This is superior to the conventional approach in which only a single-valued solution is obtained.

The further study will focus on the sensitivity analysis in a hierarchical system optimization and concurrent design in life cycle process [31]. In the presence of multiple levels, another approach of future research interest is the Stackelberg game theory approach.

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