Collaborative Reliability Analysis under the Environment of Multidisciplinary Design Optimization

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Collaborative Reliability Analysis under the Environment of Multidisciplinary Design Optimization

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Abstract: Uncertainties in multidisciplinary design optimization (MDO) have a significant influence on the whole design process of engineering systems. The most probable point (MPP) based reliability analysis is an approach that utilizes the safety index β to measure the effect of uncertainties. Collaborative optimization (CO) is a two-level optimization method specially created for large-scale distributed-analysis applications. Simulated annealing-based collaborative optimization (SA–CO) is one of the improved forms of CO that overcomes the difficulty of convergence given the existing of highly nonlinear consistency constraints. By combining the MPP-based reliability analysis method with SA–CO, we present a new collaborative reliability analysis method under the environment of MDO to deal with uncertainties existing in MDO, that is, MPP–SA–CO. Demonstrated by two typical examples, the proposed method inherits the advantages of CO. Also, accurate and efficient results are obtained by employing simulated annealing algorithmic as the system level optimizer and it features response surface instead of disciplinary optimization.

Key Words: multidisciplinary design optimization, collaborative optimization, simulated annealing-based collaborative optimization, reliability analysis.

1. Introduction

Multidisciplinary design optimization (MDO) [1] has been developed to optimize large-scale and coupled systems, where 'multidisciplinary' implies that a system involves multiple interacting disciplines. Numerous approaches have been proposed for analyzing these MDO problems, such as multidisciplinary feasible (MDF) method, individual discipline feasible (IDF) method [2], collaborative optimization (CO) [3], concurrent subspace optimization (CSSO) [4], and bi-level integrated system synthesis [5]. For these methods, CO uses separate optimization routines for each subsystem to satisfy interdisciplinary compatibility, while a system-level optimizer coordinates the tradeoffs among subsystems [6]. Traditional MDO methods generate deterministic designs, because loading, failure modes, design requirements, design variables, design parameters, objectives, and constraints are translated into deterministic parameters by the safety factor method. This can simplify the computation to some extent, but it may lead to suboptimal designs. Using excessively large safety factors makes the design conservative, while low safety factors will result in low reliability.

The advancement of computer-aided engineering has brought forward the development of simulation tools. These tools provide designers with flexible and inexpensive means to deal with complicated system analysis and design under a multidisciplinary collaborative environment [7]. However, the generated design may not be feasible because the simulated system may not fully represent the real system.

Uncertainty exists in engineering design. It has serious impact on the system performance if not properly addressed. In a MDO environment, a system consists of multiple highly coupled disciplines, which use different discipline models. The uncertainty of one discipline is propagated to another discipline by linking variables. And the final uncertainties of MDO are the accumulation of all disciplines uncertainties. Therefore, uncertainties should be considered in MDO to balance security and economical efficiency. Combining reliability analysis technique with optimization is one of the solutions to this problem.

One of the existing most probable point [8] (MPP) based reliability analysis methods under the

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environment of MDO is called MPP–IDF. In this method, the procedure of MPP-based reliability analysis method is combined with the collaborative disciplinary analysis to automatically satisfy the interdisciplinary consistency in reliability analysis. Another MPP-based method is called MPP–MDF. It applies reliability analysis technique directly with MDF method. The last two methods are collaborative methods presented by Du and Chen [9]. The MPP–MDF method requires a double loop, i.e. the inner loop performs multidisciplinary analysis (MDA) and the outer loop computes the reliability index. Thus, it must perform a MDA once MPP is found; the method is time-consuming and computationally expensive, especially for large-scale MDO problems. The proposed method avoids a complete MDA during the optimization process as it treats the coupled relations among disciplines as consistency constraints to automatically satisfy the interdisciplinary consistency requirement. In this way, the proposed method achieves interdisciplinary analysis and optimization in parallel.

Padmanabhan and Batill [10] developed the CSSO-based reliability analysis method, aiming at reducing computational costs associated with performing reliability analysis under MDO, called MPP–CSSO. It employs the decomposition strategy to search MPP. This method includes a CSSO approach and a special case of CSSO that requires only one subspace optimization. Hence, the MPP search only involves one discipline. Both the decomposition strategies require an algorithm that can generate a solution for an infeasible bounded MPP search. Because the bounds are too small, the equality constraint is infeasible.

Compared with traditional MDO, the computation of MDO under uncertainty is more complicated. Therefore, it is critical to have an efficient reliability analysis approach. CO is capable of addressing both computational and organizational complexities that exist in MDO, especially for designing large-scale complex engineering systems in a distributed design environment. CO has excellent potential compared with other MDO approaches, but it also has computational challenges because of its particular mathematical model. In this article, we present a new reliability analysis method under the environment of MDO, called MPP–SA–CO. It combines the simulated annealing-based collaborative optimization [11] (SA–CO) with the MPP-based reliability analysis method.

This article is organized as follows. Section 2 discusses reliability analysis under MDO. Section 3 proposes the SA–CO method. Section 4 proposes the collaborative reliability analysis method under the environment of MDO. In Section 5, two examples are used to illustrate the effectiveness of the proposed method. Finally, Section 6 is a summary of our conclusions.

2. Reliability Analysis under MDO

When uncertainty is considered, the mathematical expression of traditional MDO is redefined as:

$$\begin{aligned} \min \quad & f(x,y) \\ s.t. \quad & P\left\{c_i(x,y_{\cdot i}) \ge 0\right\} \ge R_i \quad i = 1,2,\ldots,n \end{aligned} \tag{1}$$

where f is the design objective, c_i the design constraint of discipline i of traditional MDO, x the vector of design variables, y the vector of interdisciplinary linking variables, $y_{\cdot i}$ the linking variables which are inputs from other disciplines except discipline i, and R_i is the reliability requirement of discipline i.

The objective of reliability analysis is to address reliability constraints in Equation (1), which is a major part of computation load. In Equation (1), reliability can be achieved theoretically by the integral of the joint probability density function. However, in practical engineering, the integrand is often high dimensional and highly nonlinear, and it is too difficult to perform the computation. To solve this difficulty [12], MPP based reliability method [9,13,21] and saddle point approximation method [22–24] were developed to solve reliability optimization problems. First-order reliability method [9] (FORM) or second-order reliability method [13] is used to approximate the reliability estimation whose core is the MPP search. Searching for MPP can be formulated as an optimization problem with an equality constraint:

$$\begin{aligned} \min \quad & \beta = \sqrt{u^T u} \\ s.t. \quad & c(u) = 0 \end{aligned} \tag{2}$$

To perform the MPP search, the input variables x are transformed into independent standard normal ones u using Rosenblatt transformation [14]; β is the safety index which is defined as the shortest distance from the origin to the surface of limit-state function [15]. After obtaining β, we can apply the FORM model and calculate R based on the relation $R = \Phi(\beta)$.

3. Simulated Annealing-based Collaborative Optimization

The CO formulation is a two-level hierarchical scheme for MDO, proposed by Kroo on the basis of IDF method. Each discipline is optimized independently subject to constraints exerted by other disciplines. The discrepancy among disciplines is harmonized by the system optimizer. Thus, consistent optimal design can be obtained by an iterative process between the system and the disciplinary levels. The method eliminates complicated MDA and it makes disciplines to be distributed and paralleled. Its structure is analogous to existing organization form of the aircraft design [16]. However, its solution usually is a local optima in the practical application due to its particular mathematical expression that the optimization formulation of the system level includes the nonlinear equality constraints, which cannot satisfy the Kuhn–Tucker conditions. One of the solutions to the problem is to apply modern design methods to CO, such as simulated annealing.

Simulated annealing is a generalization of a Monte Carlo (MC) method for examining the equations of state and frozen states of n-body systems [17]. The concept is based on the manner in which liquids solidify or metals recrystalize in the process of annealing. In an annealing process, initially at high temperature and disordered, the melt is slowly cooled so that the system at any time is approximately in thermodynamic equilibrium. As the cooling proceeds, the system becomes more ordered and approaches a 'frozen' ground state at $T=0$ (http://www.cs.sandia.gov/opt/survey/sa.html).

A combinatorial problem is similar to simulated annealing. The current state of the thermodynamic system is analogous to the current solution to the combinatorial problem. The energy equation for the thermodynamic system is analogous to the objective function and ground state to the global minimum (http://www.cs.sandia.gov/opt/survey/sa.html).

Applying simulated annealing as the system optimizer of CO and introducing response surface method instead of the disciplinary optimizers forms an improved CO, called SA–CO. Figure 1 shows the flowchart of the SA–CO method [11].

As shown in Figure 1, in order to reduce the number of exact analysis and optimization, a few new solutions are generated at each temperature level and delivered to the exact disciplinary analysis/optimization problems, and then the exact response values are obtained. These solutions, along with the exact response values, form a number of sample points. The response surface for disciplinary analysis/optimization are constructed using these sample points and applied instead of the exact disciplinary analysis/optimization. The SA method is used to find the optimal solutions for the system level objective function at a temperature level. Then, the temperature is reduced slowly and the above process is repeated at another temperature level until the convergence conditions are met. Typically, the following criteria can be used as convergence conditions:

1. Setting the stopping temperature artificially;
2. Setting the number of outer loop iterations artificially;
3. Optimal solutions satisfy with certain accuracy during the iterative calculation process;
4. Testing the stability of the system entropy.

4. Collaborative Reliability Analysis under the Environment of MDO

To illustrate the MPP–SA–CO approach, we first provide the mathematical models of the problem below.

System level optimization:

$$\begin{aligned} \min \quad & \beta = \sqrt{u^T u} + M\left(\sum_{i=1}^{N} \left|g_i^*(Z)\right|\right) \\ s.t. \quad & z_{i\min} \le z_i \le z_{i\max} \\ & Z = (u_i, y_i) \end{aligned} \tag{3}$$

where Z is the design variable at the system level, z_i the design variable of discipline i, $z_{i\min}$ and $z_{i\max}$ the lower and upper boundaries of z_i, respectively, N the number of the discipline, β the objective function, and g^* the constraint at the system level exerted by disciplinary optimization. M is the penalty factor.

Disciplinary level optimization:

$$\begin{aligned} \min \quad & g_i(u_{di}) = \sum_{j=1}^{l_i} \left(u_{ij} - z_{ij}\right)^2 + \sum_{j=1+l_i}^{l_i^*} \left(y_{ji} - z_{ij}\right)^2 \\ s.t. \quad & c_i(u_{di}) = 0 \\ & u_{di\min} \le u_{di} \le u_{di\max} \end{aligned} \tag{4}$$

where g_i is the objective function of discipline i in disciplinary level, u_{di} is design variable of discipline i and l_i its length, $u_{di\min}$ and $u_{di\max}$ are the lower and upper boundaries of u_{di}, respectively, u_{ij} the input linking variable, y_i the outputs of discipline i and l_{1i} its length, y_{ji} the output linking variable, l_i^* the length of z_i, and c_i the constraint of discipline i which is a constraint in reliability analysis. In practical engineering, $c_i(u_{di})=0$ may not occur. Given the above optimization models, the MPP–SA–CO solution strategy is shown in Figure 2.

According to the strategy and the mathematic model of MPP–SA–CO, the proposed method includes the following steps:

1. Discipline optimizer i acquires a few input variables z_i from the system optimizer to generate samples for building up response surface, treating these as fixed parameters in the discipline optimization.
2. Contribution analysis (CA) i is performed based on the design variables u_{di} provided by discipline optimizer i, returning the values of discipline output vectors y_i and design variables u_{di}. In this way, we can obtain a certain number of samples to build up the response surface for discipline i.

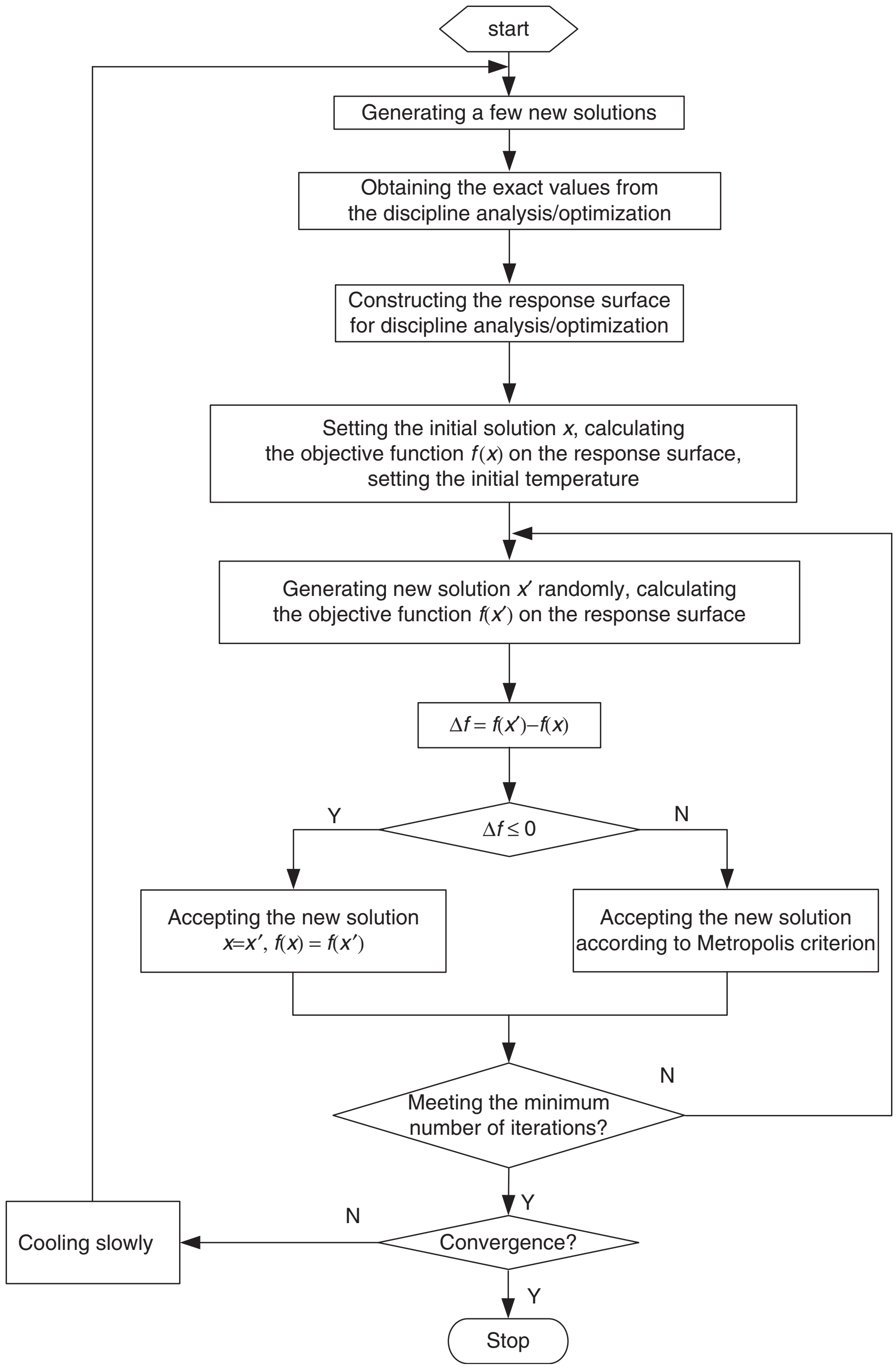

Figure 1. The flowchart of SA–CO.

3. Discipline optimizer.i is used to calculate the approximate solution of u_{di} and y_i and obtain the objective function g_i
4. The optimal value g_i of discipline i is passed to the system optimizer and used as a constraint. Next, the system optimizer obtains updated system design variables Z by adjusting the scale of domain and cooling, and redistributes them to each discipline optimizer.
5. Repeat steps (1)–(4) until the convergence condition is satisfied.

Because a multidisciplinary design problem includes many constraints, we recommend performing reliability analysis for active constraints only to improve the computation efficiency. The active constraints may be explicit functions of only a single or a few disciplines. For example, the stress constraints explicitly depend

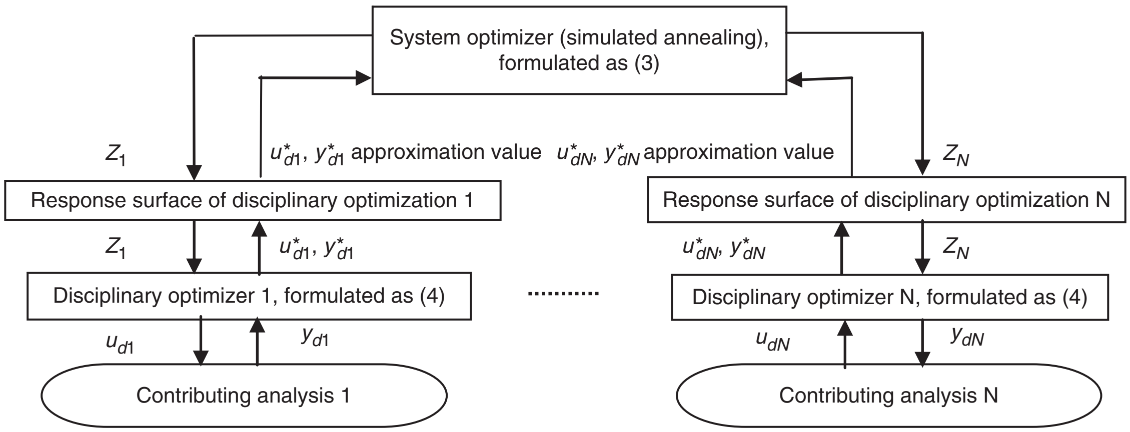

Figure 2. The mathematic model of MPP–SA–CO.

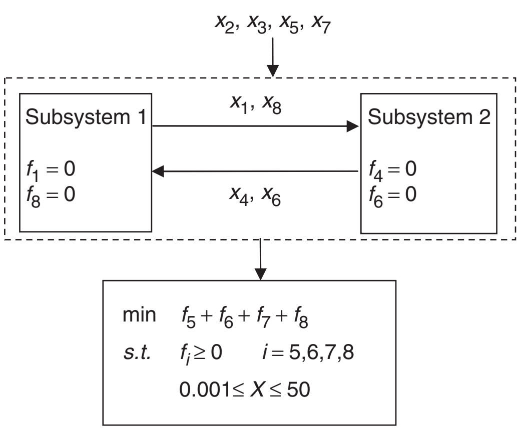

Figure 3. Heart dipole MDO model.

only on the stresses obtained from the static's discipline. Frequency constraints explicitly depend on the frequencies obtained from the dynamics discipline, etc. [10]. As we know, one discipline's outputs depend on other disciplines' inputs in a multidisciplinary problem. The active constraints are implicit functions of all disciplines' outputs. The MPP search for such an active constraint is still a MDO problem.

5. Examples

In this section, we use two examples to illustrate the proposed MPP–SA–CO approach. One example is the design of a heart dipole and the other the design of the power train for an automobile. In each example, the result from the MC method is considered as the correct solution.

5.1 Heart Dipole

5.1.1 PROBLEM DESCRIPTION AND ANALYSIS

Heart Dipole [18] is a typical MDO example proposed by NASA, which arises from the experimental electrolytic determination of the resultant dipole moment. The traditional solution is to solve the following eight nonlinear equations. However, heart dipole can be converted into a test problem for MDO methods by defining a system with two subsystems. The system diagram in Figure 3 shows the data dependencies. The original problem is given as follows:

$$
\begin{aligned}
&f_1(X) = x_1 + x_2 - d_{mx}\\
&f_2(X) = x_3 + x_4 + d_{my}\\
&f_3(X) = x_5x_1 + x_6x_2 - x_7x_3 - x_8x_4 + d_A\\
&f_4(X) = x_7x_1 + x_8x_2 + x_5x_3 + x_6x_4 - d_B\\
&f_5(X) = x_1x_5^2 - x_1x_7^2 - 2x_3x_5x_7 + x_2x_6^2 - x_2x_8^2\\
&\quad - 2x_4x_6x_8 - d_C\\
&f_6(X) = x_3x_5^2 - x_3x_7^2 + 2x_1x_5x_7 + x_4x_6^2 - x_4x_8^2\\
&\quad - 2x_2x_6x_8 + d_D\\
&f_7(X) = x_1x_5^3 - 3x_1x_5x_7^2 + x_3x_7^3 - 3x_3x_7x_5^2 + x_2x_6^3\\
&\quad - 3x_2x_6x_8^2 + x_4x_8^3 - 3x_4x_8x_6^2 + d_E\\
&f_8(X) = x_3x_5^3 - 3x_3x_5x_7^2 + x_1x_7^3 - 3x_1x_7x_5^2 + x_4x_6^3\\
&\quad - 3x_4x_6x_8^2 + x_2x_8^3 - 3x_2x_8x_6^2 - d_F
\end{aligned}
\tag{5}
$$

where $d_{mx}, d_{my}, d_A, d_B, d_C, d_D, d_E, d_F$ are constants.

Heart dipole problem can be regarded as a MDO problem containing two disciplines, involving four design variables x_2, x_3, x_5, x_7, and four linking variables x_1, x_4, x_6, x_8. The objective of the problem is to minimize the sum of f_5, f_6, f_7, f_8 subject to constraints.

Table 1. Results of reliability analysis for limit-state function f_5, f_6, f_7, f_8.

Method	f_5	f_6	f_7	f_8
MC				
β	0.00159776	3.09023	0.00138661	0.00064458
R	0.50064	0.999	0.500558	0.500259
MPP–CO–SA				
β	0.00159763	3.38718	0.00138648	0.000644565
R	0.500637	0.999647	0.500553	0.500257

The constraints require f_5, f_6, f_7, f_8 should be greater than 0 and the design variables should not to exceed their bounds.

5.1.2 CALCULATION AND RESULT ANALYSIS

MDF, one of MDO approaches, is adopted to perform deterministic optimization, and its result is used as initial values for MPP–SA–CO. The design variables x_3, x_5 are regarded as random variables and the constraints f_5, f_6, f_7, f_8 are treated as limited state functions. The reliability index β of the limited state functions is calculated by assuming the random variables to obey normal distribution, whose coefficient of variation is 0.1. The optimization expression for limited state function f_5 is given as follows, and those for f_6, f_7, f_8 is similar to f_5.

System level optimization:

$$\begin{aligned} &\min\ \beta = \sqrt{u_3^2 + u_5^2} + M\left(\left|g_1^*(Z)\right| + \left|g_2^*(Z)\right|\right) \\ &Z = (u_3, u_5, x_1, x_4, x_6, x_8) \end{aligned} \tag{6}$$

where u_3, u_5 are standard normal space vectors corresponding to random variables x_3, x_5, Z is the design variable, and $M = 100$.

Subsystem level optimization 1:

$$\begin{aligned} \min\quad & g_1 = \left(z_{11} - u_3^1\right)^2 + \left(z_{12} - u_5^1\right)^2 + \left(z_{13} - x_1^1\right)^2 \\ & + \left(z_{14} - x_8^1\right)^2 \\ s.t.\quad & f_5 = 0 \end{aligned} \tag{7}$$

where u_3^1, u_5^1 are the design variables for subsystem 1, and x_1^1, x_8^1 are obtained from discipline analysis 1.

Subsystem level optimization 2:

$$\begin{aligned} \min\ & g_2 = \left(z_{21} - u_3^2\right)^2 + \left(z_{22} - u_5^2\right)^2 + \left(z_{23} - x_4^2\right)^2 \\ & + \left(z_{24} - x_6^2\right)^2 \\ s.t.\ & f_5 = 0 \end{aligned} \tag{8}$$

where u_3^2, u_5^2 are the design variables for subsystem 2, and x_4^2, x_6^2 are obtained from discipline analysis 2.

Simulated annealing arithmetic is adopted in the system optimizer, which assumes that $t_0 = 100$, $t_f = 0.0001$, $t_{k+1} = 0.8t_k$, $L_k = 50$. Modified method of feasible directions is adopted in the subsystem optimizers and response surface model used to approximate subsystem optimization model, which is built with five samples.

The reliability R and reliability index β for all limit-state functions are shown in Table 1 and examined by MC. Table 1 also displays MPP–SA–CO results, showing a good agreement with that of MC in accuracy except for f_6. With cooling, the sample values are closer to the optimal solution, and the accuracy of response surface is better. As for efficiency, it significantly reduces the number of CA owing to the employment of approximation technique.

5.2 Automobile Power Train Design

5.2.1 PROBLEM DESCRIPTION AND ANALYSIS

With the development of the economy and traffic, energy supply becomes so tight that improving automobile power efficiency and reducing fuel consumption is one of the important research areas. However, in the process of research and development for automobile, engine manufacturers put emphasis on improving engine performance and decreasing fuel consumption, while the chassis manufacturers focus on increasing transmission efficiency [19]. In such a way, even if individual performance measures are all satisfactory, the performance of the whole automobile may not be the best.

From the theory of gas engine and automobile, increased power decreases the load rate of engine leading to bad fuel economy [20]. Therefore, the best design can be expected when both power and fuel economy are considered simultaneously.

Balancing key components of an automobile power train in a reasonable manner is one of the approaches to improve automobile power and fuel economy. A simple MDO model for some automobile power train is built, with the product of the final-drive ratio and the transmission ratio as a design variable. Its objective is to optimize power and fuel economy simultaneously.

Objective Function

Power Objective Function. When the driving gear is fixed, the difference between the performances of the real power train and the ideal power train, called loss rate of driving power ΔF, reflects power performance. The lower its value, the better the dynamic performance of the automobile. The loss rate of driving power ΔF is

$$\Delta F = \left(0.377 M_P n_P \eta_T \ln(v_{n+1}/v_1) - \sum_{j=1}^{n} \sum_{k=0}^{5} b_k \left(n_{j+1}^{k+1} - n_j^{k+1}\right)\right) / 0.377 M_P n_P \eta_T \ln(v_{n+1}/v_1) \tag{9}$$

where M_P,n_P are the torque and rotating speed of engine maximum power, respectively, η_T the transmission efficiency of power train, $b_k = 0.377\eta_T a_k/k+1$, a_k the fitting coefficient of engine external characteristic, and n_j,n_{j+1} the speed range of the jth gear.

Fuel Economy Objective Function. To measure the overall fuel economy, measurements are first performed at different working conditions. Multi-working condition cycling test is composed of acceleration, deceleration, and cruising, and its computation is shown as in http://www.cs.sandia.gov/opt/survey/sa.html. The example is based on four working conditions of a passenger car, whose fuel economy objective is $Q = \sum_{i=1}^{4} Q_i$, where Q is the overall fuel consumption and Q_i the oil consumption of every working condition.

When there are two objective functions to be optimized, a commonly used approach is to transform them into a single objective function using weighting factors, as shown below:

$$\min\ f(x) = \alpha_1 \Delta F + \alpha_2 Q \tag{10}$$

where α_1,α_2 are the weights for power loss and fuel consumption, respectively.

Constraint Function

Requirement Of Automobile Power. Direct gear maximum power factor $D_{0\max}$ is calculated by the following formulation, which shows the grade ability and the acceleration ability of direct gear or top gear.

$$D_{0\max} = \left(M_{e\max} I_0 \eta_T / R - C_D A v^2/21.15\right)/G \tag{11}$$

where $M_{e\max}$ is the engine maximum torque, v the speed at maximum torque for direct gear, I_0 the final-drive ratio, C_D the wind resistance coefficient, A the front face area, R the rolling radius of the tyre, and G the automobile weight. From Equation (11), it is known that enhancing direct gear power factor can increase acceleration ability, but can decrease maximum speed. This improves the overall top gear-transmission ratio, reduces engine load rate, and decreases automobile fuel economy. Therefore, attention to both power and economy is needed in calculating $D_{0\max}$. $D_{0\max} \geq 0.03$ for passenger car by experience.

Gear I maximum power factor $D_{\mathrm{I}\max}$ is calculated by the following formulation, which shows maximum grade ability:

$$D_{\mathrm{I}\max} = \left(M_{e\max} I_0 I_{\mathrm{I}} \eta_T / R - C_D A v_b^2/21.15\right)/G \tag{12}$$

where I_{I} is gear I-transmission ratio and v_b the maximum speed of engine gear I maximum torque.

Adhesion condition and the maximum driving force which must be less than or equal to ground adhesive force for automobile are checked computations according to the following formulation after calculating gear I maximum power factor.

$$M_{e\max} I_{\mathrm{I}} I_0 \eta_T / R \leq z_\varphi \varphi \tag{13}$$

where z_φ is the normal reaction of driving wheel and φ the road adhesion coefficient.

Requirement Of Rate Interval Among Each Gear. The middle gear transmission radio is calculated according to geometric series. In general, the radio interval between the adjacent gears should be below 1.8.

Requirement Of Fuel Economy. Engine load rate is calculated according to the requirement of engine's normal working condition:

$$U = \left[\left(Gf + C_D A v_a^2/21.15\right)(v_a/3600)/(M_e n_e \eta_T/9549)\right] \times 100\% \tag{14}$$

where U is the engine load rate, whose range is $0.7 \leq U \leq 0.9$, f the rolling resistance coefficient, v_a the running speed, M_e the engine effective torque, and n_e the engine rotation speed.

Table 2. Results of reliability analysis for limit-state function g_1,g_2,g_3,g_4

Method	g_1	g_2	g_3	g_4
MC				
β	3.13586	3.04872	2.84532	1.72463
R	0.999108	0.998887	0.997753	0.9577167
MPP–SA–CO				
β	3.13293	3.04357	2.84279	1.72275
R	0.999099	0.998836	0.997728	0.95755475

CALCULATION AND RESULT ANALYSIS

When design variables have minute fluctuation, power character and economy are influenced. Decreased power character can lead to reduced safety, and decreased fuel economy will result in the loss of energy sources. Therefore, design variables x_1, x_2, x_3, x_4, x_5 are assumed as random variables. Power requirements $g_1 = D_{0\max} - 0.03$, $g_2 = D_{1\max} - 0.2$, $g_3 = M_{e\max} I_1 I_0 \eta_T / R - z_\varphi \varphi$, and economy requirement $g_4 = U - 0.7$ are regarded as limited state functions to perform reliability analysis. Reliability index β of limited state functions is calculated by assuming the random variables to obey normal distribution, whose coefficient of variation is 0.1. The initial value (x_1,x_2,x_3,x_4,x_5) is {50,20.4808,16.758,10.26,6.84}. The whole automobile weight is 1,030,005 N, aerodynamic resistance coefficient 3.7 N·s^2/m^2, rolling resistance coefficient 0.012, transmission efficiency of power train 0.95, maximum torque of engine 353.2 N·m, maximum speed corresponding to maximum torque 1300 rpm, the rolling radius of tyre 0.49 m, and normal reaction force of driving wheel 68,670 N.

The reliability analysis formulations of each limited state function is given as follows

$$\begin{aligned} \min \quad & \beta = \sqrt{u_1^2 + u_2^2 + u_3^2 + u_4^2 + u_5^2} \\ s.t. \quad & g_1 = 0 \end{aligned} \tag{15}$$

$$\begin{aligned} \min \quad & \beta = \sqrt{u_1^2 + u_2^2 + u_3^2 + u_4^2 + u_5^2} \\ s.t. \quad & g_2 = 0 \end{aligned} \tag{16}$$

$$\begin{aligned} \min \quad & \beta = \sqrt{u_1^2 + u_2^2 + u_3^2 + u_4^2 + u_5^2} \\ s.t. \quad & g_3 = 0 \end{aligned} \tag{17}$$

$$\begin{aligned} \min \quad & \beta = \sqrt{u_1^2 + u_2^2 + u_3^2 + u_4^2 + u_5^2} \\ s.t. \quad & g_4 = 0 \end{aligned} \tag{18}$$

According to the proposed method, CO frame is built for Equations (15)–(18). Simulated annealing arithmetic is adopted in system optimizer assuming $t_0 = 100$, $t_f = 0.0001$, $t_{k+1} = 0.8t_k$, $L_k = 50$. Modified method of feasible directions is adopted in subsystem optimizer and the response surface model is used to approximate subsystem optimization model which is built with five samples.

The reliability R and reliability index β for all limit-state function are shown in Table 2, respectively, and examined by MC. The MPP–SA–CO results displayed in Table 2 show a good agreement with those of MC from the point of accuracy. If the initial value is optimum for traditional optimization, it is also optimum for reliability-based optimization. In other words, when design variables have fluctuations, optimum is still feasible. This insures the reliability for power and fuel economy.

6. Conclusions

Since the MPP-based reliability analysis under MDO employs existing MDO approaches, we perform MPP search by the means of SA–CO. In accordance with CO, its mathematic model splits into system-level optimization and disciplinary-level optimization. We adopt simulated annealing as optimizer for system-level optimization, and the use response surface in disciplinary-level optimization. Accuracy and efficiency of the new optimization method has been proved by the results of two MDO problems.

MPP-based reliability analysis is studied on the basis of existing MDO approaches, which lead to holdback of its development. In the future, the focus of research on the reliability analysis under MDO will be devoted to study MDO approaches on one hand, and find other more accuracy and efficiency reliability analysis methods on the other hand. For example, first-order saddlepoint approximation and second-order saddlepoint approximation are employed to analyze uncertainty, which are introduced by saddlepoint approximation method. Besides, the non-probabilistic reliability methods can be an alternative way to the reliability analysis under MDO when insufficient data are available.

Nomenclature

f = Objective function
c_i = Constraint function in discipline i
x = Vector of design variables
y = Vector of interdisciplinary linking variables
$y_{\cdot i}$ = Input linking variables to discipline i from other disciplines
R_i = Reliability requirement for discipline i
β = Safety index
Z = Design variables in system level
z_i = Design variable of discipline i
N = Number of disciplines
g^* = Constraint at system level exerted by disciplinary optimization
M = Penalty factor

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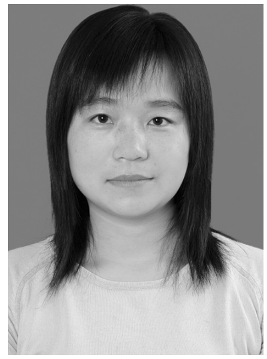

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