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Enhanced sequential optimization and reliability assessment for reliability-based design optimization †

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Abstract

Reliability-based design optimization (RBDO) has been receiving increasing attention for achieving high safety and reliability in engineering design. Sequential optimization and reliability assessment (SORA), as one of the efficient single-loop methods, decouples an RBDO problem into sequential deterministic optimization and reliability analysis. An enhanced SORA (ESORA) method is proposed with the aim of further improving the computational efficiency for RBDO, considering both cases of constant and varying variances of random design inputs while keeping the single-loop framework. Vehicle side impact example is used to test and compare the efficiency of the proposed method with existing approaches.

Keywords: Reliability-based design optimization; Sequential optimization and reliability assessment; Performance measure approach; Reliability analysis; Single-loop methods

1. Introduction

Reliability-based design optimization (RBDO) provides an approach to achieve reliable decision when considering the randomness of design variables and parameters which maybe come from manufacture, environment and so on. The typical mathematical formulation of RBDO is as follows:

$$\begin{aligned} &\min_{\mathbf{d},\boldsymbol{\mu}_\mathbf{X}} f(\mathbf{d},\boldsymbol{\mu}_\mathbf{X},\boldsymbol{\mu}_\mathbf{P}) \\ &s.t.\ \Pr(G_i(\mathbf{d},\mathbf{X},\mathbf{P})\le 0)\ge \Phi(\beta_t),\ \ i=1\sim h \\ &\quad \mathbf{d}^L\le\mathbf{d}\le\mathbf{d}^U,\ \boldsymbol{\mu}_\mathbf{X}^L\le\boldsymbol{\mu}_\mathbf{X}\le\boldsymbol{\mu}_\mathbf{X}^U \end{aligned} \tag{1}$$

where $\mathbf{d}$ is a vector of deterministic design variables, $\boldsymbol{\mu}_\mathbf{X}$ indicates a vector of mean values of random variables $\mathbf{X}=\{X_1,X_2,\cdots,X_n\}$ while $\boldsymbol{\mu}_\mathbf{P}$ represents a vector of mean values of random parameters $\mathbf{P}=\{P_1,P_2,\cdots,P_m\}$. $f(\cdot)$ is the objective function. $G_i(\cdot)$, $i=1\sim h$ are performance functions, and $\Pr(G_i(\cdot)\le 0)$ is the probability of success. $\Phi(\beta_t)$ is the target reliability and $\Phi(\cdot)$ is the cumulative distribution function (CDF) of the standard normal random variable. The superscripts 'L' and 'U' denote the lower and upper boundaries, respectively. In this formulation, $\mathbf{d},\boldsymbol{\mu}_\mathbf{X}$ are design variables to be determined.

Solving the RBDO problem directly will involve double loops: the outer loop is to minimize the objective function while conducting reliability analysis in the inner loop. To efficiently deal with an RBDO problem, one method is to improve the efficiency of reliability analysis including modifying the probability constraint as reliability index approach (RIA) and performance measure approach (PMA) [1-3] and enhancing the efficiency of algorithm in finding the most probable point (MPP) [4-6]. Although the PMA can efficiently decrease computation in reliability analysis, the computation in solving the large-scale RBDO problem using double loop approach is still unaffordable. Approaches of two new classes for RBDO are proposed [7-11]. In the first class, the RBDO problem is decoupled into sequential deterministic optimization and reliability analysis [7-9]. When constructing the deterministic constraints in the deterministic optimization, the strategy of constraint shift is adopted in Ref. [7]; the SORA proposed in Ref. [8] adopts the following strategy: to each probability constraint, utilizing its MPP of previous cycle to obtain the shift vector of each random design variable. In the second class, the RBDO problem is converted into a deterministic optimization by eliminating the reliability analysis (performing in the inner loop) through the KKT condition [10, 11].

Our objective here is to enhance the efficiency of SORA considering both cases of constant and varying variances of random design variables while keeping the single-loop struc-

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ture. The efficiency of the proposed method is testified using an engineering example.

This paper is organized as follows. The enhanced SORA (ESORA) is proposed in Section 2. An engineering example is used to illustrate the efficiency of the proposed method in Section 3, followed by the conclusions in Section 4.

2. Enhanced SORA for RBDO problems

First, we assumed that each performance function is explicit so that its expression of gradient can be obtained. When the random variables follow the normal distributions, the MPP in X-space from U-space can be gotten by

$$\begin{aligned} X^* &= \mu_X + \sigma_X \cdot U_X^* \\ P^* &= \mu_P + \sigma_P \cdot U_P^* \,. \end{aligned} \tag{2}$$

Based on Eq. (2), the following formulation holds in Cycle k:

$$\begin{aligned} X_j^{*,(i),k} &= \mu_{X_j}^k + \sigma_{X_j} \cdot U_{X_j}^{*,(i),k}, j = 1 \sim n \\ P_j^{*,(i),k} &= \mu_{P_j} + \sigma_{P_j} \cdot U_{P_j}^{*,(i),k}, j = 1 \sim m \,. \end{aligned} \tag{3}$$

The SORA approximates the MPP of Cycle k in the deterministic optimization as:

$$\begin{aligned} X_j^{*,(i)} &\approx \mu_{X_j}^k - s_j^{(i),k} = \mu_{X_j}^k - (\mu_{X_j}^{k-1} - X_j^{*,(i),(k-1)}) \\ &= \mu_{X_j}^k + \sigma_{X_j} \cdot U_{X_j}^{*,(i),(k-1)}, \; j = 1 \sim n; \\ P_j^{*,(i)} &\approx P_j^{*,(i),(k-1)} \\ &= \mu_{P_j} + \sigma_{P_j} \cdot U_{P_j}^{*,(i),(k-1)}, \; j = 1 \sim m \,. \end{aligned} \tag{4}$$

From Eqs. (3) and (4), the SORA approximates the MPP in the U-space in the kth Cycle as:

$$\begin{aligned} \mathbf{U}_\mathbf{X}^{*,(i),k} &\approx \mathbf{U}_\mathbf{X}^{*,(i),(k-1)} \\ \mathbf{U}_\mathbf{P}^{*,(i),k} &\approx \mathbf{U}_\mathbf{P}^{*,(i),(k-1)} \,. \end{aligned} \tag{5}$$

Using the PMA method, the formulation of finding the MPP of a probability constraint is:

$$\begin{aligned} G_p &= \max_{\mathbf{U_X}, \mathbf{U_P}} \; G(\mathbf{U_X}, \mathbf{U_P}) \\ s.t. \; & \| \mathbf{U_X}, \mathbf{U_P} \|_2 = \beta_t \,. \end{aligned} \tag{6}$$

Following the same way used in Ref. [10], based on Eq. (6) the relationship between the MPP $\mathbf{U}_\mathbf{X}^*, \mathbf{U}_\mathbf{P}^*$ and the gradient $\nabla G(\mathbf{U_X}, \mathbf{U_P}) |_{\mathbf{U}_\mathbf{X}^*, \mathbf{U}_\mathbf{P}^*}$ in the U-space is:

$$\begin{aligned} U_{X_j}^* &= \frac{\left. \partial G_\mathbf{U}(\mathbf{U_X}, \mathbf{U_P}) / \partial U_{X_j} \right|_{\mathbf{U}_\mathbf{X}^*, \mathbf{U}_\mathbf{P}^*}}{\| \nabla G_\mathbf{U}(\mathbf{U}_\mathbf{X}^*, \mathbf{U}_\mathbf{P}^*) \|_2} \cdot \beta_t, j = 1 \sim n; \\ U_{P_j}^* &= \frac{\left. \partial G_\mathbf{U}(\mathbf{U_X}, \mathbf{U_P}) / \partial U_{P_j} \right|_{\mathbf{U}_\mathbf{X}^*, \mathbf{U}_\mathbf{P}^*}}{\| \nabla G_\mathbf{U}(\mathbf{U}_\mathbf{X}^*, \mathbf{U}_\mathbf{P}^*) \|_2} \cdot \beta_t, j = 1 \sim m; \end{aligned} \tag{7}$$

and the relationship of gradient in the U-space and in the X-space is:

$$\begin{aligned} \frac{\partial G_\mathbf{U}(\mathbf{U_X}, \mathbf{U_P})}{\partial U_{X_j}} &= \frac{\partial G_{\mathbf{X},\mathbf{P}}(\mathbf{d}, \mathbf{X}, \mathbf{P})}{\partial X_j} \cdot \sigma_{X_j}, \; j = 1 \sim n; \\ \frac{\partial G_\mathbf{U}(\mathbf{U_X}, \mathbf{U_P})}{\partial U_{P_j}} &= \frac{\partial G_{\mathbf{X},\mathbf{P}}(\mathbf{d}, \mathbf{X}, \mathbf{P})}{\partial P_j} \cdot \sigma_{P_j}, \; j = 1 \sim m \,. \end{aligned} \tag{8}$$

From Eqs. (2), (7) and (8), at the MPP in the X-space $\mathbf{X}^*, \mathbf{P}^*$ the following formulation holds:

$$\begin{aligned} X_j^* &= \mu_{X_j} + \sigma_{X_j} \cdot \frac{b_{X_j}}{\| \mathbf{b} \|_2} \cdot \beta_t, j = 1 \sim n; \\ P_j^* &= \mu_{P_j} + \sigma_{P_j} \cdot \frac{b_{P_j}}{\| \mathbf{b} \|_2} \cdot \beta_t, j = 1 \sim m; \end{aligned} \tag{9}$$

where

$$\begin{aligned} b_{X_j} &= \left. \frac{\partial G(\mathbf{d}, \mathbf{X}, \mathbf{P})}{\partial X_j} \right|_{\mathbf{X}^*, \mathbf{P}^*} \cdot \sigma_{X_j}, \mathbf{b_X} = [b_{X_1}, b_{X_2}, \cdots, b_{X_n}]; \\ b_{P_j} &= \left. \frac{\partial G(\mathbf{d}, \mathbf{X}, \mathbf{P})}{\partial P_j} \right|_{\mathbf{X}^*, \mathbf{P}^*} \cdot \sigma_{P_j}, \mathbf{b_P} = [b_{P_1}, b_{P_2}, \cdots, b_{P_m}]; \\ \mathbf{b} &= [\mathbf{b_X}, \mathbf{b_P}] \,. \end{aligned}$$

Eq. (9) holds for each cycle, and the MPP in Cycle k can be obtained by

$$\begin{aligned} X_j^{*,k} &= \mu_{X_j}^k + \sigma_{X_j} \cdot \frac{b_{X_j}^k}{\| \mathbf{b}^k \|_2} \cdot \beta_t, j = 1 \sim n; \\ P_j^{*,k} &= \mu_{P_j} + \sigma_{P_j} \cdot \frac{b_{P_j}^k}{\| \mathbf{b}^k \|_2} \cdot \beta_t, j = 1 \sim m; \end{aligned} \tag{10}$$

where

$$\begin{aligned} b_{X_j}^k &= \left. \frac{\partial G(\mathbf{d}, \mathbf{X}, \mathbf{P})}{\partial X_j} \right|_{\mathbf{X}^{*,k}, \mathbf{P}^{*,k}} \cdot \sigma_{X_j}, \mathbf{b}_\mathbf{X}^k = [b_{X_1}^k, b_{X_2}^k, \cdots, b_{X_n}^k]; \\ b_{P_j}^k &= \left. \frac{\partial G(\mathbf{d}, \mathbf{X}, \mathbf{P})}{\partial P_j} \right|_{\mathbf{X}^{*,k}, \mathbf{P}^{*,k}} \cdot \sigma_{P_j}, \mathbf{b}_\mathbf{P}^k = [b_{P_1}^k, b_{P_2}^k, \cdots, b_{P_m}^k]; \\ \mathbf{b}^k &= [\mathbf{b}_\mathbf{X}^k, \mathbf{b}_\mathbf{P}^k] \,. \end{aligned} \tag{11}$$

When constructing the deterministic constraints at Cycle k, the gradient at the actual MPP cannot be obtained because a reliability analysis has not been performed. The gradient at the actual MPP is approximated in this way:

For the random design variables with constant variances, from Eq. (5)

$$\frac{\mathbf{X}^{*,k} - \boldsymbol{\mu}_\mathbf{X}^k}{\boldsymbol{\sigma}_\mathbf{X}} \approx \frac{\mathbf{X}^{*,(k-1)} - \boldsymbol{\mu}_\mathbf{X}^{(k-1)}}{\boldsymbol{\sigma}_\mathbf{X}}$$

which is equivalent to

$$\mathbf{X}^{*,k} \approx \boldsymbol{\mu}_{\mathbf{X}}^{k} - \boldsymbol{\mu}_{\mathbf{X}}^{(k-1)} + \mathbf{X}^{*,(k-1)} \tag{12}$$

and the MPP of random parameter is:

$$\mathbf{P}^{*,k} \approx \mathbf{P}^{*,(k-1)} . \tag{13}$$

By substituting Eqs. (12) and (13) into Eq. (11), the approximation of the gradient at the MPP is obtained. At the first cycle $(k=1)$, the gradient at the actual MPP is approximated as the gradient at the mean values of random variables and parameters.

For the random design variables with varying variances, from Eqs. (7)-(11) the following formulation can be obtained:

$$\begin{aligned} X_j^{*,k} &= \mu_{X_j}^k + \sigma_{X_j}^k \cdot \frac{b_{X_j}^k}{\|\mathbf{b}^k\|_2} \cdot \beta_t, j = 1 \sim n; \\ P_j^{*,k} &= \mu_{P_j} + \sigma_{P_j} \cdot \frac{b_{P_j}^k}{\|\mathbf{b}^k\|_2} \cdot \beta_t, j = 1 \sim m; \end{aligned} \tag{14}$$

where

$$\begin{aligned} b_{X_j}^k &= \left.\frac{\partial G(\mathbf{d},\mathbf{X},\mathbf{P})}{\partial X_j}\right|_{\mathbf{X}^{*,k},\mathbf{P}^{*,k}} \cdot \sigma_{X_j}^k, \mathbf{b}_{\mathbf{X}}^k = [b_{X_1}^k, b_{X_2}^k, \cdots, b_{X_n}^k]; \\ b_{P_j}^k &= \left.\frac{\partial G(\mathbf{d},\mathbf{X},\mathbf{P})}{\partial P_j}\right|_{\mathbf{X}^{*,k},\mathbf{P}^{*,k}} \cdot \sigma_{P_j}, \mathbf{b}_{\mathbf{P}}^k = [b_{P_1}^k, b_{P_2}^k, \cdots, b_{P_m}^k]; \\ \mathbf{b}^k &= [\mathbf{b}_{\mathbf{X}}^k, \mathbf{b}_{\mathbf{P}}^k] . \end{aligned} \tag{15}$$

For the random design variables with varying variances (expressed by $\sigma = r \cdot \mu$, r is the constant coefficient of variation), from Eq. (5)

$$\frac{\mathbf{X}^{*,k} - \boldsymbol{\mu}_{\mathbf{X}}^k}{\boldsymbol{\sigma}_{\mathbf{X}}^k} \approx \frac{\mathbf{X}^{*,(k-1)} - \boldsymbol{\mu}_{\mathbf{X}}^{(k-1)}}{\boldsymbol{\sigma}_{\mathbf{X}}^{(k-1)}}$$

which is equivalent to

$$\mathbf{X}^{*,k} \approx \boldsymbol{\mu}_{\mathbf{X}}^k + \frac{\boldsymbol{\mu}_{\mathbf{X}}^k}{\boldsymbol{\mu}_{\mathbf{X}}^{(k-1)}} \cdot (\mathbf{X}^{*,(k-1)} - \boldsymbol{\mu}_{\mathbf{X}}^{(k-1)}) . \tag{16}$$

By incorporating Eqs. (13) and (16) into Eq. (15), the gradient at the MPP is obtained. At the first cycle, the gradient at the actual MPP is approximated as the gradient at the mean values of random variables and parameters.

The discussion above is based on the normal random variables and parameters. For non-normal random variables and parameters, the Rackwitz-Fiessler' two-parameter equivalent normal method can be used to obtain the mean value and variance of equivalent normal distribution at a point of interest [9, 10].

Table 1. Optimal solution of vehicle side impact with constant variances.

Design variables	Orig. SORA	ESORA
μ_1	0.8141	0.8141
μ_2	1.3500	1.3500
μ_3	0.7278	0.7278
μ_4	1.5000	1.5000
μ_5	1.5344	1.5344
μ_6	1.2000	1.2000
μ_7	0.4000	0.4000
Objective	29.8918	29.8919
Constraints		
G_1	-9.0437×10^{-9}	-1.3203×10^{-10}
G_2	-2.0797	-2.0797
G_3	-1.4827	-1.4827
G_4	-0.0557	-0.0558
G_5	-0.0875	-0.0877
G_6	-0.0163	-0.0163
G_7	1.4780×10^{-7}	-1.3174×10^{-7}
G_8	-0.3763	-0.3763
G_9	3.5952×10^{-8}	8.5986×10^{-7}
G_{10}	-0.5221	-0.5221
Cycles	4	3
NFE	3102	2301

When the performance functions are all linear functions, because the gradient of each performance function is constant, the original RBDO problem is completely transformed into a deterministic optimization problem with Eqs. (10) and (14). In other words, the optimum of this deterministic optimization problem is the optimal solution of the original RBDO problem.

3. Numerical examples

In this section, a vehicle side impact example from Ref. [13] is used to illustrate the efficiency of the proposed method. In the vehicle side impact problem, there are seven random design variables and four random parameters, and the objective is to minimize the vehicle weight. Herein, $\mu_8 = 0.3450$, $\mu_9 = 0.1920$ is assumed. The RBDO formulation is [13]:

$$\begin{aligned} &\min f(\boldsymbol{\mu}_{\mathbf{X}},\boldsymbol{\mu}_{\mathbf{P}}) = 1.98 + 4.9\mu_1 + 6.67\mu_2 + 6.98\mu_3 \\ &\qquad + 4.01\mu_4 + 1.78\mu_5 + 2.73\mu_7 \\ &\text{s.t.} \quad P(G_i(X) \le 0) \ge R_i, \ i = 1 \sim 10 \\ &\mu_i^L \le \mu_i \le \mu_i^U, \ i = 1 \sim 7 \\ &\mu_8 = 0.3450, \mu_9 = 0.1920 \\ &\mu_{10}, \mu_{11} = 0.0 . \end{aligned}$$

The target reliability is $0.99865 = \Phi(3)$ for each probability constraint.

Table 1 lists the optimal solutions obtained by the Orig.

Table 2. Optimal solution of vehicle side impact with varying variances.

Design variables	$r_i = 0.05$		$r_i = 0.1$	
	Orig. SORA	ESORA	Orig. SORA	ESORA
μ_1	0.9669	0.9669	1.4414	1.4415
μ_2	1.3500	1.3500	1.3500	1.3500
μ_3	0.7604	0.7604	0.9154	0.9154
μ_4	1.5000	1.5000	1.5000	1.5000
μ_5	1.6293	1.6293	1.9798	1.9798
μ_6	1.2000	1.2000	1.2000	1.2000
μ_7	0.4000	0.4000	0.4000	0.4000
Objective	31.0367	31.0368	35.0681	35.0685
Constraints				
G_1	8.5253×10^{-8}	-1.4×10^{-8}	8.4853×10^{-7}	5.95×10^{-11}
G_2	-2.7523	-2.7523	-3.7294	-3.7294
G_3	-1.9984	-1.9983	-2.6137	-2.6137
G_4	-0.0636	-0.0636	-0.0760	-0.0760
G_5	-0.0918	-0.0920	-0.0973	-0.0973
G_6	-0.0169	-0.0169	-0.0071	-0.0071
G_7	1.6936×10^{-7}	-1.6×10^{-5}	6.1154×10^{-7}	-3.9×10^{-6}
G_8	-0.4248	-0.4248	-0.4895	-0.4895
G_9	9.7829×10^{-7}	-3.5×10^{-7}	2.5110×10^{-7}	-3.5×10^{-9}
G_{10}	-0.4513	-0.4513	-0.3269	-0.3269
Cycles	8	4	10	4
NFE	4173	2570	5502	2674

SORA and the ESORA with the constant variances as $\sigma_{1-4,6,7} = 0.03, \sigma_5 = 0.05, \sigma_{8,9} = 0.006$ and $\sigma_{10,11} = 10$. The starting points are the same for both methods as $[1\ 0.9\ 1\ 1\ 1.75\ 0.8\ 0.8]$. The same strategy and condition are adopted for both methods: the convergent criterion is $G_i \le 10^{-6}\ i = 1 \sim 10$ and 0.01% for the value of objective function. From Table 1, the cycles and NFE needed in ESORA are all less than those of the Orig. SORA which indicates that the ESORA is more efficient than the Orig. SORA.

Table 2 lists the optimal solutions of Orig. SORA and ESORA with varying variances. The starting points are same for both methods as $[1\ 0.9\ 1\ 1\ 1.75\ 0.8\ 0.8]$. The convergent criterion is $G_i \le 10^{-6}\ i = 1 \sim 10$ and 0.01% for the value of objective function. For both cases of $r_i = 0.05$ and $r_i = 0.1$, the cycles and NFE needed in the ESORA are all much less than those of Orig. SORA especially when the value of constant coefficient of variation increases, which indicates that the ESORA is much more efficient than the Orig. SORA.

4. Conclusions

SORA is one of the most efficient single loop methods for RBDO. In this paper, an enhanced SORA (ESORA) is proposed with the aim of further improving the computational efficiency, considering both cases of constant and varying variances, while keeping the single loop framework. In the ESORA, when the performance functions are all linear, the original RBDO problem is completely transformed into a deterministic optimization problem. When the performance functions are not all linear, in the deterministic optimization, the gradient at the actual MPP is approximated using the actual MPP and the mean values of random variables of previous cycle, and the mean values of random variables of current cycle while the gradient is approximated at the mean value of the random design variables and parameters at the first cycle.

As demonstrated in the vehicle side impact example, the same starting points and the optimization method are utilized, the cycles and NFE of ESORA are all much less than those of the original SORA, especially when the value of constant coefficient of variation increases, which indicates that the ESORA is much more efficient than the original SORA.

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