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RELIABILITY BASED OPTIMAL PREVENTIVE MAINTENANCE POLICY OF SERIES-PARALLEL SYSTEMS

To reduce the maintenance cost and improve the effectiveness of the maintenance activities in series-parallel systems, a preventive maintenance (PM) decision model for series-parallel systems subject to reliability was developed. This model considered the effect of failure maintenance on PM cycle and the restriction of system reliability in maintenance decision making, thus can help decision-maker to arrange appropriate and effective maintenance activities. Finally, an example was given to validate the proposed model.

Keywords: *Series-parallel systems, preventive maintenance, reliability based maintenance.*

1. Introduction

PM can improve machine availability and minimize related maintenance cost by arranging maintenance activities before system failure. It's an important policy to guarantee reliability of system. Because of its significance during whole life cycle of machine, researches have been conducted to establish its model [3,5,8]. After Barlow presented the simple PM model [1], many optimization models have been established in recent years. Such as Tsai presented periodic PM of a system with deteriorated components [9], and Duarte proposed an algorithm to solve the previous problem for equipment in order to ensure its availability [4].

However, these models mentioned above were established to minimize maintenance cost without considering system reliability. In addition, with the increase of system complexity, serial or parallel model cannot describe the system perfectly. Instead, compound structure system is widely used in practical engineering. This paper established a PM model which meets reliability restriction in series-parallel systems. Genetic Algorithm (GA) is applied to obtain the optimal parameters. The application of this model can be used for on-site maintenance scheduling.

This paper is organized as follows. Section 2 illustrates the detailed process of the construction of the PM decision model under concrete background and premises. In section 3, an example is presented to validate and analyze the PM model for series-parallel systems under reliability restriction. Section 4 is the conclusions of the paper.

2. PM Decision Model

2.1. Series-parallel systems model

The series-parallel system [6] is illustrated as Figure 1.
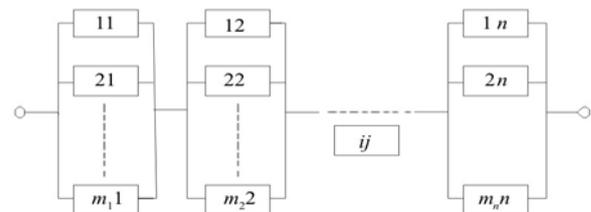
Fig. 1. Serial-parallel system

2.2. Basic assumptions

In order to construct and analyze the model, we assume that [2,7]:

1) Failure maintenance will be taken immediately after the occurrence of failure within the preventive maintenance cycle T_{PM}, and it will result in the beginning of next PM cycle.
2) Preventive maintenance will alter and reduce the failure rate in some degree. The effect is decided by age reduction factor α_i and failure acceleration factor A_i.
3) The time in which a subsystem is not available due to PM activity is negligible compared with the time elapsed between consecutive activities.

2.3. Maintenance cost model

Based on the assumptions mentioned above，the total cost during the whole system running time T_H is:

$$C_{Total} = \sum_{i=1}^{m} \sum_{j=1}^{n} \left(C(T_{PM})_i \times r_p + C_{f_i} N(i,j) + \varphi_i C_d(i) \right) \quad (1)$$

Where m is the number of subsystem in series-parallel systems, n is the total PM times of subsystem i during T_H. $C(T_{PM})$ is the PM cost of each component in series-parallel systems in per PM cycle, C_f is the cost of a failure maintenance per component in per PM cycle, $r(T_{PM})$ is probability of a PM activity in one preventive cycle, $N(i,j)$ is failure times in the i subsystem before the jth preventive maintenance, $C_d(i)$ is the system delay loss due to subsystem i maintenance activity, which can be calculated by $LT_M(i)$ (L means the production delay loss in unit time, and $T_M(i)$ means delay time when subsystem i does preventive maintenance).

Actually, it may not affect the system when one or two components in one subsystem breakdown in the serial-parallel mode, even if all the components in the same subsystem need maintenance activities, the maintenance can be arranged in the nonproductive time to reduce production loss. When the subsystem is a serial structure, system delay correlation factor φ_i is equal to 1 and it will cause productive loss, otherwise equal to 0[6].

2.4. Failure rate and failure times

Dedopoulos presented that preventive maintenance can neither renew the system nor maintain the same failure rate [3], therefore, age reduction model was considered. Later, age reduction factor α_i was proposed to reflect the effect to the failure rate after PM. Based on this factor, we present failure acceleration factor A_i which reflects the changing speed of failure rate [6].

Assume i subsystem's PM cycle as $T_{PM}(i)$, after jth maintenance activity, its failure rate has been reduced to the status $\alpha_i T_{PM}(i)$ before this PM. Consider that: PM may influence failure rate curvature, so failure acceleration factor A_i is used. The recursion relationship of failure rate λ_i, before the jth preventive maintenance, can be expressed like this:

$$\begin{aligned} \lambda_1(t) &= \lambda(t) \\ \lambda_2(t) &= A_i \lambda_1(t + T_{PM}(i) - \alpha_i T_{PM}(i)) \\ &\vdots \\ \lambda_j(t) &= A_i \lambda_{j-1}(t + T_{PM}(i) - \alpha_i T_{PM}(i)) \end{aligned}$$

Simplify:

$$\lambda_j(t) = \left(\prod_{i=1}^{i=j-1} A_i \right) \lambda(t + (j-1)(T_{PM}(i) - \alpha_i T_{PM}(i))) \quad (2)$$

It should be noted that:
1) The range of α_i is $0 < \alpha_i < 1$, which describes the degree of age reduction of the ith subsystem. The larger α_i is, the more effective the preventive maintenance makes.
2) A_i which represents the ith subsystem failure acceleration factor should satisfy $A_i > 0$. $A_i > 1$ means preventive maintenance accelerates the failure rate, $A_i = 1$ means no effect and $0 < A_i < 1$ represents the deceleration of the rate.

Before the jth PM, failure times in the ith subsystem can be written as:

$$N(i,j) = \int_0^{T_{PM}(i)} \lambda(t)dt =$$
$$= \int_0^{T_{PM}(i)} \left(\prod_{i=1}^{i=j-1} A_i \right) \lambda\left(t + (j-1)(T_{PM}(i) - \alpha_i T_{PM}(i))\right) dt \quad (3)$$

2.5. Reliability restriction

During the implementation of PM, we must make sure that the system should be reliable enough so as to accomplish the production tasks. Subsystem's reliability is:

$$R_i(t) = \exp(-\int_0^t \lambda_i(t)dt) \quad (4)$$

According to Equation (2), (4), when running into time t after jth PM activity, the ith subsystem's reliability is:

$$R_i(t) = \exp(-\int_0^{t+j(T_{PM}(i)-\alpha_i T_{PM}(i))} \lambda_i(t)dt) \quad (5)$$

Thus, the system reliability $R(T)$ at time T can be written as,

$$R(T) = \prod_{j=1}^{n} \left\{ 1 - \prod_{i=1}^{m_j} \left(1 - R_{ij}(t)\right) \right\} \quad (6)$$

Assume the reliability requirement for ith subsystem is R_{i0} and system reliability is R_0, the reliability constraints of the optimization model is,

$$R_i(t) \geq R_{i0} \quad (7)$$

$$R(T) \geq R_0 \quad (8)$$

and preventive maintenance time restriction is

$$T_{PM}(i) \leq T_{PM0} \quad (9)$$

2.6. Optimization model

According to above analysis, the objective function is:

$$\min f(T) = \min(C_{Total}(T)) =$$
$$= \min \sum_{i=1}^{m} \sum_{j=1}^{n} \left(C(T_{PM})_i \times r_p + C_{f_i} N(i,j) + \varphi_i C_d(i) \right) \quad (10)$$

s.t. $R_i(t) \geq R_{i0}$

s.t. $R(T) \geq R_0$

s.t. $T_{PM}(i) \leq T_{PM0}$

The design variable is

$$\overline{T}_{PM} = [T_{PM}(1), T_{PM}(2), T_{PM}(3), ..., T_{PM}(m)]^T$$

where m is the number of subsystems.

It should be indicated that the optimization model is a nonlinear problem. Simplification of the system is necessary. In this paper, we will only analyze the minimal divided model of the system, as is prevented by Figure 2.

3. Example

Assume the complicated system can be divided into the structure as shown in Figure 2. Failure rate of every subsystem follows Weibull distribution as Eq. (11).

$$\lambda(t) = \frac{m}{\eta}\left(\frac{t}{\eta}\right)^{m-1} \tag{11}$$
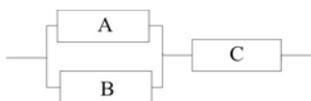
Fig. 2. Serial-parallel model

3.1. System initialization

Initialization of subsystem is shown in Table 1.

Tab. 1. System initialization

Sub-system	m	η	C_r	C_{pm}	a	A	φ
A	2.2	220	200	600	0.95	1.0	0
B	2.2	220	200	600	0.95	1.0	0
C	2	200	240	650	0.9	0.99	1

T_H=600.0Hours, C_L=800, R_{10}=R_{20}= R_{30}=0.8, R_0=0.8.

3.2. Calculation

There are many local optimum solutions when the model is complex. To obtain the global optimum solution and avoid dropping into the local ones, we utilized the Genetic Algorithm (GA) to solve the problem and validated the result by Direct Search Toolbox. The flowchart of GA is shown in Figure 3.

3.3. Result analysis

Figure 4 describes the relation between reliability and PM cycle while Figure 5 gives the minimal PM cost that can satisfy reliability requirement.

When the system and its subsystems can satisfy reliability requirement in each PM cycle, we say that the system satisfies reliability requirement. At this moment, the optimal maintenance cycle with minimum maintenance cost is T_{PM}=[75,75,54.55]^T. This can be regarded as the best preventive strategy with the minimal cost 26342.8389.

4. Conclusions

In this paper, we proposed a PM model for a series-parallel systems under consideration of reliability restriction. This makes it possible to guarantee more reliable operation for machines and devices. An example is given and the result show that the proposed model has good performance for maintenance decision-making. In the future research, two questions need further investigation:
1) With the increase of system's complexity, decision model becomes more complicated. In addition, selection of a suitable method to obtain the global optimum solution is also necessary.
2) We should consider some factors (such as the criticality of each subsystem) in the process of reliability allocation.
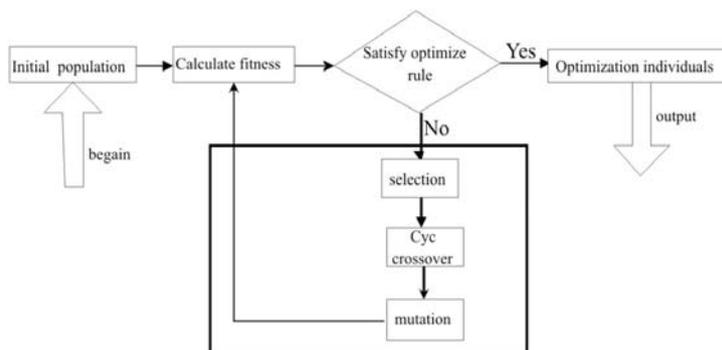
Fig. 3. Flowchart of GA
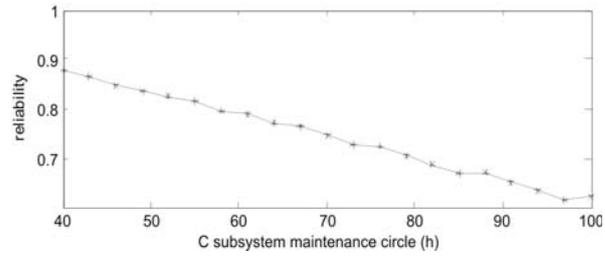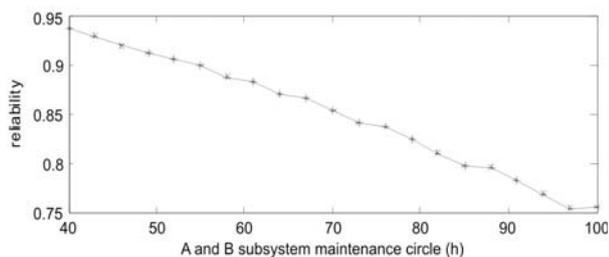
Fig. 4. Relation between reliability and PM
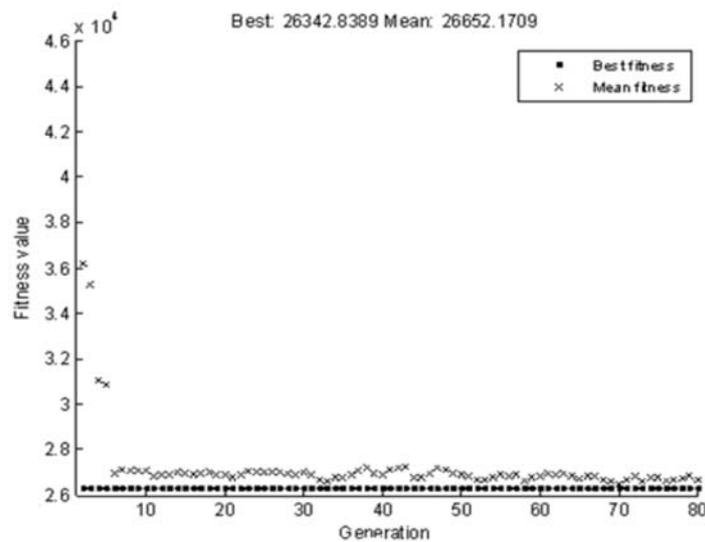
Fig. 5. Minimal PM cost

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