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Comment

Comment on "A framework to practical predictive maintenance modeling for multi-state systems" by Tan C.M. and Raghavan N. [Reliab Eng Syst Saf 2008;93(8):1138–50]

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ABSTRACT

"System-perspective" proposed by "A framework to practical predictive maintenance modeling for multi-state systems" by Tan C.M. and Raghavan N. [A framework to practical predictive maintenance modeling for multi-state systems. Reliab Eng Syst Saf 2008;93(8):1138–50] is a very useful method to evaluate and optimize the maintenance strategy for complex systems, especially for multi-state systems (MSS). The commented paper proposes an innovative process and modeling method to present imperfect maintenance effects on MSS, but there exist some incorrect points and misunderstandings. In this paper, these problems are pointed out and are attempted to be corrected under the original framework of the commented paper.

1. Introduction

Refer to "A framework to practical predictive maintenance modeling for multi-state systems," Reliab Eng Syst Saf 2008;93(8):1138–50 by Cher Ming Tan and Nagarajan Raghavan [1].

The commented paper attempts to propose a simple practical framework for predictive maintenance (PdM)-based scheduling of multi-state systems (MSS). PdM schedules were derived from a "system-perspective" using the failure time of the overall system estimated from its expected performance degradation trend. PdM is regarded not "as good as new" to restore MSS to its original performance. Restoration factor (RF) was introduced to quantitatively measure the quality of maintenance work on system performance under the PdM policy. The mean performance rate

$$E(G_s) = \left[\sum_{i=1}^{N_s} p_i(t) g_i\right] \tag{1}$$

is adopted to present the degradation trend, corresponding to instant time t. Thus, the time to next failure (TTF), as mentioned in the paper, for an MSS during the kth operation cycle was estimated by solving

$$G_k(t) - W = 0, \tag{2}$$

where W is user minimum demand, and MSS can be regarded as failure when its performance rate falls below W.

The commented paper suggests a good concept and approach that MSS will maintain when its system performance rate does not satisfy the requirement, which is very common in practical engineering. This is because, when some elements fail in MSS, it may be more expensive to disassemble to restore a single failed element, and maintenance will usually be carried out when the "system-perspective" performance does not meet the demand. So, this problem that is referred to in the commented paper is worth discussion.

However, there exist some incorrect points in the commented paper. The remainder of this paper is organized as follows: Section 2 will discuss the incorrect points existing in calculating the TTF of MSS under user demand, and employ the Markov reward model to obtain TTF, and compare these results with those in the commented paper. Section 3 will discuss the introduced concept of RF, and the misunderstandings that arise under this concept. Section 4 will conditionally propose a virtual age maintenance model, which is denied by the commented paper. Section 5 will give a brief conclusion.

2. Incorrect points in formulation of TTF

The commented paper suggests employing the mean performance rate, corresponding to instant time t, to present the degradation trend. PdM action is carried out when mean performance rate falls below user demand W. Actually, calculating

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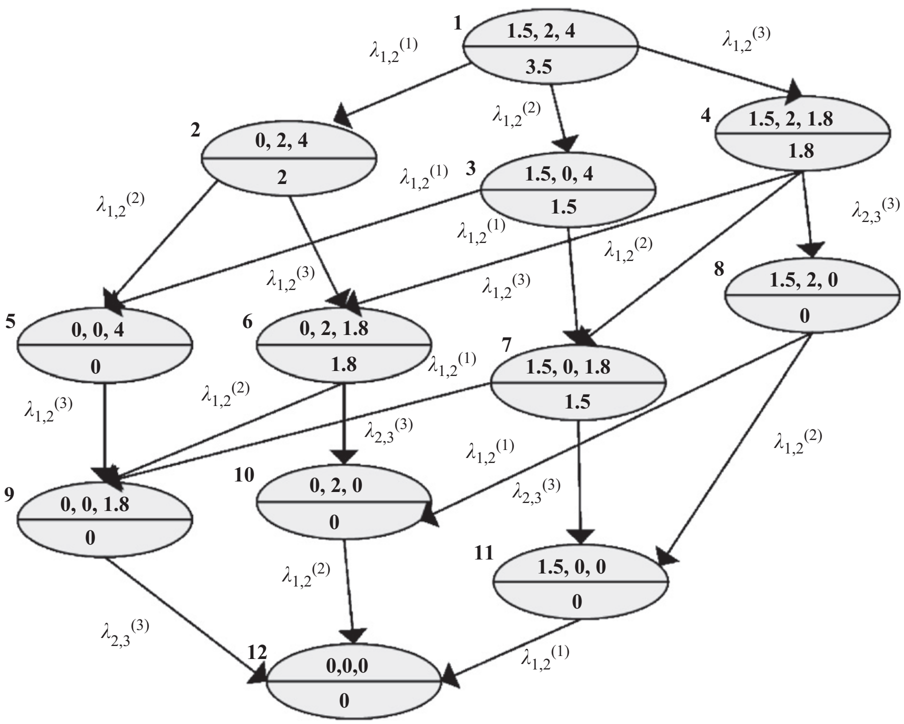

Fig. 1. State-space diagram for the studied MSS.

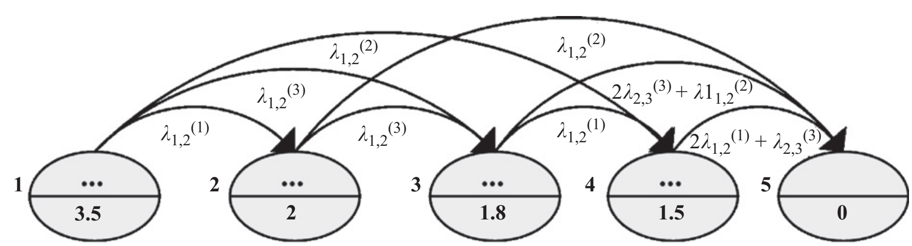

Fig. 2. Simplified state-space diagram of MSS.

Table 1
Transition intensities of MSS

State	1	2	3	4	5
1		$\lambda_{1,2}^{(1)}$	$\lambda_{1,2}^{(3)}$	$\lambda_{1,2}^{(2)}$	0
2	0		$\lambda_{1,2}^{(3)}$		$\lambda_{1,2}^{(2)}$
3	0	0		$\lambda_{1,2}^{(2)}$	$2\lambda_{2,3}^{(3)}+\lambda_{1,2}^{(2)}$
4	0	0	0		$2\lambda_{1,2}^{(1)}+\lambda_{2,3}^{(3)}$
5	0	0	0	0	

TTF of MSS is absolutely different from the methods of modeling optimal maintenance policy and variables decision in a continuous degrading system [2–5], because MSS degrades with state jumps and is a discrete degradation system. Moreover, the degradation of MSS cannot be measured by its mean performance, and is measured by its probability in discrete states, corresponding to different performance rates.

Generally, the mean time to failure of non-repairable MSS can be calculated by the Markov reward model [6]. To illustrate the incorrect conclusions in the commented paper, the presented studied case is studied again following the Markov reward model [7]. The total state-space diagram of the MSS with three elements is presented in Fig. 1.

In order to simplify the state-space diagram, the states with the same system performance rate are united into one state, and the simplified diagram is shown in Fig. 2 with its transition intensities tabulated in Table 1.

The corresponding transition intensities matrix can be denoted by

$$\boldsymbol{a} = |a_{i,j}| = \begin{pmatrix} a_{1,1} & \cdots & a_{1,5} \\ \vdots & \ddots & \vdots \\ a_{5,1} & \cdots & a_{5,5} \end{pmatrix} \tag{3}$$

and satisfies $a_{i,i} = -\sum_{j=1, j\neq i}^{k} a_{i,j}$.

If user demand is 3.0, states 2–5 can be regarded as unacceptable states, and the mean time to unacceptable states is equivalent to TTF or the mean time sojourning in state 1. Thus, all the unacceptable states (states 2–5) are united into absorbing state 2. According to the Markov reward theory, the Markov reward matrix is given by

$$\mathbf{r} = |r_{i,j}| = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \tag{4}$$

and the corresponding transition intensities matrix is given by

$$\boldsymbol{a} = |a_{i,j}| = \begin{pmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{pmatrix} = \begin{pmatrix} -(\lambda_{1,2}^{(1)} + \lambda_{1,2}^{(3)} + \lambda_{1,2}^{(2)}) & \lambda_{1,2}^{(1)} + \lambda_{1,2}^{(3)} + \lambda_{1,2}^{(2)} \\ 0 & 0 \end{pmatrix}. \tag{5}$$

One obtains long-run expected reward equation as

$$\mathbf{0} = \boldsymbol{u} + \boldsymbol{a}\boldsymbol{V}(\boldsymbol{t}), \tag{6}$$

where $u_i = r_{i,i} + \sum_{j \neq i, j=1}^{5} a_{i,j} r_{i,j}$. Then, the expected long-run reward equation according to Eq. (6) is given by

$$\begin{cases} 0 = r_{1,1} + a_{1,2}V_2 + a_{1,1}V_1 \\ \quad = 1 + (\lambda_{1,2}^{(1)} + \lambda_{1,2}^{(3)} + \lambda_{1,2}^{(2)})V_2 - (\lambda_{1,2}^{(1)} + \lambda_{1,2}^{(3)} + \lambda_{1,2}^{(2)})V_1, \\ 0 = r_{2,2} + a_{2,1}V_1 + a_{2,2}V_2 = 0 + 0 + 0 \end{cases} \tag{7}$$

with initial condition $V_2 = 0$. If the MSS starts its evolution from the absorbing state 2, it can never leave this state, and then no additional reward can be accumulated. Therefore, the TTF or mean time sojourning in state 1 is

$$\mathrm{TTF} = V_1 = \frac{1}{\lambda_{1,2}^{(1)} + \lambda_{1,2}^{(3)} + \lambda_{1,2}^{(2)}} = \frac{1}{2.0 + 2.0 + 3.5} = 0.1333 \text{ year}. \tag{8}$$

In the commented paper, the TTF under demand $W = 3$ is approximately equal to 0.04 year, which is dramatically different from the above result. Sequentially, considering the user demand $W = 2.5$, the acceptable state is still only state 1, and TTF is the same as for $W = 3$, but applying the proposed method in the commented paper, the corresponding TTF becomes longer (nearly 0.09 year).

In fact, TTF is the average time that the system is sojourning in the acceptable state, but is not equal to the average time that mean performance rate trend satisfy the demand. This is because the MSS will be repaired when it falls into the unacceptable state, and will not continue to operate anymore; the expected performance rates in unacceptable states cannot sum to present the trend. Therefore, the TTF is determined by the sojourning time in the states whose performance rates are no less than the demand. This is why TTF is the same for $2.0 < W \leqslant 3.5$, which is different from the approach proposed by the commented paper.

For demand $W = 2.0$, states 3–5 are combined into absorbing state 3, and the reward matrix is given by

$$\mathbf{r} = |r_{i,j}| = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{9}$$

and the transition intensities matrix is given by

$$\boldsymbol{a} = |a_{i,j}| = \begin{pmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,1} & a_{2,2} & a_{2,3} \\ a_{3,1} & a_{3,2} & a_{3,3} \end{pmatrix} = \begin{pmatrix} -(\lambda_{1,2}^{(1)} + \lambda_{1,2}^{(2)} + \lambda_{1,2}^{(3)}) & \lambda_{1,2}^{(1)} & \lambda_{1,2}^{(2)} + \lambda_{1,2}^{(3)} \\ 0 & -(\lambda_{1,2}^{(3)} + \lambda_{1,2}^{(2)}) & \lambda_{1,2}^{(3)} + \lambda_{1,2}^{(2)} \\ 0 & 0 & 0 \end{pmatrix}. \tag{10}$$

The long-run expected reward equation is as follows:

$$\begin{cases} 0 = r_{1,1} + a_{1,3}V_3 + a_{1,2}V_2 + a_{1,1}V_1 = 1 + (\lambda_{1,2}^{(3)} + \lambda_{1,2}^{(2)})V_3 + \lambda_{1,2}^{(1)}V_2 - (\lambda_{1,2}^{(1)} + \lambda_{1,2}^{(3)} + \lambda_{1,2}^{(2)})V_1, \\ 0 = r_{2,2} + a_{2,1}V_1 + a_{2,2}V_2 + a_{2,3}V_3 = 1 + 0V_1 - (\lambda_{1,2}^{(3)} + \lambda_{1,2}^{(2)})V_2 + (\lambda_{1,2}^{(3)} + \lambda_{1,2}^{(2)})V_3, \\ 0 = r_{3,3} + a_{3,1}V_1 + a_{3,2}V_2 + a_{3,3}V_3 = 0 + 0 + 0, \end{cases} \tag{11}$$

with initial condition $V_3 = 0$, and gives

$$\mathrm{TTF} = V_1 = \frac{1 + \lambda_{1,2}^{(1)}/(\lambda_{1,2}^{(3)} + \lambda_{1,2}^{(2)})}{\lambda_{1,2}^{(1)} + \lambda_{1,2}^{(3)} + \lambda_{1,2}^{(2)}} = 0.1818 \text{ year}, \tag{12}$$

which is obviously different from the result of the commented paper ($\mathrm{TTF} \approx 0.149$ year).

3. Misunderstandings in concept of RF

In the commented paper, RF is introduced to represent the percentage recovery of the system's mean performance in the kth operation cycle (after the kth maintenance action) relative to its mean performance during the previous $(k-1)$th operation cycle. The system mean performance during the kth operation cycle is given as

$$G_k(t) = G_{k-1}(t)\mathrm{RF}[k-1]. \tag{13}$$

According to above equation, if one assumes $t = 0$, the system is certainly at its best at state 1, whose performance rate is denoted by g_1; the mean performance at $t = 0$ is equal to g_1 at state 1. After imperfect PdM actions, the mean performance rate is formulated as

$$G_2(t) = G_1(t)\mathrm{RF}[1], \tag{14}$$

where RF[1] is a random variable, and $0 < \mathrm{RF}[1] < 1$. When $t = 0$, one may have

$$G_2(0) = g_1^{(2)} = g_1^{(1)}\mathrm{RF}[1] = G_1(0)\mathrm{RF}[1], \tag{15}$$

where $g_j^{(i)}$ denotes the performance rate in state j in the ith PdM cycle. This means that the performance rate of best state 1 decreases after repair. And then, one has

$$g_i^{(k)} = g_i \prod_{r=1}^{k-1} \mathrm{RF}[r], \tag{16}$$

which means that the original performance rate at each state becomes lower when $0 < \mathrm{RF}[r] < 1$. Actually, in practical engineering it may be possible that after maintenance the performance rates will derate. However, there is a conflict with MSS definition in [6–10]. Levitin and Lisnianski defined that an MSS has many possible states, and the state is divided according to its possible discrete performance rates. If RF approach is employed, after maintenance, the possible performance rates are different from these before maintenance, and the repaired MSS can be regarded as a new MSS with different performance rates at each state. It is difficult to evaluate whether the transition intensity is the same with those two MSSs.

Therefore, although RF is a good concept and approach to describe the maintenance quality, it is still unsuitable to apply directly to MSS without definition conflict. More useful and rational approaches need to be discussed to analyze the maintenance degree and system reliability in MSS.

4. Denial of virtual age model

The authors of the commented paper claim that Kijima's virtual age model [11,12], which is frequently used to describe the effect of maintenance quality on the effective restored age of the binary state system, cannot be used in MSS to describe the quality of maintenance, and a studied case is presented to illustrate its problems. Firstly, without considering the fundamental problem in directly using the virtual age model in MSS by the method of mean performance rates trend, the authors just employ type II virtual age model

$$\mathrm{Age}[k] = (\mathrm{Age}[k-1] + \mathrm{TTF}[k-1])(1 - \mathrm{RF}[k-1]), \tag{17}$$

which comes from [12]

$$V_n = (V_{n-1} + X_n)a. \tag{18}$$

The authors say the sum Age[$k-1$]+TTF[$k-1$] is approximately constant at the beginning of every operation cycle, and the time to replacement (TTR) is infinite. Actually, the result is based on an assumption that RF[$k-1$] ($k = 2,3,\ldots$) are almost the same in each cycle. If the RF[$k-1$] is increasing with k, the result does not exist. On the other hand, even if RF[$k-1$] ($k = 2,3,\ldots$) are almost the same, Kijima's type I virtual age model can be adopted to describe maintenance quality. Kijima's type I virtual age model is formulated as [11]

$$V_n = V_{n-1} + X_n a, \tag{19}$$

and, the virtual age model of MSS is given by

$$\mathrm{Age}[k] = \mathrm{Age}[k-1] + \mathrm{TTF}[k-1](1 - \mathrm{RF}[k-1]). \tag{20}$$

Infinite TTR can be avoided when Age[k] is monotonically increasing, and TTF is monotonically decreasing, corresponding to k. In fact, whether to employ type I or II model is decided by the practical problem and statistical results.

Furthermore, directly employing the virtual age model as Eqs. (17) and (20) under the mean performance rate trend presents some problems. Assuming that, after the PdM, the virtual age of MSS is not zero, the current mean performance rate is given by

$$E(G_2(t)) = E(G_1(\mathrm{Age}[1] + t)). \tag{21}$$

The above equation can be expressed as

$$E(G_2(t)) = E\left(\sum_{i=1}^{N_s} p_i(\mathrm{Age}[1] + t) g_i\right). \tag{22}$$

This denotes that MSS may have some probability at every possible state after PdM, even the unacceptable states and the worst state. It is impossible that after maintenance, the MSS is still at unacceptable states or even becomes worse. Thus, this illustrates that it is irrational to combine mean performance trend with any type of virtual age model.

To employ the concept of virtual age, the following approach to modeling the maintenance quality of MSS is proposed, similar to binary state system. Assuming that there are N_{sa} acceptable states whose performance rates satisfy the user demand, the reliability of MSS without maintenance is formulated as

$$R(t) = A(t) = \sum_{i=1}^{N_{sa}} p_i(t) \tag{23}$$

and failure probability function is given as

$$F(t) = 1 - R(t) = 1 - \sum_{i=1}^{N_{sa}} p_i(t) = \sum_{i=N_{sa}+1}^{N_s} p_i(t). \tag{24}$$

The failure probability of MSS after kth PdM is formulated, according to virtual age model, as

$$\begin{aligned} F_{k+1}(t) &= \Pr\{T \leqslant t | T > \mathrm{Age}[k]\} \\ &= \frac{\Pr\{\mathrm{Age}[k] < T \leqslant t\}}{\Pr\{T > \mathrm{Age}[k]\}} \\ &= \frac{F(t) - F(\mathrm{Age}[k])}{1 - F(\mathrm{Age}[k])} \end{aligned} \tag{25}$$

and then the cumulative probability function of reliability is formulated as

$$\begin{aligned} R_{k+1}(t) &= 1 - F_{k+1}(t) \\ &= \frac{R(t)}{R(\mathrm{Age}[k])}. \end{aligned} \tag{26}$$

When $t = t'$+Age[k] is substituted,

$$R_{k+1}(t' + \mathrm{Age}[k]) = \frac{R(t' + \mathrm{Age}[k])}{R(\mathrm{Age}[k])} = \frac{\sum_{i=1}^{N_{sa}} p_i(t' + \mathrm{Age}[k])}{\sum_{i=1}^{N_{sa}} p_i(\mathrm{Age}[k])}. \tag{27}$$

The TTF in the (k+1)th PdM cycle is equal to the integral of $R_{k+1}(t)$ as

$$\mathrm{TTF}_{k+1} = \int_0^\infty R_{k+1}(t' + \mathrm{Age}[k])\,\mathrm{d}t'. \tag{28}$$

Applying the method in the illustrative case in the commented paper, it is assumed that $W = 1.8$ and only state 1 is acceptable. For simplicity, Kijima's type I virtual age model is employed, and parameter $a = 0.90$. The TTF in each cycle is presented in Fig. 3.

According to the above illustrative case, the TTF is becoming shorter and shorter after PdM. The replacement criterion that the initial mean performance rate of MSS after PdM is lower than the demand in the commented paper is not proper anymore. MSS is regarded as restoring to its acceptable states in our proposed model; in other words, after PdM, it is as if that MSS has worked at virtual age Age[k] without falling into unacceptable states. The approach for MSS is similar to the approach to a binary state system. The expected profit per unit time in each PdM cycle is usually regarded as the decision criterion; for shorter TTF it will make lower expected profit per unit time. An optimal PdM cycle should be obtained to maximize the expected profit per unit time in the life cycle of MSS.

However, it should be noted that for exponential failure distribution, the virtual age model may have some flaws, because the conditional probability function after maintenance action is the same as a new system (called memoryless property). Thus, when $2.0 < W \leqslant 3.5$, the TTF in each cycle is the same for the failure of MSS following the exponential distribution proved by Eq. (5), and so TTR is infinite for this reason, which is different from the commented paper. Therefore, virtual age model is not very suitable for MSS, especially its degradation process following

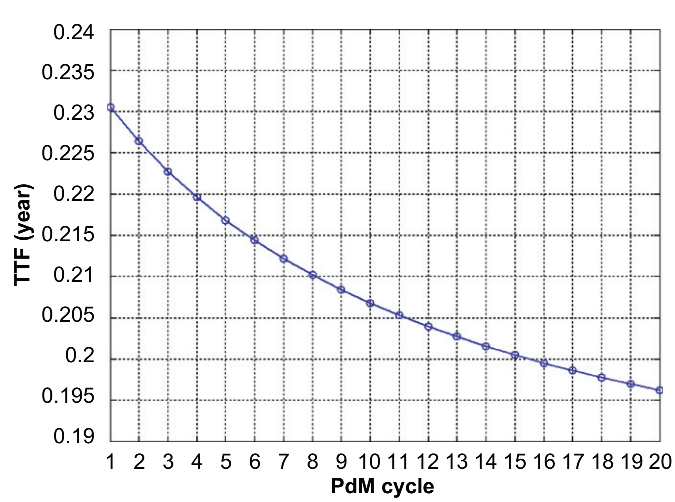

Fig. 3. TTF vs. PdM cycle under type I virtual age model.

exponential distribution, but is proven efficient in systems with increasing failure rate (IFR).

5. Conclusions

In this paper, the incorrect points in the commented paper are discussed. The authors of the commented paper propose an improper method to evaluate the TTF of MSS by mean performance rates trend. This paper presents the Markov reward model to assess the TTF and illustrates the different results from these two methods. Although RF is a good method to qualify the maintenance actions, it conflicts with the basic definition of MSS. Thus, the RF method is not very suitable for MSS. Moreover, virtual age model is denied by the authors of the commented paper for it cannot describe the maintenance quality. This conclusion is incorrect. Similar to binary state systems, the virtual age maintenance model for MSS is proposed by employing Kijima's types I and II model; the TTF corresponding to PdM cycle was plotted to debate with the commented paper. Actually, virtual age model is not very suitable for MSS as mentioned in this context, especially if the system degrades following exponential distribution (or non-IFR), and it is suggested that a more rational imperfect maintenance model should be proposed in further research.

Despite some errors in the commented paper, its authors present a very valuable concept of considering the maintenance schedules in "system-perspective", which is different with [9,10]. In many cases, due to the expensive disassembly, in terms of cost and time, maintenance activities are only performed when system performance does not satisfy the requirements and comprehensive recovery will be done. Therefore, the "system-perspective" method is worth further research.

Acknowledgments

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References

[1] Tan CM, Raghavan N. A framework to practical predictive maintenance modeling for multi-state systems. Reliab Eng Syst Saf 2008;93(8):1138–50.
[2] Grall A, Dieulle L, Berenguer C, Roussignol M. Continuous-time predictive-maintenance scheduling for a deteriorating system. IEEE Trans Reliab 2002;51(2):141–50.
[3] Klutke GA, Yang Y. The availability of inspected systems subject to shocks and graceful degradation. IEEE Trans Reliab 2002;51(3):371–4.
[4] Li WJ, Pham H. Reliability modeling of multi-state degraded systems with multi-competing failures and random shocks. IEEE Trans Reliab 2005;54(2):297–303.
[5] Li WJ, Pham H. An inspection-maintenance model for systems with multiple competing processes. IEEE Trans Reliab 2005;54(2):318–27.
[6] Lisnianski A, Levitin G. Multi-state system reliability assessment, optimization, application. Singapore: World Scientific; 2003.
[7] Lisnianski A. Extended block diagram method for a multi-state system reliability assessment. Reliab Eng Syst Saf 2007;92(12):1601–7.
[8] Levitin G. A universal generating function approach for the analysis of multi-state systems with dependent elements. Reliab Eng Syst Saf 2004;84(3):285–92.
[9] Levitin G, Lisnianski A. Optimization of imperfect preventive maintenance for multi-state systems. Reliab Eng Syst Saf 2000;67(2):193–203.
[10] Levitin G, Lisnianski A. Joint redundancy and maintenance optimization for multi-state series-parallel systems. Reliab Eng Syst Saf 1999;64(1):33–42.
[11] Kijima M, Morimura H, Suzuki Y. Periodical replacement problem without assuming minimal repair. Eur J Oper Res 1988;37:194–203.
[12] Kijima M. Some results for repairable systems with general repair. J Appl Prob 1989;26:89–102.