Zhonglai WANG
Hong-Zhong HUANG
Xiaoping DU

PROJEKTOWANIE NIEZAWODNOŚCIOWE Z WYKORZYSTANIEM KILKU STRATEGII UTRZYMANIA

RELIABILITY - BASED DESIGN INCORPORATING SEVERAL MAINTENANCE POLICIES

Tradycyjna optymalizacja projektowania niezawodnościowego (RBDO) minimalizuje funkcję celu opisującą koszty w zależności od ograniczeń niezawodności. Ograniczenia niezawodności oparte są na modelach fizycznych, takich jak symulacja z wykorzystaniem metody elementów skończonych, których używa się do określania stanu komponentu lub systemu. Stąd niezawodność oznacza tu tzw. niezawodność fizyczną. Ograniczenia niezawodności są zazwyczaj statyczne i nie wyjaśniają problemów związanych z cyklem życia produktu. W niniejszej pracy zaproponowano kilka modeli optymalizacji projektowania niezawodnościowego wykorzystujących kilka strategii utrzymania. Koszt cyklu życia produktu w omawianych modelach został zminimalizowany przy jednoczesnym spełnieniu wymogów niezawodności i dostępności podczas cyklu życia produktu. Do obliczenia czasowo zależnej niezawodności wykorzystano metodę analizy niezawodności pierwszego rzędu (FORM). Możliwość praktycznego wykorzystania proponowanych modeli zilustrowano przykładem.

Słowa kluczowe: *Optymalizacja projektowania niezawodnościowego, cykl życia, eksploatacja, metoda analizy niezawodności pierwszego rzędu.*

Traditional reliability-based design optimization (RBDO) minimizes a cost-type objective function subject to reliability constraints. The reliability constraints are based on physical models, such as finite element simulation, which are used to specify the state of a component or a system. Hence the reliability is the so-called physical reliability. The reliability constraints are usually static without accounting for product lifecycle issues. In this work, several reliability-based design optimization models incorporating several maintenance policies are proposed. The product lifecycle cost is minimized while the constraints of product lifecycle reliability or availability are satisfied. The First Order Reliability Method (FORM) is employed to calculate the time dependent reliability. An engineering example is used to illustrate the proposed models.

Keywords: *Reliability-based design optimization, lifecycle, maintenance, first order reliability method.*

1. Introduction

Optimization has been widely applied in engineering design because it produces optimal design solutions quickly and inexpensively. With the recognition of the effects of uncertainty, optimization under uncertainty has also been increasingly used [2, 6, 17, 21, 30]. Examples of uncertainty include variations in loading, material properties, dimensions, operation conditions, and even the lack of knowledge.

Reliability-based design optimization (RBDO) [1, 7, 15, 25, 29] is a typical methodology of optimization under uncertainty. RBDO ensures that reliability requirement be satisfied at desired levels through reliability constraints. Since a higher reliability implies a higher cost, RBDO seeks a good balance between reliability and cost by minimizing a cost-type objective function subject to reliability constraints. Different from statistics-based reliability in reliability engineering, RBDO involves the so-called physics-based reliability [16]. The reason is that the state of a component (or a system) can be modeled by physical models, such as finite element analysis and dynamics simulation. Hence reliability can be evaluated based on the physical models given the distributions of the inputs to the physical models.

It is well known that reliability is a function of time. In many cases, reliability decreases with time due to performance degradation. In traditional RBDO, however, reliability is usually static. Only the initial reliability before the product is put into operation is considered. Time-dependent reliability constraints are considered in a few studies, including [11, 23], but product lifecycle issues, such as the lifecycle cost and maintenance are rarely accounted for. The importance of reliability in product lifecycle is widely recognized today. Frangopol and Maute provide a brief review of the lifecycle reliability-based optimization with emphasis on civil and aerospace structures [9].

The performance of many products degrades with time during their serviceable lifecycle. For restoring and maintaining product performance, maintenance is necessary. Maintenance, as an important intervention to a product, can extend the serviceable life effectively. Maintenance can be categorized into corrective maintenance and preventive maintenance. When a failure occurs, corrective maintenance takes place. Even if no failure occurs, maintenance can also be implemented, and this type of maintenance is preventive. According to the depth of maintenance, the types of maintenance can be further classified into six categories: improved, complete, imperfect, minimal, worse, and worst maintenances [22].

Since traditional RBDO considers reliability upfront in the design stage before a physical product is made, it may not be easy to consider maintenance. While maintenance generates additional operation costs, appropriate maintenance strategies specified in the design stage will reduce the total product lifecycle cost significantly. Accounting for product lifecycle, such as maintenance, will therefore further the benefits of RBDO. Exploratory work in this area has been reported [26]. The method combines RBDO with structural reliability analysis for aging structure optimization design. Many studies have been done on system optimal optimization based on statistical tool [14, 18-20].

Our focus in this study is to develop a new RBDO method based on physical models that includes time-dependent reliability constraints, as well as maintenance consideration. With the new method, the product lifecycle cost is minimized and reliability and availability requirements are satisfied. Three design optimization models are established according to the depth of maintenance.

The organization of this paper is as follows. In Section 2, time-dependent reliability-based design optimization is introduced. The proposed design optimization models are given in Section 3. In Section 4, an engineering example is used to demonstrate the proposed models. Conclusions and future work are summarized in Section 5.

2. Time-Dependent Reliability-Based Design Optimization

The general reliability-based design optimization is modeled as [8]:

$$\begin{aligned} &\underset{\mathbf{d},\boldsymbol{\mu}_X}{\text{minimize}} \quad \text{Cost}(\mathbf{d},\mathbf{X},\mathbf{P}) \\ &\text{subject to} \quad \Pr\{g_i(\mathbf{d},\mathbf{X},\mathbf{P}) \ge 0)\} \ge [R_i] \quad i=1,2,...,n_g \\ &\qquad \mathbf{d}^L \le \mathbf{d} \le \mathbf{d}^U, \boldsymbol{\mu}_X^L \le \boldsymbol{\mu}_X \le \boldsymbol{\mu}_X^U \end{aligned} \tag{1}$$

In the above model, $\mathbf{d}$ is the vector of deterministic design variables, for example, the number of teeth of a gear. $\mathbf{X}$ is the vector of random design variables, whose mean values $\boldsymbol{\mu}_X$ are to be determined. For example, the width and diameter of a gear could be the random design variables. $\mathbf{P}$ is the vector of random parameters. Random parameters $\mathbf{P}$ are out of designers' control and are sometimes called noise factors. For example, the random temperature is a random parameter for a gear design problem. $\Pr\{\cdot\}$ denotes a probability. $\Pr\{g_i(\mathbf{d},\mathbf{X},\mathbf{P}) \ge 0\} \ge [R_i]$ means that the probability of satisfying the constraint $g_i(\mathbf{d},\mathbf{X},\mathbf{P}) \ge 0$ should be greater than or equal to the desired reliability $[R_i]$. Such a probability is obviously the reliability associated with constraint $g_i(\mathbf{d},\mathbf{X},\mathbf{P}) \ge 0$. n_g is the number of constraint functions. $\mathbf{d}^L$ and $\mathbf{d}^U$ are the lower and upper bounds of $\mathbf{d}$, respectively, and $\boldsymbol{\mu}_X^L$ and $\boldsymbol{\mu}_X^U$ are the lower and upper bounds of $\boldsymbol{\mu}_X$, respectively.

Reliability $\Pr\{g_i(\mathbf{d},\mathbf{X},\mathbf{P}) \ge 0\}$ can be calculated by integrating the joint probability density function of $(\mathbf{X},\mathbf{P})$ over the safe region defined by $g_i(\mathbf{d},\mathbf{X},\mathbf{P}) \ge 0$. The equation is given by:

$$\Pr\{g_i(\mathbf{d},\mathbf{X},\mathbf{P}) \ge 0\} = \int_{g_i(\mathbf{d},\mathbf{X},\mathbf{P}) \ge 0} f_{\mathrm{X,P}}(\mathbf{x},\mathbf{p})\mathrm{d}\mathbf{x}\mathrm{d}\mathbf{p} \tag{2}$$

where $f_{\mathrm{X,P}}(\mathbf{x},\mathbf{p})$ is the joint probability density function of $(\mathbf{X},\mathbf{P})$. Since the integration boundary $g_i(\mathbf{d},\mathbf{X},\mathbf{P}) = 0$ is usually nonlinear and multidimensional, there is rarely a closed-form solution to Eq. (2). In principle, Monte Carlo simulation [12] can be used to compute the probability for any constraint functions, dimensionality, and distributions. However, if the reliability is high, the number of simulations required will be prohibitively high, and the simulation process will be computationally expensive. To have a good balance between accuracy and efficiency, engineers usually use the First Order Reliability Method (FORM). The central idea of FORM is to linearize the constraint function $g_i(\mathbf{d},\mathbf{X},\mathbf{P})$ at its limit state (integration boundary) $g_i(\mathbf{d},\mathbf{X},\mathbf{P}) = 0$ at a point that has the highest probability density. Then the probability can be easily calculated. In this paper, we assume all the random variables in $(\mathbf{X},\mathbf{P})$ are independent. With this assumption, the three steps involved in FORM are as follows.

Step 1: Transform random variables $\mathbf{Z} = (\mathbf{X},\mathbf{P})$ to standard normal random variables $\mathbf{U} = (\mathbf{U}_X,\mathbf{U}_P)$. The transformation is given by:

$$F_{Z_j}(z_j) = \Phi(u_j), j = 1,2,\cdots,n_X + n_P \tag{3}$$

where $F_{Z_j}(z_j)$ is the cumulative distribution function (CDF) of random variable Z_j, $\Phi(u_j)$ is the CDF of the standard normal variable U_j, n_X is the number of random design variables, and n_P is the number of random parameters. After the transformation, the constraint function becomes $g_i(\mathbf{d},\mathbf{U}_Z) = g_i(\mathbf{d},\mathbf{U}_X,\mathbf{U}_P)$, where: $\mathbf{U}_Z = (\mathbf{U}_X,\mathbf{U}_P)$.

Step 2: Search the Most Probable Point (MPP) in the U-space. The MPP is the point with the shortest distance from the origin to the constraint boundary $g_i(\mathbf{d},\mathbf{U}_X,\mathbf{U}_P) = 0$. The shortest distance is denoted by β_i and is referred to as a reliability index. Because of the shortest distance to the origin, the MPP has the highest probability density on the constraint boundary $g_i(\mathbf{d},\mathbf{U}_X,\mathbf{U}_P) = 0$. The accuracy loss of reliability calculation is minimized when $g_i(\mathbf{d},\mathbf{U}_X,\mathbf{U}_P)$ is linearized at the MPP. The model for the MPP search is given by the minimization problem shown below:

$$\begin{cases} \underset{\mathbf{U}_X,\mathbf{U}_P}{\text{minimize}} \quad \beta = \|\mathbf{U}_X, \mathbf{U}_P\| \\ \text{subject to} \quad g_i(\mathbf{d},\mathbf{U}_X,\mathbf{U}_P) = 0 \end{cases} \tag{4}$$

Step 3: Compute reliability. After the constraint function is linearized at the MPP, reliability is easily calculated by:

$$\Pr\{g_i(\mathbf{d},\mathbf{X},\mathbf{P}) \ge 0\} = \Phi(\beta_i) \tag{5}$$

When the product performance is time dependent, the optimization model in Eq. (1) can be rewritten as:

$$\begin{aligned} &\underset{\mathbf{d},\boldsymbol{\mu}_X}{\text{minimize}} \quad \text{Cost}\left[\mathbf{d}(t),\mathbf{X}(t),\mathbf{P}(t)\right] \\ &\text{subject to} \quad \Pr\{g_i[\mathbf{d},\mathbf{X}(t),\mathbf{P}(t),t] \ge 0)\} \ge [R_i] \quad i=1,2,...,n_g \\ &\qquad \mathbf{d}^L \le \mathbf{d} \le \mathbf{d}^U, \boldsymbol{\mu}_X^L \le \boldsymbol{\mu}_X \le \boldsymbol{\mu}_X^U \end{aligned} \tag{6}$$

where the objective and the constraint functions are time dependent.

Reliability is then calculated by:

$$\begin{aligned} &\underset{\mathbf{d},\boldsymbol{\mu}_X}{\text{minimize}} \quad \text{Cost}\left[\mathbf{d}(t),\mathbf{X}(t),\mathbf{P}(t)\right] \\ &\text{subject to} \quad \Pr\{g_i[\mathbf{d},\mathbf{X}(t),\mathbf{P}(t),t] \ge 0)\} \ge [R_i] \quad i=1,2,...,n_g \\ &\qquad \mathbf{d}^L \le \mathbf{d} \le \mathbf{d}^U, \boldsymbol{\mu}_X^L \le \boldsymbol{\mu}_X \le \boldsymbol{\mu}_X^U \end{aligned} \tag{7}$$

FORM is also applicable for solving the above equation at any instant of time t. The MPP search model in Eq. (4) is rewritten as:

$$\begin{cases} \underset{\mathbf{U}_X,\mathbf{U}_P}{\text{minimize}} & \|\mathbf{U}_X,\mathbf{U}_P\| \\ \text{subject to} & g_i(\mathbf{d},\mathbf{U}_X,\mathbf{U}_P,t)=0 \end{cases} \tag{8}$$

Since FORM uses an iterative process to search the MPP, the entire RBDO involves a double-loop procedure as shown in Fig. 1.

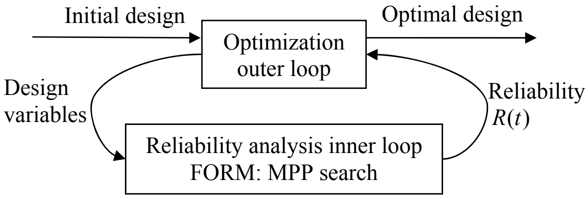

Fig. 1. Time-dependent RBDO – a double loop procedure

3. RBDO with Maintenance Consideration

In this Section, the three proposed RBDO models are presented. The models are based on product lifecycle with maintenance consideration. The models differ from each other in terms of the depth of maintenance.

3.1. Nonrepairable products

Products can be divided into nonrepairable products and repairable products. If a product is nonrepairable, the RBDO model is similar to the model in Eq. (6). Since the product lifecycle cost depends on how long the product is in operation, we also add the product life T_d as a design variable. The RBDO model then becomes:

$$\begin{aligned} &\underset{\mathbf{d},\boldsymbol{\mu}_X,T_d}{\text{minimize}} \quad \text{Cost}\left[\mathbf{d}(t),\mathbf{X}(t),\mathbf{P}(t),T_d\right] \\ &\text{subject to} \quad \Pr\left\{g_i[\mathbf{d},\mathbf{X}(T_d),\mathbf{P}(T_d),T_d]\geq 0)\right\}\geq\left[R_i\right] \quad i=1,2,...,n_g \\ &\qquad \mathbf{d}^L\leq\mathbf{d}\leq\mathbf{d}^U,\boldsymbol{\mu}_X^L\leq\boldsymbol{\mu}_X\leq\boldsymbol{\mu}_X^U,T_d^L\leq T_d\leq T_d^U \end{aligned} \tag{9}$$

Since reliability decreases with time, we only need to worry about the reliability at the end of the product lifecycle. Therefore, the reliability constraints are only for the time instant when $t = T_d$, and the MPP search in Eq. (6) is conducted at $t = T_d$. Conventional RBDO methods are applicable for solving the design problem in Eq. (6). The efficiency of solving the above RBDO problem is similar to that of solving a static RBDO model in Eq. (1). Since no repair is involved, the cost of maintenance is not included in the objective function.

3.2. Repairable products

When products are repairable, the RBDO model will be more complicated. In this subsection, two RBDO models for repairable products are proposed according to the depth of maintenance. The first model is for perfect maintenance, and the second model is for the minimum maintenance.

3.2.1. RBDO model with perfect preventive and corrective maintenance

Preventive maintenance consists of planned maintenance actions, which aim at the prevention of failures. In many preventive maintenance models, a product (system) is assumed to be as good as new after each preventive maintenance action [31]. In this sense, preventive maintenance is said to be perfect. On the other hand, corrective maintenance consists of maintenance actions that involve the repair or replacement of components which have failed or broken down. After corrective maintenance, the system may or may not be as good as new [31]. A maintenance model where both preventive maintenance and corrective maintenance are perfect is discussed in [3]. The model in [3] is adopted in this subsection.

Fig. 2 shows a general maintenance model with both preventive maintenance and corrective maintenance. In the figure, τ denotes the time to preventive maintenance since last maintenance. T is the time to failure since last maintenance. T_{PM} and T_{CM} are times of performing the preventive maintenance and corrective maintenance, respectively. If $\tau < T$, preventive maintenance is performed. Corrective maintenance is conducted when failure occurs during the time interval $[0,\tau]$. In this subsection, the performance and reliability of a product are restored to the as-good-as-new state after preventive and corrective maintenance.

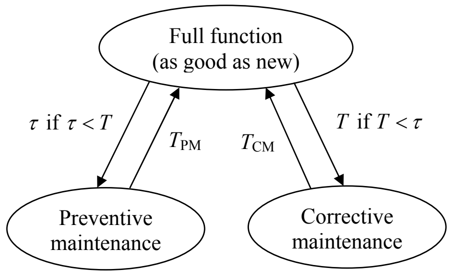

Fig. 2. Product states under preventive and corrective maintenance

With the as-good-as-new assumption, the lifecycle of a product could be long. Then it is reasonable to use the steady-state availability. A cycle is defined as the duration between two maintenance actions (either preventive or corrective maintenance). The uptime and downtime during one cycle can be respectively expressed by:

$$T_{up} = R(\tau)\tau + [1 - R(\tau)]T \tag{10}$$

and

$$T_{down} = R(\tau)T_{PM} + [1 - R(\tau)]T_{CM} \tag{11}$$

The mean uptime is given by:

$$E(T_{up}) = R(\tau)\tau + [1 - R(\tau)]E(T) \tag{12}$$

where $E(\cdot)$ stands for an expectation operation.

The mean downtime is given by:

$$E(T_{down}) = R(\tau)\mathrm{E}(T_{PM}) + [1 - R(\tau)]E(T_{CM}) \tag{13}$$

The steady-state availability is then calculated by:

$$A = \frac{E(T_{up})}{E(T_{up}) + E(T_{down})} = \frac{R(\tau)\tau + [1 - R(\tau)]E(T)}{R(\tau)\tau + [1 - R(\tau)]E(T) + R(\tau)E(T_{PM}) + [1 - R(\tau)]E(T_{CM})} \quad (14)$$

$R(\tau)$ can be calculated by FORM through the MPP search in Eq. (8). The mean time to failure $E(T)$ can be computed through the reliability function $R(t)$. The equation is given by [24]:

$$E(T) = \int_0^{+\infty} R(t)dt \quad (15)$$

As mentioned previously, an analytical expression of $R(t)$ rarely exists. Numerical methods are therefore necessary to computer $E(T)$. If Simpson's rule is applied [10], $E(T)$ is approximated by:

$$E(T) = \frac{t_N}{3N}\left[R(t_0) + 4R(t_1) + 2R(t_2) + 4R(t_3) + \dots + 2R(t_{N-2}) + 4R(t_{N-1}) + R(t_N)\right] \quad (16)$$

where N is the number of sample points, and $R(t_i)$ ($i = 1,2,...,N$) are calculated by FORM.

Then the mean cycle T_C and the mean maintenance cost C_M are respectively computed by:

$$T_C = R(\tau)\tau + [1 - R(\tau)]E(T) + R(\tau)E(T_{PM}) + [1 - R(\tau)]E(T_{CM}) \quad (17)$$

and

$$C_C = R(\tau)C_P + [1 - R(\tau)]C_r \quad (18)$$

where C_P and C_r are the cost to perform a preventive maintenance and a corrective maintenance, respectively.

Based on the maintenance analysis above, the RBDO model is given by:

$$\begin{aligned} &\underset{\mathbf{d},\boldsymbol{\mu}_X,\tau}{\text{minimize}} && \text{Cost}\left[\mathbf{d}(t), \mathbf{X}(t), \mathbf{P}(t), \tau\right] \\ &\text{subject to} && A \geq [A] \\ &&& \mathbf{d}^L \leq \mathbf{d} \leq \mathbf{d}^U, \boldsymbol{\mu}_X^L \leq \boldsymbol{\mu}_X \leq \boldsymbol{\mu}_X^U, \tau^L \leq \tau \leq \tau^U \end{aligned} \quad (19)$$

where $[A]$ is the desired steady-state availability. The time to preventive maintenance since last maintenance τ is also one of the design variables. The objective function is the total product lifecycle cost, which is equal to the cost without maintenance (the cost in Eq. (6)) plus the maintenance cost in Eq. (18). This model allows us to determine the optimal design variables and optimal preventive maintenance period so that the product lifecycle cost is minimized with sufficient availability above an expected level.

It is noted that the reliability requirement is not directly incorporated in this RBDO model. It is possible to add a reliability constraint to the model; however, adding a reliability requirement may not be necessary. The reason is that high availability implies high reliability. As indicated in Eq. (14), higher reliability also results in higher availability.

Since the MPP search needs to be performed at the time of preventive maintenance τ and at a number of sample points for the mean time to failure in Eq. (16), the efficiency of solving the RBDO model is lower than that of solving the RBDO model for nonrepairable products in Section 3.1.

3.2.2. RBDO model with minimal maintenance

In the last subsection, the performance of a product is assumed to be restored to as good as new after maintenance. In this subsection, the reliability of a product after maintenance is assumed to remain the same as that before its failure. The maintenance actions are therefore imperfect and actually are minimal. It is also assumed further that no preventive replacement or preventive maintenance is implemented. The maintenance model for this situation is given in [3] and is provided below:

$$\Pr\{N(t+s) - N(s) = n\} = \frac{\exp\{-[(Z(t+s) - Z(s)]\}[Z(t+s) - Z(s)]^n}{n!} \quad (20)$$

where $\{N(t), t \geq 0\}$ is assumed to be a non-homogeneous Poisson process (NHPP), which specifies the probability of the number of failures being equal to n during the time period $[s,s+t]$. $Z(t)$ is the cumulative hazard function and is given by:

$$Z(t) = \int_0^t z(\tau)d\tau \quad (21)$$

where $z(t)$ is the hazard function and can be derived from the reliability function,

$$z(t) = -\frac{1}{R(t)}\frac{d[R(t)]}{dt} \quad (22)$$

where reliability $R(t)$ can be obtained by FORM through the model in Eq. (8).

The quality of a product can be controlled by the probability of failure during the lifecycle, and the maintenance time is neglected. Then the RBDO model is given by:

$$\begin{aligned} &\underset{\mathbf{d},\boldsymbol{\mu}_X,T_\mathrm{d}}{\text{minimize}} && \text{Cost}\left[\mathbf{d}(t), \mathbf{X}(t), \mathbf{P}(t), T_\mathrm{d}\right] \\ &\text{subject to} && \Pr\{N(T_d) \leq n\} \geq [R] \\ &&& \mathbf{d}^L \leq \mathbf{d} \leq \mathbf{d}^U, \boldsymbol{\mu}_X^L \leq \boldsymbol{\mu}_X \leq \boldsymbol{\mu}_X^U, T_\mathrm{d}^L \leq T_\mathrm{d} \leq T_\mathrm{d}^U \end{aligned} \quad (23)$$

In this model, the probability of no more than n failures is considered in the constraint, where $[R]$ is the desired reliability. The objective function specifies the product lifecycle cost per unit time including the maintenance cost. The product lifecycle T_d is also one of the design variables.

Based on Eqs. (20) and (21), the reliability in the RBDO model is given by:

$$\Pr\{N(T_d) \leq n\} = \frac{\exp[-Z(T_d)]Z(T_d)^n}{n!} \quad (24)$$

where:

$$Z(T_d) = \int_0^{T_d} z(\tau)d\tau \quad (25)$$

and $z(t)$ can be obtained from Eq. (22), in which reliability $R(t)$ is computed by FORM.

4. Case Study

This case study is the design of a liquefied petroleum gas storage tank for home use [5].

4.1. Problem statement

The parameters of the tank are shown in Fig. 3, where h is the thickness, H is the radius, L is the height, and P_b is the bursting pressure.

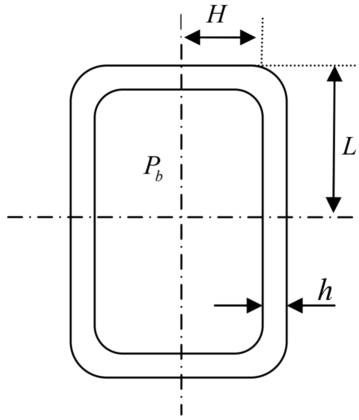

Fig. 3. The pressure tank

When the hoop stress exceeds the material ultimate strength, the tank is considered not functioning. The limit-state (constraint) function is given by

$$g(t) = S_U - \frac{P_b H}{r h(t)}\left(1 - \frac{H^2}{2L^2}\right) \tag{26}$$

where S_U is the material ultimate strength, and r is the ratio of bursting pressure and internal pressure. The thickness of the tank is assumed to corrode in a stochastic manner due to the external environment. The thickness is given by $h(t) = h_0 - 0.05t$, where h_0 is the random initial thickness.

In this example, the deterministic design variables **d** include parameters associated with maintenance and product lifetime. The distributions of the random design variables and the random parameters are given in Table 1. COV in Table 1 is the coefficient of variation, which is defined by the ratio of the standard deviation and the mean. The bounds of design variables are given in Table 2.

Tab. 1. Distributions of stochastic variables

Variables		Mean	COV	Distribution
X	H (mm)	μ_H	0.1	Normal
	L (mm)	μ_L	0.1	Normal
	h_0 (mm)	μ_{h_0}	0.1	Normal
P	S_U (MPa)	387.0	0.05	Normal
	P_b (MPa)	14.495	0.1	Normal

Tab. 2. Bounds of design variables

Variables	Lower bound	Upper bound
μ_H (mm)	142.65	174.35
μ_L (mm)	213.975	261.524
μ_{h_0} (mm)	1.5	3.8
T_d (month)	10.0	100.0
τ (month)	10.0	100.0

4.2. RBDO when the tank is nonrepairable

In this subsection, the pressure tank is considered as nonrepairable. The RBDO model for nonrepairable products in Eq. (9) is therefore used. The cost per unit time is treated as the objective function. The initial cost is determined by the cost of material, which is directly proportional to the volume of the tank and is evaluated at the initial thickness. The cost is given by:

$$C_I = 2\pi\rho(2\mu_{h_0}\mu_H\mu_L - 2\mu_{h_0}^2\mu_H - \mu_{h0}^2\mu_L + \mu_{h_0}^3 + \mu_{h_0}\mu_H^2)/T_d \tag{27}$$

where ρ is a cost coefficient and ρ= 0.01\$/mm^3.

The design model is given by:

$$\begin{aligned} &\underset{\mu_{h_0},\mu_H,\mu_L,T_d}{\text{minimize}} \quad C = 2\pi\rho(2\mu_{h_0}\mu_H\mu_L - 2\mu_{h_0}^2\mu_H - \mu_{h0}^2\mu_L + \mu_{h_0}^3 + \mu_{h_0}\mu_H^2)/T_d \\ &\text{subject to} \quad \Pr\left\{g(T_d) = S_U - \frac{P_b H}{r h(T_d)}\left(1 - \frac{H^2}{2L^2}\right) \geq 0\right\} \geq [R] \\ &\qquad 1.5 \leq \mu_{h_0} \leq 3.8,\ 142.65 \leq \mu_H \leq 174.35, \\ &\qquad 213.975 \leq \mu_L \leq 261.524,\ 10.0 \leq T_d \leq 100.0 \end{aligned} \tag{28}$$

Two cases are considered. In case 1, the required reliability at the lifetime T_d is $[R]$ = 0.95, while in case 2, $[R]$ = 0.98. FORM is used for reliability analysis, and Sequential Quadratic Programming (SQP) is used for optimization. The optimal results are shown in Table 3, where R is the actual reliability defined by $\Pr\left\{g(T_d) = S_U - \frac{P_b H}{r h(T_d)}\left(1 - \frac{H^2}{2L^2}\right) \geq 0\right\}$. The results indicate that the higher the reliability requirement, the higher the lifecycle cost.

Tab. 3. Design results without maintenance

Case	$[R]$	R	μ_{h_0} (mm)	μ_H (mm)	μ_L (mm)	T_d (month)	C (\$/month)
1	0.95	0.95	3.11	142.65	213.98	100.0	156.12
2	0.98	0.98	3.25	142.65	213.98	100.0	163.14

The reliability in terms of time during the product lifecycle is plotted in Fig. 4. It can be seen that the design results ensure that the reliability of the tank is always greater than or equal to the desired reliability during the entire product lifecycle.

4.3. RBDO with perfect preventive and corrective maintenance

In the above subsection, we assumed the tank to be nonrepairable. In this subsection, we assume that the tank is repairable with perfect preventive and corrective maintenance. Two cases are considered where the desired availability in each case is 0.95 and 0.98, respectively. The mean times of preventive maintenance and corrective maintenance are 1 month and 3 months, respectively; namely, T_{CM} = 1 month, and T_{PM} = 3 months. The cost functions are given by:

$$C_I = 2\pi\rho(2\mu_{h_0}\mu_H\mu_L - 2\mu_{h_0}^2\mu_H - \mu_{h0}^2\mu_L + \mu_{h_0}^3 + \mu_{h_0}\mu_H^2) \tag{29}$$

$$C_M = R(\tau)C_P + [1 - R(\tau)]C_r \tag{30}$$

$$C = C_I + C_M \tag{31}$$

where C_I is the initial cost, which is directly proportional to the volume of the tank. C_M is the maintenance cost where C_r = \$10000 is the cost to perform one corrective maintenance, and C_P = \$1000 is the cost to perform one preventive maintenance. C is the total cost during a cycle.

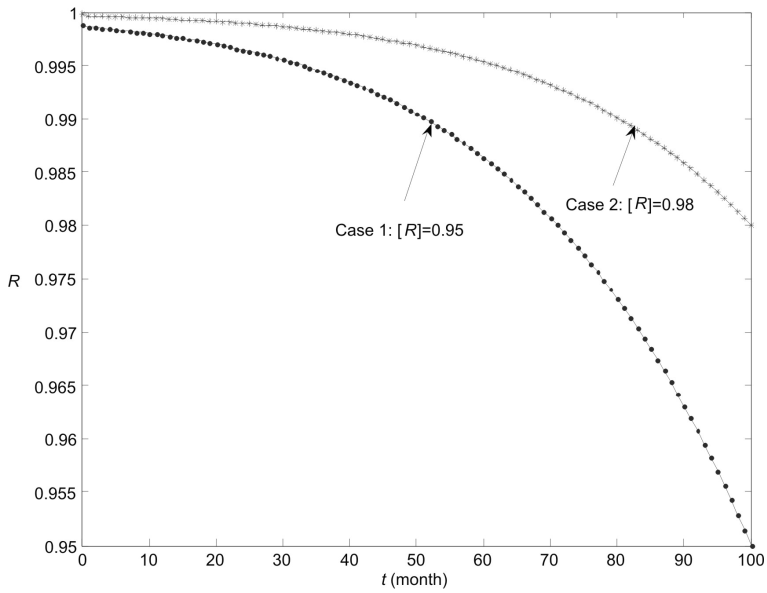

Fig. 4. Relationship between reliability and time

The design model is given by:

$$\underset{\mu_{h_0},\mu_H,\mu_L,\tau}{\text{minimize}} \quad C = 2\pi\rho(2\mu_{h_0}\mu_H\mu_L - 2\mu_{h_0}^2\mu_H - \mu_{h0}^2\mu_L + \mu_{h_0}^3 + \mu_{h_0}\mu_H^2)$$
$$+ R(\tau)C_P + [1 - R(\tau)]C_r$$
$$\text{subject to} \quad A \geq [A] \tag{32}$$
$$1.5 \leq \mu_{h_0} \leq 3.8,\ 142.65 \leq \mu_H \leq 174.35,$$
$$213.975 \leq \mu_L \leq 261.524,\ 10.0 \leq \tau \leq 100.0$$

The design results are provided in Table 4. It can be seen that the greater the desired availability, the longer the time to preventive maintenance since last maintenance, and the higher the cost as well. It is also seen that at the optimal design the actual availability A is exactly at the required availability $[A]$.

4.4. RBDO with minimal maintenance

Now we assume that the tank is repairable with minimal maintenance, and therefore the model in Eq. (23) is used. In this example, the maintenance time is negligible and only one failure is allowed (n = 1 in Eq. (24)). With this assumption, Eq. (24) can be rewritten as:

$$Pr\{N(T_d) \leq 1\} = \exp[-Z(T_d)] + \exp[-Z(T_d)]Z(T_d) \tag{33}$$

Then the probability of failure can be computed by:

$$\Pr\{N(T_d) = 1\} = \exp[-Z(T_d)]Z(T_d) = R(T_d)\ln[-R(T_d)] \tag{34}$$

The corrective maintenance cost is hence given b:y

$$C_M = R(T_d)\ln[-R(T_d)]C_r \tag{35}$$

The total cost per unit time is then given by:

$$C = (C_I + C_M)/T_d \tag{36}$$

where C_I is the initial cost in Eq. (27).

The design model is given by:

$$\underset{\mu_{h_0},\mu_H,\mu_L,T_d}{\text{minimize}} \quad C = [2\pi\rho(2\mu_{h_0}\mu_H\mu_L - 2\mu_{h_0}^2\mu_H - \mu_{h0}^2\mu_L + \mu_{h_0}^3 + \mu_{h_0}\mu_H^2)$$
$$+ R(T_d)\ln(-R(T_d))C_r]/T_d$$
$$\text{subject to} \quad \Pr\{N(T_d) - N(0) \leq 1\} \geq R \tag{37}$$
$$1.5 \leq \mu_{h_0} \leq 3.8,\ 142.65 \leq \mu_H \leq 174.35,$$
$$213.975 \leq \mu_L \leq 261.524,\ 10.0 \leq T_d \leq 100.0$$

The problem is solved for two cases where the desired reliability for each case is 0.98 and 0.999, respectively. The results are given in Table 5.

From Table 5, we see that H, L, and T_d have a small impact on reliability while h_0 has a large impact on reliability. The cost increases when the desired reliability increases.

5. Conclusions

To reduce product lifecycle cost, it is necessary to account for reliability upfront in the parameter design stage. In this work, we propose several reliability-based design optimization (RBDO) models with the consideration of product lifecycle cost and maintenance. The product lifecycle cost including the maintenance cost is minimized. Reliability or availability is considered in design constraints. The First Order Reliability

Tab. 4. Design results under perfect preventive and corrective maintenance

Case	[A]	A	μ_{h_0} (mm)	μ_H (mm)	μ_L (mm)	τ (month)	C($)
1	0.95	0.95	2.64	142.65	213.98	24.20	15047.0
2	0.98	0.98	2.86	142.65	213.98	59.32	15972.0

Tab. 5. Design results under minimal maintenance

Case	[R]	R	μ_{h_0} (mm)	μ_H (mm)	μ_L (mm)	T_d (month)	C ($/month)
1	0.98	0.98	2.85	142.65	213.98	100.0	143.46
2	0.999	0.999	3.13	142.65	213.98	100.0	157.13

Method (FORM) is employed to calculate the time-dependent reliability.

This work is just a preliminary study on accounting for product lifecycle reliability in the early design stage. The idea has been implemented through a simple engineering design problem as shown in Section 4. In this study, we concentrate on the feasibility of incorporating lifecycle reliability during the parameter optimization design. The computational implementation is not our focus. As shown in Fig. 1, solving the proposed RBDO models in a straightforward manner involves a double-loop procedure. Even though this procedure is applicable to black-box objective and constraint functions (such as finite element analysis), the computation may be very expensive. To alleviate the computational burden, efficient RBDO methods should be used. Those methods include single-loop methods [4, 13, 28] and sequential-loop methods [8, 27].

Our future work will be the treatment of general stochastic processes and the introduction of more maintenance strategies and cost models. The improvement of computational efficiency is also one of the future research directions.

****

This work is partially supported by the National Natural Science Foundation of China under the contract number 50775026, and the Open Project Program of the State Key Laboratory of Mechanical Transmission, Chongqing University, China under contract number 200802. We would also like to thank the support from the China Scholarship Council for the first author for his stay at MST as a Visiting Scholar.

****

6. References

1. Allen M, Maute K. Reliability-based design optimization of aeroelastic structures. Structural and Multidisciplinary Optimization 2004; 27: 228-242.
2. Cafeo J A, Donndelinger J A, Lust R V, Mourelatos Z P. The need for nondeterministic approaches in automotive design: A Business Perspective, Engineering Design Reliability Handbook, eds. E. Nilolaidis, D. M. Chiocel, and S. Singhal. CRC Press: Washington D.C, 2005.
3. Cassady C R, Maillart L M. Introduction to repairable system modeling. Las Vegas, 2008; 2008 Annual Reliability and Maintainability Symposium Tutorial.
4. Chiralaksanakul A, Mahadevan S. First-order approximation methods in reliability-based design optimization. ASME Journal of Mechanical Design 2005; 127: 851-857.
5. Dai S H, Wang M O. Reliability Analysis in Engineering Applications. Van Nostrand Reinhold: New York, 1992.
6. Du X, Sudjianto A, Chen W. An integrated framework for optimization under uncertainty using inverse reliability strategy. ASME Journal of Mechanical Design 2004; 126: 562-570.
7. Du X, Huang B. Reliability-based design optimization with equality constraints. International Journal for numerical methods in engineering 2007; 72: 1314-1331.
8. Du X, Chen W. Sequential optimization and reliability assessment for probabilistic design. ASME Journal of Mechanical Design 2004; 126: 225-233.
9. Frangopol D M, Maute K. Life-cycle reliability-based optimization of civil and aerospace structures. Computers & Structures 2003; 81: 397-410.
10. Hoffman J D. Numerical Methods for Engineers and Scientists. Marcel Dekker: New York, 1987.
11. Kuschel N, Rackwitz R. Structural Optimization under Time-Variant Reliability Constraints. Cracow, 1998; 8th IFIP WG 7.5 Working Conference.
12. Laumakis P J, Harlow G. Structural reliability and Monte Carlo simulation. International Journal of Mathematical Education in Science and Technology 2002; 33: 377-387.
13. Liang J, Mourelatos Z P, Tu J. A single-loop method for reliability-based design optimization. International Journal of Product Development (IJPD) 2008; 5: 76-92.
14. Lin D, Zuo M J, Yam R C M, Meng M Q H. Optimal system design considering warranty, periodic preventive maintenance, and minimal repair. Journal of the Operational Research Society 2000; 51: 869-874.
15. Mahadevan S, Rebba R. Inclusion of model errors in reliability-based optimization. ASME Journal of Mechanical Design 2006; 128: 936-944.
16. Mahadevan S. Physics-based reliability models, Reliability-Based Mechanical Design, eds. T. A. Cruse. Marcel Dekker Inc.: New York, 1997.
17. Mahadevan S. Modeling and simulation for design under uncertainty, Modeling and Simulation-Based Life Cycle Engineering. Spon Press: New York, 2002.
18. Monga A, Zuo M J. Optimal design of series-parallel systems considering maintenance and salvage values. Computers and Industrial Engineering 2001; 40: 339-348.
19. Monga A, Zuo M J. Optimal system design considering maintenance and warranty. Computers and Operations Research 1998; 25: 691-705.
20. Monga A, Zuo M J, Toogood R. Reliability based design of systems considering preventive maintenance and minimal repair. International journal of Reliability, Quality and Safety Engineering 1997; 4: 55-71.
21. Noor A K. Perspectives on nondeterministic approaches, Engineering Design Reliability Handbook, eds. E. Nilolaidis, D. M. Chiocel, and S. Singhal. CRC Press: Washington D.C, 2005.
22. Pham H, Wang H Z. Imperfect maintenance. European Journal of Operational Research 1996; 94: 425-438.

23. Rackwitz R. Reliability analysis - past, present and future. Notre Dame, 2000; 8th ASCE Joint Specialty Conference on Probabilistic Mechanics and Structural Reliability.
24. Tobias P A, Trindade D C. Applied Reliability. Chapman & Hall/CRC: New York, 1995.
25. Tu J, Choi K K, Young H P. A new study on reliability-based design optimization. ASME Journal of Mechanical Engineering 1999; 121: 557-564.
26. Vittal S, Hajela P. Probabilistic design and optimization of aging structures. New York, 2004; 10th AIAA/ISSMO Multidisciplinary Analysis and Optimization.
27. Wu Y T, Shin Y, Sues R, Cesare M. Safety-factor based approach for probabilistic-based design optimization. Seattle Washington 2001; 42nd AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics and Materials Conference and Exhibit.
28. Yang R J, Gu L. Experience with approximate reliability-based optimization methods. Structural and Multidisciplinary Optimization 2003; 26: 152-159.
29. Youn B D, Choi K K. Selecting probabilistic approaches for reliability-based design optimization. AIAA Journal 2004; 42: 124-131.
30. Zang T A, Hemsch M J, Hilburger M W, Kenny S P, Luckring J M, Maghami P, Padula S L, Stroud W J. Needs and opportunities for uncertainty-based multidisciplinary design methods for aerospace vehicles. NASA/TM-2002-211462 2002.
31. Zhang Y L. A geometric-process repair-model with good-as-new preventive repair. IEEE Trans. on Reliability 2002; 51: 223-228.

Zhonglai WANG, Ph.D. candidate
Prof. Hong-Zhong HUANG, Ph.D.
School of Mechanical, Electronic, and Industrial Engineering
University of Electronic Science and Technology of China
Chengdu, Sichuan, 611731, P. R. China
The State Key Laboratory of Mechanical Transmission
Chongqing University
Chongqing, 400044, P. R. China
e-mail: hzhuang@uestc.edu.cn

Prof. Xiaoping DU, Ph.D.
Department of Mechanical and Aerospace Engineering
Missouri University of Science and Technology
1870 Miner Circle, Rolla, Missouri, 65409 – 4494, U.S.A.