IIE Transactions
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/uiie20

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Version of record first published: 05 Jul 2010.

To cite this article: Yu Liu, Yanfeng Li, Hong-Zhong Huang, Ming J. Zuo & Zhanquan Sun (2010): Optimal preventive maintenance policy under fuzzy Bayesian reliability assessment environments, IIE Transactions, 42:10, 734-745

To link to this article: http://dx.doi.org/10.1080/07408170903539611

IIE Transactions (2010) **42**, 734–745

ISSN: 0740-817X print / 1545-8830 online
DOI: 10.1080/07408170903539611

Optimal preventive maintenance policy under fuzzy Bayesian reliability assessment environments

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Received June 2005 and accepted November 2009

Reliability assessment is an important issue in reliability engineering. Classical reliability-estimating methods are based on precise (also called "crisp") lifetime data. It is usually assumed that the observed lifetime data take precise real numbers. Due to the lack, inaccuracy, and fluctuation of data, some collected lifetime data may be in the form of fuzzy values. Therefore, it is necessary to characterize estimation methods along a continuum that ranges from crisp to fuzzy. Bayesian methods have proved to be very useful for small data samples. There is limited literature on Bayesian reliability estimation based on fuzzy reliability data. Most reported studies in this area deal with single-parameter lifetime distributions. This article, however, proposes a new method for determining the membership functions of parameter estimates and the reliability functions of multi-parameter lifetime distributions. Also, a preventive maintenance policy is formulated using a fuzzy reliability framework. An artificial neural network is used for parameter estimation, reliability prediction, and evaluation of the expected maintenance cost. A genetic algorithm is used to find the boundary values for the membership function of the estimate of interest at any cut level. The long-run fuzzy expected replacement cost per unit time is calculated under different preventive maintenance policies, and the optimal preventive replacement interval is determined using the fuzzy decision making (ordering) methods. The effectiveness of the proposed method is illustrated using the two-parameter Weibull distribution. Finally, a preventive maintenance strategy for a power generator is presented to illustrate the proposed models and algorithms.

Keywords: Preventive maintenance, T-age replacement, Bayesian estimation, fuzzy lifetime data, fuzzy reliability, fuzzy parameter estimation, reliability assessment, fuzzy value ranking, neural network, genetic algorithm

1. Introduction

Many of the analysis methods used to estimate the reliability of a product depend on access to a large amount of lifetime data. In these methods, the parameters of the lifetime distribution are assumed to be constant but unknown, and sample statistics are used as the estimators of these parameters. This requires a relatively large amount of lifetime data. Such methods have been used in a variety of fields and have solved many practical problems successfully. However, with the progress of modern industrial technologies, product development cycles have become shorter and shorter while the lifetime of products has become longer and longer. As a result, it is time-consuming, costly, and sometimes impossible to obtain sufficient lifetime observations (Tanaka *et al.*, 1983; Park and Kim, 1990; Kenarangui, 1991). In many engineering applications, there may be very few available data points, at times only one or two observations. In such cases, it is impossible to estimate lifetime distribution parameters using conventional statistical analysis methods. The Bayesian approach, in which the parameters of the lifetime distribution are considered to be random variables themselves, has been developed to overcome this difficulty. This enables an engineer to systematically combine the subjective data based on experts' knowledge and intuitive judgment with the objective data from observations, thereby obtaining a balanced estimate, and that estimate is updated as more information and data become available. Therefore, it can be used even when there are only a few lifetime data points. There is extensive literature on classic Bayesian reliability analysis and many practical problems have been solved successfully using this method (Mahadevan *et al.*, 2001; Akama, 2002).

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In engineering applications, randomness is not the only type of uncertainty that can exist in a system. Fuzziness can be found in numerous practical situations and means that exact observations on a system are impossible (Fruehwirth-Schnatter, 1992; Cai, 1996). In this situation lifetime distributions need to be defined using fuzzy concepts. Tanaka *et al.* (1983), Onisawa (1988), Singer (1990), Cai *et al.* (1991, 1993, 1995), Capelle and Kerre (1995), Huang (1995, 1996, 1997), Cai (1996), Utkin and Gurov (1996), Cremona and Gao (1997), Huang *et al.* (2004), and Huang *et al.* (2006) have all attempted to define reliability in terms of fuzzy set theory. The applications of fuzzy set theory to reliability include fault tree analysis, failure modes and effects analysis, optimization of probist reliability, life testing, structural reliability, software reliability, human reliability, and profust, posbist, and posfust reliability theories (Cai, 1996). In addition, fuzzy multi-state system reliability theory, an extension of fuzzy binary state reliability theory, has started to receive attention (Ding and Lisnianski, 2008; Liu *et al.*, 2008; Liu and Huang, 2010).

Fuzzy Bayesian reliability assessment, in which fuzzy parameters are assumed to be fuzzy random variables with fuzzy prior distributions, was developed by Wu (2004a, 2004b, 2004c). Chou and Yuan (1987), Viertl and Gurker (1995), and Viert (1997) have also used fuzzy set theory in Bayesian reliability analysis. Their research, however, has only focused on single-parameter distributions such as the exponential, binomial, and Poisson distributions. In Huang *et al.* (2006), we proposed a numerical method that was able to determine the membership functions of parameters and the reliability functions of multi-parameter distributions using a fuzzy Bayesian approach. The membership functions were determined using neural networks and a genetic algorithm. This article extends that work in that we discuss the creation of a preventive maintenance strategy using a fuzzy Bayesian reliability assessment framework with the aim of investigating the impact of fuzzy lifetime data on maintenance decision making. Popova and Wu (1999) applied fuzzy set theory to renewal processes and formulated the long-run fuzzy average reward per unit time and the T-age replacement policy using a fuzzy cost structure. Motivated by their work, we provide an expression for the long-run fuzzy expected replacement cost per unit time using a fuzzy Bayesian reliability function, which is computed based on fuzzy lifetime data. Neural networks are implemented to approximate the complicated multiple integral functions, and a genetic algorithm is used as a global optimization tool to find the maximum and minimum boundaries of the membership functions for the fuzzy reliability indices at arbitrary cut levels. Two fuzzy decision methods are introduced to determine the optimal preventive maintenance strategy in a fuzzy environment.

The remainder of this article is organized as follows. Section 2 briefly introduces the Bayesian approach. Section 3 proposes Bayesian parameter estimation for multi-parameter lifetime distributions and presents a detailed discussion for the case of a Weibull distribution. Preventive maintenance policy is defined and the long-run expected replacement cost per unit time is formulated in Section 4. The definitions of a fuzzy number and fuzzy lifetime data are introduced in Section 5. Section 6 discusses Bayesian parameter estimation for multi-parameter distributions using fuzzy data, neural networks, and genetic algorithms. Section 7 presents the two adopted fuzzy decision methods. A case study of a power generator is given in Section 8 to illustrate the effectiveness of the proposed models and algorithms. Conclusions are drawn in Section 9.

2. The Bayesian approach

For a continuous random variable, $X, (X \sim f(\cdot\,|\theta), \theta \in \Theta)$ with continuous parameter space, Θ; *a priori* density, $\pi(\cdot)$, of the parameter θ; and observation space, M_X, of X; Bayes' theorem for exact data observations, $x_1, x_2, \ldots, x_n$, is

$$\pi(\theta\,|x_1, x_2, \ldots, x_n) = \frac{l(\theta; x_1, x_2, \ldots, x_n)\,\pi(\theta)}{\int_\Theta l(\theta; x_1, x_2, \ldots, x_n)\,\pi(\theta)\,\mathrm{d}\theta}, \quad \forall\theta \in \Theta \tag{1}$$

where $l(\theta; x_1, x_2, \ldots, x_n)$ is the likelihood of the observations with a given parameter, θ. When the lifetime dataset is complete with exact observations, the likelihood function is given by

$$l(\theta; x_1, x_2, \ldots, x_n) = \prod_{i=1}^{n} f(x_i\,|\theta), \quad \forall\theta \in \Theta. \tag{2}$$

Mainly because of its use of subjective prior beliefs, the use of Bayes' theorem for statistical inference has been controversial within the field of statistics for many years. It is important to note that the prior distribution cannot be specified arbitrarily, because the prior distribution has a large impact especially when the amount of observed data is limited. Several effective methods have been proposed to determine *a priori* distributions, such as non-informative priors, conjugate priors, Jeffreys' priors, empirical Bayesian priors, maximum entropy priors, bootstrap priors, and random weight priors (Berger, 1985; Press, 1989; Smith, 1998).

3. Bayesian parameter estimation for multi-parameter lifetime distributions

In this section, we summarize the literature on Bayesian parameter estimation and Bayesian reliability prediction using precise lifetime data. In Section 6, we will discuss our proposed method for Bayesian parameter estimation, Bayesian reliability prediction, and the expected maintenance cost evaluation using fuzzy lifetime data.

Given the lifetime probability density function (pdf), $f(x\,|\theta)$; the sample data, $D = (x_1, x_2, \ldots, x_n)$; the parameter space, Θ; and *a priori* distribution, $\pi(\theta)$, one can determine the posterior distribution of parameter θ.

According to Bayes' theorem, one has

$$\pi(\theta \mid D) = \pi(\theta \mid x_1, x_2, \ldots, x_n) \propto \pi(\theta)\, l(\theta; x_1, x_2, \ldots, x_n). \tag{3}$$

After the posterior distribution of the parameter, $\pi(\theta \mid D)$, is determined, reliability indices can be estimated. There are two important indices in Bayesian reliability analysis. One is the mathematical expectation of the estimated parameter of the lifetime distribution:

$$\hat{\theta} = \int_{-\infty}^{+\infty} \pi(\theta \mid D)\theta \mathrm{d}\theta. \tag{4}$$

The other is the updated reliability function:

$$R(t \mid D) = \int_t^{\infty} \int_{\Theta} f(x \mid \theta) \pi(\theta \mid D)\, \mathrm{d}\theta \mathrm{d}x. \tag{5}$$

We will now summarize the detailed equations for updating $\hat{\theta}$ and $R(t|D)$ for a specific multi-parameter distribution, namely, the two-parameter Weibull distribution (Huang *et al.*, 2006).

The pdf of the Weibull distribution with two parameters, m and η, is written as

$$f(x|m,\eta) = \frac{m}{\eta}\left(\frac{x}{\eta}\right)^{m-1} \exp\left(-\left(\frac{x}{\eta}\right)^m\right), \quad 0 < m, \eta < +\infty. \tag{6}$$

According to the non-informative method (Cremona and Gao, 1997), the *a priori* distribution of the parameters of the Weibull distribution is given by

$$\pi(m,\eta) \propto \frac{1}{m\eta}. \tag{7}$$

The likelihood function of the observations from the Weibull distribution is

$$l(m,\eta; x_1, x_2, \ldots, x_n) = \left(\frac{1}{\eta}\right)^{mn} m^n \prod_{i=1}^{n} x_i^{m-1} \times \exp\left(-\left(\frac{1}{\eta}\right)^m x_i^m\right). \tag{8}$$

The joint posterior distribution of the parameters of the Weibull distribution is

$$\pi(m,\eta|x_1,x_2,\ldots,x_n) = \frac{(1/\eta)^{mn+1} m^{n-1} \prod_{i=1}^n x_i^{m-1} \exp\left(-(1/\eta)^m x_i^m\right)}{\int_0^{+\infty}\int_0^{+\infty} (1/\eta)^{mn+1} m^{n-1} \prod_{i=1}^n x_i^{m-1} \exp\left(-(1/\eta)^m x_i^m\right) \mathrm{d}m\mathrm{d}\eta}. \tag{9}$$

The marginal posteriori distributions of the parameters m and η are, respectively, written as

$$\pi(m|D) = \int_0^{+\infty} \pi(m,\eta)\mathrm{d}\eta = \frac{\int_0^{+\infty} (1/\eta)^{mn+1} m^{n-1} \prod_{i=1}^n x_i^{m-1} \exp\left(-(1/\eta)^m x_i^m\right) \mathrm{d}\eta}{\int_0^{+\infty}\int_0^{+\infty} (1/\eta)^{mn+1} m^{n-1} \prod_{i=1}^n x_i^{m-1} \exp\left(-(1/\eta)^m x_i^m\right) \mathrm{d}m\mathrm{d}\eta}, \tag{10}$$

$$\pi(\eta|D) = \int_0^{+\infty} \pi(m,\eta)\mathrm{d}m = \frac{\int_0^{+\infty} (1/\eta)^{mn+1} m^{n-1} \prod_{i=1}^n x_i^{m-1} \exp\left(-(1/\eta)^m x_i^m\right) \mathrm{d}m}{\int_0^{+\infty}\int_0^{+\infty} (1/\eta)^{mn+1} m^{n-1} \prod_{i=1}^n x_i^{m-1} \exp\left(-(1/\eta)^m x_i^m\right) \mathrm{d}m\mathrm{d}\eta}. \tag{11}$$

The expected values of the parameters m and η are, respectively, derived as

$$\hat{m} = \int_0^{+\infty} \pi(m|D)m\mathrm{d}m = \frac{\int_0^{+\infty} m \int_0^{+\infty} (1/\eta)^{mn+1} m^{n-1} \prod_{i=1}^n x_i^{m-1} \exp\left(-(1/\eta)^m x_i^m\right) \mathrm{d}\eta\mathrm{d}m}{\int_0^{+\infty}\int_0^{+\infty} (1/\eta)^{mn+1} m^{n-1} \prod_{i=1}^n x_i^{m-1} \exp\left(-(1/\eta)^m x_i^m\right) \mathrm{d}m\mathrm{d}\eta}, \tag{12}$$

$$\hat{\eta} = \int_0^{+\infty} \pi(\eta|D)\eta\mathrm{d}\eta = \frac{\int_0^{+\infty} \eta \int_0^{+\infty} (1/\eta)^{mn+1} m^{n-1} \prod_{i=1}^n x_i^{m-1} \exp\left(-(1/\eta)^m x_i^m\right) \mathrm{d}m\mathrm{d}\eta}{\int_0^{+\infty}\int_0^{+\infty} (1/\eta)^{mn+1} m^{n-1} \prod_{i=1}^n x_i^{m-1} \exp\left(-(1/\eta)^m x_i^m\right) \mathrm{d}m\mathrm{d}\eta}. \tag{13}$$

The updated reliability function of the Weibull distribution then becomes:

$$R(t|D) = \frac{\int_t^{+\infty}\int_0^{+\infty}\int_0^{+\infty} x^{m-1}\left(\frac{1}{\eta}\right)^{mn+m+1} m^n \exp\left(-\left(\frac{1}{\eta}\right)^m x^m\right) \prod_{i=1}^n x_i^{m-1} \exp\left(-\left(\frac{1}{\eta}\right)^m x_i^m\right) \mathrm{d}\eta\mathrm{d}m\mathrm{d}x}{\int_0^{+\infty}\int_0^{+\infty} \left(\frac{1}{\eta}\right)^{mn+1} m^{n-1} \prod_{i=1}^n x_i^{m-1} \exp\left(-\left(\frac{1}{\eta}\right)^m x_i^m\right) \mathrm{d}\eta\mathrm{d}m}, \tag{14}$$

and the other reliability indices, such as mean time to failure, failure rate function, and conditional survival function, can be derived based on Equation (14).

4. Preventive maintenance policy

Systems suffer from deterioration with age and unexpected damage after they are put into operation. Maintenance is carried out to keep a system in or restore it to an acceptable operating condition for the fulfillment of service requirements. A large number of reports in the literature discuss maintenance policy, because multiple maintenance actions are involved in the lifecycle of systems (Pham and Wang, 1996; Cassady *et al.*, 2001). A survey of maintenance policy is given in Wang (2002). To study the impact of fuzzy lifetime data on maintenance strategy optimization, this article uses the preventive age replacement policy (also known as "T-age replacement policy" in Popova and

Wu (1999)) to demonstrate our proposed methodology, because it is simple and widely employed in industry (Wang, 2002).

Before formulation, some basic assumptions should be noted.

1. c_p is the cost of each preventive replacement, and c_f is the cost of each corrective (or failure) replacement. Without loss of generality, we assume $c_f \gg c_p$, since the corrective maintenance is unplanned and often involves additional cost, such as product loss and unplanned transportation.
2. $f(t)$ is the probability density function of the lifetime of the system.
3. The maintenance policy is to perform a preventive replacement once the system has reached a specified age, T_p, or carry out corrective replacement when failure occurs within the preventive replacement interval.
4. The objective is to determine the optimal preventive replacement age, T_p^*, of the system to minimize the long-run expected total replacement cost per unit time.

Thus, two possible cycles of operations are involved: one cycle determined by the system reaching its planned preventive replacement age, T_p, and the other determined by the system ceasing to operate due to a failure occurring before the planned replacement time. The long-run expected replacement cost per unit time, $C_s\ (T_p)$, is

$$C_s(T_p) = \frac{\text{Total expected replacement cost per cycle}}{\text{Expected cycle length}} = \frac{E(C(T_p))}{E(L(T_p))}, \quad (15)$$

where the total expected replacement cost per cycle is

$$E(C(T_p)) = c_p R(T_p) + c_f[1 - R(T_p)], \quad (16)$$

and the expected cycle length is

$$E(L(T_p)) = \int_0^{T_p} t f(x)\mathrm{d}x + \int_{T_p}^{\infty} T_p f(x)\mathrm{d}x = T_p R(T_p) + \int_0^{T_p} t f(t)\mathrm{d}t = \int_0^{T_p} R(t)\mathrm{d}t. \quad (17)$$

Thus, we have:

$$C_s(T_p) = \frac{c_p R(T_p) + c_f[1 - R(T_p)]}{\int_0^{T_p} R(t)\mathrm{d}t}. \quad (18)$$

According to Equations (14) and (17), the updated expected cycle length using Bayesian estimation is integrated as

$$E(L(T_p)) = \int_0^{T_p} R(t|D)dt = \frac{\int_0^{T_p}\int_t^{+\infty}\int_0^{+\infty}\int_0^{+\infty} x^{m-1}\left(\frac{1}{\eta}\right)^{mn+m+1} m^n \exp\left(-\left(\frac{1}{\eta}\right)^m x^m\right)\prod_{i=1}^n x_i^{m-1}\exp\left(-\left(\frac{1}{\eta}\right)^m x_i^m\right)\mathrm{d}\eta\mathrm{d}m\mathrm{d}x\mathrm{d}t}{\int_0^{+\infty}\int_0^{+\infty}\left(\frac{1}{\eta}\right)^{mn+1} m^{n-1}\prod_{i=1}^n x_i^{m-1}\exp\left(-\left(\frac{1}{\eta}\right)^m x_i^m\right)\mathrm{d}\eta\mathrm{d}m}, \quad (19)$$

and $R(T_p)$ in Equation (16) can be derived by setting $t = T_p$ in Equation (14).

As illustrated in Sections 3 and 4, though the lifetime data are non-fuzzy, the equations for obtaining updated estimates of the parameters, the reliability function of the multi-parameter distribution, and the expected cycle length are quite complicated. They involve multiple integrals. Numerical methods can be used to evaluate these equations.

5. Fuzzy lifetime data

In order to model fuzzy observed lifetime data, a generalization of real numbers is necessary. A lifetime observation will be represented by a fuzzy number, $\tilde{x}$. A fuzzy number is a subset, denoted by $\tilde{x}$, of the set of real numbers (denoted by $\Re$) and is characterized by the so-called membership function, $\mu_{\tilde{x}}(\cdot)$. Fuzzy numbers satisfy the following constraints (Zadeh, 1965, 1978).

1. $\mu_{\tilde{x}} : \Re \to [0, 1]$ is Borel-measurable.
2. $\exists x_0 \in \Re : \mu_{\tilde{x}}(x_0) = 1$.
3. The so-called λ-cut level ($0 < \lambda \le 1$), defined as: $B_\lambda(\tilde{x}) = \{x \in \Re : \mu_{\tilde{x}}(x) \ge \lambda\}$, are all closed intervals; i.e., $B_\lambda(\tilde{x}) = [a_\lambda^L, b_\lambda^U]$, $\forall\lambda \in (0, 1]$. This means that the membership function has to be a unimodal function with a maximum. For example, a strictly concave function is a unimodal function with a maximum.

According to the presented definition of a fuzzy number, an exact (non-fuzzy) number can be treated as a special case of a fuzzy number. For a non-fuzzy real observation, $x_0 \in \Re$, the corresponding membership function is $\mu_{\{x_0\}}(x_0) = 1$. For a non-fuzzy interval observation, $[c, d]$, the corresponding membership function is $\mu_{[c,d]}(x) = 1$ for $c \le x \le d$. For a fuzzy lifetime observation, the L-R (Left-Right) type membership functions are commonly used (Huang, 1995; Ding and Lisnianski, 2008; Ding *et al.*, 2008; Liu *et al.*, 2008; Liu and Huang, 2010). The triangular and trapezoidal membership functions are special cases of the *L-R* type membership functions.

Given fuzzy lifetimes, $\tilde{x}_1, \tilde{x}_2, \ldots, \tilde{x}_n$, we have the following corresponding membership functions, $\mu_{\tilde{x}_1}(\cdot), \mu_{\tilde{x}_2}(\cdot), \ldots, \mu_{\tilde{x}_n}(\cdot)$. We call $\tilde{\mathbf{x}} = (\tilde{x}_1, \tilde{x}_2, \ldots, \tilde{x}_n)$ a fuzzy sample or a fuzzy vector where each $\tilde{x}_i$ for $i = 1, 2, \ldots, n$ can be considered as a realization of the associated fuzzy number $\tilde{x}$. If the definition domain of $\tilde{x}$ is M, then the definition domain of $\tilde{\mathbf{x}}$ is M^n.

To the best of our knowledge, no study except Huang *et al.* (2006) has been reported on estimating the parameters or the reliability function of multi-parameter distributions when the lifetime data are fuzzy values. In the following section, we will introduce the approach proposed in Huang *et al.* (2006) for this purpose.

6. Bayesian parameter estimation for multi-parameter distributions using fuzzy data

Given fuzzy lifetime data points, $\tilde{x}_1, \tilde{x}_2, \ldots, \tilde{x}_n$, with their corresponding membership functions, $\mu_{\tilde{x}_1}(\cdot), \mu_{\tilde{x}_2}(\cdot), \ldots, \mu_{\tilde{x}_n}(\cdot)$, the Bayesian point estimates of the parameters and the reliability function are fuzzy numbers where the fuzziness depends on the fuzziness of the n observed lifetime data points.

Consider a general fuzzy function, $\tilde{f}(\tilde{x}_1,\tilde{x}_2,\ldots,\tilde{x}_n)$, for which the membership functions of the n fuzzy arguments are known. We need to find the membership function of $\tilde{f}(\tilde{x}_1,\tilde{x}_2,\ldots,\tilde{x}_n)$. When the explicit membership function of $\tilde{f}(\tilde{x}_1,\tilde{x}_2,\ldots,\tilde{x}_n)$ is difficult to determine directly using the extension principle of fuzzy set theory (Zadeh, 1965, 1978), we proposed the following approach to generating the membership function numerically in Huang *et al.* (2006). The approach guarantees that the membership function of $\tilde{f}(\tilde{x}_1,\tilde{x}_2,\ldots,\tilde{x}_n)$ is unimodal with a maximum.

Step 1. Let λ change from zero to one with an increment size that satisfies the precision requirement.

Step 2. For each fixed value of λ selected in Step 1, find the maximum value of the function, $f(x_1, x_2, \ldots, x_n)$, such that $\mu_{\tilde{x}_i}(x) \geq \lambda$ for all $i = 1, 2, \ldots, n$. Denote this maximum value by $f_{\mathrm{R}}(\lambda)$.

Step 3. For the same fixed value, λ, used in Step 2, find the minimum value of the function $f(x_1, x_2, \ldots, x_n)$ such that $\mu_{\tilde{x}_i}(x) \geq \lambda$ for all $i = 1, 2, \ldots, n$. Denote this minimum value by $f_{\mathrm{L}}(\lambda)$.

Because of the unimodality requirement of a membership function, one has

$$\mu_{\tilde{f}}(f) \geq \lambda \quad \text{for} \quad f_{\mathrm{L}}(\lambda) \leq f \leq f_{\mathrm{R}}(\lambda),$$

where $f_{\mathrm{L}}(\lambda)$ and $f_{\mathrm{R}}(\lambda)$ are, respectively, the lower and upper boundary values of the function, $\tilde{f}$, at cut level, λ.

Through the iterative procedure given above, we are able to numerically obtain the membership function of $\tilde{f}(\tilde{x}_1, \tilde{x}_2, \ldots, \tilde{x}_n)$. Reducing the step size of λ increases the accuracy of the membership function of $\tilde{f}$ at the expense of increased computation time.

From the equations given in Section 3, we can see that multiple integrals are required for Bayesian parameter estimation, reliability prediction for non-fuzzy lifetime data following multi-parameter distributions, and estimation of the expected cycle length. Evaluation of such multiple integrals is time-consuming, and yet we have to evaluate the membership functions of the parameters and the reliability function of such multi-parameter distributions using fuzzy lifetime data. It is obvious that determination of the membership functions of the estimated parameters and the reliability function requires repeated evaluation of such integrals. A more efficient method of evaluating these integrals is needed.

Artificial neural networks have been successfully used in many areas of science and engineering (Aourid and Do, 1995), as well as the reliability field (Liu *et al.*, 2003; Rajpal *et al.*, 2006). Cybenko (1989) and Aourid and Do (1995) pointed out that the finite linear combination of sigmoidal functions used in neural networks can approximate any continuous function of n variables with support in the unit hypercube to any degree of accuracy. Liu *et al.* (2003) used neural networks to approximate a complicated utility function involving multiple integrals for optimal design of a continuous-state system. As a result, we propose to use a neural network to approximate $f(x_1,x_2,\ldots,x_n)$. A feed-forward neural network with a single hidden layer is used. The back-propagation learning scheme is adopted.

To approximate the functions for point estimates of parameters, we need to approximate Equations (12) and (13). From these equations, we can see that input data for the neural network are $(x_1, x_2, \ldots, x_n)$ and the output data for the neural network are m and η. Because the parameters may take real values, the transfer function of the output layer is chosen to be a linear function.

To find the Bayesian reliability function, we need to approximate Equation (14). We can see that the input data of this neural network is $(t, x_1, x_2, \ldots, x_n)$. The single output is the value of reliability, R. Since the reliability is between zero and one, we choose the transfer function of the output layer to be the log-sigmoid function:

$$\mathrm{logsig}\,(n) = \frac{1}{1 + \exp(-n)}. \tag{20}$$

The transfer function used in the hidden layer is the tansigmoid function:

$$\mathrm{tansig}\,(n) = \frac{1 - \exp(-2n)}{1 + \exp(-2n)}. \tag{21}$$

To evaluate the long-run expected replacement cost per unit time according to Equation (18), we need to approximate Equation (19) by a neural network using the input data $(T_{\mathrm{p}}, x_1, x_2, \ldots, x_n)$, and a linear function is chosen as the transfer function of the output layer.

Although function f has been approximated by a neural network, we still need to follow the proposed procedure to find the maximum and the minimum values of f subject to constraints for each selected λ value. This is a constrained optimization problem. Because the objective function is simulated by a neural network, we cannot use the gradient projection method or the feasible-direction method to search for the optimal solution. In addition, it is difficult to find the global optimal solution by using direct search methods. As a result, we use a genetic algorithm to solve the optimization problem in the process of determining the membership function of $\tilde{f}$, since it has a good global optimization capability (Leuitin, 2006; Tian *et al.*, 2008).

To find the upper bound for the fuzzy reliability at cut level λ, we need to solve the following optimization problem:

$$\begin{aligned} &\max && R(t\,|x_1, x_2, \ldots, x_n), \\ &\text{subject to:} && \mu_{\tilde{x}_i}(x) \geq \lambda, \quad i = 1, 2, \ldots, n, \\ &&& 0 \leq \lambda \leq 1, \end{aligned} \tag{22}$$

where $\mu_{\tilde{x}_i}(x)$ is the membership function of lifetime data point, $\tilde{x}_i$. The maximum value obtained for the reliability is denoted by $R_{\max}$. To find the lower bound for the fuzzy reliability at cut level λ, we need to solve the following optimization problem:

$$\begin{aligned} &\min && R(t\,|x_1, x_2, \ldots, x_n), \\ &\text{subject to:} && \mu_{\tilde{x}_i}(x) \geq \lambda, \quad i = 1, 2, \ldots, n, \\ &&& 0 \leq \lambda \leq 1, \end{aligned} \tag{23}$$

and the minimum reliability value obtained is denoted by $R_{\min}$.

To find the upper bound of the fuzzy estimate of the parameter, $\tilde{\theta}$, at cut level λ, we need to solve the following optimization problem:

$$\begin{aligned} &\max && \hat{\theta}(x_1, x_2, \ldots, x_n), \\ &\text{subject to:} && \mu_{\tilde{x}_i}(x) \geq \lambda, \quad i = 1, 2, \ldots, n, \\ &&& 0 \leq \lambda \leq 1, \end{aligned} \tag{24}$$

and the maximum value obtained for $\tilde{\theta}$ is denoted by $\theta_{\max}$.

To find the lower bound for the fuzzy estimate of the parameter, $\tilde{\theta}$, at cut level λ, we need to solve the following optimization problem:

$$\begin{aligned} &\min && \hat{\theta}(x_1, x_2, \ldots, x_n) \\ &\text{subject to:} && \mu_{\tilde{x}_i}(x) \geq \lambda, \quad i = 1, 2, \ldots, n, \\ &&& 0 \leq \lambda \leq 1, \end{aligned} \tag{25}$$

and the minimum value obtained for $\tilde{\theta}$ is denoted by $\theta_{\min}$. For the Weibull distribution, θ denotes m or η.

To find the upper bound for the long-run fuzzy expected replacement cost per unit time, $\tilde{C}_s(T_p)$, at cut level λ, we need to solve the following optimization problem:

$$\begin{aligned} &\max && C_s\left(T_p, x_1, x_2, \ldots, x_n\right) \\ &\text{subject to:} && \mu_{\tilde{x}_i}(x) \geq \lambda, \quad i = 1, 2, \ldots, n, \\ &&& 0 \leq \lambda \leq 1, \end{aligned} \tag{26}$$

and the maximum value obtained for $\tilde{C}_s(T_p)$ is denoted by $C_s(T_p)_{\max}$.

To find the lower bound of the long-run fuzzy expected replacement cost per unit time, $\tilde{C}_s\left(T_p\right)$, at cut level λ, we need to solve the following optimization problem:

$$\begin{aligned} &\min && C_s\left(T_p, x_1, x_2, \ldots, x_n\right), \\ &\text{subject to:} && \mu_{\tilde{x}_i}(x) \geq \lambda, \quad i = 1, 2, \ldots, n, \\ &&& 0 \leq \lambda \leq 1, \end{aligned} \tag{27}$$

and the minimum value obtained for $\tilde{C}_s(T_p)$ is denoted by $C_s(T_p)_{\min}$.

The implementation of the genetic algorithm involves the following steps:

1. Representation: A binary vector (or real-coded vector) is used as a chromosome to represent the specific realizations of the n fuzzy lifetimes and the time instant. The length of the chromosome depends on the domain of the variables and the required precision. An initial population of solutions is created.
2. Fitness function: The fitness function is used to measure the solutions in terms of their fitness. The objective function (for maximization problems) or the reciprocal of the objective function (for minimization problems) is taken as the fitness function.
3. Crossover: The probability of crossover is selected based on the fitness of chromosomes. If the fitness is greater, the chance of being selected is greater. One-point crossover is used in our study.
4. Mutation: The probability of mutation is initialized at the beginning. Mutation alters one or more genes with the probability equal to the probability of mutation. The method used for mutation is random point mutation. Mutation of a binary string is the process of changing one of the genes between zero and one (or its feasible range for real-coded genetic algorithms).
5. Selection: The expanded population is composed of chromosomes generated through crossover and mutation as well as the original population of chromosomes. The chromosomes with higher fitness values are selected as the parent generation in the next iteration of the algorithm.
6. Stopping criteria: The maximum number of iterations and the change in population fitness value are used in combination to determine when to stop the optimization process.

7. Fuzzy decision making

There are two fuzzy decision-making methods introduced in this section to determine the optimal solution in the fuzzy environment.

7.1. *Method I: fuzzy number distance*

This method was first introduced by Murakami *et al.* (1983) and updated and extended by Cheng (1998). It ranks fuzzy numbers by the distance from the origin to the centroid point of fuzzy alternative $\tilde{A}$.

Let $\bar{x}$ denote the coordinate on the horizontal axis corresponding to the centroid of $\tilde{A}$ and let $\bar{y}$ denote such coordinate on the vertical axis. A trapezoid fuzzy number, $\tilde{A} = [a, b, c, d]$, with its membership function, $\mu_{\tilde{A}}$, given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x-a}{b-a}, & a \le x \le b, \\ 1, & b \le x \le c, \\ \dfrac{x-d}{c-d}, & \text{c} \le x \le d, \\ 0, & \text{otherwise} \end{cases} \tag{28}$$

and let $\mu^{\mathrm{L}}_{\tilde{A}}(x) = \mu_{\tilde{A}}(x)(x \in [a, b])$ and $\mu^{\mathrm{R}}_{\tilde{A}}(x) = \mu_{\tilde{A}}(x)(x \in [c, d])$. The inverse functions of $\mu^{\mathrm{L}}_{\tilde{A}}(x)$ and $\mu^{\mathrm{R}}_{\tilde{A}}(x)$ are written as: $g^{\mathrm{L}}_{\tilde{A}} = a + (b-a)\mu_{\tilde{A}}$ and $g^{\mathrm{R}}_{\lambda} = d + (c-d)\mu_{\tilde{A}}$. Thus, the centroid point $(\bar{x}, \bar{y})$ of $\tilde{A}$ is defined as

$$\bar{x} = \frac{\int_a^b x\mu^{\mathrm{L}}_{\tilde{A}}\mathrm{d}x + \int_b^c x\mathrm{d}x + \int_c^d x\mu^{R}_{\tilde{A}}\mathrm{d}x}{\int_a^b \mu^{\mathrm{L}}_{\tilde{A}}\mathrm{d}x + \int_b^c 1\mathrm{d}x + \int_c^d \mu^{\mathrm{R}}_{\tilde{A}}\mathrm{d}x}, \tag{29}$$

$$\bar{y} = \frac{\int_0^1 \left(\mu_{\tilde{A}} g^{\mathrm{L}}_{\tilde{A}}\right)\mathrm{d}\mu_{\tilde{A}} + \int_0^1 \left(\mu_{\tilde{A}} g^{\mathrm{R}}_{\tilde{A}}\right)\mathrm{d}\mu_{\tilde{A}}}{\int_0^1 g^{\mathrm{L}}_{\tilde{A}}\mathrm{d}\mu_{\tilde{A}} + \int_0^1 g^{R}_{\tilde{A}}\mathrm{d}\mu_{\tilde{A}}}. \tag{30}$$

The distance index between the centroid point $(\bar{x}, \bar{y})$ and the origin is defined as

$$R(\tilde{A}) = \sqrt{(\bar{x})^2 + (\bar{y})^2}, \tag{31}$$

and the ranking of the fuzzy alternatives follows Definition 1 according to the distance index.

Definition 1.

1. If $R(\tilde{A}) < R(\tilde{B})$, then $\tilde{A} < \tilde{B}$.
2. If $R(\tilde{A}) = R(\tilde{B})$, then $\tilde{A} = \tilde{B}$.
3. If $R(\tilde{A}) > R(\tilde{B})$, then $\tilde{A} > \tilde{B}$.

Thus, a better decision can be made under the fuzzy environment, and this method can rank several alternatives simultaneously.

7.2. *Method II: the Liou-Wang method (Liou and Wang, 1992)*

More than two alternatives can be ranked simultaneously by this method, and an optimism index, β, is used to capture the optimistic attitude of the decision maker.

For a fuzzy number $\tilde{A}$, the interval at the λ-cut level is denoted by $\tilde{A}_\lambda = [A^{\mathrm{L}}_\lambda, A^{U}_\lambda]$; the method defines the left integral value of $\tilde{A}$ as

$$I_{\mathrm{L}}(\tilde{A}) = \int_0^1 A^{\mathrm{L}}_\lambda \mathrm{d}\lambda, \tag{32}$$

and the right integral value as

$$I_{\mathrm{R}}(\tilde{A}) = \int_0^1 A^{\mathrm{U}}_\lambda \mathrm{d}\lambda. \tag{33}$$

Thus, the total integral value of the fuzzy number, $\tilde{A}$, is

$$I^{\beta}(\tilde{A}) = \beta I_{\mathrm{R}}(\tilde{A}) + (1-\beta) I_{\mathrm{L}}(\tilde{A}), \tag{34}$$

where $\beta(\beta \in [0, 1])$ is an optimism parameter selected by the decision maker; for example, $\beta = 0.5$ reflects a neutral attitude.

According to the total integral value, $I^{\beta}(\tilde{A})$, a fuzzy decision can be ordered by following Definition 2.

Definition 2.

1. If $I^{\beta}(\tilde{A}) < I^{\beta}(\tilde{B})$, then $\tilde{A} < \tilde{B}$.
2. If $I^{\beta}(\tilde{A}) = I^{\beta}(\tilde{B})$, then $\tilde{A} = \tilde{B}$.
3. If $I^{\beta}(\tilde{A}) > I^{\beta}(\tilde{B})$, then $\tilde{A} > \tilde{B}$.

Following these two ranking methods, the optimal preventive replacement policy can be determined by ranking the long-run fuzzy expected replacement cost per unit time.

8. Illustrative example

To demonstrate our proposed method, the case of a power generator is presented in this section. The lifetime distribution considering the fuzzy lifetime data is analyzed first, and then the optimal preventive age replacement policy is determined.

8.1. *Lifetime distribution analysis*

The lifetime of the power generator is modeled by a two-parameter Weibull distribution. The pdf of the Weibull distribution is given in Equation (6). Suppose that we have obtained five fuzzy failure data points, $\tilde{x}_1, \tilde{x}_2, \ldots, \tilde{x}_5$, with membership function given respectively as

$$\mu_{\tilde{x}_1}(x) = \begin{cases} (x-3.2)/0.3, & 3.2 \le x \le 3.5, \\ (3.8-x)/0.3, & 3.5 < x \le 3.8. \end{cases}$$

$$\mu_{\tilde{x}_2}(x) = \begin{cases} (x-3.7)/0.3, & 3.7 \le x \le 4.0, \\ (4.2-x)/0.2, & 4.0 < x \le 4.2. \end{cases}$$

$$\mu_{\tilde{x}_3}(x) = \begin{cases} (x-4.2)/0.3, & 4.2 \le x \le 4.5, \\ (4.8-x)/0.3, & 4.5 < x \le 4.8. \end{cases}$$

$$\mu_{\tilde{x}_4}(x) = \begin{cases} (x-4.6)/0.2, & 4.6 \le x \le 4.8, \\ (5.0-x)/0.2, & 4.8 < x \le 5.0. \end{cases}$$

$$\mu_{\tilde{x}_5}(x) = \begin{cases} (x-5.0)/0.2, & 5.0 \le x \le 5.2, \\ (5.4-x)/0.2, & 5.2 < x \le 5.4. \end{cases}$$

where the unit time is a year.

To estimate the parameter m the transfer function of the output layer of the neural network was selected to be a linear function. With Equation (12), we obtained 40 training data points. Other parameter values for the used neural network were

number of hidden nodes = 10;
number of input nodes = 5;
learning rate = 0.01;

acceptable training error = 0.000 001;
training epochs = 5000.

The program was developed in Matlab. Through training we obtained the interconnection weights and the bias weights.

Then the genetic algorithm was used to determine the membership function of the fuzzy parameter estimation, $\tilde{\hat{m}}$. The real-coded genetic algorithm approach was adopted, and we used the following parameter values for the genetic algorithm:

length of chromosome = 5; size of population = 80; fitness function: the parameter value when it is to be maximized and the reciprocal of the parameter value when it is to be minimized; probability of crossover = 0.6; probability of mutation = 0.05; iteration epochs = 500.

Using the genetic algorithm, the parameter estimates at any cut level can be calculated. The membership function of the fuzzy parameter Bayesian estimate, $\tilde{\hat{m}}$, can be obtained by connecting these estimates with different membership function values, as shown in Fig. 1.

The output layer transfer function of the neural network model for estimation of the parameter η was selected to be a linear function. Using Equation (13), we also obtained 40 data points for training the neural network. The other parameter values for training the neural networks were the same as those in the neural network for estimating parameter m. Through training, we obtained all the interconnection weights and the bias weights.

The genetic algorithm was then used to determine the membership function of the fuzzy parameter estimation, $\tilde{\hat{\eta}}$. The parameter values for the genetic algorithm in this case for $\tilde{\hat{\eta}}$ were the same as those used for the parameter $\tilde{\hat{m}}$. The fitness function was determined in the same manner.

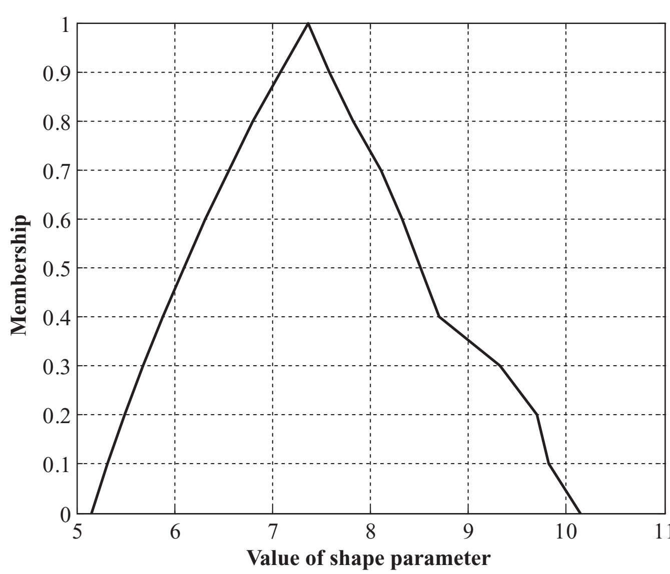

Fig. 1. Membership function of the parameter $\tilde{\hat{m}}$.

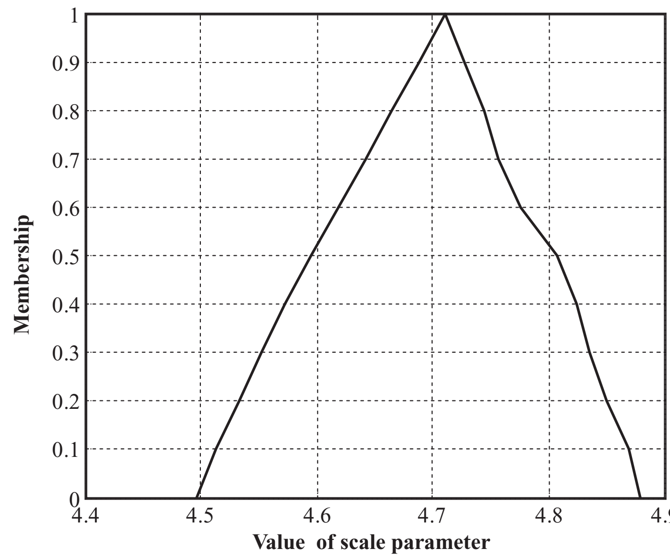

Fig. 2. Membership function of the parameter $\tilde{\hat{\eta}}$.

Using the genetic algorithm, the parameter estimate at any cut level can be calculated. The membership function of the fuzzy parameter Bayesian estimation, $\tilde{\hat{\eta}}$, can be obtained by connecting these estimates with different degrees of membership, as illustrated in Fig. 2.

Using Equation (14), we calculated the reliability function at time t using precise data, $x_1, x_2, \ldots, x_n$. This way, we could also obtain a training data set for the neural network. The parameter values used in our training of this neural network for the reliability function were

number of samples = 60; number of input nodes = 6; number of hidden nodes = 12; learning rate = 0.02; acceptable training error = 0.000 001; training epochs = 5000.

Through training, we obtained the interconnection weights and the bias weights of the neural network.

The genetic algorithm was used to determine the membership function of fuzzy reliability, $\tilde{R}(t|\tilde{D})$. The following parameter values for the genetic algorithm were used:

length of chromosome = 5; size of population = 80; fitness function: the reliability function when it is to be maximized and the reciprocal of the reliability function when it is to be minimized; probability of crossover = 0.6; probability of mutation = 0.05; iteration epochs = 500.

Using the genetic algorithm, we also calculated the reliability values at time point t using different cut level values. Connecting these reliability values with different membership function values at the same time point, t, as plotted in Fig. 3, makes it possible to obtain the membership function for estimating fuzzy Bayesian reliability. Figure 3 illustrates

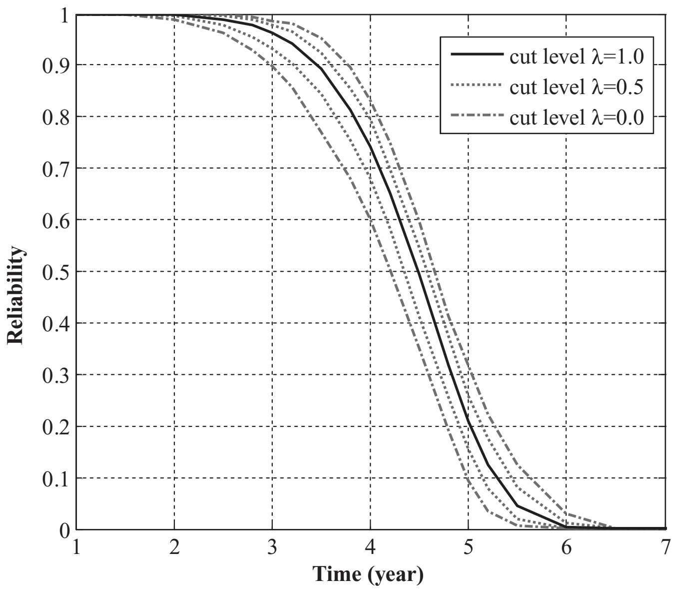

Fig. 3. Reliability at different λ cut levels.

a single curve for the crisp case where $\lambda = 1$, two dash-dot curves as the boundaries of the fuzzy value under the condition $\lambda = 0$, and two dot curves for the case $\lambda = 0.5$.

The membership function of the reliability at the time point of 4 years is shown in Fig. 4.

8.2. *Fuzzy analysis of the long-run expected replacement cost*

It is assumed that the preventive replacement activity is performed on the power generator at fixed time intervals, T_p, and that the corresponding maintenance costs for

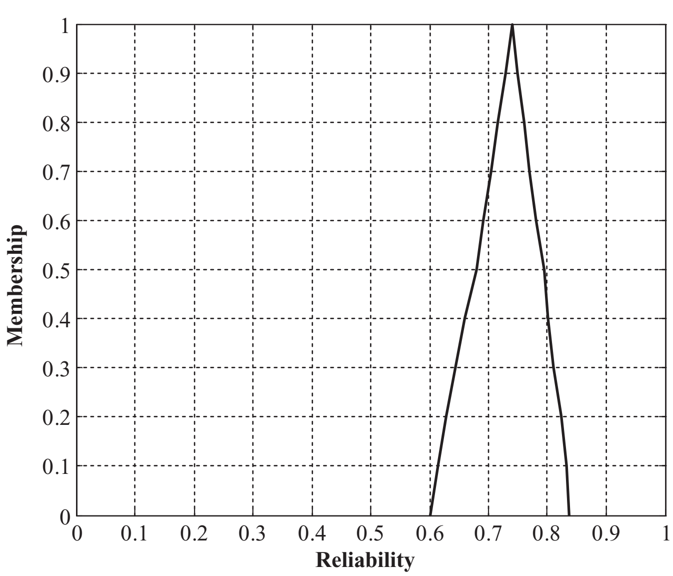

Fig. 4. Membership function of reliability at $t = 4.0$ years.

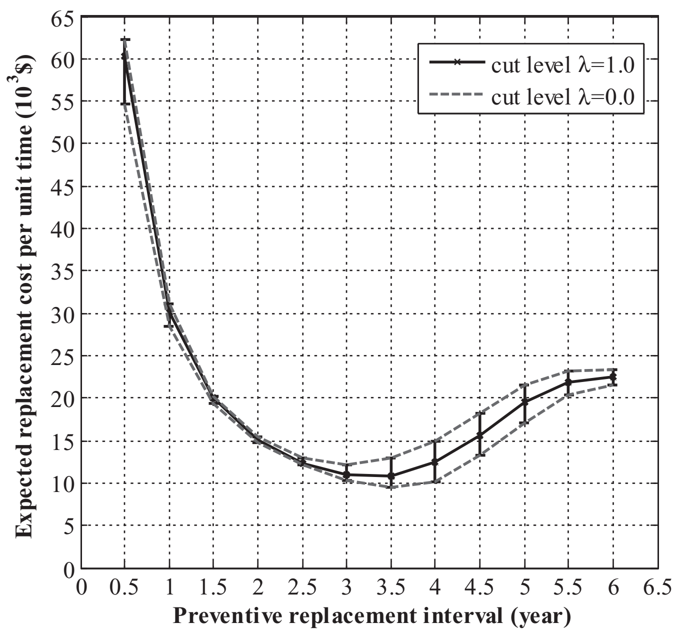

Fig. 5. The long-run fuzzy expected replacement cost per unit time at different discrete T_p values.

different actions are $c_f = \$100 \times 10^3$ and $c_p = \$30 \times 10^3$, respectively.

Randomly generating some preventive replacement interval values, T_p, makes it possible to calculate the updated expected cycle length, $E(L[T_p])$, using precise failure data and the complex multiple integrals as presented in Equation (19). This gives us a set of training data for the neural network of:

number of samples = 60; number of input nodes = 6; number of hidden nodes = 14; learning rate = 0.02; acceptable training error = 0.000 001; training epochs = 5000.

The long-run expected replacement cost per unit time, $C_s(T_p)$, can be obtained through Equation (18). Actually, we used two trained neural networks to approximate the $R(T_p)$ and $E(L[T_p])$ in Equation (18), respectively. The membership of the long-run fuzzy expected replacement cost per unit time for different preventive replacement intervals, T_p, was estimated at any cut level through the genetic algorithm. The parameters for the genetic algorithm were

length of chromosome = 5; size of population = 80; fitness function: the long-run expected replacement cost per unit time when it is to be maximized and the reciprocal of the long-run expected replacement cost per unit time when it is to be minimized; probability of crossover = 0.6; probability of mutation = 0.05; iteration epochs = 800.

Using the genetic algorithm, the membership function of long-run expected replacement cost per unit time under different preventive intervals was obtained, as were the membership functions of estimated parameter and

Table 1. Fuzzy decision comparison using method I

	Preventive replacement interval (years)		
	3.0	*3.5*	*4.0*
$\bar{x}_{\tilde{C}_s}$	11.068	10.958	12.508
$\bar{y}_{\tilde{C}_s}$	0.4573	0.4624	0.4661
$R(\tilde{C}_s)$	11.077	10.968	12.517

fuzzy reliability. We obtained the possible expected replacement cost per unit time for different preventive interval strategies using half-year increments. The results are shown in Fig. 5, where the solid line denotes the case in which the failure data are crisp; i.e., $\lambda = 1.0$. The two dash curves are the upper and lower boundaries of possible values for the maintenance cost under the fuzzy environment; i.e., λ is set to zero. The figure shows that the gap between the upper and lower boundaries varies for different strategies. A larger gap exists when $T_p = 0.5$ year and $T_p > 3.5$ years, indicating that the fuzzy failure data creates significant levels of uncertainty to the average maintenance cost value.

In order to determine the optimal preventive replacement interval, T_p^*, the introduced fuzzy order algorithms in Section 7 were used to rank the fuzzy values. From Fig. 5, we selected the cases $T_p =$ 3.0, 3.5, 4.0 years as possible alternatives. The corresponding memberships for these three cases are given in Fig. 6.

The ranking results are as follows.

1. Method I.

Equations (29), (30), and (31) give us the centroid point coordinates and the distances as tabulated in Table 1.

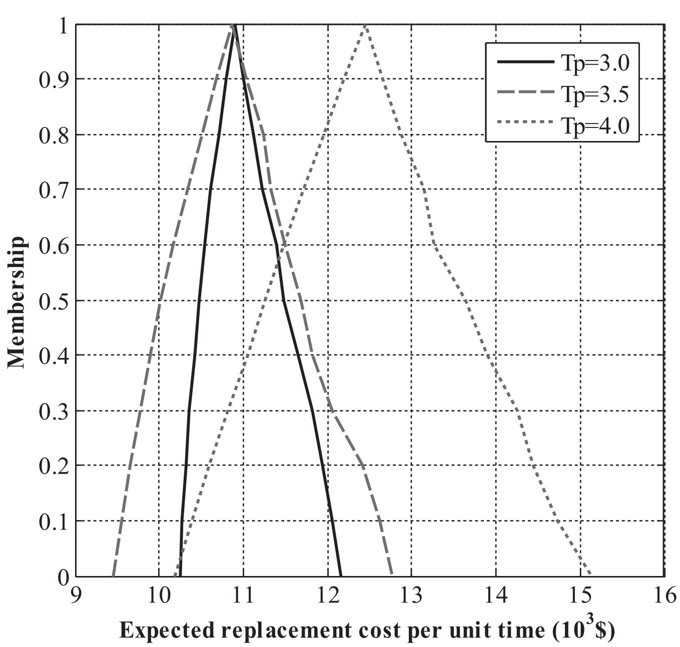

Fig. 6. Membership functions of the expected replacement cost per unit time for different replacement policies.

Table 2. Fuzzy decision comparison using method II

	Preventive replacement interval (years)		
	3.0	*3.5*	*4.0*
$I_L(\tilde{C}_s)$	5.754	5.484	6.104
$I_R(\tilde{C}_s)$	6.278	6.380	7.409
$I^{0.5}(\tilde{C}_s)$	6.016	5.932	6.757

The order of the fuzzy alternatives is $\tilde{C}_s(T_p = 3.5) < \tilde{C}_s(T_p = 3.0) < \tilde{C}_s(T_p = 4.0)$, and, according to this method, the optimal interval for performing the preventive replacement is 3.5 years.

2. Method II.

Equations (32), (33), and (34) with the optimism parameter $\beta = 0.5$ give the alternative left, right and total integrals as tabulated in Table 2.

The order of the fuzzy alternatives is $\tilde{C}_s(T_p = 3.5) < \tilde{C}_s(T_p = 3.0) < \tilde{C}_s(T_p = 4.0)$; that is, the same result as that given by the previous method is providing the identical optimal preventive replacement interval (3.5 years) as method I.

3. Other perspectives.

Although the illustrated ranking methods show $T_p = 3.5$ years to be the optimal option (the fuzzy ranking method producing the minimum expected maintenance cost), other decision criteria may yield different results. For example, from Fig. 6, one can observe that the case in which $T_p =$ 3.5 years has a larger range of possible values (between \$9.45 × 10^3 and \$12.77 × 10^3) than the case in which $T_p =$ 3.0 years (between \$ 10.25 × 10^3 and \$12.07 × 10^3). In other words, one can conclude that the fuzzy environment exerts less influence on the case in which $T_p = 3.0$ years than the case in which $T_p = 3.5$ years. In industry, decision makers may select $T_p = 3.0$ years (or even $T_p = 2.5$ years) as their final solution though it costs more, because it is more robust under uncertain conditions.

9. Conclusions

This article discusses an optimal preventive maintenance strategy under a fuzzy Bayesian environment. The methodology deals with not only determining the membership function of the fuzzy estimates of the parameters and the reliability functions of multi-parameter distributions but also examining maintenance policy and approaches to fuzzy decision making. An artificial neural network is used to obtain approximate solutions for fuzzy parameter estimation, reliability prediction, and evaluation of long-run fuzzy expected replacement cost per unit time. The genetic algorithm is used to find the boundary values for the membership functions at any cut level. The methods provide a way of determining the membership functions of the parameter estimates and the reliability functions of multi-parameter distributions, which are very difficult to obtain using conventional methods involving multiple integrals. The membership function of the long-run fuzzy expected replacement cost per unit time is obtained in the same fashion, and two fuzzy ranking methods are implemented to determine the optimal solution. Fuzzy uncertainty is inevitable in practice, and the methodology proposed in this article makes risk analysis and maintenance decision making in a fuzzy uncertainty environment more tractable. In addition, it can provide engineers with more useful information about the state of a system, thus enabling them to make better decisions about safety and economic issues.

Acknowledgements

This research was partially supported by the National Natural Science Foundation of China under contract number 50775026 and the Natural Sciences and Engineering Research Council of Canada. The authors also wish to acknowledge the constructive comments from reviewers and the editor.

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