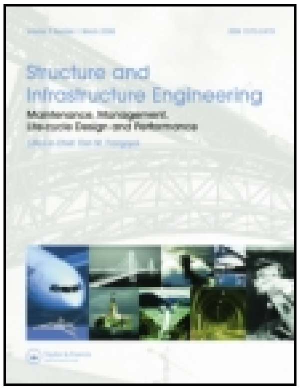

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An optimal sequential preventive maintenance policy under stochastic maintenance quality

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An optimal sequential preventive maintenance policy under stochastic maintenance quality

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This paper presents a sequential imperfect preventive maintenance policy for a degradation system. Two kinds of activity, called continuous preventive maintenance (PM) and minimal repair, are simultaneously considered when arranging discrete imperfect preventive maintenance schedules. In order to obtain the maximum benefit in a finite lifetime, an expected benefit model is formulated based on maximal/equal cumulative-hazard rate constraints, and the optimal PM intervals are obtained using a genetic algorithm (GA). It is usually difficult to determine fixed maintenance quality after performing maintenance activities. This problem is addressed in the present paper by assuming that the reduction factor is a stochastic variable following probability distributions at fixed times. It is more rational to describe the fluctuation and trend of quality of discrete preventive maintenance during a lifetime; this makes optimisation results more robust and insensitive to the randomness of the crucial parameters in imperfect PM models. A numerical case is presented to illustrate the proposed model and some discussions are summarised.

Keywords: degradation system; sequential preventive maintenance; imperfect maintenance; reduction factor; genetic algorithm

1. Introduction

Many of today's system have become increasingly complicated. The importance of ensuring systems operate properly requires a level of reliability capable of avoiding failure, reducing costly breakdown and ensuring security. Preventive maintenance (PM) is necessary to restore/keep a repairable system functional. For this reason, research on optimal preventive maintenance policies is a hot issue, and one that is very important for academic research and industrial application. PM seeks to provide maximum system reliability and safety with a minimum of maintenance resources (cost, time, staff, etc.). How to assess the quality/effect of preventive maintenance and how to arrange its activities to meet one's satisfaction have been studied in practice since the 1960s.

Researchers generally focus on the feasibility and cost of maintaining a repairable system. A huge number of maintenance policies have been proposed in the literature (see Murthy and Hwang 1996, Dedopoulos and Smeers 1998, Ben-Daya and Alghamdi 2000, Tsai *et al*. 2001). In particular, in aviation or the semiconductor industry, because a fault in an airplane can be disastrous and any flaw in a semiconductor product line may seriously affect product quality and lead to an enormous cost increase, system reliability should be emphasised. The optimisation of maintenance policies is, therefore, worthy of study. Furthermore, repairable systems commonly have a finite and fixed lifetime (see Nakagawa and Mizutani 2009), so it is crucial that effective PM strategies need to be found to achieve maximum benefits in each system's finite lifespan.

Generally, maintenance activity falls into two main categories: corrective (unplanned) and preventive (planned), corresponding to corrective maintenance (CM) and preventive maintenance (PM), respectively (see Pham and Wang 1996). To distinguish the degree or quality of maintenance, Pham and Wang (1996) classified it as perfect, minimal, imperfect, worse and worst; these five classifications are now widely accepted. Generally speaking, maintenance does not make a system 'as good as new', but younger. It is commonly held that maintenance restores an item to somewhere between 'as good as new' (perfect) and 'as bad as old' (minimal), the sort of imperfect maintenance that settles for keeping an item pretty much in the condition it was. Much of the literature discusses modelling imperfect maintenance and its application; a comprehensive survey of this is presented by Pham and Wang (1996). The maintenance quality/degree that describes the effect (such as age reduction or failure rate decrement) obtained by maintenance actions is

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a critical factor in imperfect maintenance optimisation problems. Existing research usually regards the parameters related to maintenance quality/degree as fixed values; these values need to be determined based on a large volume of maintenance data (see Ben-Daya and Alghamdi 2000, Tsai *et al.* 2001). Wu and Clements-Croome (2005) arrived at an optimal maintenance policy in finite time by assuming that the parameters that affect the quality of maintenance follow a certain probability distribution. It is, however, more rational to consider the relative parameters affecting maintenance quality as stochastic variables, because the mean of maintenance quality is monotonic, decreasing as the system ages.

This problem is tackled in the present paper by assuming that the age reduction factor, which represents the quality of an imperfect preventive maintenance action, is a stochastic variable following a certain probability distribution at a certain time. The approach suitably describes the fluctuation and reduction-factor trend as the system ages. For the purpose of maximising profit, this paper considers the optimisation of preventive maintenance policy in a finite lifetime. Our maintenance quality model is introduced in detail and an optimal mathematical model is developed in §2. The maximal cumulative-hazard strategy and the equal cumulative-hazard strategy are then investigated. In §3, the basic procedure of a genetic algorithm (GA) is introduced, and the representing and decoding processes are presented. A numerical case is given in §4 to illustrate the proposed model and algorithm, and the results are tabulated to quantify the difference between the maximal cumulative-hazard and equal cumulative-hazard strategies. A brief conclusion is given in §5.

2. Mathematical formulation

We begin our formulation with the following assumptions:

(1) Maintenance is performed on a degradation system with a finite lifespan. In other words, as time passes, the failure rate of a system will monotonically increase as the reliability decreases (if no PM intervenes).
(2) Discrete and continuous PM actions are simultaneously carried out in the life cycle. CM is performed immediately upon failure during discrete PM intervals, and is regarded as minimal repair that restores the system to 'as bad as old' condition (without changing the current failure rate and effective age). The duration of CM can be ignored.
(3) It is assumed that the cost for discrete PM actions is a fixed value, but not a decision variable.
(4) The salvage value of the system decreases as the number of failures increases.

2.1. Continuous preventive maintenance

Actually, maintenance staff often perform minimal PM during operation time, clearing, calibrating, lubricating, locking tight, checking, etc. If, however, a system is subjected to such inspection and service within very short intervals, relative to the time between failures, one can regard these actions as occurring continuously over time. Without reducing the operation time of the system, they can lower the occurrence of failures; even prolong the life of the system. In other words, these actions have effects that slow down the failure rate. Referring to Murthy and Hwang (1996), if $u(t)$ represents the maintenance effort rate (such as work hours per unit time) expended at time t, and the original failure rate function, $r(t)$, of the system is a differentiable function, then the derivative function, $r'(t)$, of failure rate function, $r(t)$, under continuous PM can be expressed as:

$$\frac{\mathrm{d}r'(t)}{\mathrm{d}t} = v_0(t) - bu(t) \tag{1}$$

and

$$0 < bU < v_0(0). \tag{2}$$

In these equations, $v_0(t)$ represents the differential function of failure rate without considering continuous PM effort, U represents the maximum continuous PM effort and b is a constant. We can see from Equation (2) that the failure rate function, $r'(t)$, increases monotonically, even if continuous PM is performed. As a result, the continuous PM cost, C, after a working hour, t, is given by:

$$C = c_c \int_0^t u(t)\mathrm{d}t, \tag{3}$$

where c_c represents the cost per unit of maintenance effort.

2.2. Imperfect discrete preventive maintenance

In this paper, discrete PM actions are carried out at scheduled time intervals, $\pi_i (i = 1,2, \ldots ,N\text{–}1)$. It is assumed that the system will be resold after the interval

π_N. Generally speaking, a PM action has an imperfect effect on the effective age of the system. Imperfect discrete PM does not affect the failure rate function; it just reduces the effective age of the system (see Kijima *et al*. 1988, Kijima 1989). Assuming that it costs t_p time at each imperfect discrete PM action and the reduction factors after each discrete PM are represented by $Y_1, Y_2, \ldots Y_n$, the effective age, ω_1^-, just before the first discrete PM and the effective age, ω_1^+, just after the first PM can be written as:

$$\omega_1^- = \pi_1 \text{ and } \omega_1^+ = (1 - Y_1)\phi_1\pi_1, \tag{4}$$

where ϕ_i represents the environment effect factor at each epoch. Thus, the effective age before and after the second and third discrete PM actions can be derived as:

$$\begin{aligned} \omega_2^- &= \omega_1^+ + \pi_2 = (1 - Y_1)\phi_1\pi_1 + \pi_2, \\ \omega_2^+ &= (1 - Y_2)(\omega_1^+ + \phi_2\pi_2) \\ &= (1 - Y_2)((1 - Y_1)\phi_1\pi_1 + \phi_2\pi_2) \\ &= (1 - Y_1)(1 - Y_2)\phi_1\pi_1 + (1 - Y_2)\phi_2\pi_2, \end{aligned} \tag{5}$$

$$\begin{aligned} \omega_3^- &= \omega_2^+ + \pi_3 = (1 - Y_1)(1 - Y_2)\phi_1\pi_1 \\ &\quad + (1 - Y_2)\phi_2\pi_2 + \pi_3, \\ \omega_3^+ &= (1 - Y_3)(\omega_2^+ + \phi_3\pi_3) \\ &= (1 - Y_1)(1 - Y_2)(1 - Y_3)\phi_1\pi_1 \\ &\quad + (1 - Y_2)(1 - Y_3)\phi_2\pi_2 + (1 - Y_3)\phi_3\pi_3. \end{aligned} \tag{6}$$

If that is the case, the effective age before and after the nth imperfect discrete PM action can be formulated iteratively as:

$$\omega_n^- = \sum_{j=1}^{n-1}\left[\prod_{i=j}^{n-1}(1 - Y_i)\phi_j\pi_j\right] + \pi_n$$

and

$$\omega_n^+ = \sum_{j=1}^{n}\left[\prod_{i=j}^{n}(1 - Y_i)\phi_j\pi_j\right]. \tag{7}$$

The cumulative failure rate in each discrete PM interval can be determined as:

$$E(n_i) = \int_{\omega_{i-1}^+}^{\omega_i^-} r(t)\mathrm{d}t, \tag{8}$$

where the failure rate function, $r(t)$, can be obtained by the integral of Equation (1).

2.3. *Stochastic maintenance quality model*

Much published work focuses on modelling imperfect maintenance; there is, for example, the (p,q) rule by Nakagawa (1979), the types I and II age reduction models in Kijima *et al*. (1988) and Kijima (1989), the geometric process model by Zhang *et al*. (2002), the quasi-renewal process model in Wang and Pham (2006) and a survey of imperfect maintenance models by Pham and Wang (1996). These models measure maintenance quality/degree through relevant parameters that are regarded as fixed values and can be estimated by field data and statistics (see Dedopoulos and Smeers 1998). As mentioned in Wu and Clements-Croome (2005), in real-world environments, it is usually difficult to specify precisely the quality of a maintenance action. This is due to differences in improvement between individual systems, even in response to the same maintenance action and also to a lack of maintenance data. This uncertainty makes it hard to optimise maintenance policy, because some optimisation results are very sensitive to the parameters related to maintenance quality and to the selected imperfect maintenance models (see Bartholomew-Biggs *et al*. 2009). It can, therefore, be more practical to obtain the parameters combining estimations of experts in real applications, and assume that these parameters vary in a given interval according to certain probability distributions. For example, many maintenance engineers working with building service systems do not specify the return date as being 2 years after maintenance; rather, they specify intervals between 1 and 3 years in accordance with a uniform probability distribution (see Evans *et al*. 1998).

This paper extends the work of Wu and Clements-Croome (2005) by assuming that the reduction factors in Equations (4) to (7) are stochastic variables. It is reasonable to consider the average effect of discrete PM degradation using the system age increment. The degradation path of reduction factors and the probability distribution corresponding to certain working times can be estimated by domain experts using maintenance data. This approach makes the optimisation results more robust and insensitive to the randomness of the crucial parameters in imperfect PM models.

If the reduction factor, Y_i, of the ith discrete PM action follows a probability distribution denoted by $F_i\,(y \mid t)$ at time t, and if the density function of Y_i is given by:

$$f_i(y|t) = \frac{\mathrm{d}F_i(y|t)}{\mathrm{d}y}, \tag{9}$$

the probability distribution can be estimated by domain experts and statistical results, and the expected value of the reduction factor at time t can be obtained:

$$E(Y_i|t) = \int_{y_i^{\mathrm{L}}}^{y_i^{\mathrm{U}}} y f_i(y|t)\mathrm{d}y_i, \qquad (10)$$

where y_i^{U} and y_i^{L} are the upper and lower boundaries of Y_i, respectively.

This paper assumes that the possible values for the reduction factor fall within $[y_i^{\mathrm{L}}, y_i^{\mathrm{U}}]$ at time t and follow a normal distribution. To correlate quality degradation of discrete PM with a system's age, we propose using the following mean value of reduction factors:

$$\partial\mu(t)/\partial t < 0, \qquad (11)$$

where $\mu(t) = E(Y_i|t)$ and $\mu(t)$ is a monotonic decreasing function, respect to t. This indicates that the quality of discrete PM worsens with age of system.

2.4. Salvage value

Consider that the system is resold immediately at the end of its life cycle, T, and that the resale price of the system is given by:

$$S_{\mathrm{A}}(T) = c_{\mathrm{m}}\exp(-A_0 T - A_1 N(T)), \qquad (12)$$

where c_{m} is the price once it is purchased and offered for resale without further use. The resale price is lower than the original purchase price, c_0, and A_0, A_1 are positive constants. $N(T)$ is the expected number of failures in a finite life cycle and is given by:

$$N(T) = \sum_{i=1}^{N} E(n_i), \qquad (13)$$

where $E(n_i)$ is obtained using Equation (8). This indicates that the failure history and the use of the system will affect the resale price simultaneously.

2.5. Expected profit

Based on the hypothesis and models given above, the total expected profit from the equipment in its life cycle can be calculated by:

$$\max P = r(T-(N-1)t_{\mathrm{p}}) + S_{\mathrm{A}}(T) - c_{\mathrm{p}}(N-1) - c_{\mathrm{c}} \sum_{i=1}^{N}\int_{\omega_{i-1}^{+}}^{\omega_i^{-}} u(t)\mathrm{d}t - c_{\mathrm{r}}\sum_{i=1}^{N} E(n_i) - c_0$$

$$\text{subject to} \sum_{i=1}^{N}\pi_i + (N-1)t_{\mathrm{p}} = T, \qquad (14)$$

where the discrete PM scheduled time intervals, $\pi_1, \pi_2, \pi_3 \ldots\ldots \pi_N$ and N are decision variables, and r, c_{p} and c_{r} denote the output profit per unit time, the fixed discrete PM cost, and the CM cost, respectively.

Most of the literature regards the expected cost per unit time or sum of cost to be an optimisation criterion, but does not consider some important constraints (see Jiang and Ji 2002). To make the formulation more realistic, limitations and constraints, such as minimal allowed reliability/availability and maximal cumulative hazard, should be considered for practical applications. Two types of constraint, defined as the maximal cumulative-hazard strategy and the equal cumulative-hazard strategy, are presented in the following subsections.

2.5.1. *Maximal cumulative-hazard strategy*

Since a system becomes weaker as its age increases, it is practical to restrict to an acceptable level, called the maximal cumulative-hazard rate, in each discrete PM interval. This level can keep the system working constantly in a desired state in each discrete PM cycle. This paper postulates that the maximal allowed cumulative-hazard rate is H_{s} in each discrete PM interval; the additional constraint for optimisation formulation in Equation (14) is given by

$$H_i \le H_{\mathrm{s}}, i = 1, 2, ..., N, \qquad (15)$$

where

$$H_i = \int_{\omega_{i-1}^{+}}^{\omega_i^{-}} r(t)\mathrm{d}t = E(n_i). \qquad (16)$$

2.5.2. *Equal cumulative-hazard strategy*

Owing to the fact that the failure rate of a system quickly increases in the course of the degradation process, systems are hard to maintain and their PM intervals become shorter in their later stages. If, however, an equal cumulative-hazard-rate level is given at each discrete PM interval, subsequently, appropriate discrete PM intervals can be determined, and the number of decision variables can be reduced directly. Thus, it becomes easier to obtain the optimal value. The additional constraints can be shown by

$$H_{\mathrm{equal}} = H_i = H_{i+1} = H_{i+2}, ..., = H_N. \qquad (17)$$

Based on Equations (16) and (17) and the decision variables in Equation (14), the number of sequential discrete PM intervals, N, and the corresponding scheduled time intervals (π_1,π_2,π_3. . . . . .π_N) are directly determined by the optimal constraint variable, H^*_{equal}. The algorithm for calculating N* and $\pi^*_1, \pi^*_2, \pi^*_3......\pi^*_{N^*}$ are presented in Figure 1, based on the optimal H^*_{equal} obtained.

3. The GA optimisation technique

Equation (14) formulates a complicated non-linear programming problem. An exhaustive examination of all possible solutions is not realistic due to time limitation. Meta-heuristic algorithms, such as the GA, Tabu search, the simulated annealing algorithm, and ant colony optimisation (ACO) are efficient and effective approaches to searching for the optimal solution (or approximate optimal solution) in combinational and non-linear programming problems that do not require derivative information to determine the next direction of the search. These approaches also adapt well to other problems. The GA employed in this paper to solve the presented problem is briefly introduced in the following sections.

3.1. The GA

The GA is one of the most widely used of the evolutionary searching methods that were inspired by the optimisation procedure that exists in nature and biological phenomena. Because of its advantages, the GA has become the most popular universal tool for solving various optimisation problems. It has been successfully applied to an abundance of optimisation problems in reliability engineering (see Lisnianski and Levitin 2003, Levitin 2005, 2006) and maintenance (see Levitin and Lisnianski 1999, 2000).

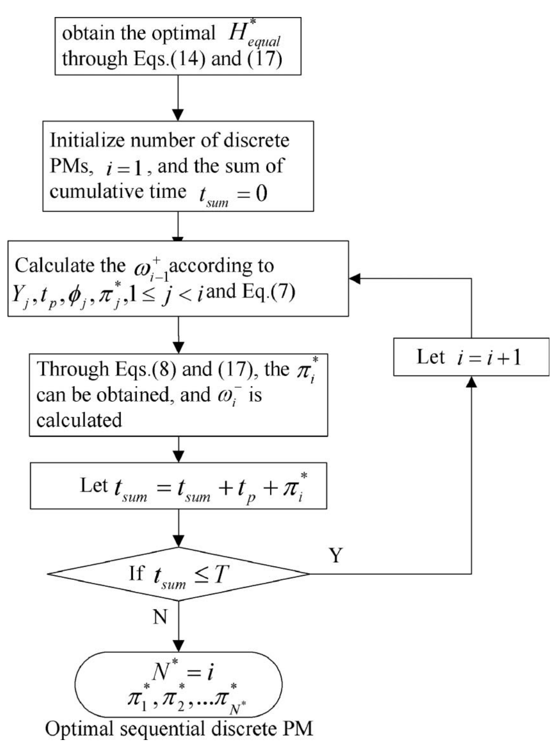

Figure 1. Determining the optimal discrete PM sequence under the equal cumulative-hazard strategy.

Basically, the GA operates with solutions represented by ‘chromosomes’. Selection procedures are employed to maintain diversity in the population (the set of solutions). Unlike other non-meta-heuristic algorithms, the GA deals with the solutions of each generation without considering derivative information, and iteratively leads the population trend to global optimal points with tractable manipulations. Detailed information on the GA can be found in books (e.g. see Lisnianski and Levitin 2003, Levitin 2005). The basic procedure of the GA is given below.

At first, an initial population is randomly generated, consisting of N_s individuals (a singular solution) with S length strings. The new offspring (new solution) for the next generation is obtained during the genetic cycle using genetic operators of crossover and mutation and a specified selection strategy based on the fitness of each individual. Fitness can be regarded as the corresponding results of each solution; in minimisation problems, the smaller the result, the greater the fitness of the individual. Through the selection strategy, the crossover operator produces a new solution from parent populations and facilitates the inheritance of some properties from the parents by the offspring. Mutation operators swap the initial order of strings located in two randomly chosen positions; this results in slight changes to the offspring’s structure. The diversity of each generation is maintained, and premature convergence to a local optimum is avoided through this random jump approach. By decoding the solutions, the fitness of each individual is evaluated and a selection procedure is performed based on the individual’s fitness. Individuals with better fitness have a greater chance of being selected to join the next generation. The iterative process terminates when solutions in the population meet some criteria, such as: (1) the genetic cycles repeat N_c times, (2) the variance of fitness in populations is less than ε_1 and (3) the variance of average fitness in subsequent populations is not more than ε_2, where ε_1 and ε_2 are two user-specified values. The final population contains the best solution achieved, as well as different approximate optimal solutions that may be useful to the decision maker.

To apply the GA to a specific problem, solution representation and decoding are important procedures that must be defined. The penalty function approach is employed to handle infeasible solutions.

3.2. Solution representation

For the optimisation problem (14) considering a maximal cumulative-hazard strategy with an additional constraint (15), the decision variables are the scheduled time intervals, $\pi_1,\pi_2,\pi_3.\ldots.\ldots.\pi_N$ and the total number of discrete PM intervals, N. Nevertheless, the optimal number, N^*, is difficult to be determined before optimisation, and the length of string for each individual solution is uncertain. Given that $N_{\max}$ represents the maximum possible number of PMs in a finite time span, T, an individual solution is represented by real number strings $\pi = \{\pi_1,\pi_2,\pi_3,\ldots, \pi_{N_{\max}}\}$. Each π_i should vary in the range (1,T). Furthermore, the upper limit, T, for π_i can be reduced by considering the constraint (15). At the very least, it will be equal to$T' = H_1^{-1}(H_s)$, where $H_1^{-1}(\cdot)$ denotes the inverse function of $H_1(\cdot)$.

3.3. Solution decoding

The following procedure determines the fitness value for an arbitrary individual solution represented by the real number strings $\pi = \{\pi_1,\pi_2,\pi_3,\ldots,\pi_{N\max}\}$.

(1) Initialise the variables $T_{\text{sum}} = 0$, indicating the cumulative time, and that $N_{\text{sum}} = 1$, representing the total number of discrete PM intervals, then go to step 2;
(2) Let $T_{\text{sum}} = T_{\text{sum}} + \pi_{N_{\text{sum}}}$ and go to step 3;
(3) Check whether $T_{\text{sum}} < T$ is satisfied; if $T_{\text{sum}} < T$ go to step 4, otherwise go to step 6;
(4) Let $T_{\text{sum}} = T_{\text{sum}} + t_{\text{p}}$ and go to step 5;
(5) Check whether $T_{\text{sum}} < T$ is satisfied; if $T_{\text{sum}} < T$ go to step 7, otherwise go to step 6;
(6) $\pi'_{N_{\text{sum}}} = T - \sum_{i=1}^{N_{\text{sum}}-1}\left(\pi'_i + t_{\text{p}}\right)$; go to step 8;
(7) $\pi'_{N_{\text{sum}}} = \pi_{N_{\text{sum}}}$ and $N_{\text{sum}} = N_{\text{sum}} + 1$; go to step 2;
(8) The decoding solution is $\pi' = \{\pi'_1,\pi'_2,\pi'_3,\ldots, \pi'_{N_{\text{sum}}},0,\ldots 0\}$, and the number of discrete PM intervals is equal to N_{sum}; go to step 9;
(9) According to the solution strings $\pi' = \{\pi'_1, \pi'_2,\pi'_3,\ldots, \pi'_{N_{\text{sum}}},0,\ldots 0\}$ and Equation (14), the corresponding fitness of the individual solution is obtained.

4. A numerical case

Suppose the failure time of a degradation system follows a two-parameter Weibull distribution with the shape parameter being $m = 2.2$ and the scale parameter being $\eta = 100$. The finite lifetime, T, of each system is equal to 1000 hours, and the duration of a discrete PM action, t_{p}, is equal to 5 hours. The continuous preventive maintenance effect is given by $u(t) = t^{0.2}$. The reduction factor follows a normal distribution within the range (0.3,1.0), where $y^L = 0.3$ and $y^U = 1.0$. Assume the mean value of the normal distribution is:

$$\mu(t) = \exp\left(-\frac{t}{T}\right). \tag{18}$$

The expected value of the reduction factors at certain times is tabulated in Table 1, which indicates that the reduction factor decreases as time passes.

Assume that the fixed cost of a discrete PM is \$1200 per unit time. CM costs are higher than PM costs for unplanned interruptions (see Evans *et al.* 1998), therefore, $c_{\text{r}}/c_{\text{p}} = 40$. The values of other parameters are presented in Table 2.

Based on the GA optimisation method and the mathematical models presented in the previous section, the optimal results corresponding to different maximal cumulative-hazard constraints are those tabulated in Table 3.

Similarly, Table 4 presents the optimal results obtained under different equal cumulative-hazard constraints using the enumeration method.

One can see that the optimal number of discrete PMs is equal to 35 under the equal cumulative-hazard strategy, and that $H_{\text{equal}} = 0.125$ is the best strategy (with maximal profit \$370130) in these alternative strategy listed in Table 3 and 4. Through the process shown in Figure 1, the sequential time epoch for the discrete PM actions are 40.70, 34.38, 32.89, 31.69, 30.60, ... , 16.70, 16.42, 16.16, 15.90, 15.66 (hours). The time interval between two discrete PMs decreases progressively, which is determined by the degrading nature of the system.

Profit decreases when over-maintenance (non-optimal maintenance policy) is performed under the same constraint. As shown in Tables 3 and 4, the optimal results under the two strategies are very different, even with the same constraint values. When

Table 1. Expected reduction factors at certain times.

t	10	100	500	990
$E(Y_i\|t)$	0.9900	0.9048	0.6065	0.3716

Table 2. Relevant parameters.

r	A_0	A_1	b	c_0
800	10^{-4}	0.05	10^{-5}	150000
c_{m}	c_{p}	c_{c}	c_{r}	ϕ_i
120000	1200	10	48000	1

Table 3. Optimal results under the maximal cumulative-hazard strategy.

H_s	Optimal N^*	Maximum P^*
0.3	34	359670
0.2	35	362420
0.15	40	352750
0.125	42	347070
0.1	48	338410
0.05	62	313510

Table 4. Optimal results under the equal cumulative-hazard strategy.

H_{equal}	Optimal N^*	Maximum P^*
0.3	24	337300
0.2	29	355760
0.15	33	361550
0.125	**35**	**370130**
0.1	39	362770
0.05	51	332230

the cumulative-hazard rate constraint is larger than 0.2 ($H_s, H_{\text{equal}} \geq 0.2$), the maximal cumulative-hazard strategy apparently provides larger profits than does the equal strategy. This indicates that the equal cumulative-hazard strategy is less adequate. On the other hand, when the cumulative-hazard rate constraint is less than 0.125 ($H_s, H_{\text{equal}} \leq 0.125$), although there exists an over-maintenance problem under the equal cumulative-hazard rate strategy, better results can also be obtained. When the hazard rate constraint is strict, greater maintenance frequency is required to meet the constraint, and thus the number of variables increases. This makes the solution process time-consuming and the global optimal result difficult to obtain; however, as mentioned in the previous section, the equal cumulative-hazard strategy greatly reduces the number of variables and makes the solution process quicker and easier to perform.

5. Conclusions

This paper discusses a sequential PM policy for a degradation system with a finite lifetime. Considering that uncertainty is a crucial problem in maintenance modelling, the stochastic reduction factor is proposed to describe the randomness and trend of the PM quality. This is reasonable and useful because it makes the optimisation results more robust and insensitive to any uncertainty incurred due to a lack of data. Two constraint strategies are proposed and compared in an illustrative case, which concludes that, although the maximal cumulative-hazard rate strategy is more efficient than the equal constraint strategy, it is almost impossible to obtain the global optimal solution, even through the GA method. The larger the number of discrete PMs involved, the more difficult it is to obtain the global optimal solution. Thus, the equal cumulative-hazard rate is tractable and outperforms the maximal strategy.

Further work may expand the model and release some restrictions, such as how to set up the relationship between the reduction factor and the corresponding PM cost. The relationship between maintenance cost and reliability is also an interesting subject, as is its modelling and sensitive analysis. Furthermore, many PM models in the literature do not consider the constraints from maintenance resources (such as available cost time and staff, see Cassady *et al.* (2001)), but this is a more practical problem. The selection of sets of efficient PM activities under a limited budget is also worth consideration.

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