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JOINT OPTIMIZATION OF REDUNDANCY AND MAINTENANCE STAFF ALLOCATION FOR MULTI-STATE SERIES-PARALLEL SYSTEMS

OPTYMALIZACJA ŁĄCZONA ALOKACJI NADMIAROWOŚCI ORAZ ALOKACJI PRACOWNIKÓW SŁUŻB UTRZYMANIA RUCHU W WIELOSTANOWYCH SYSTEMACH SZEREGOWO-RÓWNOLEGŁYCH

Multi-state system (MSS), as a kind of complex system consisting of elements with different performance levels, widely exists in engineering practices. In this paper, redundancy and maintenance staff allocation problems for repairable MSS with series-parallel configuration are considered simultaneously. The traditional redundancy allocation problem (RAP) for MSS always assumes that maintenance resources are unlimited. However in many practical situations, maintenance resources are limited due to the budget and/or time. To maximize the system availability under a certain demand, there are two feasible ways: (1) designing an optimal system configuration with available elements, and (2) allocating more maintenance staffs to reduce waiting time for repair. With the assistance of Markov queue model, the availabilities of identical version elements with the pre-assigned number of maintenance staffs can be evaluated. The universal generation function (UGF) is employed to assess the availability of entire MSS under a certain demand. Two optimization formulas considering the limited maintenance resources are proposed. One regards the limitation of maintenance resources as a constraint, and the other considers minimizing the total system cost including both the system elements and maintenance staff fees. The system redundancy and staffs allocation strategies are jointly optimized under required availability. A numerical case is presented to illustrate the efficiency of the proposed models. The Firefly Algorithm (FA), which is a recently developed metaheuristic optimization algorithm, is employed to seek the global optimal strategy.

Keywords: *multi-state series-parallel systems, redundancy allocation problem (RAP), maintenance staff allocation, queue theory, universal generation function (UGF), firefly algorithm (FA).*

Systemy wielostanowe (multi-state systems, MSS), stanowiące typ złożonych systemów zbudowanych z elementów o różnym poziomie wydajności, znajdują szerokie zastosowanie w praktyce inżynierskiej. W prezentowanej pracy podjęto rozważania łączące zagadnienia alokacji nadmiarowości oraz alokacji pracowników służb utrzymania ruchu w naprawialnych systemach MSS o konfiguracji szeregowo-równoległej. Tradycyjnie ujmowane zagadnienie alokacji nadmiarowości (redundancy allocation problem, RAP) w systemach MSS zawsze zakłada, że środki obsługi są nieograniczone. Jednakże w wielu sytuacjach praktycznych, środki obsługi mogą być ograniczone budżetem i/lub czasem. Istnieją dwa możliwe sposoby maksymalizacji gotowości systemu przy określonym zapotrzebowaniu użytkowników: (1) zaprojektowanie optymalnej konfiguracji systemu z wykorzystaniem dostępnych elementów oraz (2) alokowanie większej liczby pracowników obsługi w celu zmniejszenia czasu oczekiwania na naprawę. Dostępność jednakowych wersji elementów przy wcześniej określonej liczbie pracowników obsługi oceniano za pomocą modelu kolejek Markowa. Uniwersalną funkcję generacyjną (UGF) wykorzystano do oceny gotowości całego systemu MSS przy określonym zapotrzebowaniu. Zaproponowano dwa równania optymalizacyjne uwzględniające ograniczone środki obsługi. W jednym z nich ograniczoność środków obsługi potraktowano jako ograniczenie (constraint), natomiast drugie równanie dotyczyło minimalizacji całkowitych kosztów systemu włącznie z kosztami elementów systemu oraz płacą pracowników służb utrzymania ruchu. Strategie alokacji nadmiarowości systemu oraz alokacji pracowników poddano jednoczesnej optymalizacji z uwzględnieniem wymaganej gotowości. Wydajność proponowanych modeli zilustrowano przykładem numerycznym. Poszukiwania optymalnej strategii globalnej prowadzono przy pomocy niedawno opracowanego metaheurystycznego algorytmu optymalizacyjnego znanego jako algorytm świetlika (Firefly Algorithm, FA).

Słowa kluczowe: *wielostanowe systemy szeregowo równoległe, zagadnienie alokacji nadmiarowości (RAP), alokacja pracowników służb utrzymania ruchu, teoria kolejek, uniwersalna funkcja generacyjna (UFG), algorytm świetlika (FA).*

1. Introduction

Redundancy allocation problem (RAP), aims at providing redundancy at various stages of a system and determining an optimal system level configuration while considering the tradeoff between system reliability/performance and resources, has received increasing attention in reliability engineering as of late.

The reported works on RAP mainly focus on the problems of determining the optimal redundancy level for various stages, and/or selecting a set of components available in the market to maximize system reliability under some constraints, such as volume, weight and cost budget. They involve not only single/multi- objective optimization problem, but also binary/multi- state system configuration. For binary state systems, Coit et al. [4] introduced redundancy allocation optimization problem for binary state series-parallel systems. Coit and Konak [3] proposed a multiple weighted objectives heuristic algorithm to determine the optimal redundancy allocation solution for binary series-parallel systems. Other algorithms such as dynamic programming, integer programming, tabu search, and annealing simulation method, ant colony optimization approach were also proposed to solve the RAP in the context of binary state systems [6]. As an extension from binary state systems to MSSs, much research pays intensive attentions on RAP for MSSs. Lisnianski and Levitin [10] introduced RAP formulation for multi-state series-parallel systems, and the configuration of MSSs was determined by selecting an optimal set of components (elements) available in market. Tian et al. [20] proposed to jointly determine the optimal components state distribution of multi-state series parallel systems and its optimal redundancy level for each stage (defined as reliability-RAP). They formulated a multi-objective optimization problem, and physical programming was employed to solve the problem. Taboada et al. [19] proposed multi-objective multi-state genetic algorithm (MOMS-GA) to determine the optimal redundancy solution set under multi-criterions (cost, weight, availability etc.). Nourelfath et al. [17] developed an integrated model to jointly optimize the redundancy levels and imperfect preventive maintenance strategy for MSSs. A comprehensive survey of current advance in RAP can be referred to Ref.[6].

Most existing RAPs in literature assume that the maintenance resources are unlimited [18]. However, as stated in Ref.[1,12], the maintenance strategy suffers resources limitations in industrial applications, such as staffs, maintenance cost, and time, etc. Nourelfath and Dutuit [15] first proposed to solve the RAP considering repair policy. They discussed the issue when the number of repair teams is less than the number of reparable elements. A heuristic algorithm was applied to determine system configuration under availability constraint. Once a preliminary solution was found, stochastic Petri nets were used to model the different repair policies to evaluate the true system availability under limited repair teams. The optimization process was divided into two steps where the initial solution of the second step is conditional based on results from the first step. Nourelfath and Kadi [16] studied the same problem, dependencies resulting from maintenance teams sharing were taken into account. Universal generating function combined with Markov model was employed to calculate the element availability under limited maintenance staffs. They also employed a heuristic approach at the first step to optimize the system structure without considering the limitation of maintenance resources. To satisfy the constraint under limited maintenance staffs, system structure and maintenance resource allocation strategies are further optimized based on the solution obtained in the preceding step. Apparently, two-step optimizing process employed in the previous literature [15,16] cannot guarantee achieving the global optimal solution. Furthermore, maintenance activities were approximated via Markov queue model for each subsystem in Ref.[16]. Divergence may exist when the failure and repair rates are distinct between components within the same subsystem.

In this paper, the RAP of multi-state systems incorporating with staff allocation is studied. MSS is defined as a system being able to perform its task at different performance levels caused by degradation of components and parts in the system and/or the failure of some elements (more definition and applications of MSSs are referred to Ref.[10]). Different from the previous literature, elements available in market are chosen while the corresponding repair staffs are also allocated to maintain their availability. Markov queue model and universal generating function method are also employed to evaluate the availability [16] , and two optimization problems (PII and PIII) are proposed. The system configuration and the staff allocation strategy are optimized simultaneously through the firefly algorithm.

The remainder of this paper is organized as follows. The joint optimization problems are formulated in Section 2. In Section 3, the Markov queue model as well as the universal generating function method is presented to evaluate the availability of element and system with limited repair staffs. Section 3 briefly introduces the firefly algorithm and its implementation in the proposed optimization problems. A numerical case is given in Section 4 to verify the efficiency and effectiveness of the proposed models, and it is followed by a brief conclusion in Section 5.

2. Problem formulation

2.1. Definition of MSS

The MSS was primarily introduced in the middle of the 1970's by Murchland [14], and later discussed in Ref.[10]. According to the definition in Ref.[10], a system that possesses a finite number of performance rates is called an MSS. For example, if a flash memory chip fails in a computer system, the system can continues operate, but with derated memory capacity. This kind of system has a range of performance levels from its perfect functioning state to complete failure. There are many different situations in which a system should be treated as an MSS:

1) A system consisting of different units that have a cumulative performance effect on the entire system.
2) A system consisting of elements with variable performance due to deterioration (fatigue, partial failures etc.) and repairs actions.
3) A system with continuous performance deterioration is oftentimes simplified into the one with multiple discrete performance rates via state combination to reduce the computation burden [11-13].

In order to analyze the behavior of an MSS, one has to know the characteristics of its elements. Any system element j can have k_j different states corresponding to the performance rates, which is represented by the set:

$$\mathbf{g}_j = \{g_{j1}, g_{j2}, ..., g_{jk_j}\}\ , \tag{1}$$

where g_{ji} is the performance rate of element j in state i , $i \in \{1, 2, ..., k_j\}$.

The performance rate $G_j(t)$ of element j at any instant $t \geq 0$ is a random variable that takes its values from $\mathbf{g}_j : G_j(t) \in \mathbf{g}_j$. Therefore for the time interval $[0, T]$ where T is the MSS operation period, the performance rate of element j is defined as a stochastic process. The probabilities associated with different states of the system element j at any instant t can be represented by the set:

$$\mathbf{p}_j = \{p_{j1}(t), p_{j2}(t), ..., p_{jk_j}(t)\}\ , \tag{2}$$

where $p_{ji}(t)$ represents the probability that $G_j(t) = g_{ji}$. The state probabilities should satisfy the condition $\sum_{i=1}^{k_j} p_{ji}(t) = 1$ for any $t \geq 0$. Because the elements' states at any instant time t compose the complete group of mutually exclusive events.

The output performance of the entire MSS is defined as a stochastic process based on the system structure function:

$$G_s(t) = \phi(G_1(t), ..., G_N(t)) \tag{3}$$

where $G_i(t)$ is the performance stochastic process of the i^{th} element, and $\phi(\cdot)$ is system structure function. Thus, the probabilities associated with the different system state can be denoted by the set:

$$\mathbf{p(t)} = \{p_{s1}(t), p_{s2}(t), \ldots, p_{sK}(t)\} \tag{4}$$

where $p_{si}(t) = \Pr\{G_s(t) = g_{si}\}$, and K the number of possible system states, and g_{si} is corresponding performance at the i^{th} system state.

The system availability at time instant t for arbitrary demand W is given by:

$$A(t,W) = \Pr(G_s(t) \ge W) = \sum_{i=1}^{K} p_{si}(t) 1(F(g_{si},W) \ge 0), \tag{5}$$

where $1(x)$ is unity function: $1(TRUE)=1$, $1(FALSE)=0$, and $F(g_{si},W) = g_{si} - W$. If the demand is a random variable with M possible values, the availability of the MSS can be computed by:

$$A(t,W) = \sum_{i=1}^{M} q_i(t) \Pr(G_s(t) \ge W_i) = \sum_{i=1}^{M} q_i(t) \sum_{j=1}^{K} p_{sj}(t) 1(F(g_{sj}(t), W_i) \ge 0) \tag{6}$$

where W_i is possible user demand and $q_i(t)$ is corresponding probability at time t.

For a stationary system or a long time horizon, the instantaneous state probability at time t can be replaced by stationary state probability. Equations (5) and (6) will represent the stationary availability of a MSS.

2.2. Optimal design formulation

Before proposing optimization formulations, some basic assumptions are presented as follows:

(1) The MSS contains N_s subsystems connected in series. N_{is} versions of elements are available in market to be chosen for the i^{th} subsystem, and elements in the same subsystem are connected in parallel. Systems with this sort of configuration are called series-parallel MSSs.

(2) The elements in each subsystem are binary capacity elements. A binary capacity element i has two performance rates: nominal $g_{i1} \ne 0$ and $g_{i2} = 0$ for failure state.

(3) The elements available in market can be allocated to specified subsystems in a MSS. Thus, the MSS configuration is determined by choosing a set of element to assign to specified subsystems properly.

(4) The number of repair staffs is less than the number of elements in a MSS. One staff is just able to repair one version of element at a time. More elements of the same version exist in a MSS, more staffs are needed to keep a high element availability.

(5) The objective is to minimize the system cost under availability constraint.

The earlier reported works on RAP often ignore the limitation of repair staff (assumption 4). Thus, the optimization problem **PI** is formulated as (without considering constraints on weight, volume, etc.)

PI:

$$\begin{aligned} \min \quad & C_s = \sum_{i=1}^{N_s} \sum_{j=1}^{N_{is}} c_{ij} m_{ij} \\ s.t. \quad & A \ge A_0 \\ & m_{ij}^L \le m_{ij} \le m_{ij}^U \end{aligned} \tag{7}$$

where c_{ij} and m_{ij} represents the cost of the j^{th} version element and the corresponding number being used in the i^{th} subsystem, respectively. m_{ij}^L and m_{ij}^U are lower and upper bounds, and A_0 is the lower bound of availability constraint.

When considering the assumption 4, two types of optimization problems are proposed as follows. In the first type of problem, the staff cost is regarded as an additional constraint. The optimization formulation **PII** is given by

PII:

$$\begin{aligned} \min \quad & C_s = \sum_{i=1}^{N_s} \sum_{j=1}^{N_{is}} c_{ij} m_{ij} \\ s.t. \quad & A \ge A_0 \\ & C_{staff} = \sum_{i=1}^{N_s} \sum_{j=1}^{N_{is}} c'_{ij} n_{ij} \le C_{s0} \\ & m_{ij}^L \le m_{ij} \le m_{ij}^U \\ & m_{ij} \ge n_{ij} \end{aligned} \tag{8}$$

where n_{ij} and c'_{ij} represent the number of repair staffs for the j^{th} version element and cost of per staff in the i^{th} subsystem, respectively.

In the second type of problem, the system construction cost incorporating with repair staff cost is treated as the objective. The problem **PIII** is formulated as

PIII:

$$\begin{aligned} \min \quad & C_s = \sum_{i=1}^{N_s} \sum_{j=1}^{N_{is}} (c_{ij} m_{ij} + c'_{ij} n_{ij}) \\ s.t. \quad & A \ge A_0 \\ & m_{ij}^L \le m_{ij} \le m_{ij}^U \\ & m_{ij} \ge n_{ij} \end{aligned} \tag{9}$$

2.3. Elements availability evaluation

As stated in previous section, we assume that every staff is just able to repair one version of element allocated in each subsystem. Suppose that, there exist m elements of version j in the i^{th} subsystem and n $(n \le m)$ staffs available to repair failed elements as good as new. These elements are characterized by their identical failure rate λ_{ij} and repair rate μ_{ij}. With the assumption that an element just can be repaired by a staff at a time, it can be modeled through M/M/n queue theory as shown in Figure 1.

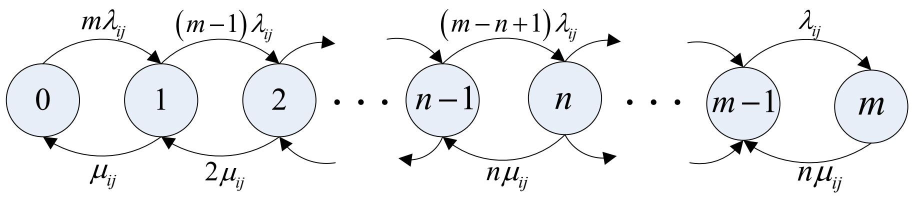

Fig. 1. The Markov diagram of M/M/n queue

The Markov transition intensity matrix is given by

$$Q=\begin{bmatrix} -m\lambda_{ij} & m\lambda_{ij} & & & & & & \\ \mu_{ij} & -[(m-1)\lambda_{ij}+\mu_{ij}] & (m-1)\lambda_{ij} & & & & & \\ & 2\mu_{ij} & -[(m-2)\lambda_{ij}+2\mu_{ij}] & (m-2)\lambda_{ij} & & & & \\ & & & \ddots & & & & \\ & & & n\mu_{ij} & -[(n-1)\lambda_{ij}+n\mu_{ij}] & (n-1)\lambda_{ij} & & \\ & & & & \ddots & & & \\ & & & & n\mu_{ij} & -(\lambda_{ij}+n\mu_{ij}) & \lambda_{ij} \\ & & & & & n\mu_{ij} & -n\mu_{ij} \end{bmatrix} \quad (10)$$

The stationary distribution can be derived by solving the corresponding Chapman-Kolmogorov equation:

$$\mathbf{P_{ij}}Q=\mathbf{0}\ , \quad (11)$$

where the vector $\mathbf{P_{ij}}=\{p_{ij}^0,p_{ij}^1,...,p_{ij}^m\}$ represented discrete probability distribution. The single p_{ij}^k is given by:

$$p_{ij}^k=\begin{cases} \dfrac{m!}{k!(m-k)!}(\dfrac{\lambda_{ij}}{\mu_{ij}})^k p_{ij}^0, & 0\le k\le n \\ \dfrac{1}{n!n^{k-n}}\dfrac{m!}{(m-k)!}(\dfrac{\lambda_{ij}}{\mu_{ij}})^k p_{ij}^0, & n<k<m \\ \dfrac{1}{n!}\dfrac{m}{n^{m-n}}(\dfrac{\lambda_{ij}}{\mu_{ij}})^m p_{ij}^0, & k=m \end{cases}\ , \quad (12)$$

where

$$p_{ij}^0=\left[\sum_{k=0}^{n-1}C_m^k(\frac{\lambda_{ij}}{\mu_{ij}})^k+\sum_{k=n}^{m}\frac{1}{n!}\frac{1}{n^{k-n}}\frac{m!}{(m-k)!}(\frac{\lambda_{ij}}{\mu_{ij}})^k\right]^{-1}, \quad (13)$$

and p_{ij}^k denotes the probability that k elements of version j in the i^{th} subsystem are available in a MSS. The rest $m-k$ elements are being repaired or waiting repair.

2.4. Universal generating function (UGF)

The UGF representing the probability mass function of a discrete random variable is defined by a polynomial form [7, 10, 21]. In the case of multi-state systems, UGF represents the random performance variable G_j of the elements and it is given by:

$$u_j(z)=\sum_{i=1}^{k_j}p_{ji}z^{g_{ji}}\ , \quad (14)$$

where the variable G_j has k_j possible values and $p_{ji}=\Pr\{G_j=g_{ji}\}$.

Therefore, in order to obtain the UGF of systems with arbitrary structure, one has to apply the composition operators $\otimes$ as follows recursively:

$$\begin{aligned} U_s(z)&=\otimes\{u_1(z),...,u_N(z)\} \\ &=\otimes\left\{\sum_{i_1=1}^{k_1}p_{1i_1}z^{g_{1i_1}},...,\sum_{i_N=1}^{k_N}p_{Ni_N}z^{g_{Ni_N}}\right\} \\ &=\sum_{i_1=1}^{k_1}...\sum_{i_N=1}^{k_N}\left(\prod_{j=1}^{N}p_{ji_j}z^{\phi\left(g_{1i_1},...,g_{Ni_N}\right)}\right) \end{aligned}\ . \quad (15)$$

This polynomial represents all of the possible mutually exclusive combination of relating probabilities of each combination corresponding to the value of function $\phi(g_{1i_1},...,g_{Ni_N})$ which is determined by the system structure and performance rates combination property. For example, in the case of flow transmission system with two elements connected in series, one has:

$$\phi(G_1,G_2)=\min\{G_1,G_2\}\ , \quad (16)$$

and for the case where the two elements connected in parallel, the function is given by:

$$\phi(G_1,G_2)=G_1+G_2\ . \quad (17)$$

2.5. UGF of elements

In order to evaluate the reliability of a MSS with limited repair staffs, the UGF of elements availability is formulated at first. Assume that, in a flow transmission system, there are m_{ij} elements of version j and n_{ij} repairmen for these elements in the i^{th} subsystem. According to the Markov model in section 2.3, there exist totally $m_{ij}+1$ state for these elements. The UGF of the m_{ij} elements is formed as follows:

$$u_{ij}(z)=\sum_{k=0}^{m_{ij}}p_j^k z^{(m_{ij}-k)g_{j1}}\ , \quad (18)$$

where p_j^k is the probability that there are k elements of version j failed, which is achieved through the queue algorithm proposed in section 2.3. $(m_{ij}-k)g_{j1}$ is the corresponding performance rates at that state.

Thus, the UGF of each subsystem can be calculated through composition operation of the UGF of each version of elements. Then the UGF of MSS is achieved with iteratively operation as mentioned in section 2.4. Subsequently, the availability of the MSS under specified user demand can be determined according to Eqs. (5) and (6).

3. Firefly algorithm

Equations (7-9) are complicated non-linear programming problems. An exhaustive examination of all possible solutions is not realistic due to the limitation of computational time. Meta-heuristic algorithms such as Genetic Algorithm (GA), Tabu Search (TS), Simulated Annealing (SA) algorithm, and Ant Colony Optimization (ACO), Particle Swarm Optimization (PSO), Firefly Algorithm (FA), Bat Algorithm(BA), are computationally efficient approaches to seek global optimal solution (or approximate optimal solution) in tough and complex optimization problems. The most attractive feature of these algorithms is that they are inspired by behaviors of biological systems and/or physical systems in nature. Also they possess intelligence to find global optimal solution even without derivative information. FA, a recently developed metaheuristic optimization algorithm, is employed in this paper to solve the proposed optimization problems since the superiority of FA over some other metaheuristic optimization algorithms was reported in Refs.[22-24]. The basic principle and its implementation in our problems are briefly introduced in the following sections.

3.1. Basic principle of firefly algorithm

Firefly algorithm inspired by the flashing behavior of fireflies was recently put forth by Yang [22-24]. The fundamental functions of flashing light of fireflies are to communicate (like attracting mating partners) and to attract potential prey. Inspired by this nature, the firefly algorithm was developed by idealizing some of the flashing characteristics of fireflies and representing each individual solution of optimization problem as a firefly in population. Three major idealized rules are [2, 5, 22]: (1) all fireflies in the population are unisex so that any individual firefly will be attracted at other fireflies; (2) for any pair of fireflies, the less bright one will move towards the brighter one. The attractiveness of a firefly is proportionally related to the brightness which decreases with increasing distance between two fireflies; (3) the brightness of a firefly is proportionally related to the value of objective function in the similar way to the fitness in genetic algorithm. The procedure of implementing the FA for a maximum optimization problem is summarized by the pseudo code shown in Figure 2 [5, 22-24].

Begin
Objective function $f(\boldsymbol{x})$, $\boldsymbol{x}=(x_1,\ldots,x_d)$
Define parameter of FA (*light absorption coefficient* γ)
Generate initial population of fireflies $\boldsymbol{x}_i$ (i=1.2, …n)
Determine brightness I_i *at* $\boldsymbol{x}_i$ *by objective function* $f(\boldsymbol{x}_i)$
Set counter t=1
while (t < *MaxIteration*)
for $i = 1 : n$ *all fireflies in population*
for $j = 1 : n$ *all fireflies in population*
If ($I_j > I_i$)
Move firefly i ($\boldsymbol{x}_i$) *towards j* ($\boldsymbol{x}_j$) *via Eq.(20)*
Evaluate value of objective function for firefly i ($\boldsymbol{x}_i$), *and update brightness* I_i *and attractiveness*.
end if
end for j
end for i
Rank fireflies by brightness and find the current best; $t = t + 1$
end while
Get the final best and postprocess results
End

Fig. 2. The pseudo-code of the FA

In the firefly algorithm, for simplicity, the attractiveness of a firefly is related to brightness I_i of the firefly which in turn is associated with the output of objective function $f(\mathbf{x}_i)$. For example, in the maximum optimization problems, the brightness I_i of the firefly i at location xi can be chosen as $I_i \infty f(\mathbf{x}_i)$. In nature, brightness decreases with the distance from its source, and light dims due to media. Therefore, the attractiveness β of one firefly to another is relative, and it should possess monotonically decreasing pattern with respect to the distance r_{ij} between firefly i and firefly j. In addition, the light absorption coefficient γ is also introduced to quantify the degree of light absorption. Several forms have been proposed to characterize the functional relationship of attractiveness β with respect to the distance r and light absorption coefficient γ. The following Gaussian form is used in the study:

$$\beta(r) = \beta_0 e^{-\gamma r^2} \tag{19}$$

where β_0 is the attractiveness at $r = 0$, and it equals to brightness. The light absorption coefficient γ can be either varied with respect to t or fixed [2, 22]. The distance r is defined as the Cartesian distance between a pair of fireflies $r_{ij} = \|\mathbf{x}_i - \mathbf{x}_j\|$. Firefly i will move towards firefly j if firefly j possesses a greater brightness than firefly i, and the movement is determined by [5]

$$\mathbf{x}_i^{t+1} = \mathbf{x}_i^t + \beta_0 e^{-\gamma r^2}(\mathbf{x}_j^t - \mathbf{x}_i^t) + \alpha \boldsymbol{\varepsilon}_i \tag{20}$$

where $\mathbf{x}_i^t$ denotes the location of firefly i at the t^{th} iteration. The second term is due to the attraction from firefly j, and the last term is random movement. α is the randomization parameter and $\boldsymbol{\varepsilon}_i$ is a vector of random numbers drawn from a standard normal distribution representing the partial randomness of movement.

The iterative optimization process terminates once it meets some criterions, such as: 1) the number of iterations reaches the preset maximum value, and 2) the variance of average brightness in subsequent population is not more than ε_2, etc.

To apply the FA to a specific optimization problem, solution representation is an important procedure which must be defined. Penalty function approach can be employed to handle infeasible solutions.

3.2. Solution representation

For the optimization problem in Eqs. (8) and (9), the individual solution i is represented by the location $\mathbf{x}_i$ of firefly i as

$\mathbf{x}_i = \{\underbrace{x_1, x_2, \ldots, x_E}_{\text{redundancy number}}, \underbrace{x_{E+1}, \ldots, x_{2E}}_{\text{staff number}}\}$, where s_1 to s_E represent the number of redundancy for element of each version; and s_{E+1} to s_{2E} represent the number of staffs for element of each version. For example, a MSS consists of two subsystems. There exist two versions of elements in market for each subsystem. A specific individual solution $\mathbf{x}_i = \{\underbrace{1,2,0,3}_{part\,1}, \underbrace{1,1,0,2}_{part\,2}\}$ denotes that the MSS contains one version 1 element and two version 2 elements in subsystem 1, and three version 2 elements in subsystem 2. The repair staffs for each version of elements are 1, 1, 0, and 2, respectively. Since $\mathbf{x}_i$ only contain integers, the real numbers generated in the initial population and movements during iteration process have to be rounded off. In addition, on the ground that the number of staffs should be not greater than the number of redundancy of the corresponding element. Therefore in steps of initialization and movements, the number of staffs (part 2 of $\mathbf{x}_i$) of each individual solution $\mathbf{x}_i$ will be continually generated until its value is not greater than to the corresponding number of redundancy (part 1 of $\mathbf{x}_i$).

4. An illustrative case

Consider a flow transmission MSS consisting of four subsystems connected in series, and there are three versions of binary capacity element available in market for each subsystem. The parameters of these elements are tabulated in Table 1, as well as the cost for the corresponding repair staffs. The possible user demands at different levels are presented in Table 2 with the associated probabilities.

We solve the optimization problems **PI**, **PII**, and **PIII** under the same availability constraint $A_0 = 0.90$ and bounds ($m_{ij}^L = 0$, $m_{ij}^U = 5$). In our study, the firefly algorithm is performed to search a good solution in a computationally efficient manner. From our experimental tests, the values of parameters in FA are set as: $\alpha = 0.6$, $\gamma = 1.0$. The FA program is executed 10 times, and the optimal solution among

Table 1. The characters of element availability in market

	Ver. 1	Ver. 2	Ver. 3		Ver. 1	Ver. 2	Ver. 3
Subsystem 1				**Subsystem 3**			
Performance	120	85	65	Performance	130	100	75
λ_{1i}	0.0067	0.007	0.0065	λ_{3i}	0.0125	0.012	0.013
μ_{1i}	0.02	0.025	0.018	μ_{3i}	0.05	0.052	0.046
Cost ($)	1.5	1.2	0.9	Cost ($)	0.8	0.7	0.5
Staff Cost ($)	6.0	3.0	2.0	Staff Cost ($)	2.5	1.5	3.5
Subsystem 2				**Subsystem 4**			
Performance	100	95	65	Performance	125	95	65
λ_{2i}	0.0129	0.0135	0.012	λ_{4i}	0.00658	0.007	0.068
μ_{2i}	0.03	0.04	0.035	μ_{4i}	0.032	0.03	0.035
Cost ($)	3.5	2.5	2.0	Cost ($)	5.0	4.2	3.5
Staff Cost ($)	3.5	5.5	2.0	Staff Cost ($)	1.5	3.5	2.5

Table 2. The user demands

Demand (rate)	200	160	120	80
Probability	0.25	0.4	0.25	0.1

these results will be chosen as the final optimal result. The corresponding optimal solutions are tabulated in Tables 3, 4, and 5, respectively, where system configuration is listed in the column "Structure", and staff allocation strategy in the "Staff" column. For example, in Table 4, the solution "Subsys 1: 1-1-2-2-2" in the first row of the "Structure" column, denotes the subsystem 1 consists of 2 version 1 elements and 3 version 2 elements, while "1-2" in the "Staff" column of Table 4 represents to allocate 1 staff for version 1 elements and 2 for version 2.

In order to satisfy the availability constraint in problem **PI**, it requires at less 19 repair staffs with the cost equal to \$51, and the total cost is equal to \$88.7.

In problem **PII**, we considering that the number of repair staffs

*Table 3. Optimal results for problem **PI***

Availability	Cost ($)	Structure
0.90011	37.7	Subsys 1: 1-1-2-2-2
		Subsys 2: 2-2-2-3-3-3
		Subsys 3: 1-2-2-2-3
		Subsys 4: 1-1-2

should be lower than 10, we set $c'_{ij} = 1$ and $C_{s0} = 10$, and the optimal results is presented in Table 4.

The staff cost in this case is equal to \$34, and thus the total cost is equal to \$78. If we assume the repair staffs are unlimited (at least 22 staffs) for this system configuration, the corresponding system availa-

*Table 4. Optimal results for problem **PII***

Availability	Cost($)	Structure	Staff
0.9008	44	Subsys 1: 1-2-2-2-2	1-2
		Subsys 2: 2-2-2-2-3-3	2-1
		Subsys 3: 1-1-1-1-2-2-2	1-1
		Subsys 4: 1-1-2-2	1-1

bility is equal to 0.9468. It is indicated that the limited resource makes approximately 4.93% reduction in availability.

The result for problem **PIII** is given in Table 5 and the total cost incorporating with staff cost are regarded as the objective to be minimized.

It shows that the cost for the elements is equal to \$44.8, which has

*Table 5. Optimal results for problem **PIII***

Availability	Total Cost ($)	Structure	Staff Cost ($)	Staff
0.90021	69.3	Subsys 1: 2-2-2-2-3-3	24.5	2-1
		Subsys 2: 2-2-3-3-3-3		1-2
		Subsys 3: 1-1-1-2-2-2-2		1-1
		Subsys 4: 1-1-1-1		2

nearly 1.82% increases compared to **PII** in the cost of elements. This result has approximately 11.15% reduction in total cost compared to **PII** under the same availability constraint. The result for **PII** (\$78) is better than **PI** (\$88.7). The optimal result in **PIII** has the least total cost (\$69.3) when considering the staff cost jointly.

5. Conclusions

In this paper, a joint optimization problem of redundancy and maintenance staff allocation for MSSs is proposed. The limited maintenance resource is a common issue in practices as emphasized by many researchers, but existing literature seldom discusses the RAP considering limited resources. Nourelfath et al. [15,16] first discussed the staff allocation in RAP through Petri nets and Markov model where maintenance strategies were considered within each module. As an extension of previous research, this paper allows the maintenance staff to be allocated for each version of elements. Without introducing any approximating approach in modeling as Ref. [16] , the M/M/n queue model and the UGF method are proposed to evaluate the availability of elements and system, respectively. Two optimization formulations concerning limitations from maintenance resources are introduced. The firefly algorithm, a recent developed metaheuristic algorithm, is employed to solve the resulting combinational optimization problems. Compared with the multi-step optimization approach proposed in Refs. [15, 16], the proposed approach which solve the optimization problem just in one step is more effective to achieve the global optimal solution. Also, as observed from our study, results from the proposed methods outperform the ones from traditional RAP without considering the limited maintenance staffs. However, this paper only studys the situation where the maintenance staffs allocate within the same subsystem and version of elements. Allocating maintenance staffs across the entire system is still an open research issue subjected to discussions in the future work.

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