A diagnostics design decision model for products under warranty

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Abstract

Warranty obligations in product service involve additional costs to the manufacturers. Building diagnostic features into a product can reduce the warranty costs because of fewer visits needed by warranty service representatives (through self-guided user repairs for simple problems) and a shorter service time (diagnosing time cut short) for each failure. However, this approach increases the product cost and is worthwhile only if the reduction in the expected warranty cost is more than the additional increased costs of the diagnostics system. Models are developed to evaluate diagnostics design decisions for products with warranty service from the manufacturers' point of view. The roles and influence of the key parameters and decision variables based on the consideration of uncertainty are explored. Numerical examples are given to illustrate the proposed models.
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1. Introduction

Products have become more and more complex and customers are often uncertain about the performance of new products. A warranty policy is a seller's assurance to a buyer that the product is or shall be as represented. Warranty agreement is considered to be a contractual agreement between buyers and sellers upon the sale of the product and can be viewed as a signal to convey information regarding product reliability. Now most industrial and durable consumer goods are provided with warranty services and warranty has served as an important marketing tool (Blischke and Murthy, 1996). Different aspects of warranty have been studied by researchers from diverse disciplines. Blischke and Murthy (1996) deal with several of these issues including warranty and law, warranty cost models, warranty and marketplace, warranty and engineering, warranty and management, etc. Recently, Murthy and Djamaludin (2002) reviewed the related literature over the last 10 years.

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Nomenclature

$F_1(t)$	cumulative distribution function for the first time to failure of the product
$f_1(t)$	failure density function associated with $F_1(t)$
$r_1(t)$	hazard function associated with $F_1(t)$
$F_2(t)$	cumulative distribution function for the first time to failure of the diagnostic equipment
$f_2(t)$	failure density function associated with $F_2(t)$
$r_2(t)$	hazard function associated with $F_2(t)$
P_1	product unit sale price without diagnostic equipment (marketing variable)
P_2	product unit sale price with diagnostic equipment (marketing variable)
W	duration of warranty period (marketing variable)
$\omega(W)$	expected warranty cost during warranty period for each unit sold
c_{M1}	unit production cost of the product without diagnostic equipment
c_{M2}	unit production cost of the diagnostic equipment
c_D	research and design cost of the diagnostic equipment
c_{r1}	expected repair cost of a product warranty claim for type-1 failure not including diagnosing cost
c_{r2}	expected repair cost of a product warranty claim for type-2 failure not including diagnosing cost
c_{r3}	expected cost of servicing a warranty claim for the diagnostic equipment
c_{r4}	expected diagnosing cost of a product failure without diagnostic equipment
$S_1(t)$	expected number of warranty repairs per unit of product over $[0,t]$
$S_2(t)$	expected number of warranty repairs per unit of diagnostic equipment over $[0,t]$
α	proportion of type-1 failures which can be repaired by consumers guided by diagnostics system
L	product life cycle
$Q_1(L)$	cumulative sales volume for product without diagnostic equipment over $[0,L]$
$Q_2(L)$	cumulative sales volume for product with diagnostic equipment over $[0,L]$
G	expected gain from a diagnostics design over the product life cycle

Warranty servicing incurs additional costs to the manufacturer and thus has an impact on profits. These costs, in fact, are unpredictable future costs, which typically range from 2% to as much as 15% of net sales (Murthy and Djamaludin, 2002). The expected warranty cost depends on the product reliability. Thus it can be influenced by engineering decisions made with regard to product design, development and manufacturing. Optimal decision-making at the design stage must take into account the manufacturing and warranty costs. Many researchers have investigated optimization strategies that link engineering issues with warranty, such as reliability improvement through redundancy and development, maintenance, testing policies, burn-in, etc., to either maximize the manufacturers' profit or to minimize the total cost. Illustrative examples can be found in Murthy and Nguyen (1987), Hussain and Murthy (1995, 1998, 2000, 2003), Mi (1997), Monga and Zuo (1998), Pohl and Dietrich (1999) and Shue and Chien (2005). Warranty costs can be minimized through optimal servicing strategies and effective warranty logistic management, such as replace–repair strategy, cost repair limit strategy, warranty reserving, spares and repairs demand, inventory management, etc. (see for example, in Murthy and Djamaludin (2002), Nguyen and Murthy (1989), Jack and Van der Duyn Schouten (2000), Iskandar et al. (2005), Murthy and Nguyen (1988), Zuo et al. (2000), Buczkowski et al. (2005) and Ja et al. (2002)). In Majeske et al. (1997), Wu and Meeker (2002), Yang and Cekecek (2004) and Ward and Christer (2005), product design changes under warranty are evaluated using warranty field data. The design decisions are motivated by seeking reduced warranty costs.

Product support denotes a set of goods and services to ensure the continued satisfactory use of a product. It creates additional values for customers and manufacturers/service providers alike. Designing the product with redundancy adding, automated diagnostic mechanisms, using modular components and providing loaners are a few examples of product support strategies (see for example, in Markeset and Kumar (2003, 2004), Karmarkar and Kubat (1983, 1987), Hegde and Kubat (1989) and Dussault et al. (1985)). Markeset and Kumar (2004) examined issues related to dimensioning of product support for advanced industrial products

on the basis of a case study conducted in a manufacturing company that produces automated production line systems. They analyzed factors/parameters/issues influencing product support strategies for industrial products and explored measures to control them. Also, they presented an approach for design and development of product support and maintenance concepts for industrial systems in a multinational environment in Markeset and Kumar (2003). Taken in isolation, the value of loaners in product support was studied by Karmarkar and Kubat (1983). They developed cost models for evaluation of a modular design, and discussed the distinction between integrated modules and replaceable subassemblies in Karmarkar and Kubat (1987).

Adding a diagnostic mechanism is a typical product support strategy. An example of a diagnostic mechanism which can be considered as a product support strategy is the type where a fault message is displayed indicating the cause of failure as soon as the product fails. Then, the user or a professional service provider can fix the failures according to the messages shown. Often, industrial and office equipment is marketed by emphasizing the built-in diagnostics capability for locating the cause of failure (Hegde and Kubat, 1989). A typical example of equipment with such built-in diagnostic features is a modern photocopier. Many researchers have studied diagnostics technology from different perspectives (Nolan, 1996; Laskey and Barry, 1974; Chitore et al., 1998; Wiegmann and Douglas, 2002; Venkatasubramanian, 2005; Hegde and Kubat, 1989; Dussault et al., 1985). Nolan (1996) introduced basic diagnostics technology and outlined how the concurrent engineering tool set (CETS) technology can be used to foster a capability to design, manage, and deploy a 100% diagnostics capability for systems. Venkatasubramanian (2005) presented a broad overview of the various approaches to automated fault diagnosis and described the state-of-the-art efforts in terms of industrial applications in intelligent diagnostics and prognostics. The control systems paradigm is used to manage products' entire lifecycles to avoid costly failures or degradation in performance.

For products under warranty, those with good built-in diagnostic equipment typically require fewer visits from service representatives and a reduced service time (shorter diagnosis time) for each failure, resulting in a reduction of warranty costs. However, the addition of diagnostics will increase the product's production cost. Whether to build a diagnostic feature into a new product is an important decision to make early in a new product design. Very little work has been done in this area. Hegde and Kubat (1989) developed models to study the tradeoffs involved in designing a product with added diagnostics technology. They investigated the impact of diagnostics design in reducing the service cost and the effectiveness of diagnostics as a strategy to reduce the customer downtime cost. Dussault et al. (1985) reported the progress of a continuing study seeking to develop management tools to support diagnostic design decision-making. They discussed current work in defining diagnostic tradeoffs and costs for an integrated diagnostics design program. A baseline for development of tools for allocating diagnostic resources was presented.

The product design decision on building in diagnostics features largely depends on whether the reduction in the expected warranty cost is more than the additional costs of adding the diagnostics. In this paper, we develop trade-off models that can help product designers decide whether the use of diagnostics is an effective support strategy from the product warranty service point of view. Most uncertainties in warranty analysis arise because of the variability and nondeterministic nature of the parameters involved in the design. This paper examines model analysis, robustness of diagnostic design decisions and several parameters' influence on design decisions. The models capture the tradeoffs involved in the reduction of warranty costs and the increased production cost in a new product design. The outline of the paper is as follows. In Section 2 we give the mathematical details of the model formulations. Section 3 explores the roles and influence of key parameters and decision variables. Section 4 presents an illustrative numerical example. Finally, in Section 5, we conclude with a brief discussion of the possible extensions for future investigation.

2. Model formulation

2.1. Warranty cost analysis

When an item is returned for rectification because of failure under warranty, the manufacturer has to pay the transportation cost, warranty handling costs, material and labor cost, etc. Following Blischke and Murthy (1996), we aggregate all these costs into a single cost termed "repair cost" for each claim. Because some of the

costs are uncertain, the repair cost is a random variable. Let c_r denote the expected value of this cost. The warranty cost depends on the repair cost per claim and the number of repairs during the warranty period. The expected warranty cost for each unit, $\omega(W)$ is given by

$$\omega(W) = c_r S(W), \tag{1}$$

where $S(W)$ is the expected number of failures over $[0, W]$.

The subsequent failures depend on the action taken to rectify a claim under warranty. The number of failures under warranty is a random variable and Blischke and Murthy (1996) examined the following three cases:

Case (i): Replace failed item by a new one. If the replacement time is negligible in relation to the mean time between failures, the expected number of failures over $[0,t]$, $S(t)$, is given by $M(t)$, the ordinary renewal function associated with product's cumulative distribution function $F(t)$, which is given by

$$M(t) = F(t) + \int_0^t M(t - x)f(x)\,dx. \tag{2}$$

Case (ii): Imperfect repair. The repaired item follows a new distribution function $G(t)$, which is different from $F(t)$. In this case the expected number of failures over $[0,t]$ (under the assumption that repair times are negligible), $S(t)$, is given by the delayed renewal function $M_d(t)$ which can be obtained by solving the following integral equation:

$$M_d(t) = F(t) + \int_0^t M_G(t - x)f(x)\,dx, \tag{3}$$

where $M_G(t)$ is the ordinary renewal function associated with $G(t)$.

Case (iii): Minimal repair. In this case the hazard function after repair is the same as that just before the failure. Under the assumption that repair times are negligible, failures over time occur according to a NHPP process with the intensity function given by the hazard function. The expected number of failures over $[0,t]$, $S(t)$, is given by

$$S(t) = \int_0^W r(t)\,dt. \tag{4}$$

2.2. Diagnostics system modeling

A good built-in diagnostic system provides clarity and convenience for maintenance and diagnosis. It helps increase the customers' satisfaction and the reputation of the company. The diagnostic equipment can reduce the warranty costs by guiding the user to rectify simple problems or the service provider to fix complex faults. On the other hand, the built-in diagnostic features increase the design, development, manufacturing cost, and warranty servicing cost for failures in the diagnostic system itself. It is worthwhile only if the reduction in the expected warranty cost of the item is more than the additional increased costs.

Let $F_1(t)$ be the cumulative distribution function for the first time to failure of the product. Let $f_1(t)$ and $r_1(t)$ denote the density and the hazard functions associated with $F_1(t)$, respectively. The diagnostic system incurs a design cost of c_D and manufacturing cost of c_{M2}. The diagnostic system as a unique equipment will fail over time and needs additional warranty service. Let $F_2(t)$ be the cumulative distribution function for the first time to failure of the diagnostic system. Let $f_2(t)$ and $r_2(t)$ denote the density and the hazard functions associated with $F_2(t)$.

Suppose that the product failures can be classified into two types. A type-1 failure can be rectified by the consumer following the guidance of the diagnostics equipment and a type-2 failure must be repaired by the warranty service provider. Let α be the proportion of type-1 failures. Let c_{r1} and c_{r2} denote the expected repair cost of a warranty claim for a type-1 failure and a type-2 failure not including the diagnosis cost, respectively. Let c_{r3} be the expected repair cost of a warranty claim for the diagnostics equipment. Let c_{r4} denote the expected diagnosing cost of a failure without diagnostics equipment. The warranty cost of the diagnostic equipment is the same as shown in Section 2.1 (refer to Eqs. (1)–(4)) for the product.

For normal products (without diagnostics) under warranty, the sum of the unit production and warranty servicing costs is given by

$$\alpha c_{r1} S_1(W) + (1 - \alpha)c_{r2} S_1(W) + c_{r4} S_1(W) + c_{M1}, \tag{5}$$

where $S_1(W)$ is the expected number of warranty repairs per unit normal product over $[0, W]$ and c_{M1} denotes the unit production cost of the product without diagnostic equipment.

For smart products (with diagnostics), the sum of the production and warranty serving costs is given by

$$c_{r3} S_2(W) + (1 - \alpha)c_{r2} S_1(W) + c_{M1} + c_{M2}, \tag{6}$$

where $S_2(W)$ is the expected number of warranty repairs per unit diagnostic equipment over $[0, W]$ and c_{M2} is the unit production cost of the diagnostic equipment.

Based on Eqs. (5) and (6), one can identify which design philosophy minimizes the total producer costs.

For a single unit manufactured product with the built-in diagnostics option, the reduction in the expected warranty cost includes the type-1 failure warranty costs, the failure diagnosing cost minus the warranty cost of the diagnostics system. Thus it is given by

$$\alpha c_{r1} S_1(W) - c_{r3} S_2(W) + c_{r4} S_1(W). \tag{7}$$

Eq. (7) is positive for a diagnostics led decision, which means that the diagnostics design is advantageous. Suppose that the sales of the products without diagnostics and with diagnostics are $Q_1(L)$ and $Q_2(L)$ units, respectively, the gain in return outweighs the additional increased diagnostics equipment costs if

$$\begin{aligned} Q_2(L)[P_2 - c_{r3} S_2(W) - (1 - \alpha)c_{r2} S_1(W) - c_{M1} - c_{M2}] \\ - Q_1(L)[P_1 - \alpha c_{r1} S_1(W) - (1 - \alpha)c_{r2} S_1(W) - c_{r4} S_1(W) - c_{M1}] > c_D, \end{aligned} \tag{8}$$

where c_D denotes the design cost of the diagnostic equipment.

Eq. (8) is the basic diagnostics design decision model. The diagnostics design is viable if Eq. (8) is satisfied. The parameters of Eq. (8) need to be assessed when one makes design decisions. This paper explores the role and influence of the key parameters and presents the decision models in Section 3. Before we move on to Section 3, we first consider the special case when $Q_1(L)$ and $Q_2(L)$ are identical. This assumption is reasonable for special equipment which have specific users.

Now, if we assume the same accumulated sales, i.e., $Q_1(L) = Q_2(L) = Q(L)$, the decision model can be rewritten as follows:

$$Q(L)[P_2 - c_{r3} S_2(W) - c_{M2} - P_1 + \alpha c_{r1} S_1(W) + c_{r4} S_1(W)] > c_D. \tag{9}$$

The expression inside the square brackets of Eq. (9) must be positive for a diagnostic design decision to be worth contemplating.

The gain of a diagnostics design is obtained by assessing the benefits and is given by

$$\begin{aligned} G = Q_2(L)[P_2 - c_{r3} S_2(W) - (1 - \alpha)c_{r2} S_1(W) - c_{M1} - c_{M2}] \\ - Q_1(L)[P_1 - \alpha c_{r1} S_1(W) - (1 - \alpha)c_{r2} S_1(W) - c_{r4} S_1(W) - c_{M1}] - c_D. \end{aligned} \tag{10}$$

When $Q_1(L) = Q_2(L) = Q(L)$, the gain of a diagnostics design is given by

$$G = Q(L)[P_2 - c_{r3} S_2(W) - c_{M2} - P_1 + \alpha c_{r1} S_1(W) + c_{r4} S_1(W)] - c_D. \tag{11}$$

Let us consider a product with $W = 1$ year and $L = 3$ years. Let the product and the diagnostic equipment have exponential lifetime distributions with parameters $r_1 = 0.2$ and $r_2 = 0.1$, respectively. Thus $F_1(t) = 1 - e^{-0.2t}$ and $F_2(t) = 1 - e^{-0.1t}$. We consider Case (iii) where failed items are repaired minimally and let the expected costs of each minimal repair have $c_{r1} = \$10$, $c_{r2} = \$60$, $c_{r3} = \$20$, $c_{r4} = \$5$. From Eq. (4), we have the expected number of failures over $[0,W]$, $S_1(W) = r_1 W = 0.2$ for the product and $S_2(W) = r_2 W = 0.1$ for the designed diagnostic equipment. Let the unit production cost of the product without diagnostic equipment be $c_{M1} = \$200$ and the unit production cost of the diagnostic equipment be $c_{M2} = \$60$. We assume the same total sales volume, i.e., $Q_1(L) = Q_2(L) = Q(L) = 50\,000$. Let the unit sale prices be $P_1 = \$440$ and $P_2 = \$500$. The design cost of diagnostics equipment is estimated to be \$5000. Let the proportion of type-1

failure of the product be $\alpha = 0.8$. We then have from Eq. (11), the expected gain of the diagnostics design is

$$Q(L)[P_2 - c_{r3}S_2(W) - c_{M2} - P_1 + \alpha c_{r1}S_1(W) + c_{r4}S_1(W)] - c_D = \$25\,000.$$

For a product diagnostics design being viable, G must be larger than zero. It is evident that providing the term in the square brackets in Eq. (10) or (11) is positive, the benefit of diagnostic equipment design increases as $Q_2(L)$ increases. Thus the sales volume is of great influence to the expected gain and diagnostic design decision.

3. Analysis of key parameters and decision variables

From Eq. (8), we know that the diagnostics design decision depends on many parameters and variables. The design decision can vary considerably among various services and situations. In this section, we will examine these parameters and variables' influence on the decision-making.

3.1. Sales volume led diagnostics design

The sales volume is of importance to the profits and depends on product price, quality, post-sale servicing, etc. Assuming all products produced can be sold, if there is some possible flexibility in the sales volume $Q(L)$ lower than an upper bound $\bar{Q}(L)$, there is a critical sales level $Q^*(L)$ above which a diagnostics design is viable and advantageous.

Proposition 1. *If the benefits of built-in diagnostics system outweigh the additional increased costs of diagnostic equipment, the diagnostics design is viable. For a diagnostics design to be viable, the sales volume $Q_2(L)$ must be greater than $Q_2^*(L)$, where*

$$Q_2^*(L) = \frac{Q_1(L)[P_1 - \alpha c_{r1}S_1(W) - (1-\alpha)c_{r2}S_1(W) - c_{r4}S_1(W) - c_{M1}] + c_D}{P_2 - c_{r3}S_2(W) - (1-\alpha)c_{r2}S_1(W) - c_{M1} - c_{M2}}. \tag{12}$$

If $Q_1(L) = Q_2(L) = Q(L)$, then

$$Q^*(L) = \frac{c_D}{P_2 - c_{r3}S_2(W) - c_{M2} - P_1 + c_{r1}\alpha S_1(W) + c_{r4}S_1(W)}. \tag{13}$$

Eqs. (12) and (13) are derived directly from Eqs. (8) and (9), respectively.

If the product is profitable, i.e.

$$P_1 - \alpha c_{r1}S_1(W) - (1-\alpha)c_{r2}S_1(W) - c_{r4}S_1(W) - c_{M1} > 0,$$
$$P_2 - c_{r3}S_2(W) - (1-\alpha)c_{r2}S_1(W) - c_{M1} - c_{M2} > 0, \tag{14}$$

then there is always a production volume that would make diagnostics design viable, but is only possible when $Q_2^*(L) < \bar{Q}_2(L)$ or $Q^*(L) < \bar{Q}(L)$, respectively. The viable range of the sales volume for a diagnostics design to be worthwhile is shown in Fig. 1, which also shows the relationship between the gain of a diagnostics design G and product sales volume $Q(L)$.

3.2. Warranty period led diagnostics design

The warranty cost depends on the product reliability and warranty strategy. The warranty period is one of the most important factors of a warranty policy and has major influence on the warranty design. The length of warranty is often viewed as the signal of the product quality and reliability. The longer the warranty period is, the more possible the diagnostics design is advantageous. For a product, there is a critical warranty period W^* above which a diagnostics design is advantageous.

Proposition 2. *If the benefits of the diagnostics design outweigh the additional increased costs of the diagnostic equipment, the diagnostics design is viable. With other parameters fixed, there is a critical warranty period W^**
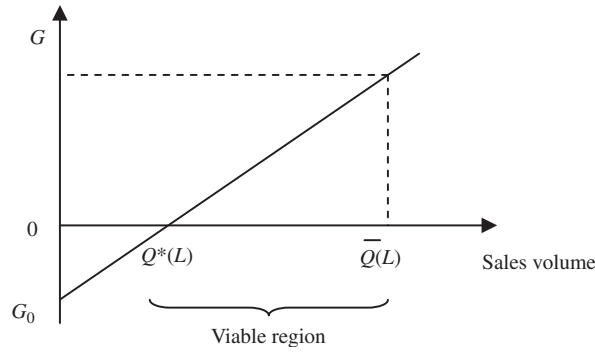
Fig. 1. Gain of a diagnostics design as a function of sales volume.

above which the diagnostics design is advantageous where W^* satisfies

$$Q_2(L)[P_2 - c_{r3}S_2(W^*) - (1 - \alpha)c_{r2}S_1(W^*) - c_{M1} - c_{M2}]$$
$$- Q_1(L)[P_1 - \alpha c_{r1}S_1(W^*) - (1 - \alpha)c_{r2}S_1(W^*) - c_{r4}S_1(W^*) - c_{M1}] - c_D = 0. \tag{15}$$

If $Q_1(L) = Q_2(L) = Q(L)$, then

$$Q(L)[P_2 - c_{r3}S_2(W^*) - c_{M2} - P_1 + \alpha c_{r1}S_1(W^*) + c_{r4}S_1(W^*)] - c_D = 0. \tag{16}$$

Eqs. (15) and (16) are derived based on Eqs. (10) and (11), respectively. The warranty period W^* can be evaluated from Eqs. (1)–(4) and (15), (16). For example, when the product and the diagnostics equipment follow exponential lifetime distributions and $Q_1(L) = Q_2(L) = Q(L)$, we consider Case (iii) where minimal repair is provided to failed items. From Eqs. (1), (4) and (16), we have

$$Q(L)[P_2 - c_{r3}r_2 W^* - c_{M2} - P_1 + \alpha c_{r1}r_1 W^* + c_{r4}r_1 W^*] - c_D = 0. \tag{17}$$

We can get

$$W^* = \frac{(c_D/Q(L)) - p_2 + c_{M2} + p_1}{\alpha c_{r1}r_1 + c_4 r_1 - c_{r3}r_2}, \tag{18}$$

where r_1 and r_2 are the failure rate of the product and the diagnostics equipment, respectively. When $W > W^*$, the diagnostics design is advantageous. Eq. (7) plays a key role because it makes the expression inside the square brackets of Eq. (17) increasing in W, which makes the diagnostics design more viable when the warranty period increases.

3.3. Proportion of type-1 failures α

The proportion of type-1 failures which can be repaired by the consumer, α, is very important in the decision of adopting the diagnostics design.

Proposition 3. *If the benefits of the diagnostics design outweigh the additional increased costs of the diagnostics design, the diagnostics design is viable. For such a design to be viable, the proportion of type-1 failures α must satisfy*

$$\begin{cases} \alpha^* < \alpha < 1 & \text{if } Q_2(L)c_{r2}S_1(W) + Q_1(L)[c_{r1}S_1(W) - c_{r2}S_1(W)] > 0, \\ 0 < \alpha < \alpha^* & \text{if } Q_2(L)c_{r2}S_1(W) + Q_1(L)[c_{r1}S_1(W) - c_{r2}S_1(W)] < 0, \end{cases} \tag{19}$$

where

$$\alpha^* = \frac{c_D - Q_2(L)[P_2 - c_{r3}S_2(W) - c_{r2}S_1(W) - c_{M1} - c_{M2}] + Q_1(L)[P_1 - c_{r2}S_1(W) - c_{r4}S_1(W) - c_{M1}]}{Q_2(L)c_{r2}S_1(W) + Q_1(L)[c_{r1}S_1(W) - c_{r2}S_1(W)]}. \tag{20}$$

If $Q_1(L) = Q_2(L) = Q(L)$, *for the diagnostics design to be viable, the proportion of type-1 failures α must be greater than α^* where*

$$\alpha^* = \frac{c_D - Q(L)[P_2 - c_{r3}S_2(W) - c_{M2} - P_1 + c_{r4}S_1(W)]}{Q(L)c_{r1}S_1(W)}. \tag{21}$$

Eqs. (19)–(21) are derived directly from Eqs. (8) and (9).

To examine the robustness of the diagnostics decision to the parameter α, the proportion of type-1 failures may be modeled by a distribution function $H(\alpha)$. We have the expected gain from a diagnostics design over the life cycle, G, i.e.,

$$\begin{aligned}
G = \int_\alpha \{ & Q_2(L)[P_2 - c_{r3}S_2(W) - (1-\alpha)c_{r2}S_1(W) - c_{M1} - c_{M2}] \\
& - Q_1(L)[P_1 - \alpha c_{r1}S_1(W) - (1-\alpha)c_{r2}S_1(W) - c_{r4}S_1(W) - c_{M1}] \} \, dH(\alpha).
\end{aligned} \tag{22}$$

If this expression is larger than the expected diagnostics design costs c_D, the diagnostics design is viable.

3.4. Uncertainty of the diagnostics design cost c_D

From the developed model and previous analysis, the decision clearly depends upon the cost of the diagnostics design, c_D, which is assumed to be known. In the formulations presented earlier, we have assumed an expected cost of diagnostics design c_D. However, c_D is usually unknown until the diagnostics design is completed. In practice, there are different ways for estimating c_D which depend upon the nature of the product. Ward and Christer (2005) suggest that in the case of expensive products, a characteristic form of re-design cost D^* can be expressed as $D^* = B + 100P_1$, where P_1 is the unit production cost and B a fixed base cost. Liu et al. (2006) assume that the design cost is a function of the reliability parameter λ, and is modeled as $c_D^*(\lambda) = a + b\lambda^{-m}$, where a is the fixed (setup) cost, λ is the product reliability parameter and a, b, $m > 0$. This implies that as λ decreases (the product becomes more reliable), the design and development cost increases.

To examine the robustness of the diagnostics design decision to design cost risk, the diagnostics cost may be modeled by a distribution function $F(c_D)$. In this paper, we conduct an analysis of the expected gain and the risk of a wrong decision. In this case, the expected benefit of a diagnostics design is given by a modified form of Eq. (10), namely

$$\begin{aligned}
G = \int_{c_D} \{ & Q_2(L)[P_2 - c_{r3}S_2(W) - (1-\alpha)c_{r2}S_1(W) - c_{M1} - c_{M2}] \\
& - Q_1(L)[P_1 - \alpha c_{r1}S_1(W) - (1-\alpha)c_{r2}S_1(W) - c_{r4}S_1(W) - c_{M1}] - c_D \} \, dF(c_D)
\end{aligned} \tag{23}$$

which can be rewritten as

$$\begin{aligned}
G = & \, Q_2(L)[P_2 - c_{r3}S_2(W) - (1-\alpha)c_{r2}S_1(W) - c_{M1} - c_{M2}] \\
& - Q_1(L)[P_1 - \alpha c_{r1}S_1(W) - (1-\alpha)c_{r2}S_1(W) - c_{r4}S_1(W) - c_{M1}] - \int_{c_D} c_D \, dF(c_D).
\end{aligned} \tag{24}$$

Eq. (24) indicates that the form of the expected cost function remains as before, but the expected design cost of the diagnostics equipment is replaced by a statistical expected value. For example, if we assume that the diagnostics equipment cost follows a uniform distribution over $[\bar{c}_D/2, 2\bar{c}_D]$, it creates a bias in diagnostics design costs beyond the expected. In this case, the initial expected design cost $\bar{c}_D$ in Eq. (10) becomes $5\bar{c}_D/4$, which implies that the term at the left-hand side of Eq. (8) now needs to be larger to justify a diagnostics design so as to weaken the decision risk.

The risk of making a bad diagnostics design decision is the probability that the diagnostics design costs outweigh the benefits, i.e.

$$
\begin{aligned}
&1 - Pr\{Q_2(L)[P_2 - c_{r3}S_2(W) - (1-\alpha)c_{r2}S_1(W) - c_{M1} - c_{M2}] \\
&\quad - Q_1(L)[P_1 - \alpha c_{r1}S_1(W) - (1-\alpha)c_{r2}S_1(W) - c_{r4}S_1(W) - c_{M1}] \geqslant c_D\} \\
&= F(Q_2(L)[P_2 - c_{r3}S_2(W) - (1-\alpha)c_{r2}S_1(W) - c_{M1} - c_{M2}] \\
&\quad - Q_1(L)[P_1 - \alpha c_{r1}S_1(W) - (1-\alpha)c_{r2}S_1(W) - c_{r4}S_1(W) - c_{M1}]).
\end{aligned}
\tag{25}
$$

As in the previous example, we assume that the diagnostics cost follows a uniform distribution over $[\bar{c}_D/2, 2\bar{c}_D]$. Then the risk of making a bad diagnostics design decision is given by

$$
\frac{2\{Q_2(L)[P_2 - c_{r3}S_2(W) - (1-\alpha)c_{r2}S_1(W) - c_{M1} - c_{M2}] - Q_1(L)[P_1 - \alpha c_{r1}S_1(W) - (1-\alpha)c_{r2}S_1(W) - c_{r4}S_1(W) - c_{M1}])\} - c_D^*}{3c_D^*}.
\tag{26}
$$

3.5. Estimation of failure distributions $F_1(t)$ and $F_2(t)$

The failure distributions of the product and the diagnostics equipment have impact on the design decision. Reasonable characterization of product reliability at the design stage or early prototype stage is non-trivial. Selecting an appropriate representation of the lifetime distribution that best meets certain criteria for obtaining the minimal cost warranty policy is one of the critical issues in warranty analysis. A method of estimating $F_1(t)$ and $F_2(t)$ is by estimating subjectively an appropriate distribution based upon field data from other similar products. Over time, as the development progresses through design, planning and testing, the knowledge increases to a level that ultimately should stabilize. Then the estimation can be updated and a good estimation can be obtained. Methods are needed for obtaining failure distributions that are trustworthy.

4. Numerical example

As in the previous example discussed in Section 2, we consider a product with $L = 3$ years. Failed items are repaired minimally. We set the parameters as follows:

$$L = 3 \text{ year}, \quad W = 1 \text{ year}, \quad c_{r1} = \$10, \quad c_{r2} = \$60,$$
$$c_{r3} = \$20, \quad c_{r4} = \$5, \quad c_{M1} = \$200, \quad c_{M2} = \$60,$$
$$Q_1(L) = 50\,000, \quad Q_2(L) = 50\,000, \quad P_1 = \$440,$$
$$P_2 = \$500, \quad c_D = \$20\,000.$$

Two cases of the lifetime distributions for the product and diagnostic equipment are postulated. In case 1, let the product and the diagnostic equipment follow the exponential lifetime distributions with parameters $r_1 = 0.3$ and $r_2 = 0.15$, respectively. So that $F_1(t) = 1 - e^{-0.3t}$ and $F_2(t) = 1 - e^{-0.15t}$.

In case 2, we assume that the diagnostics equipment follows the exponential lifetime distribution with parameter $r_2 = 0.4$ and the product follows a Weibull lifetime distribution, i.e. the density function is given by

$$
f(t) = \begin{cases} \frac{m}{\eta}\left(\frac{t-\gamma}{\eta}\right)^{m-1} \exp\left[-\left(\frac{t-\gamma}{\eta}\right)^m\right], & x \geqslant \gamma, \\ 0, & x < \gamma \end{cases}
\tag{27}
$$

with parameters $m = 1.5$, $\gamma = 0$, and $\eta = 1$. Then the hazard function is given by $r_1(t) = 1.5t^{0.5}$.

From Eqs. (1) and (4), we get $S_1(W) = \int_0^W r_1(t)\,dt = 1$.

The expected values of gain according to Eq. (11) are shown in Table 1 for different α values chosen for the example. From Table 1, we can see that the expected gain of a diagnostics design increases as α increases. When case 1 is considered, the diagnostics design is not viable (according to Eq. (21)) when $\alpha < 0.63$. When case 2 is considered, the diagnostics design is not viable when $\alpha < 0.34$. We conclude that the parameter α has important influence on the expected gain of the diagnostics design. From this example, we know that the lifetime distributions are also very influential. The bigger the hazard rate of the product is, the more likely the

Table 1
The expected gain G for different parameters α

Case 1: $F_1(t)$ and $F_2(t)$ are exponential distribution		Case 2: $F_1(t)$ is Weibull distribution, $F_2(t)$ is exponential distribution	
α	Expected gain value	α	Expected gain value
0.3	−50000	0.3	−20000
0.5	−20000	0.5	80000
0.7	10000	0.7	180000
0.9	40000	0.9	280000

diagnostics design is viable. When a manufacturer is considering a diagnostics design but the new product and diagnostics equipment have little failure information available, the manufacturer should seek expert opinion to estimate the failure parameters. The manufacturer can then decide whether or not to make a diagnostics design based on subjective estimates.

If the product and the diagnostic equipment failure distributions are exponential with parameters $r_1 = 0.3$ and $r_2 = 0.15$ and $\alpha = 0.6$, Eq. (18) demonstrates that there is a critical warranty period $W^* = 1.33$ years above which a diagnostics design is advantageous. If the diagnostic costs follow a uniform distribution over [10 000,40 000], Eq. (26) demonstrates that the risk of making a bad diagnostics design decision is 17%.

5. Conclusions

Diagnostics design as a product support strategy can reduce the total warranty cost by providing guidance for the user to rectify most common types of failures. This approach increases the production cost for the item under warranty. Thus the diagnostic design decision is important for new product design. But very little work has been done in this area. Warranty analysis involves uncertainty because of the variability and nondeterministic nature of many parameters. This paper develops diagnostics design decision models for a product under warranty from a manufacturer's point of view and investigates the robustness of diagnostics design decision to the key parameters and decision variables. These models can help product design managers (designers) in deciding whether to build in the diagnostics equipment for the product under warranty in design of new products.

There still exist several challenging issues to be explored. The diagnostics design conditions given in Section 3 are necessary, but not sufficient. The conditions that Eq. (7) is larger than zero and the product is profitable may be necessary, too. The sales volume, $Q(L)$, has been assumed deterministic. In practice, the product sales volume is dependent on the product class, quality, price, post-sale service, and the market competition and so on. Therefore the sales volume, $Q(L)$, is uncertain and can only be estimated. Furthermore, if a diagnostic design proves successful, it is possible that the accumulated sales volume can be increased. These considerations can be incorporated to further refine our proposed models in this paper.

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