Identification of characteristic components in frequency domain from signal singularities

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In rotating machinery condition monitoring, identification of characteristic components is fundamental in many engineering applications so as to obtain fault sensitive features for fault detection and diagnosis. This paper proposed a novel method for the identification of characteristic components in frequency domain based on singularity analysis. In this process, Lipschitz exponent function is constructed from the signal through wavelet-based singularity analysis. In order to highlight the periodic phenomena, autocorrelation transform is employed to extract the periodic exponents and Fourier transform is used to map the time-domain information into frequency domain. Case study with rolling element bearing vibration data shows that the proposed has very excellent capability for the identification of characteristic components compared with traditional methods. © *2010 American Institute of Physics.* [doi:10.1063/1.3361039]

I. INTRODUCTION

In condition monitoring process of rotating machinery (e.g., gearbox and bearing), identification of characteristic components is fundamental in many engineering applications so as to obtain more sensitive features for fault detection and diagnosis. Traditional methods directly utilize Fourier transform (FT) to obtain spectrum of signal, which provides a global description of characteristic components in frequency domain. However, fault-related signatures usually demonstrate nonstationarity, and the FT has strong limitations because of its spectral resolution and loss of temporal information after transformation.[1] One way to deal with this problem is called "windowing" and the short-time FT (STFT), also called windowed FT, has numerous applications in speech, vibration, and image processing. The window length is an important parameter, which determines time and frequency resolutions in STFT. However, according to Heisenberg's uncertainty principle,[2] it is impossible to achieve high resolution in both time and frequency domains simultaneously.

In order to overcome such problems in nonstationary signal processing, the idea of signal decomposition has been accepted and discrete wavelet transform (DWT) is such a popular method that can decompose signal into different scales corresponding to different frequency bands.[3] For example, Prabhakar *et al.*[4] applied DWT for rolling bearing race fault detection and found that the impulses caused by bearing failure appear periodically with a time period corresponding to characteristic defect frequencies. Wang *et al.*[5] proposed a health evaluation method based on wavelet decomposition. In the process of wavelet decomposition, an index was defined to choose the optimal detail signal and a health index named frequency spectrum growth index was proposed for description of machine health condition. In fact, research on wavelet has attracted intensive attentions and a lot of theoretical and practical methods (such as wavelet packet,[6] wavelet lifting,[7] etc.) have been developed and applied.

It should be noted that wavelet-based singularity analysis is another trend in the family of wavelet applications. The idea of this comes from the fact that most fault-related signatures of signal are often carried by singularity points.[8] Wavelet function can be properly chosen to have n vanishing moments so as to remove polynomial trend in signal, which masks weak singularities in vibration signal. Therefore, wavelet-based method is an appropriate way in signal singularity detection. Reference 8 illustrates that wavelet modulus maxima method is a standard way to detect singularity points and provides numerical procedures to compute Lipschitz exponent of singularity. Sun and Tang[9] applied wavelet transform modulus maxima to detect abrupt changes in the vibration signal. Miao *et al.*[10] proposed a new kurtosis based health index by calculating the kurtosis of Lipschitz exponent and used this method to perform maintenance decision-making.

The purpose of this paper is to develop a novel method for the identification of characteristic components in frequency domain based on singularity analysis with wavelet, which can be regarded as a new way for feature extraction and fault identification. In this process, continuous wavelet transform (CWT) is applied to original vibration signal for singularity detection. Then, Lipschitz exponent function based on wavelet transform modulus maxima is defined through approximate estimation of Lipschitz exponent. In order to highlight the periodic components, correlation analysis of Lipschitz exponent function is performed and FT is employed to reveal fault-related signatures in frequency domain.

The organization of the paper is as follows. Section II

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gives a brief description of wavelet-based singularity analysis. A novel method based on Lipschitz exponent function and correlation analysis for identification of fault characteristic frequency is proposed in Sec. III. Section IV further investigates and validates the proposed method with rolling element bearing vibration data under different load and rotating speed conditions. In the end, conclusions are summarized in Sec. V.

II. WAVELET BASED SINGULARITY ANALYSIS

A. Description of singularity with Lipschitz exponent

Signals collected from rotating machinery may contain nonstationary transient components caused by faults. These transient phenomena can be regarded as singularities in signal and a wavelet-based singularity analysis is an effective way for fault detection and identification.[10] Theoretically, Lipschitz exponent, also called Holder exponent, can quantitatively describe function regularity.[8] If and only if there exist two constants A and $h_0>0$, and a polynomial $P_n(x)$ of order n, such that

$$|f(x)-P_n(x-x_0)|\leq A|x-x_0|^{\alpha} \quad \text{for} \quad |x-x_0|<h. \tag{1}$$

We call Lipschitz regularity of $f(x)$ and x_0, the superior bound of all values α such that $f(x)$ is Lipschitz α at x_0. Here, $f(x)\in L^2(R)$. The polynomial $P_n(x)$ is often associated with Taylor's expansion of $f(x)$ at x_0. Function $f(x)$ that is continuously differentiable at a point is Lipschitz 1 at this point. If the Lipschitz regularity α of $f(x)$ at $x=x_0$ satisfies $n<\alpha<n+1$, then $f(x)$ is n times differentiable at x_0 but its nth derivative is singular at x_0 and α characterizes this singularity. Here n is a positive integer.

To measure the Lipschitz with Eq. (1), a classical way is to look at the asymptotic decay of the amplitude of function $f(x)$'s FT. However, it only provides a global regularity measurement because FT cannot localize information in temporal domain. That is to say, it cannot deal with nonstationary transient signals. On the other hand, wavelet transform is a better choice because of its compact support.[8] In case the wavelet $\psi(x)$ has a compact support, the value of wavelet coefficient $Wf(s,x)$ depends on the value of $f(x)$ in a neighborhood, of size proportional to the scale s. Besides, we must impose the wavelet $\psi(x)$, which has enough vanishing moments, to gain Lipschitz exponent α. For instance, a wavelet $\psi(x)$ with $n+1$ vanishing moments, if and only if for all positive integers $k<n+1$, is that

$$\int_{-\infty}^{+\infty} x^k\psi(x)dx=0, \quad \text{for} \quad 0\leq k<n+1. \tag{2}$$

It is obvious that the wavelet with $n+1$ vanishing moments is orthogonal to the polynomials of up to order n. Then, the wavelet transform of $f(x)$ using $\psi(x)$ at the location x_0 can eliminate those polynomials up to order n. Furthermore, a wavelet with $n+1$ vanishing moments can be written as the $n+1$th order derivative of the signal $f(x)$ smoothed by a smoothing function $\theta(x)$ in the form

$$Wf(s,x)=f(x)^*\psi_s(x)=s^n\frac{d^n}{dx^n}(f^*\bar{\theta}_s)(x)$$

with

$$\bar{\theta}_s(x)=(1/s)\theta(1/s). \tag{3}$$

Then, it is possible to examine any rate of change in the signal amplitude by selecting a suitable wavelet function because the wavelet transform is a smoothed derivative of the signal at various scales.

It is proved in Ref. 8 that if the function $f(x)$ is Lipschitz α at x_0, $n<\alpha<n+1$, then there exists a constant A such that for all points x in the neighborhood of x_0 and any scale s,

$$|Wf(s,x)|\leq A(s^{\alpha}+|x-x_0|^{\alpha}). \tag{4}$$

The local Lipschitz exponent of $f(x)$ at x_0 depends on the decay of $|Wf(s,x)|$ at fine scales in the neighborhood of x_0. The decay can be measured through local maxima. Define modulus maxima as any point (s_0,x_0) such that $|Wf(s,x)|$ is a local maximum at $x=x_0$. Singularities can be identified by the presence of modulus maxima. If there exists a scale $s_0>0$ and a constant C, such that for $x\in(a,b)$ and $s<s_0$, all the modulus maxima of $|Wf(s,x)|$ belong to a cone defined by

$$|x-x_0|\leq Cs. \tag{5}$$

Then at each modulus maxima (s,x) in the cone defined by Eq. (5),

$$|Wf(s,x)|\leq Bs^{\alpha}, \tag{6}$$

which is equivalent to

$$\log_2|Wf(s,x)|\leq \log_2 B+\alpha\log_2 s. \tag{7}$$

Here, $B=A(1+C^{\alpha})$.

B. Correlation analysis

Correlation is a mathematical tool often used in signal processing for analyzing functions or time series, such as time-domain signals. Autocorrelation is the correlation of a signal with itself, which is very useful to find periodic patterns in a signal. For example, it can determine the presence of a periodic signal which has been buried under noise. Various autocorrelation measures have been proposed and the mostly used are Bartlett (positive definite but biased) and Blackman–Tukey (unbiased but not guaranteed to be positive definite), which are defined as follows. If $s(n)$, $n=0,1,2,\ldots,N-1$ is a real data record of length N, then the Bartlett autocorrelation measure is

$$r_s(m)=\frac{1}{N}\sum_{n=0}^{N-m-1}s(n)s(n+m), \quad m=0,1,2,\ldots,(N-1), \tag{8}$$

while the Blackman–Tukey one is

$$r_s(m) = \frac{1}{N-m}\sum_{n=0}^{N-m-1} s(n)s(n+m), \quad m = 0,1,2,\ldots,(N-1). \tag{9}$$

Including additive noise $\nu(n)$, the mixed signal becomes $x(n)=s(n)+\nu(n)$, and the autocorrelation of the mixed signal is described as follows:

$$r_x(m) = \frac{1}{N}\sum_{n=0}^{N-m-1}[s(n)+\nu(n)][s(n+m)+\nu(n+m)]$$
$$= r_s(m) + r_{\nu s}(m) + r_{s\nu}(m) + r_\nu(m), \tag{10}$$
$$m = 0,1,2,\ldots,(N-1).$$

Here, $r_{\nu s}(m)$ and $r_{s\nu}(m)$ are cross-correlations, which are the correlations of two different signals. Generally, the value of cross-correlation between random noise and signal is so small that it can be ignored. Therefore, in case $s(n)$ is a periodic signal, its autocorrelation can be simplified as

$$r_x(m) = r_s(m) + r_\nu(m), \quad m = 0,1,2,\ldots,(N-1). \tag{11}$$

Here, suppose $\nu(n)$ is the white noise, then $r_\nu(m)$ is the autocorrelation of the white noise, which is an impulse function $\delta(m)$ at $m=0$. So $s(n)$ can be estimated through $r_x(m)$. If $\nu(n)$ is not the white noise, $r_\nu(m)$ attenuates quickly as m increases. It also can be seen that the trend of $r_x(m)$ determines whether $s(n)$ is contained in the mixed signal.

III. IDENTIFICATION OF CHARACTERISTIC COMPONENTS

A. Lipschitz exponent estimation

Rolling element bearings suffer from faults due to fatigue and severe working conditions. In this section, we will present a novel method to extract fault-related signatures (namely, bearing fault characteristic frequencies) from signal singularities. In this process, CWT is employed to obtain wavelet modulus maxima of original vibration signal. From Eq. (7), it is known that wavelet transform provides an asymptotic way to estimate Lipschitz α. In this paper, modulus maxima of CWT produce a two-dimensional time-scale matrix, in which one dimension indicates a different time point x in the signal, and the other one denotes a different frequency scale s. For simplification, we estimate α as follows:

$$\alpha = \frac{\log_2|Wf(s,x)| - \log_2 B}{\log_2 s}. \tag{12}$$

The above simplification has been applied by Robertson *et al.*[11] in structural health monitoring and Miao *et al.*[10] in gearbox fault diagnosis. Here, we take each column which represents the frequency spectrum of the signal at certain time point x and calculate Lipschitz α using Eq. (12). Therefore, Lipschitz exponent function LEP(x) can be obtained.

In this research, we use a method called total least square (TLS) to estimate the slope α. Consider a data set, $(x_1,y_1),(x_2,y_2),(x_3,y_3),\ldots,(x_n,y_n)$, and we hope to fit them with a line $ax+by-c=0$, which goes through a point (x_0,y_0). Now we suppose that the line goes through the center of data set. The center of data set is given as follows:

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i, \quad \bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i, \tag{13}$$

so, we can get

$$c = a\bar{x} + b\bar{y}. \tag{14}$$

The other form is

$$a(x-\bar{x}) + b(y-\bar{y}) = 0, \tag{15}$$

where $[a,b]^T$ is called normal vector and $m=-a/b$ is called slope.

The TLS method minimizes the square of the distance between given data and line equation as follows:

$$d^2 = \frac{(ap+bq-c)^2}{a^2+b^2} = \frac{[a(p-x_0)+b(q-y_0)]^2}{a^2+b^2}, \tag{16}$$

where (p,q) represents a plane coordinate. Therefore, if there are n plane points, this square distance can be given as

$$D(a,b,\bar{x},\bar{y}) = \sum_{i=1}^{n}\frac{[a(x_i-\bar{x})+b(y_i-\bar{y})]^2}{a^2+b^2}. \tag{17}$$

It is known that

$$D(a,b,\bar{x},\bar{y}) \le D(a,b,x_0,y_0). \tag{18}$$

In the following statement, we solve the problem on how to minimize D. Here we use matrix to illustrate this procedure. Suppose 2×1 unit vector $t=(a^2+b^2)^{-1/2}[a,b]^T$ and $n\times 2$ matrix

$$M = \begin{bmatrix} x_1-\bar{x} & y_1-\bar{y} \\ x_2-\bar{x} & y_2-\bar{y} \\ \vdots & \vdots \\ x_n-\bar{x} & y_n-\bar{y} \end{bmatrix} \tag{19}$$

D can be given as

$$D(a,b,\bar{x},\bar{y}) = \|Mt\|_2^2 = \left\| \begin{bmatrix} x_1-\bar{x} & y_1-\bar{y} \\ x_2-\bar{x} & y_2-\bar{y} \\ \vdots & \vdots \\ x_n-\bar{x} & y_n-\bar{y} \end{bmatrix} \frac{1}{\sqrt{a^2+b^2}} \begin{bmatrix} a \\ b \end{bmatrix} \right\|_2^2. \tag{20}$$

M^TM is a 2×2 real, symmetrical, and semidefinite matrix. Decompose this matrix using matrix theory.

$$M^TM = U\delta U^T = [u_1,u_2]\begin{bmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{bmatrix}\begin{bmatrix} u_1^T \\ u_2^T \end{bmatrix}, \tag{21}$$

where σ_1 is greater than or equal to σ_2.

$$D(a,b,\bar{x},\bar{y}) = \|Mt\|_2^2 = t^TM^TMt = t^TU\delta U^Tt = (U^Tt)^T\delta(U^Tt)$$
$$= \sigma_1^2|(U^Tt)_1|^2 + \sigma_2^2|(U^Tt)_2|^2$$
$$\ge \sigma_2^2[|(U^Tt)_1|^2 + |(U^Tt)_2|^2]$$
$$= \sigma_2^2\|U^Tt\|_2^2. \tag{22}$$

According to matrix theory, we know that $\|U^Tt\|_2$ is equal to

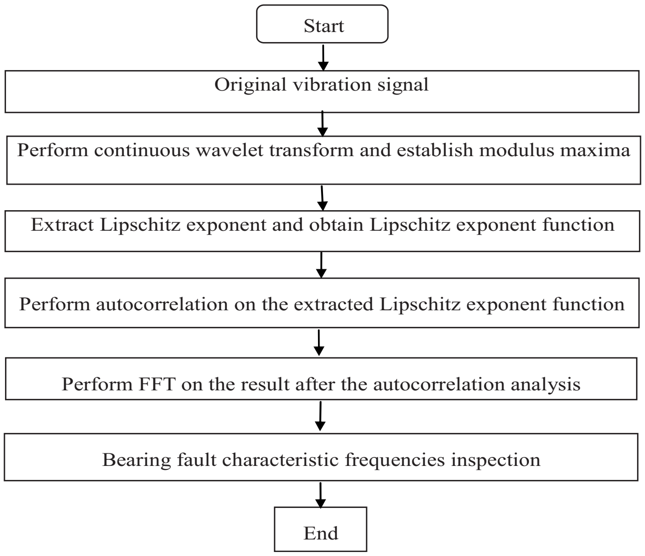


FIG. 1. The flowchart of the proposed method.

$\|t\|_2$ and Euclidean norm of t is equal to 1. So we can further simplify Eq. (22) as follows:

$$D(a,b,\bar{x},\bar{y}) \geq \sigma_2^2 \tag{23}$$

On the other hand, when t is equal to u_2, we can obtain following equation:

$$D(a,b,\bar{x},\bar{y}) = t^T M^T M t = t^T [u_1,u_2] \begin{bmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{bmatrix} \begin{bmatrix} u_1^T \\ u_2^T \end{bmatrix} t$$
$$= \sigma_1^2 \|u_1^T t\|_2^2 + \sigma_2^2 \|u_2^T t\|_2^2 = \sigma_2^2, \tag{24}$$

where Euclidean norms of t, u_1, and u_2 are all equal to 1. Equation (24) illustrates that the minimum value of D can be obtained, if t is equal to u_2. With TLS, we can realize the estimation of Lipschtiz exponent through the slope α.

B. Fault characteristic frequency extraction

In order to extract the periodic information, we apply the autocorrelation transform of Lipschitz exponent function using Eq. (8) or Eq. (9) and the autocorrelation of Lipschitz exponent function is $r_{LP}(t)$. It is noted that, in this paper, we set $m=N-1$, where N is the length of signal. Then we normalize the autocorrelation signal as follows:

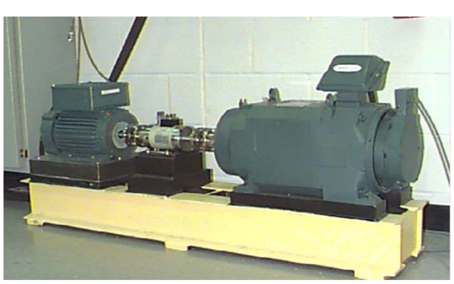


FIG. 2. (Color online) Test rig of bearings.

TABLE I. Motor bearing specification (inches).

Inside diameter	0.9843
Outside diameter	2.0472
Thickness	0.5906
Ball diameter	0.3126
Pitch diameter	1.537

$$r'_{LP}(t) = \frac{r_{LP}(t) - \overline{r_{LP}(t)}}{\sigma}, \tag{25}$$

where $r_{LP}(t)$ is the autocorrelation of Lipschitz exponent function, $\overline{r_{LP}(t)}$ is the mean value of $r_{LP}(t)$, and σ denotes the standard deviation of $r_{LP}(t)$.

To identify characteristic frequency, FT is employed to map the time-domain signal of $r'_{LP}(t)$ into frequency domain.

$$ES(f) = \left| \int_{-\infty}^{+\infty} r'_{LP}(t) e^{-2ift\pi} dt \right|. \tag{26}$$

Here, $ES(f)$ denotes absolute value of FT amplitude of $r'_{LP}(t)$. Therefore, we can identify bearing faults via bearing fault characteristic frequencies.

Finally, we can summarize the proposed method in this paper and Fig. 1 shows the flow chart of it.

IV. CASE STUDY

In this paper, the real motor bearing data picked up with a sampling frequency of 12 kHz by an accelerometer placed at the six o'clock position at the drive end of the motor housing is used to validate the proposed method. The test rig is shown in Fig. 2. Single point faults were introduced to normal bearings using electrodischarge machining with a fault diameter of 0.007 in. and the fault depth is 0.0011 in. The specification of bearings is shown in Table I. The shaft rotation speed f_r varies from 1730 to 1797 rpm. The characteristic frequencies of the bearing are calculated by the following formulas:[12]

$$f_I = 5.4152 \times f_r, \tag{27}$$

$$f_O = 3.5848 \times f_r. \tag{28}$$

Here, f_I and f_O are inner race fault characteristic frequency and outer race fault characteristic frequency, respectively. There are a total of 12 data sets including four normal bearings, four inner race fault bearings, and four outer race fault bearings under different rotation speeds and work loads, and Table II shows their corresponding characteristic frequencies. In order to enhance the computation efficiency, each data with 1 s is selected for analysis.

TABLE II. Fault characteristic frequencies of motor bearing under different rotation speeds and motor loads.

Motor load (HP)	Motor speed (rpm)	Inner race f_I (Hz)	Outer race f_O (Hz)
0	1797	162.1	107.3
1	1772	160.0	105.9
2	1750	157.9	104.6
3	1730	155.7	103.1

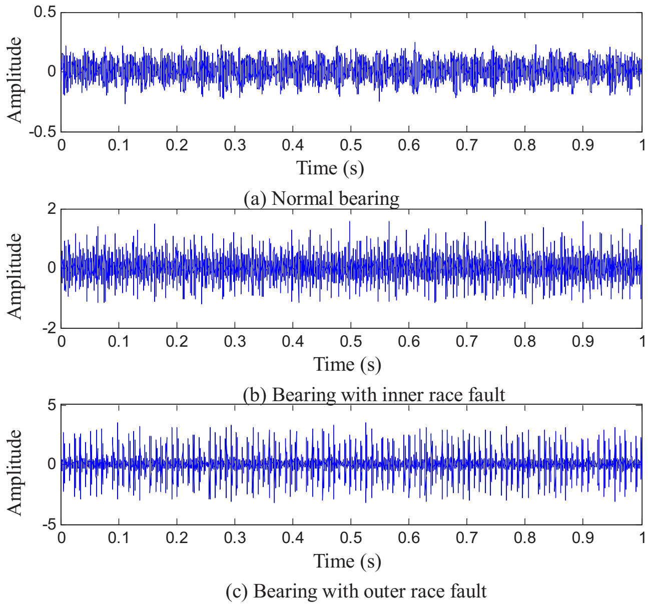


FIG. 3. (Color online) The original signals under motor speed 1797 rpm and load HP 0.

A. Validation of fault characteristic identification under motor speed 1797 rpm and load HP 0

In this section, we used vibration signals including normal bearing data, inner race fault data, and outer race fault data under motor speed 1797 rpm and load HP 0 to validate the proposed method. Figure 3 shows the original vibration signals of normal data, inner race fault data and outer race fault data.

Wavelet transform with the derivative of Gaussian function as wavelet function has been applied to extract Lipschitz exponent functions for all original vibration signals. Lipschitz exponent functions of these three data are shown in Fig. 4. X-axis represents time. Y-axis represents the value of Lipschitz exponent function over time.

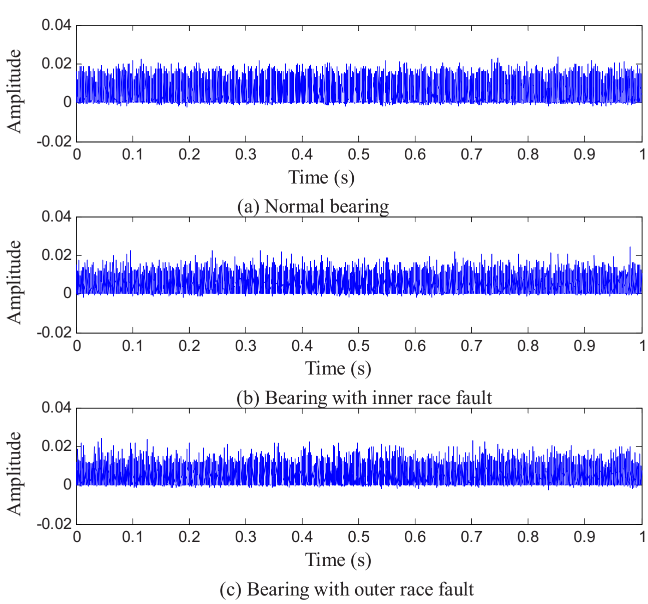


FIG. 4. (Color online) Lipschitz exponent functions under motor speed 1797 rpm and load HP 0.

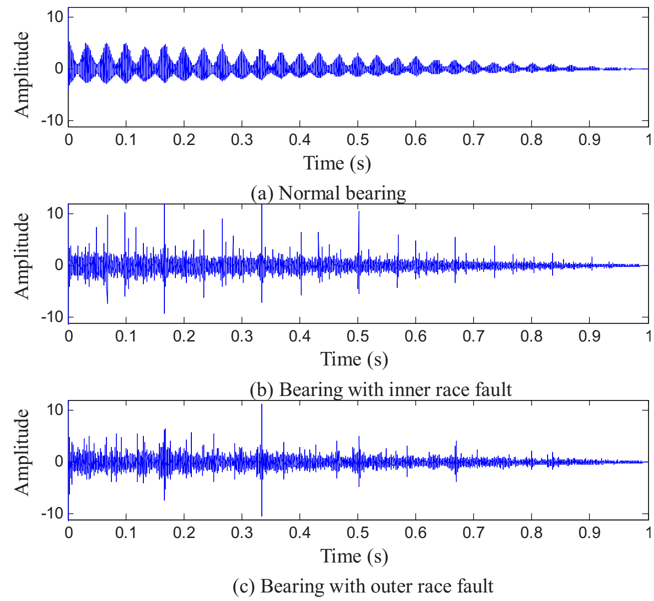


FIG. 5. (Color online) Autocorrelation of Lipschitz exponent functions under motor speed 1797 rpm and load HP 0.

In order to highlight the periodic effect of Lipschitz exponent function, autocorrelation transform is performed and their normalizations are shown in Fig. 5. The observation from Fig. 5 indicates that the periodic signals may exist and FT is used to map these time-domain signals into frequency domain. As a result, fault-related signatures, namely, bearing fault characteristic frequencies, are detected in Fig. 6. When bearing is under normal condition, Fig. 6(a) shows no fault signatures. However, bearings with inner race faults and outer race faults can be found via their corresponding fault characteristic frequencies in Figs. 6(b) and 6(c), respectively. It is obvious that the proposed method can identify fault-related characteristic components in frequency domain.

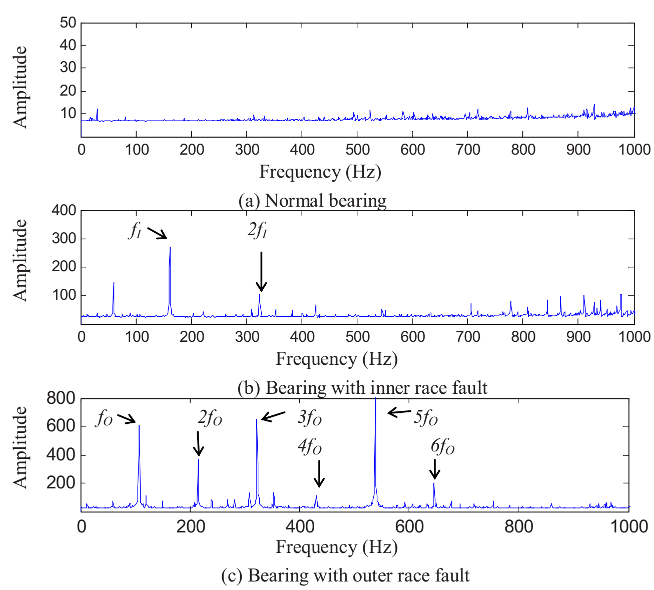
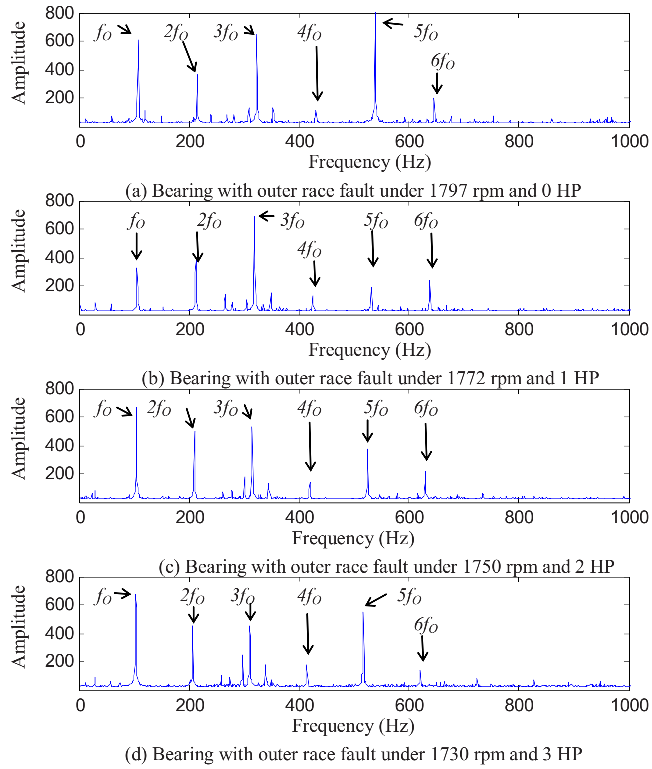
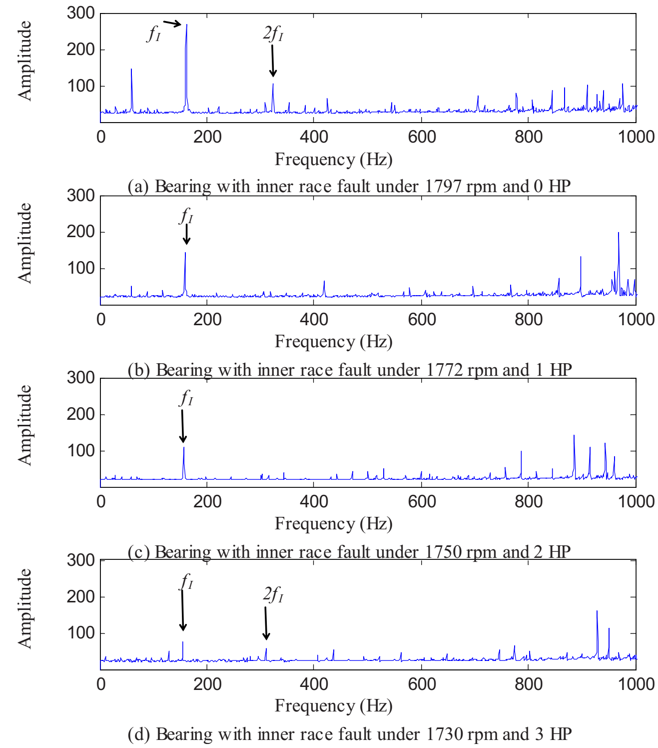


FIG. 6. (Color online) Identification of fault-related characteristic components under motor speed 1797 rpm and load HP 0.



(a) Bearing with outer race fault under 1797 rpm and 0 HP
(b) Bearing with outer race fault under 1772 rpm and 1 HP
(c) Bearing with outer race fault under 1750 rpm and 2 HP
(d) Bearing with outer race fault under 1730 rpm and 3 HP

FIG. 7. (Color online) Identification of characteristic components of bearing with outer race fault under different speeds and loads.



(a) Bearing with inner race fault under 1797 rpm and 0 HP
(b) Bearing with inner race fault under 1772 rpm and 1 HP
(c) Bearing with inner race fault under 1750 rpm and 2 HP
(d) Bearing with inner race fault under 1730 rpm and 3 HP

FIG. 8. (Color online) Identification of characteristic component of bearing with inner race fault under different speeds and loads.

B. Validation of fault characteristic identification under different working conditions

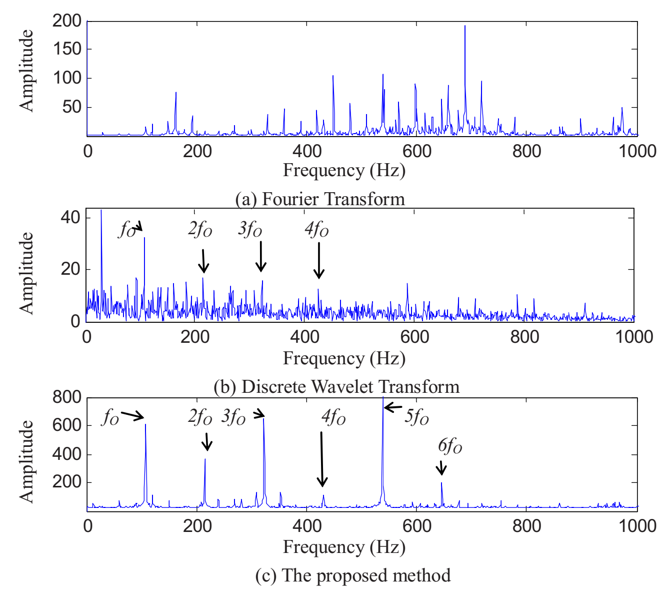
In this section, we consider the influence of different working conditions, including different rotation speeds and different loads. Figure 7 shows the analysis results using vibration data of bearing with outer race fault under different working conditions. It is obvious that the proposed method can clearly identify outer race fault characteristic frequency f_O and its harmonics. In addition, Fig. 8 gives the analysis results using bearing with inner race fault under different working conditions, which also shows good capability for the identification of characteristic component f_I.

C. Comparison with other methods for fault characteristic frequency identification

For comparison study, vibration data under motor speed 1797 rpm and load HP 0 are analyzed by FT, DWT, and the proposed method in this section, respectively. When we apply DWT to decompose the signal, two tough problems needs to be clarified. One is that which wavelet should be chosen for signal decomposition. Another is the selection of decomposition level. In this section, Daubechies-9 wavelet with four decomposition levels is selected for signal analysis and synthesis with DWT. The comparison results are shown in Figs. 9 and 10 for bearing with outer race fault and inner race fault, respectively.

From Fig. 9, both the DWT and the proposed method can identify the fault characteristic frequency f_O and its harmonics while FT cannot. This is due to the fact that FT itself is a globe mapping, which may weaken the transient signatures after transformation. Moreover, the proposed method is better than DWT because comparison between Figs. 9(b) and 9(c) shows that the characteristic components are more distinct with the proposed method [see Fig. 9(c)]. The proper selection of decomposition level and wavelet basis is still a problem with DWT.

Figure 10 shows the analysis results of bearing with inner race fault using three different methods. In this figure, inner race fault characteristic components can be identified by these methods. Compared to Fig. 10(c), both Figs. 10(a) and 10(b) contain many unrelated frequencies, especially



(a) Fourier Transform
(b) Discrete Wavelet Transform
(c) The proposed method

FIG. 9. (Color online) Comparison of characteristic component identification using vibration of bearing with outer race fault.

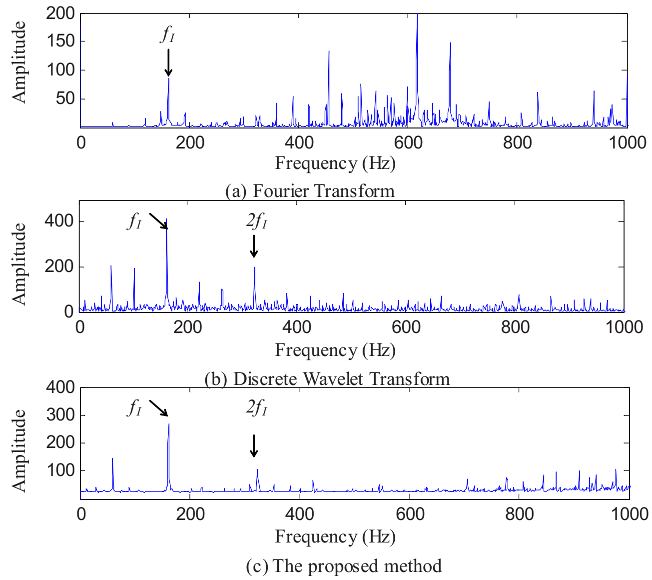


FIG. 10. (Color online) Comparison of characteristic component identification using vibration of bearing with inner race fault.

Fig. 10(a). These signatures may obscure the visual inspection. On the other hand, the proposed method provides distinct result for the identification of fault characteristic components. Therefore, we can conclude that the proposed method is better than others to extract the fault-related characteristic components.

V. CONCLUSIONS

In this paper, a novel method for the identification of characteristic components in frequency domain is proposed. In this method, Lipschitz exponent function is constructed from the signal through wavelet-based singularity analysis. In order to highlight the periodic phenomena, autocorrelation transform is employed to extract the periodic exponents. Lastly, FT is utilized to map the time-domain information into frequency domain. In the case study, the proposed method is first validated using vibration data collected from motor bearing under rotation speed 1797 rpm and load HP 0. It is obvious that the proposed method can identify fault-related characteristic components. Furthermore, the vibration signals under different loads and rotation speeds are used to verify the proposed method and the proposed method shows good capability. In the end, a comparison study among FT, DWT, and the proposed method is conducted. The analysis shows that the proposed method can provide distinct result and it has very excellent capability for the identification of characteristic components in frequency domain.

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