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Neurocomputing for Fuzzy Finite Element Analysis of Structures Based on Fuzzy Coefficient Programming

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There exist problems in fuzzy finite element methods because technique of solving fuzzy equations is not perfect. For example, computation amount is too big and both sides of the equality are not exactly equal when solutions are substituted into the original equation. The concept of monosource fuzzy number is developed to simplify the calculation process of fuzzy equations. However the source of fuzziness in practical engineering is difficult to be judged and the source of fuzzy coefficient is non-unique. Indeed, no efficient method is available to solve fuzzy finite element equations. In this paper, fuzzy coefficient programming is combined with the essence of elasticity. In other words, the force equilibrium of elastic object is the process of minimizing energy of a quadratic equation. A new fuzzy finite element solution and a new neural network algorithm of fuzzy finite element are developed. The method was proved to be efficient and feasible through circuit simulation.

Keywords: Fuzzy finite element, Fuzzy coefficient programming, Fuzzy equations, Neural network

1 INTRODUCTION

The finite element method is a very popular tool for both static and dynamic analysis of engineering systems. Its ability to predict the behavior of a structure under static or dynamic loads is not only of scientific interest, but also of practical significance. A reliable finite element analysis could make prototype production and testing obsolete, therefore significantly reducing the associated

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design validation cost. Traditional finite element approaches require crisp or well defined input parameters. For instance, given the geometry, material properties, load and boundary conditions, a crisp result can be calculated on an element-by-element basis. Unfortunately, sometimes it is very difficult to ensure the reliability of the result of a finite element analysis for realistic structures that are not precisely defined [1]. For instance, the geometric properties, as well as the effects of service conditions on physical, mechanical, and electromechanical properties of smart structures, are vaguely understood and therefore cannot be precisely defined. If the uncertainty is due to vaguely defined system characteristics, imprecision of data, insufficient information and /or subjectivity of opinion or judgement, then fuzzy set based treatment is appropriate.

The fuzzy finite element methodology is a new area of finite element analysis that began in the early 1990s. [2,3] applied fuzzy set based methods to finite element analysis of geotechnical soils and foundations, elasto-plastic finite element analysis, [4] applied fuzzy set based methods to modelling of damping in dynamic finite element analysis. [5,6] applied fuzzy set based methods to static analysis of structures. [7] proposed a fuzzy finite element method for static analysis of engineering systems using an optimisation based scheme for the numerical solution of systems of fuzzy parameters, geometry and applied loads was considered and implemented in their approach. [8] developed a fuzzy finite element method for vibration analysis of imprecisely defined systems by using a search-based algorithm. Their approach enhances the computational efficiency in fuzzy operations for identifying the system dynamic responses. [9–11] developed a finite element analysis procedure utilizing the concept of fuzzy sets through interval calculations and also computed the response of different structural systems due to geometric and loading uncertainties. Noor et al applied fuzzy finite element method to analysis of space structures [12], flexible multibody systems [13], welding residual stress fields [14], composite structures [15], tethered satellite system [16]. [17] gave a Neumann expansion for fuzzy finite element analysis. [18] proposed a fuzzy finite element approach for modelling smart structures with vague or imprecise uncertainties. [19] proposed a fuzzy arithmetical approach to the solution of finite element problems with uncertain parameters.

On the basis of the second order perturbation method of small parameter, [20], [21] provided the fuzzy functional of overall potential energy and the second order perturbation expansions, and deduced definite recursion equation of fuzzy variational principle. Fuzzy finite element recursion equation was established based on fuzzy variational principle. [22] took into account the process in which the element stiffness matrix is assembled into the total stiffness matrix, and improved the fuzzy finite element method developed by [20], [21].

The basic idea of all methods mentioned above is that: fuzziness of the coefficients was introduced into precise finite element equation, and fuzzy finite element equation was transformed to a set of precise interval equations in terms of a set of threshold values λ, solving these interval equations, then taking use of factorization theorem of fuzzy set theory to find solution of fuzzy finite element equilibrium equation [23]. In this way, we have to face a large number of interval equations. What's more is that the interval numbers are quite different from precise numbers, so that the algorithm operations are different completely. For example two interval numbers a and b, $a - b = [\underline{a} - \bar{b}, \bar{a} - \underline{b}]$. It is obvious that the result of an interval number subtracted by itself is not zero. Hitherto there is no satisfied interpretation to the interval equations.

The second idea to solve fuzzy finite element equations is that the fuzziness of coefficient in the equations is not taken into consideration at first. After the expression of variables being determined with conventional equations solution, fuzziness of coefficient is introduced and fuzzy solution can be obtained according to fuzzy operation rules [19].

Recently, the concept of fuzzy source and operation rules of monosource fuzzy numbers is developed [24]. The third idea is that the concept of monosource fuzzy numbers is introduced into equations, and fuzzy equation can be transformed to conventional equations by using the operation rules of monosource fuzzy numbers, which can largely simplify the calculation process [25].

Among the three methods mentioned above, the third one, i.e. the fuzzy finite element equation based on monosource fuzzy numbers, is easy to calculate and a precise solution can be obtained. However it is difficult to judge the source of fuzziness and the source of fuzzy coefficient is non-unique, thus restricting the use of the method. The first two methods are widely used because they have general sense in practical application. But the disadvantage of the two methods is that computation amount is too big and both sides of the equality are unequal when the solutions are substituted into the original equation. So the methods are only approximate methods.

Neural network is a complicated nonlinear dynamic system with highly parallelism. Optimization problems can be mapped into dynamical circuit with appropriate neural network. The problem can be solved within an elapsed time of circuit time-constant. The essence of neural network optimizing calculation is through constructing appropriate network structure and learning method and associating some coefficients of network with design variables and energy function of network with some objective functions of optimization. When the neural network running, energy of the network is reduced and the energy reaches minimum value when the system attains equilibrium.

The paper combines fuzzy coefficient programming [26] with the essence of elasticity, i.e. the process of force equilibrium of elastic object is the process of minimizing energy of a quadratic equation. A new fuzzy finite element solution and a new neural network algorithm of fuzzy finite element are developed. The method was proved to be efficient and feasible through the circuit simulation.

2 FUZZY COEFFICIENT PROGRAMMING

Conventional programming problem can be described as follows

$$\text{(P)} \quad \max f(\mathbf{c}, \mathbf{x})$$

$$\text{s.t.} \begin{cases} g(\mathbf{a}, \mathbf{x}) \leq 0 \\ h(\mathbf{b}, \mathbf{x}) = 0 \\ \mathbf{x} \in D \end{cases} . \tag{1}$$

where $f(\mathbf{c}, \mathbf{x})$ is a n-variable function whose variables are $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ with the coefficients

$$\mathbf{c} = (c_1, c_2, \ldots, c_n)$$

$$g(a, x) = (g_1(\mathbf{a}_1, \mathbf{x}), g_2(\mathbf{a}_2, \mathbf{x}), \ldots, g_n(\mathbf{a}_n, \mathbf{x}))'$$

where $g_i(\mathbf{a}_i, \mathbf{x})$ is a n-variable function whose variables are $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ with the coefficients

$$\mathbf{a}_i = (a_{i1}, a_{i2}, \ldots, a_{is_i})$$

$$h(\mathbf{b}, \mathbf{x}) = (h_1(\mathbf{b}_1, \mathbf{x}), h_2(\mathbf{b}_2, \mathbf{x}), \ldots, h_m(\mathbf{b}_m, \mathbf{x}))'$$

where $h_j(\mathbf{b}_j, \mathbf{x})$ is a n-variable function whose variables are $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ with the coefficients

$$\mathbf{b}_j = (b_{j1}, b_{j2}, \ldots, b_{jp_j}), \; j = 1, 2, \ldots, m.$$

D is a subset of R^n.

For many practical problems, the coefficients of programming (P) can not be known definitely. If the coefficients of (P) are fuzzy numbers, the problems are transformed to “fuzzy coefficient programming” problems, in brief FCP whose expression is:

$$\text{(P)} \quad \max f(\tilde{\mathbf{c}}, \mathbf{x})$$

$$\text{s.t.} \begin{cases} g(\tilde{\mathbf{a}}, \mathbf{x}) \leq 0 \\ h(\tilde{\mathbf{b}}, \mathbf{x}) = 0 \\ \mathbf{x} \in D \end{cases} . \tag{2}$$

where $f(\tilde{\mathbf{c}}, \mathbf{x})$ is a n-variable function whose variables are $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ with the coefficients

$$\tilde{\mathbf{c}} = (\tilde{c}_1, \tilde{c}_2, \ldots, \tilde{c}_n)$$

$$g(\tilde{\mathbf{a}}, \mathbf{x}) = (g_1(\tilde{\mathbf{a}}_1, \mathbf{x}), g_2(\tilde{\mathbf{a}}_2, \mathbf{x}), \ldots, g_n(\tilde{\mathbf{a}}_n, \mathbf{x}))'$$

where $g_i(\tilde{\mathbf{a}}_i, \mathbf{x})$ is a n-variable function whose variables are $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ with the coefficients

$$\tilde{\mathbf{a}}_i = (\tilde{a}_{i1}, \tilde{a}_{i2}, \ldots, \tilde{a}_{is_i}).$$

$$h(\tilde{\mathbf{b}}, \mathbf{x}) = (h_1(\tilde{\mathbf{b}}_1, \mathbf{x}), h_2(\tilde{\mathbf{b}}_2, \mathbf{x}), \ldots, h_m(\tilde{\mathbf{b}}_{\mathbf{m}}, x))'.$$

where $h_j(\tilde{\mathbf{b}}_j, \mathbf{x})$ is a n-variable function whose variables are $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ with the coefficients

$$\tilde{\mathbf{b}}_j = (\tilde{b}_{j1}, \tilde{b}_{j2}, \ldots, \tilde{b}_{jp_j}),\ j = 1, 2, \ldots, m.$$

All the fuzzy coefficients are called "fuzzy coefficients" of FCP. Definition and solution related to fuzzy optimization are given as follows

1) For any given degree of membership $\lambda(0 \le \lambda \le 1)$, corresponding λ-cut set of any fuzzy number $\tilde{e}$ is a closed interval (the λ -cut set of $\tilde{e}$ is defined as support set of the fuzzy number when $\lambda = 0$), i.e. $\tilde{e}(\lambda) = [e'(\lambda), e''(\lambda)]$ where real number $e'(\lambda) \le e''(\lambda)$.

2) For any fuzzy vector $\tilde{\mathbf{e}} = (\tilde{e}_1, \tilde{e}_2, \ldots, \tilde{e}_n)$, λ -cut set of $\tilde{e}_i$ is $\tilde{e}_i(\lambda) = [e'_i(\lambda), e''_i(\lambda)]$. Let $\tilde{\mathbf{e}}'(\lambda) = [e'_1(\lambda), e'_2(\lambda)), \ldots, e'_n(\lambda)]$, then $\tilde{\mathbf{e}}''(\lambda) = [e''_1(\lambda), e''_2(\lambda), \ldots, e''_n(\lambda)]$. To any value $e_i(\lambda)$ in the interval $[e'_i(\lambda), e''_i(\lambda)]$ there exists a vector $\mathbf{e}(\lambda) = [e_1(\lambda), e_2(\lambda), \ldots, e_n(\lambda)]$ accordingly. For $e_i(\lambda) \in [e'_i(\lambda), e''_i(\lambda)]$, we have $\mathbf{e}(\lambda) \in [\tilde{\mathbf{e}}'(\lambda), \tilde{\mathbf{e}}''(\lambda)]$.

3) For any given degree of membership λ and any number randomly taken from λ-cut set of each fuzzy coefficient, a general rule can be obtained:

$$(\mathrm{P}) \quad \max f(\mathbf{c}(\lambda), \mathbf{x})$$

$$\text{s.t.} \begin{cases} g(\mathbf{a}(\lambda), \mathbf{x}) \le 0 \\ h(\mathbf{b}(\lambda), \mathbf{x}) = 0 \\ x \in D \end{cases} . \tag{3}$$

Eq. (3) is called a "λ-rule" of FCP, in brief $\lambda - \mathrm{P}$, $z^*(\lambda)$ denotes the optimal value of (3) (if (3) has no feasible solution, $z^*(\lambda) = -\infty$ is defined as the optimal value). Where $\mathbf{c}(\lambda) = (c_1(\lambda), c_2(\lambda), \ldots, c_n(\lambda))$, $c_i(\lambda)$ is an arbitrary value in λ-cut set $[c'_i(\lambda), c''_i(\lambda)]$ of fuzzy number $\tilde{c}_i$; $\mathbf{a}_i(\lambda) = (a_{i1}(\lambda), a_{i2}(\lambda), \ldots, a_{\mathbf{is_i}}(\lambda))$ where $a_{iz}(\lambda)$ is an arbitrary value in λ-cut set $[a'_{iz}(\lambda), a''_{iz}(\lambda)]$ of fuzzy number $\tilde{a}_{iz}$, $z = 1, 2, \ldots, s_i$; $\mathbf{b}_j(\lambda) = (b_{j1}(\lambda), b_{j2}(\lambda), \ldots, b_{jp_j}(\lambda))$ where $b_{jw}(\lambda)$ is an arbitrary value in λ-cut set $[b'_{jw}(\lambda), b''_{jw}(\lambda)]$ of fuzzy number $\tilde{b}_{jw}$, $w = 1, 2, \ldots, p_j$.

FCP usually has many $\lambda - \mathrm{P}$ (at least one). So quadratic programming can be obtained:

$$(\mathrm{P}) \quad \max z^*(\lambda)$$

$$\text{s.t.} \begin{cases} \mathbf{c}(\lambda) \in [\mathbf{c}(\lambda)', \mathbf{c}(\lambda)''] \\ \mathbf{a}(\lambda) \in [\mathbf{a}(\lambda)', \mathbf{a}(\lambda)''] \\ \mathbf{b}(\lambda) \in [\mathbf{b}(\lambda)', \mathbf{b}(\lambda)''] \end{cases}. \tag{4}$$

Variable $\mathbf{x}$ obtained here is "λ-optimal solution" of FCP shown as (2), denoted as $\mathbf{x}(\lambda)$. And $\tilde{\mathbf{X}} = \{\mathbf{x}(\lambda) | 0 \leq \lambda \leq 1\}$ is the fuzzy optimal solution of programming (2).

It can be concluded from above that a fuzzy coefficient programming problem can be transformed to a conventional quadratic programming problem. However the problem of quadratic programming is very complicated. To some FCP that meet with some conditions, a simple solution can be obtained according to the following theorem.

Theorem [26]: if $\mathbf{x} \in D$, $f(\mathbf{c}, \mathbf{x})$ and $g(\mathbf{a}, \mathbf{x})$ are non-decline functions of c and a, $h(\mathbf{b}, \mathbf{x})$ is a non-decline continuous function, λ-optimal programming (4) and conventional programming (5) have the same optimal values.

$$(\mathrm{P}) \quad \max f(\mathbf{c}(\lambda)'', \mathbf{x})$$

$$\text{s.t.} \begin{cases} g((\mathbf{a}(\lambda))', \mathbf{x}) \leq 0 \\ h((\mathbf{b}(\lambda))', \mathbf{x}) \leq 0 \\ h((\mathbf{b}(\lambda))'', \mathbf{x}) \geq 0 \\ \mathbf{x} \in \mathbf{D} \end{cases}. \tag{5}$$

In the same way, for minimizing fuzzy optimization problem

$$(\mathrm{P}) \quad \min f(\mathbf{c}, \mathbf{x})$$

$$\text{s.t.} \begin{cases} g(\mathbf{a}, \mathbf{x}) \leq 0 \\ h(\mathbf{b}, \mathbf{x}) = 0 \\ \mathbf{x} \in D \end{cases}. \tag{6}$$

if $\mathbf{x} \in D$, $f(\mathbf{c}, \mathbf{x})$ and $g(\mathbf{a}, \mathbf{x})$ are non-decline functions of c and a respectively and $h(\mathbf{b}, \mathbf{x})$ is a non-decline continuous function of b, λ-optimal programming (6) and conventional programming (7) have the same optimal values.

$$(\mathrm{P}) \quad \min f(\mathbf{c}(\lambda)', \mathbf{x})$$

$$\text{s.t.} \begin{cases} g((\mathbf{a}(\lambda))', \mathbf{x}) \leq 0 \\ h((\mathbf{b}(\lambda))', \mathbf{x}) \leq 0 \\ h((\mathbf{b}(\lambda))'', \mathbf{x}) \geq 0 \\ \mathbf{x} \in \mathbf{D} \end{cases}. \tag{7}$$

3 FUZZY COEFFICIENT OPTIMIZATION OF FUZZY EQUILIBRIUM

The problem of elasticity finite element's calculation can be summed up to the following problem of quadratic optimization [27]:

$$\begin{aligned} &\min_{x\in\Omega} \prod = \frac{1}{2}\delta^{\mathrm{T}}\mathbf{K}\delta - \mathbf{q}^{\mathrm{T}}\delta \\ &\text{s.t} \quad \mathbf{A}\delta = \overline{\delta}(\text{in } s_u) \end{aligned}. \tag{8}$$

where **K** is global stiffness matrix, δ is nodal displacement vector, **q** is nodal load vector, **A** is constraint matrix.

There are many coefficients in practical engineering analysis, such as Poisson's ratio and elastic modulus of material, structure size, boundary conditions and load are all fuzzy. Under common conditions, nodal displacement constraints are considered to be known clearly, i.e. they have no fuzziness. Then the problem of elasticity finite element can be summed up to the following quadratic fuzzy coefficient optimization problem:

$$\begin{aligned} (\mathrm{P}) \quad &\min_{x\in\Omega} \prod = \frac{1}{2}\delta^{\mathrm{T}}\tilde{\mathbf{K}}\delta - \tilde{\mathbf{q}}^{\mathrm{T}}\delta \\ &\text{s.t} \quad \mathbf{A}\delta = \overline{\delta}(\text{in } s_u) \end{aligned}. \tag{9}$$

The equation is a quadratic programming problem. To simplify the problem with the above theorem, the equation can be changed to

$$\begin{aligned} (\mathrm{P}) \quad &\min_{x\in\Omega} \prod = \frac{1}{2}\delta^{\mathrm{T}}\tilde{\mathbf{K}}\delta + \hat{\tilde{\mathbf{q}}}^{\mathrm{T}}\delta \\ &\text{s.t} \quad \mathbf{A}\delta = \overline{\delta}. \end{aligned}. \tag{10}$$

where $\hat{\tilde{\mathbf{q}}} = -\tilde{\mathbf{q}}$.

Here $f = \frac{1}{2}\delta^{\mathrm{T}}\tilde{\mathbf{K}}\delta + \hat{\tilde{\mathbf{q}}}^{\mathrm{T}}\delta$; f is a non-decline function of $\tilde{\mathbf{K}}$ and $\hat{\tilde{\mathbf{q}}}^{\mathrm{T}}$. According to the theorem quadratic fuzzy coefficient programming is transformed to conventional programming problem:

$$\begin{aligned} (\mathrm{P}) \quad &\min \frac{1}{2}\delta^{\mathrm{T}}\tilde{\mathbf{K}}(\lambda)'\delta + \hat{\tilde{\mathbf{q}}}(\lambda)'^{\mathrm{T}}\delta \\ &\text{s.t} \quad \mathbf{A}\delta = \overline{\delta} \end{aligned}. \tag{11}$$

where λ ranges at [0,1].

Equality constraint finite element problem (11) can be transformed to unconstraint finite element problem: nodal stiffness matrix is transformed to global stiffness matrix; the element at intercross of the ith row and ith column in global stiffness matrix is deleted if the ith displacement has constraints, then a global stiffness matrix of reduced order is obtained, denoted as $\tilde{\mathbf{K}}_a(\lambda)'$;

if $\overline{\delta} = 0$, the ith component is deleted from the line vector of load, then load vector of reduced order is obtained; if $\overline{\delta} \neq 0$, the ith component is deleted from the line vector of load vectors and rectifying calculation is applied on the vectors, then line vectors of load of reduced order is obtained, denoted as $\hat{\tilde{\mathbf{q}}}_a(\lambda)^{\mathrm{T}}$. Thus constraint optimization is transformed to unconstraint optimization.

4 NEUROCOMPUTING BASED ON FUZZY COEFFICIENT OPTIMIZATION

It can be seen from above analysis that the problem of solving fuzzy finite element equations can be transformed to a set of unconstraint optimization problem by using fuzzy coefficient programming, i.e. each λ corresponds to an unconstraint optimal equations. If n values of λ are taken, n different optimization problems have to be solved. It will take lots of time to solve the complicated problem that has thousands of nodes. For neural network can obtain solution of optimum problem within an elapsed time of circuit time-constant, a neural network will be constructed to realize real-time fuzzy finite element algorithm in this section. Each $\lambda - P$ optimization problem will be calculated with the neural network. The energy function in the neural network is denoted as $E(\lambda)$, and $E(\lambda)$ is taken as objective functions of $\lambda - P$ optimization, expressed by:

$$E(\lambda) = \frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{m}\tilde{\mathbf{K}}_a(\lambda)'_{ij}\delta_i\delta_j + \sum_{i=1}^{m}\hat{\tilde{q}}_a(\lambda)'_i\delta_i. \tag{12}$$

whose dynamical equation is:

$$\frac{\mathrm{d}\delta_{\mathrm{i}}}{\mathrm{d}t} = -\frac{\mathrm{d}E(\lambda)}{\mathrm{d}\delta_i} = -\sum_{j=1}^{m}\tilde{\mathbf{K}}_a(\lambda)'_{ij}\delta_j - \hat{\tilde{\mathbf{q}}}_a(\lambda)'_i. \tag{13}$$

In view of

$$\begin{aligned}\frac{\mathrm{d}E(\lambda)}{\mathrm{d}t} &= \sum_{i=1}^{m}\frac{\mathrm{d}\delta_i}{\mathrm{d}t}\sum_{j=1}^{m}\tilde{\mathbf{K}}_a(\lambda)'_{ij}\delta_j + \sum_{i=1}^{m}\hat{\tilde{\mathbf{q}}}_a(\lambda)'_i\frac{\mathrm{d}\delta_i}{\mathrm{d}t}\\ &= -\sum_{i=1}^{m}\frac{\mathrm{d}\delta_i}{\mathrm{d}t}\left(-\sum_{j=1}^{m}\tilde{\mathbf{K}}_a(\lambda)'_{ij}\delta_j - \hat{\tilde{\mathbf{q}}}_a(\lambda)'_i\right)\\ &= -\sum_{i=1}^{m}\frac{\mathrm{d}\delta_i}{\mathrm{d}t}\frac{\mathrm{d}\delta_i}{\mathrm{d}t}\\ &= -\sum_{i=1}^{m}\left(\frac{\mathrm{d}\delta_i}{\mathrm{d}t}\right)^2 \leq 0\end{aligned}$$

When $\frac{\mathrm{d}E(\lambda)}{\mathrm{d}t} = 0$, we have

$$\frac{\mathrm{d}\delta_i}{\mathrm{d}t} = 0, i = 1, 2, \ldots, m.$$

which shows that minimum energy point is coincident with the point $\frac{\mathrm{d}\delta_i}{\mathrm{d}t} = 0$. To a quadratic optimization problem, there is a unique minimum solution when $\tilde{\mathbf{K}}(\lambda)'$ is a positive definite matrix, i.e. neural network will converge to global minimum value of the optimization.

5 NUMERICAL EXAMPLE

Following example shows the construction of neural network and its application on the solution of fuzzy finite element based on fuzzy coefficient programming.

As shown in Fig. 1, a uniform rectangle sheet, thickness $h = 1mm$, one end is fixed, $\tilde{q}$ is applied load on the other end and $\tilde{q}$ is a triangle fuzzy number (0.98,1,1.02) kN/m, length 2 m, width 1 m, Young's modulus $\tilde{E}$ is a triangle fuzzy numbers (204,206,208) $\times 10^3$MPa, Poisson's ratio $\mu = \frac{1}{3}$. Gravitation is not taken into consideration. Try to find each point's displacement.

Equations constructed are expressed by submatrix, i.e.

$$\tilde{\mathbf{K}}\boldsymbol{\delta} = \tilde{\mathbf{F}}. \tag{14}$$

where

$$\tilde{\mathbf{K}} = \begin{bmatrix} \tilde{K}_{11} & \tilde{K}_{12} & \tilde{K}_{13} & \tilde{K}_{14} \\ \tilde{K}_{21} & \tilde{K}_{22} & \tilde{K}_{23} & \tilde{K}_{24} \\ \tilde{K}_{31} & \tilde{K}_{32} & \tilde{K}_{33} & \tilde{K}_{34} \\ \tilde{K}_{41} & \tilde{K}_{42} & \tilde{K}_{43} & \tilde{K}_{44} \end{bmatrix}$$
$$= \frac{3\tilde{E}h}{32} \begin{bmatrix} 7 & 0 & -3 & 2 & 0 & -4 & 4 & 2 \\ 0 & 13 & 2 & -1 & -4 & 0 & 2 & -12 \\ -3 & 2 & 7 & -4 & -4 & 2 & 0 & 0 \\ 2 & -1 & -4 & 13 & 2 & -12 & 0 & 0 \\ 0 & -4 & -4 & 2 & 7 & 0 & -3 & 2 \\ -4 & 0 & 2 & -12 & 0 & 13 & 2 & -1 \\ 4 & 2 & 0 & 0 & -3 & 2 & 7 & -4 \\ 2 & -12 & 0 & 0 & 2 & -1 & -4 & 13 \end{bmatrix}$$

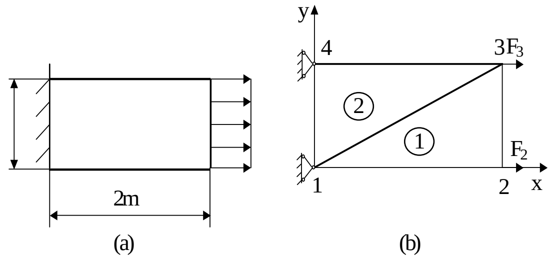

FIGURE 1
Load of rectangle sheet and finite element diagram.

$$\delta = \begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \end{bmatrix} = \begin{bmatrix} x_1 \\ y_1 \\ x_2 \\ y_2 \\ x_3 \\ y_3 \\ x_4 \\ y_4 \end{bmatrix}, \tilde{\mathbf{F}} = \begin{bmatrix} \tilde{F}_1 \\ \tilde{F}_2 \\ \tilde{F}_3 \\ \tilde{F}_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \frac{\tilde{q}}{2} \\ 0 \\ \frac{\tilde{q}}{2} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

where $\delta_1 = 0$, $\delta_4 = 0$.

For constraints $\delta_1 = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ and $\delta_4 = \begin{bmatrix} x_4 \\ y_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, according to the method to resolve finite element equations these rows and columns were deleted where displacement is 0 in $\tilde{\mathbf{K}}$, corresponding component in $\tilde{\mathbf{F}}$ is deleted, then equilibrium equation can be obtained:

$$\tilde{\hat{\mathbf{K}}}\boldsymbol{\delta} = \tilde{\hat{\mathbf{F}}}$$

$$\tilde{\hat{\mathbf{K}}} = \begin{bmatrix} \tilde{K}_{22} & \tilde{K}_{23} \\ \tilde{K}_{32} & \tilde{K}_{33} \end{bmatrix} = \frac{3\tilde{E}h}{32} \begin{bmatrix} 7 & -4 & -4 & 2 \\ -4 & 13 & 2 & -12 \\ -4 & 2 & 7 & 0 \\ 2 & -12 & 0 & 13 \end{bmatrix}$$

$$\boldsymbol{\delta} = \begin{bmatrix} \delta_2 \\ \delta_3 \end{bmatrix} = \begin{bmatrix} x_2 \\ y_2 \\ x_3 \\ y_3 \end{bmatrix}$$

$$\tilde{\hat{\mathbf{F}}} = \begin{bmatrix} \tilde{F}_2 \\ \tilde{F}_3 \end{bmatrix} = \begin{bmatrix} \frac{\tilde{q}}{2} \\ 0 \\ \frac{\tilde{q}}{2} \\ 0 \end{bmatrix}$$

The equations are substituted into (9), then the problem to solve fuzzy finite element equilibrium equations is transformed to fuzzy coefficient optimization problem:

$$\min\left(\frac{1}{2}\delta^{\mathrm{T}}\tilde{\hat{\mathbf{K}}}\delta - \tilde{\hat{F}}^{\mathrm{T}}\delta\right). \tag{15}$$

According to the above theorem and (11), fuzzy coefficient extreme value of (13) can be transformed to a set of conventional extreme value problems:

$$\min\left(\frac{1}{2}\delta^{\mathrm{T}}\hat{\mathbf{K}}(\lambda)'\delta + (-\hat{\mathbf{F}}^{\mathrm{T}})(\lambda)'\delta\right). \tag{16}$$

where $\lambda = 1 - i/n, i = 0, 1, 2, \ldots, n$.

For different λ, $\lambda - \mathrm{P}$ problem can be resolved through neural network.

Optimizing calculation of neural network is composed of the following parts:

1) integrator.
2) reverse adder.
3) feedback loop.
4) reverse controller to realize negative resistance.

Instantaneous simulation of circuit was operated on Protel98/Sim98 workstation, results are shown in Table 1.

Results of simulation are the same to the results calculated on Matlab workstation.

TABLE 1
Results of Simulation

λ	x_2	y_2	x_3	y_3
1.0	9.63	1.73	8.71	0.115
0.9	9.71	1.85	8.74	0.215
0.8	9.80	1.98	8.78	0.324
0.7	9.88	2.12	8.81	0.446
0.6	9.98	2.27	8.84	0.580
0.5	10.1	2.45	8.87	0.731
0.4	10.2	2.64	8.89	0.899
0.3	10.3	2.85	8.92	1.09
0.2	10.4	3.10	8.94	1.30
0.1	10.5	3.37	8.95	1.55
0.0	10.7	3.69	8.97	1.83

unit: 10^{-6} m

6 CONCLUSIONS

1) At present, the idea to solve fuzzy finite element equation is as following: fuzziness is introduced into precise finite element equation, then fuzzy finite element equilibrium equation is transformed to a set of crisp interval equations in terms of a set of threshold values λ, and these interval equations are resolved finally.

However so far there is no perfect method to solve these linear interval equations, so there is no efficient method to solve fuzzy finite element equation. This paper develops a new fuzzy finite element method. The method combines fuzzy coefficient programming with the essence of elasticity, i.e. the process of force equilibrium of elastic object is the process of minimizing energy of a quadratic equation.

2) The method developed in the paper has a characteristic that is for each λ value, the programming result is a clear value, which differs from the traditional method by which a calculation result becomes an interval value. Therefore, the computation result is a common set, which is composed by the fuzzy coefficient programming solution corresponding to different value λ, not a clear value for one, also not a fuzzy quantity. However, the physical meaning of value λ can be treated as a kind of measure of the structural parameter fuzziness, whether strong or weak. Take a value λ smaller, the structural parameter is more fuzzy, whereas the structural parameter is more clear. From the point of practical engineering problems, although the structural parameter takes fuzziness, a clear calculation value is required in terms of value λ in the structural design. Therefore the method developed in this paper is more valuable in engineering application.

3) Results show that the results obtained in this paper is consistent with the results obtained with conventional finite element method when $\lambda = 1$. It is shown that $x2$, $y2$, $x3$ and $y3$ increase when λ decreases, which shows that the method developed in the paper has great computing capability to the fuzzy information in Young's modulus decreasing direction. It is the concrete embodiment of fuzzy coefficient programming. The physical meaning is that these parameters which can minimize the global potential energy of system are chosen when interval equations are solved.

4) Furthermore, it is shown that $x2$, $y2$, $x3$ and $y3$ increase when λ decreases, which shows that stiffness of structure decreases when fuzzy information is introduced. From the point of safety, it is advantage to improve safety of the system. It is considered that parameter fuzziness is derived from the limitation of people's cognize. Thus, the more limited the people's cognize is, the stronger the structural parameter fuzziness is, and the safer design strategy should be adopted.

5) With the problem of finding solution becomes more complicated, parallel computation of finite element equation has become an important direction in finite element field. The proposed method introduces the idea of energy into

the calculation of fuzzy equations, which makes it possible to apply neural network optimization on fuzzy finite element field. Case study shows that the results of circuit simulation are the same as the results calculated on Matlab workstation. From the results of instantaneous analysis of circuit, it can be seen that circuit system can reach equilibrium within 200 μs, which embodies the advantage of neural network in solving real-time complicated structure problems.

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