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FUZZY MAPPING BETWEEN PHYSICAL DOMAIN AND FUNCTION DOMAIN IN DESIGN PROCESS

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The contents and meanings of mapping relationships between physical domain and function domain in different design stages, such as conceptual design, detail design and enhancement design, were analyzed. According to the analysis results, the fuzzy mapping between physical domain and function domain in different design stages was established by integrating objective information and subjective information in the design process, such as expert's knowledge, designer's preferences and customer's requirements, where the fuzzy sets, fuzzy mapping and fuzzy transition were used. The property table of structure behavior parameters was established based on the fuzzy mapping relationships. The fuzzy mapping is very important to optimize product structure, perfect product function, meet the diversified and individual requirements of customers. Finally, an example was used to illustrate the fuzzy mapping relationships between physical domain and function domain in different design stages.

Keywords: Function domain; physical domain; fuzzy mapping; fuzzy transition; property table.

1. Introduction

Axiomatic design describes the design process as the mapping activity between four domains, that is customer domain (specified by product attributes from the customer's viewpoint), functional domain (defined by design requirements), physical domain (defined by design parameters), and process domain (specified in terms of manufacturing and assembly processes and identified as process variables)[1,2,3]. Design is an optimization process to transform customer requirements into design specifications, design specifications into physical solutions, and design solutions into manufacturing processes, by which customer requirements were satisfied. Therefore, there exist some mapping relationships between functional domain and physical domain. These relationships not only reflect the importance degree of function parameters and physical parameters, but also describe the matching relationships between them[4,5,6]. However, due to the complexity of product design process, such as dynamic uncertainty, fuzzy multi-factors and constraints coupling, it is very difficult to describe quantificational mapping relationships between functional domain and physical domain. Moreover, because the design requirements and emphases of different design stages are not the same, the description of mapping relationships is also different. Fuzzy set theory was view as a stronger tool to deal with the difficulty above. Feng used fuzzy sets and fuzzy logic to develop two mapping problems between design requirement and function in detail design[7,8]. Feng developed a methodology based on fuzzy set theory to model the mapping of feature-related functions into features in detail design[9]. Catania established a fuzzy mapping system based on inference rules to solve the problem of mapping in some significant cases[10]. Hu developed fuzzy function-feature mapping in concurrent design and manufacturing consultation[11]. In this paper, the quantitative mapping relationships between functional domain and physical domain in different design stage, such as conceptual design, detail design and enhancement design, were analyzed by using fuzzy sets, fuzzy mapping and fuzzy transition. Based on the analysis results, the property table of design parameters in physical domain was established, which was viewed as reference to guide the whole process of product design.

2. Relationship between Functional Domain and Physical Domain

Functional domain is defined by design requirement, known as purpose and function of the product that takes the form of specifications. Physical domain is specified by the state diversification of product structure, i.e. structure behavior. The mapping relationships between functional domain and physical domain can be expressed by the mapping between function requirements and structure behavior of the product. Parts and components and their composing and position relationships can be described by product structure. The state of composing and position, such as axial fixation mode of axis, layout of pars, bearing style, lubricating, seal, preloading, etc, can be described by structure behavior. There exist some relationships in structure behavior, such as hierarchy, composing, time series, cause and effect, space, and so on. These relationships have strong influence on the realization of product function in different aspects and degree. So, the expression of mapping relationships between functional domain and physical domain is based on the information description of product function and structure, and it also offers a method to realize the function transform by structure behavior in different design stages[12,13].

Let Q be function requirement universe, $Q=\{q_1,q_2,\cdots,q_n\}$, and B be structure behavior parameter universe, $B=\{b_1,b_2,\cdots,b_m\}$. There exist some relationships between Q and B as follows[1]:

(i) The mapping relationship R_{qb} from Q to B is more to more, and the range of R_{qb} is equal to that of B.

(ii) One function requirement can be realized by several behaviors, and one behavior can affect or bring many function requirements.

(iii) Any behavior could bring certain function to some extent. But there exists $q \in Q$, makes $b \in B$, $\langle q,b \rangle \notin R_{qb}$. That is to say, there isn't certain behavior in B which could map to the function. Where q is called as invalid function.

In nature, the mapping activity from physical domain to functional domain is to realize the function requirements in functional domain by configuring the structure behavior parameters in physical domain. That is to realize the following mapping:

$$f : B \to Q\,. \tag{1}$$

This is a more to more mapping.

From the view of mathematics, function requirement Q is the function of behavior parameter B, and it can be called as function expression. In general, function is a multi-variable function of behavior parameters, and it can be expressed as follows:

$$f_i(b_1, b_2, \cdots, b_m) = q_i \quad (i = 1, 2, \cdots, n). \tag{2}$$

Here, b_i ($i = 1, 2, \cdots, m$) is explanatory variable, and its variation will make the corresponding variation of q. In design process, q is design goal. The design process is to realize q by optimizing design resources, which is implemented through the variation and combination optimization of b_i. Therefore, the variation of b_i shouldn't weak q, but make q be realized well.

The mapping matrix from physical domain to functional domain can be gotten by differential of function expression[2].

$$\begin{aligned}
\Delta f_1 &= \frac{\partial f_1}{\partial b_1}\Delta b_1 + \frac{\partial f_1}{\partial b_2}\Delta b_2 + \cdots + \frac{\partial f_1}{\partial b_m}\Delta b_m \\
\Delta f_2 &= \frac{\partial f_2}{\partial b_1}\Delta b_1 + \frac{\partial f_2}{\partial b_2}\Delta b_2 + \cdots + \frac{\partial f_2}{\partial b_m}\Delta b_m \\
&\quad \vdots \qquad\qquad \vdots \qquad\qquad\quad \vdots \\
\Delta f_n &= \frac{\partial f_n}{\partial b_1}\Delta b_1 + \frac{\partial f_n}{\partial b_2}\Delta b_2 + \cdots + \frac{\partial f_n}{\partial b_m}\Delta b_m
\end{aligned}$$

Let

$$A_{ij} = \frac{\partial f_i}{\partial b_j}, \ (i = 1, 2, \cdots, n\,;\ j = 1, 2, \cdots, m\,)$$

$$A = \begin{pmatrix} A_{11} & \cdots & A_{1j} & \cdots & A_{1m} \\ \vdots & & \vdots & & \vdots \\ A_{i1} & \cdots & A_{ij} & \cdots & A_{im} \\ \vdots & & \vdots & & \vdots \\ A_{n1} & \cdots & A_{nj} & \cdots & A_{nm} \end{pmatrix} = \frac{\partial(f_1, f_2, \cdots, f_n)}{\partial(b_1, b_2, \cdots b_m)}\,. \tag{3}$$

Mapping matrix A reflects the influence degree of structure behavior parameter b to function requirement q.

However, in different stages of life cycle design, the contents and meanings of mapping relationships between physical domain and function domain are different in essence. In conceptual design stage, the goal of mapping activity is to identify important function and structure behavior, their influence degree to product development and the relationships between them. Based on the analysis results, the design linchpin is to be emphasized, and the product structure is to be optimized too. In detail design stage, the intention of mapping activity is to specify the state of structure behavior parameters and its influence degree to function realization, by which the structure behavior parameters can be design reasonably. In enhancement design stage, the goal of mapping activity is to find the behavior parameters that have stronger influence on function or are very sensitive to the variation of function. By modifying and adjusting these design parameters, it can make the function be perfected more.

3. The Disadvantages of Traditional Mapping Analysis between Functional Domain and Physical Domain

The disadvantages of traditional mapping analysis between functional domain and physical domain are summarized as follows:

(i) It only can be applied to qualitative analysis of the relationships between functional domain and physical domain. But couldn't identify the important function requirements and behavior parameters and the stronger relationship between function and behavior.

(ii) It is so difficult to grasp the designer's preferences that the increasingly diversified and individual requirement of customers can't be satisfied.

(iii) It is very difficult to meet the requirements of different design stages in life cycle.

(iv) Its bad flexibility makes the interactive collaboration of design process very difficult.

Generally speaking, the importance degree of function requirement or structure behavior can be classified into two different categories: objective importance degree and subjective importance degree. Objective importance degree is one that it is taken into account by the design itself. For example, in the development process of a loader, the importance degree of operation reliability, excavation force, maximal height and distance of unload and other indices should be higher than that of weight, shape, size, and so on. Because the design modeling hinges on the objective importance degree, it can be grasped easily. Subjective importance degree is one that it is taken into account based on designer's preferences, such as psychology, taste, etc. Different people have different preferences, so it is very difficult to grasp the subjective importance degree of function requirements and behavior parameters. However, with the development of society and the improvement of customer's requirement, subjective importance degree has stronger influence on the competition capability of the product than before.

Therefore, in order to improve the competition capability of product, behavior science was introduced into design decision-making system[1]. Objective information and subjective information, such as expert's knowledge, designer's preferences and customer's requirements, have strong influence on design decision-making. When the mapping relationships between function domain and physical domain are analyzed, these objective and subjective information should be emphasized and integrated in the design decision-making process. During the design process, especially in the conceptual design stage, because design information is fuzzy, uncertain, imperfection or the design process may be not well defined, it will be difficult to use conventional mathematical models to handle the information of the design process. Fuzzy set theory provides a viable tool to solve above problems. Fuzzy sets, fuzzy mapping and fuzzy transition were used to deal with the complicated mapping relationships between function and behavior. Based on the analysis results, the important design parameters can be identified. It is very important to improve the quality of product development.

4. Fuzzy Mapping between Functional Domain and Physical Domain

4.1 *Theoretical background*[7,14,15]

Definition 4.1. Binary fuzzy relations

Let U, V be two universes. Let $\tilde{R}$ be a fuzzy relation on $U\times V$. $\tilde{R}$ is also a fuzzy set on $U\times V$. The membership function of $\tilde{R}$ is expressed by $\mu_{\tilde{R}}(u,v)$, where $u\in U$, $v\in V$.

A fuzzy relation $\tilde{R}$ on $U\times V$ can be represented as a matrix $[R]$.

$$[R] = [r_{ij}]_{m\times n}, \quad r_{ij} = \mu_{\tilde{R}}(u,v), \, (i=1,2,\cdots,m;\ j=1,2,\cdots,n). \tag{4}$$

Where the related universes U and V are finite.

The composition of two fuzzy relations $\tilde{R}$ and $\tilde{W}$ on $U\times V$ and $V\times W$, respectively, can be expressed as:

$$\mu_{\tilde{R}\circ\tilde{W}}(u,w) = \sup_{v\in V}\min\left[\mu_{\tilde{R}}(u,v),\ \mu_{\tilde{W}}(v,w)\right], \quad \forall\ u\in U, \quad \forall\ w\in W. \tag{5}$$

Definition 4.2. Level fuzzy sets

Let A be a fuzzy set, and A_α be a α-cut of A. The level fuzzy sets of A can be defined as the fuzzy sets $\tilde{A}_\alpha$, $\alpha\in(0,1]$, such that

$$\tilde{A}_\alpha = \{[u,\mu_A(u)],\ u\in A_\alpha\}. \tag{6}$$

Theorem 4.1. Given fuzzy relationship $\tilde{R}\in F(U\times V)$, fuzzy mapping can be defined only by $\tilde{R}$

$$\tilde{f} = \tilde{f}_{\tilde{R}}:\quad U\to V. \tag{7}$$

To any $u\in U$, where

$$\tilde{R}\,|\,u = \tilde{f}_{\tilde{R}}(u). \tag{8}$$

In reverse, given fuzzy relationship $\tilde{R}\in F(U\times V)$, that is $\tilde{R} = \tilde{R}_{\tilde{f}}$

To any $u\in U$, where

$$\tilde{f}(u) = \tilde{R}_{\tilde{f}}\,|\,u. \tag{9}$$

Theorem 4.2. Given $\tilde{R}\in F(U\times V)$, the fuzzy transition from U to V can be defined only, that is

$$T_{\tilde{R}}:\quad F(U)\to F(V). \tag{10}$$

To any $\tilde{A}\in F(U)$, where

$$T_{\tilde{R}}(A) = \tilde{A}\circ\tilde{R} \quad \in F(V). \tag{11}$$

Where

$$\mu_{\tilde{A}\circ\tilde{R}}(v) \overset{\Delta}{=} \bigvee_{u\in U}\left(\mu_{\tilde{A}}(u)\wedge\mu_{\tilde{R}}(u,v)\right) \quad v\in V. \tag{12}$$

For more detail notions and theory about fuzzy sets and fuzzy relations, one can see

Refs. [7,14,15].

4.2 *Conceptual design stage*

In conceptual design stage, the product is highly abstract. To design its basic structure, it needs to identify important function requirements, important structure behavior parameters and relationships between them.

Let Q_I be important function requirement set, and it is a fuzzy subset of Q, $Q_I \subseteq Q$. $\mu_I(q)$ is the degree of membership of function requirement q in Q_I. $\mu_I(q)$ can be given based on designer's subjective preference. Q_I can be denoted by

$$Q_I = \{[q, \mu_I(q)], \; q \in Q\}. \tag{13}$$

Let B_I be important structure behavior parameter set, and it is a fuzzy subset of B, $B_I \subseteq B$. $\mu_I(b)$ is the degree of membership of important structure behavior parameter b in B_I. B_I can be written as:

$$B_I = \{[b, \mu_I(b)], \; b \in B\}. \tag{14}$$

The binary fuzzy relationship in two universes Q and B is also a fuzzy set on $Q \times B$. Thus, there exist following two fuzzy relationships in $Q \times B \overset{\Delta}{=} \{(q,b) \mid q \in Q, b \in B\}$.

(i) Fuzzy relationship R^O. It describes that structure behavior b meets function requirement q. The relationship R^O depicts a general relationship between function requirements and structure behavior parameters. It doesn't depend on the subjective purpose of designer, and only reflects the real dependency of function on behavior. Its membership function is denoted by $\mu_{R^O}(q,b)$, and $\mu_{R^O}(q,b)$ can be estimated in advance by domain experts. R^O can be represented by fuzzy relationship matrix $[R^O]$ as follows:

$$[R^O] = [r_{ij}^O]_{m \times n} \quad r_{ij}^O = \mu_{R^O}(q_i, b_j) \quad (i = 1,2,\cdots,m\,;\, j = 1,2,\cdots,n). \tag{15}$$

(ii) Fuzzy relationship R^I. It describes that structure behavior parameter b is important in meeting function requirement q. Where the designer's subjective preferences are taken into account. Compared with fuzzy relationship R^O, R^I is limited to a narrow range regarding the subjective importance of the function requirement. Its membership function is denoted by $\mu_{R^I}(q,b)$. R^I can be represented by fuzzy relationship matrix $[R^I]$ as follows:

$$[R^I] = [r_{ij}^I]_{m \times n} \quad r_{ij}^I = \mu_{R^I}(q_i, b_j) \quad (i = 1,2,\cdots,m\,;\; j = 1,2,\cdots,n). \tag{16}$$

Every function requirement has corresponding structure behavior that has strong influence on it. In order to identify important function requirement, this strong influence must be avoided. So, the membership function of $\mu_{R^I}(q,b)$ can be constructed as follows[7]:

$$\mu_{R^I}(q,b) = \begin{cases} \mu_I(q) & , \quad \mu_{R^O}(q,b) \geq \mu_I(q) \\ \mu_{R^O}(q,b) & , \quad \text{otherwise} \end{cases}. \tag{17}$$

The intention of mapping activity is to identify important structure behavior parameters that have stronger influence on function from B though the fuzzy relationship R^I between functional domain and physical domain.

The structure behavior subset B_I can be formalized as a composition of Q_I and R^I.

i.e.

$$B_I = Q_I \circ R^I . \tag{18}$$

According to Theorem 4.2, one has

$$\mu_{B_I}(b) = \mu_{Q_I \circ R^I}(b) = \sup_q \min\left[\mu_I(q),\ \mu_{R^I}(q,b)\right] . \tag{19}$$

In conceptual design process, not all the problems should be treated with equally. It is some key and important problems that should be emphasized, and some unimportant should be ignored. Thus the design problem can be solved easily. Given different threshold λ ($\lambda \in [0,1]$) based on different requirements, the level fuzzy set B_λ of the different importance degree of behavior parameters can be gotten as follows:

$$B_\lambda = \left\{ b,\ \mu_{B_I}(b) \geq \lambda \right\} . \tag{20}$$

4.3 *Detail design stage*

In detail design stage, it needs to design the state of structure behavior parameters, such as grade of tolerance and fit, positioning accuracy, surface roughness, lubricating state, preloading, and so on. It needn't adopt higher grade or accuracy to meet function requirements when the moderate one can meet these requirements. Otherwise, it will make the development cost and time increase, and the design may not be the best one. Take the accuracy design of gear transmission as example, though higher accuracy is good to the performance of transmission, it needn't select higher accuracy than grade 8 because grade 8 of tolerance group II can meet design requirements. The higher the accuracy is, the higher the manufacturing cost is. Moreover, under some special circumstance, higher state may not bring higher satisfaction degree.

In this part, the following two factors should be taken into account.

(i) Designer's preferences to the degree of function satisfaction.

(ii) The influence of structure behavior parameters to function realization.

Designer's preferences (M_i) to the degree of function satisfaction and the state of structure behavior parameters (N_i) can be classified into five different categories respectively as Table 1 shown.

Table 1. The categories of designer's preferences and state of structure behavior parameters

	Categories				
i	1	2	3	4	5
M_i	highly desirable M_1	desirable M_2	tolerable M_3	undesirable M_4	highly undesirable M_5
N_i	higher N_1	high N_2	moderate N_3	low N_4	lower N_5

The function requirement that belongs to one of desirability in Table 1 can be expressed by following fuzzy set.

$$M_i^Q = \left\{ q,\ \mu_{M_i^Q}(q) \right\} \quad (i = 1,2,\cdots,5,\ q \in Q). \tag{21}$$

Where $\mu_{M_i^Q}(q)$ denoted the degree of membership of function requirement that belongs to desirability M_i.

The state of structure behavior parameter that belongs to one of categories in Table 1 can be expressed by following fuzzy set.

$$N_i^B = \left\{ b,\ \mu_{N_i^Q}(b) \right\} \quad (i = 1, 2, \cdots, 5,\ \ b \in B). \tag{22}$$

Where $\mu_{N_i^B}(b)$ denoted the degree of membership of structure behavior parameter state that belongs to class N_i.

There exists certain mapping relationship between M_i^Q and N_i^B, but this relationship is not one to one absolutely. The mapping relationship can be expressed by binary fuzzy relationship R^M in $Q \times B$. R^M denotes that the influence degree of structure behavior parameter state on function satisfaction. Its membership function is denoted by $\mu_{R^M}(q,b)$, and $\mu_{R^M}(q,b)$ can be estimated by domain experts in advance. R^M can be represented by fuzzy relationship matrix $\left[R^M\right]$ too.

N_i^B can be formalized as a composition of M_i^Q and R^M, i.e.

$$N_i^B = M_i^Q \circ R^M \quad (i = 1, 2, \cdots, 5). \tag{23}$$

According to Theorem 4.2, one has

$$\mu_{N_i^B}(b) = \mu_{M_i^Q \circ R^M}(b) = \sup_q \min\left[\mu_{M_i^Q}(q),\ \mu_{R^M}(q,b) \right]. \tag{24}$$

Thus, the state of structure behavior parameters can be defined.

4.4 *Enhancement design stage*

In enhancement design stage, the design has been shaped up basically, and the important parts and components have been also confirmed. The main task of this stage is to find faults and the structure behavior parameters that have stronger influence on function or are very sensitive to the variation of function. Maybe these parameters are not the most important, but modifying them can improve the function of product to some extent. At the same time, it is of avail to find the weakness of design, which can offer help for failure diagnosis in usage stage.

The following two factors should be taken into account:

(i) The importance degree of function requirements. It is taken into account by integrating subjective and objective perspectives.

(ii) The relationships between behavior parameters variation and function variation.

Let B_V be the behavior parameter set that its variation can result in considerable variation of corresponding function. B_V is a fuzzy subset of B, $B_V \subseteq B$. $\mu_{B_V}(b)$ is the degree of membership of b in B_V. B_V can be denoted by

$$B_V = \left\{ \left[b, \mu_{B_V}(b) \right],\ b \in B \right\}. \tag{25}$$

The fuzzy subset Q_I and its degree of membership $\mu_I(q)$ have been defined in section 3.2.

The influence degree of behavior parameter variation on function realization can be described by binary fuzzy relationship R^V in $Q \times B$. Its membership function is denoted by $\mu_{R^V}(q,b)$. R^V can be represented by fuzzy relationship matrix $\left[R^V\right]$ as follows:

$$\left[R^V\right] = \left[r_{ij}^V\right]_{m \times n} \quad r_{ij}^V = \mu_{R^V}(q_i, b_j) \quad (i = 1, 2, \cdots, m;\ \ j = 1, 2, \cdots, n). \tag{26}$$

B_V can be formalized as a composition of Q_I and R^V, i.e.

$$B_V = Q_I \circ R^V . \tag{27}$$

According to Theorem 4.2, one has

$$\mu_{B_V}(b) = \mu_{Q_I \circ R^V}(b) = \sup_q \min\left[\mu_I(q),\ \mu_{R^V}(q,b)\right] . \tag{28}$$

Given different threshold λ ($\lambda \in [0,1]$), the level fuzzy set B_λ of B_V can be gotten as follows:

$$B_\lambda = \left\{b,\ \mu_{B_V}(b) \geq \lambda\right\} . \tag{29}$$

The perturbation degree of structure behavior parameters can be classed into five different categories based on the threshold λ as shown in Table 2.

Table 2. Perturbation degree of structure behavior parameters

	Categories				
	1	2	3	4	5
λ	$\lambda \geq 0.8$	$\lambda \geq 0.6$	$\lambda \geq 0.4$	$\lambda \geq 0.2$	other
degree	bigger	big	moderate	small	smaller

Based on the analysis above, the structure behavior parameters that have stronger influence on function or are very sensitive to the variation of function can be found. These parameters should be emphasized and modified in enhancement design stage to improve product function, cut cost of correctional design, and shorten development time.

5. Establishing Property Table of Structure Behavior Parameters

According to the analysis of fuzzy mapping between physical domain and function domain in different design stages, the property table of structure behavior parameters can be established as Table 3 shown.

Table 3. Property table of structure behavior parameters

No.	Structure behavior parameters	Importance degree	State	Variation
1	Axial fixation model of axis	0.5	moderate	big
2	Radial fixation model of axis	0.9	higher	bigger
...	...	...	...	...

Property table emphasizes on the key and weak factors of product design directly. It can be used to optimize product structure, perfect product function, meet the diversified and individual requirements of customers. Because property table is based on fuzzy mapping relationships between physical domain and functional domain and subjective and objective factors are considered together, it is of avail to improve the competition capability of product.

6. Illustrative Case

Take the design of high-speed gear axis of a reducer as example to illustrate the significance of fuzzy mapping between functional domain and physical domain to product development. The function requirements and structure behavior parameters of high-speed gear axis of a reducer are listed in Table 4 and Table 5.

Table 4. Function requirements of high-speed gear axis of reducer

No.	Function requirements	No.	Function requirements
q_1	Transfer torque	q_6	Support parts on axis
q_2	Transfer thrust	q_7	Insure the working position of parts on axis
q_3	Transfer revolving motion	q_8	Reduce weight
q_4	Transfer axial motion	q_9	Bear bending moment
q_5	Transfer energy	q_{10}	Anti-vibration, anti-impact

Table 5. Structure behavior parameters of high-speed gear axis of reducer

No.	Structure behavior parameters	No.	Structure behavior parameters
b_1	Axial fixation model of axis	b_{10}	Mating of axis and bearing
b_2	Radial fixation model of axis	b_{11}	Lubricating of axis
b_3	Structure design of axis	b_{12}	Lubricating of bearing
b_4	Layout of parts on axis	b_{13}	Seal model of axle end
b_5	Axial fixation model of parts on axis	b_{14}	Surface roughness of axis
b_6	Radial fixation model of parts on axis	b_{15}	The method of reducing stress concentration
b_7	Bearing style	b_{16}	Processing of axis
b_8	Layout of bearing	b_{17}	Assemble process of axis
b_9	Mating of axis and gear		

Subjective information, such as consumer's requirements, designer's preferences, expert's knowledge, has stronger influence on the design of high-speed gear axis of a reducer. To the function requirements listed in Table 4, their $\mu_I(q)$ can be given by designer. But $\mu_I(q)$ depends on the designer's subjective intention. Different designers may assign different values of $\mu_I(q)$ even for the same function requirement. Even a requirement is very important from the objective viewpoint, a designer could assign a low value of $\mu_I(q)$. Therefore, in order to design a good product, the subjective and objective information should be taken into account together in the design process.

Based on the analysis in section 4, the relationship matrices used to identify the importance degree of function requirements in different design stages by integrating subjective and objective knowledge are as follows.

In conceptual design stage, the degree of membership $\mu_I(q)$ and fuzzy relationship matrix $[R^O]$ were given below.

$$\mu_I(q)=\begin{bmatrix}1 & 0 & 1 & 0 & 0.9 & 0.4 & 0.5 & 0.3 & 0.7 & 0.2\end{bmatrix}$$

$$\left[R^{O}\right]=\begin{bmatrix}0.2&0.8&0.8&0.7&0.3&0.7&0.5&0.6&0.6&0.5&0.4&0.6&0&0.1&0&0.2&0.5\\0.9&0.2&0.6&0.5&0.1&0.8&0.6&0.6&0.2&0.2&0&0&0&0&0&0&0\\0.2&0.9&0.7&0.7&0.1&0.7&0.6&0.6&0.7&0.7&0.5&0.7&0.2&0.3&0&0.1&0.5\\1&0.7&0.4&0.8&0.7&0&0.7&0.7&0.1&0.1&0.5&0&0&0.1&0&0&0.5\\0.3&0.7&0.6&0.8&0.2&0.8&0.5&0.5&0.8&0.6&0.5&0.6&0.2&0.2&0&0.1&0.4\\0.3&0.3&0.9&0.8&0.4&0.4&0&0&0&0&0&0&0&0&0.4&0&0.5\\0.8&0.8&0.9&0.8&0.9&0.9&0&0&0&0&0&0&0&0&0.2&0.4&0.3\\0.8&0.2&1&0.7&0.5&0.5&0&0&0&0&0&0&0&0&0.1&0.4&0.1\\0.5&0.6&0.9&0.8&0.1&0.1&0.8&0.8&0.5&0.4&0&0&0&0&0.9&0.1&0\\0.8&0.8&0.7&0.8&0.8&0.6&0.8&0.7&0.5&0.5&0.2&0.2&0.2&0&0.1&0.1&0.7\end{bmatrix}$$

In detail design stage, the degree of membership of function requirement that belongs to M_i was given by the designer based on design requirements and expressed by $\left[M_i^{Q}\right]$ ($i=1,2,\cdots,5$, $q\in Q$). Fuzzy transition relationship matrix $\left[R^{M}\right]$ was also given below.

$$\left[M_1^{Q}\right]=\begin{bmatrix}0.8&0.2&0.2&0.1&0.1&0.2&0.2&0.2&0.2&0.2\end{bmatrix}$$

$$\left[M_2^{Q}\right]=\begin{bmatrix}0.6&0.5&0.8&0.7&0.8&0.7&0.8&0.3&0.8&0.2\end{bmatrix}$$

$$\left[M_3^{Q}\right]=\begin{bmatrix}0.1&0.8&0.2&0.2&0.1&0.2&0.2&0.8&0.1&0.6\end{bmatrix}$$

$$\left[M_4^{Q}\right]=\begin{bmatrix}0&0&0&0&0&0.1&0.1&0.1&0&0.1\end{bmatrix}$$

$$\left[M_5^{Q}\right]=\begin{bmatrix}0&0&0&0&0&0&0&0&0&0\end{bmatrix}$$

$$\left[R^{M}\right]=\begin{bmatrix}0.2&0.9&0.7&0.8&0.1&0.8&0.8&0.8&0.7&0.7&0.6&0.6&0.2&0.3&0.1&0.2&0.4\\0.9&0.2&0.1&0.2&0.5&0.3&0.7&0.7&0.6&0.6&0.2&0.2&0.1&0.2&0.1&0.2&0.5\\0.2&0.8&0.7&0.7&0.7&0.3&0.8&0.8&0.7&0.7&0.6&0.6&0.2&0.3&0.1&0.2&0.3\\0.9&0.7&0.2&0.5&0.7&0.1&0.8&0.8&0.5&0.5&0.1&0.1&0.1&0.2&0.1&0.2&0.4\\0.3&0.3&0.4&0.6&0.7&0.6&0.8&0.8&0.7&0.7&0.5&0.6&0.1&0.1&0.1&0.1&0.4\\0.2&0.2&0.8&0.7&0.2&0.2&0.1&0.1&0&0&0&0&0&0&0.8&0.7&0.5\\0.7&0.7&0.8&0.7&0.6&0.6&0&0&0&0&0&0&0&0&0.7&0.6&0.6\\0.7&0.5&0.9&0.5&0.4&0.4&0&0&0&0&0&0&0&0&0.2&0.4&0.2\\0.1&0.3&0.9&0.8&0.2&0.2&0.7&0.7&0.5&0.4&0&0&0&0&0.9&0.3&0.1\\0.6&0.7&0.6&0.8&0.8&0.6&0.8&0.8&0.7&0.7&0.5&0.5&0.1&0.3&0.2&0.2&0.6\end{bmatrix}$$

In enhancement design stage, $\mu_I(q)$ and fuzzy relationship matrix $\left[R^{V}\right]$ were given below.

$$\mu_I(q)=\begin{bmatrix}1&0&1&0&0.9&0.4&0.5&0.3&0.7&0.2\end{bmatrix}$$

$$\left[R^V\right]=\begin{bmatrix} 0.3 & 0.9 & - & - & 0.2 & 0.7 & 0.8 & 0.8 & 0.6 & 0.6 & 0.4 & 0.4 & 0.2 & 0.1 & - & - & 0.3 \\ 0.9 & 0.3 & - & - & 0.2 & 0.2 & 0.8 & 0.8 & 0.7 & 0.7 & 0.1 & 0.1 & 0.1 & 0.1 & - & - & 0.4 \\ 0.7 & 0.9 & - & - & 0.6 & 0.8 & 0.8 & 0.8 & 0.7 & 0.7 & 0.5 & 0.5 & 0.1 & 0.5 & - & - & 0.5 \\ 0.9 & 0.3 & - & - & 0.2 & 0.2 & 0.8 & 0.8 & 0.7 & 0.7 & 0.1 & 0.1 & 0.1 & 0.1 & - & - & 0.4 \\ 0.5 & 0.9 & - & - & 0.7 & 0.7 & 0.8 & 0.8 & 0.8 & 0.8 & 0.5 & 0.5 & 0.1 & 0.1 & - & - & 0.6 \\ 0.3 & 0.3 & - & - & 0.3 & 0.3 & 0.2 & 0.2 & 0.2 & 0.2 & 0 & 0 & 0 & 0 & - & - & 0.7 \\ 0.8 & 0.8 & - & - & 0.8 & 0.8 & 0.1 & 0.1 & 0.1 & 0.1 & 0 & 0 & 0 & 0.1 & - & - & 0.6 \\ 0.8 & 0.2 & - & - & 0.2 & 0.2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.1 & - & - & 0.3 \\ 0.7 & 0.7 & - & - & 0.8 & 0.6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.3 & - & - & 0.3 \\ 0.8 & 0.8 & - & - & 0.7 & 0.7 & 0.8 & 0.8 & 0.7 & 0.6 & 0.6 & 0.6 & 0.1 & 0.2 & - & - & 0.7 \end{bmatrix}$$

According to Eq. (17), matrix $\left[R^I\right]$ can be obtained as follows.

$$\left[R^I\right]=\begin{bmatrix} 0.2 & 0.8 & 0.8 & 0.7 & 0.3 & 0.7 & 0.5 & 0.6 & 0.6 & 0.5 & 0.4 & 0.6 & 0 & 0.1 & 0 & 0.2 & 0.5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.2 & 0.9 & 0.7 & 0.7 & 0.1 & 0.7 & 0.6 & 0.6 & 0.7 & 0.7 & 0.5 & 0.7 & 0.2 & 0.3 & 0 & 0.1 & 0.5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.3 & 0.7 & 0.6 & 0.8 & 0.2 & 0.8 & 0.5 & 0.5 & 0.8 & 0.6 & 0.5 & 0.6 & 0.2 & 0.2 & 0 & 0.1 & 0.4 \\ 0.3 & 0.3 & 0.4 & 0.4 & 0.4 & 0.4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.4 & 0 & 0.4 \\ 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.2 & 0.4 & 0.3 \\ 0.3 & 0.2 & 0.3 & 0.3 & 0.3 & 0.3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.1 & 0.3 & 0.1 \\ 0.5 & 0.6 & 0.7 & 0.7 & 0.1 & 0.1 & 0.7 & 0.7 & 0.5 & 0.4 & 0 & 0 & 0 & 0 & 0.7 & 0.1 & 0 \\ 0.2 & 0.2 & 0.2 & 0.2 & 0.2 & 0.2 & 0.2 & 0.2 & 0.2 & 0.2 & 0.2 & 0.2 & 0.2 & 0 & 0.1 & 0.1 & 0.2 \end{bmatrix}$$

According to Eq. (19), $\mu_{B_I}(b)$ can be obtained as follows.

$$\mu_{B_I}(b)=\begin{bmatrix} 0.5 & 0.9 & 0.8 & 0.8 & 0.5 & 0.8 & 0.7 & 0.7 & 0.8 & 0.7 & 0.5 & 0.7 & 0.2 & 0.3 & 0.7 & 0.4 & 0.5 \end{bmatrix}$$

According to Eq. (24), the grade of membership of structure behavior parameter state that belongs to class N_i can be obtained and expressed by $\left[N_i^B\right]$ ($i=1,2,\cdots,5$) as follows.

$$\left[N_1^B\right]=\begin{bmatrix} 0.2 & 0.8 & 0.7 & 0.8 & 0.2 & 0.8 & 0.8 & 0.7 & 0.7 & 0.6 & 0.6 & 0.2 & 0.2 & 0.2 & 0.2 & 0.2 & 0.2 \end{bmatrix}$$

$$\left[N_2^B\right]=\begin{bmatrix} 0.7 & 0.7 & 0.8 & 0.8 & 0.7 & 0.6 & 0.8 & 0.8 & 0.7 & 0.7 & 0.6 & 0.6 & 0.2 & 0.3 & 0.8 & 0.7 & 0.6 \end{bmatrix}$$

$$\left[N_3^B\right]=\begin{bmatrix} 0.8 & 0.6 & 0.8 & 0.6 & 0.6 & 0.6 & 0.6 & 0.6 & 0.6 & 0.6 & 0.5 & 0.5 & 0.2 & 0.3 & 0.2 & 0.4 & 0.6 \end{bmatrix}$$

$$\left[N_4^B\right]=\begin{bmatrix} 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \end{bmatrix}$$

$$\left[N_5^B\right]=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

According to Eq. (28), $\mu_{B^V}(b)$ can be obtained as follows.

$$\mu_{B^V}(b)=\begin{bmatrix} 0.7 & 0.9 & - & - & 0.7 & 0.7 & 0.8 & 0.8 & 0.8 & 0.8 & 0.5 & 0.5 & 0.2 & 0.5 & - & - & 0.6 \end{bmatrix}$$

Based on the calculation results above, the property table of structure behavior parameters was established and listed in Table 6 according to Table 1 and Table 2.

Here, "- " denotes that the structure parameter has been confirmed, and it shouldn't be modified or adjusted more in enhancement design stage.

Property table can be used as reference to guide the whole design process, where both the designer's subjective preferences and objective design requirements were taken into account.

Table 6. Property table of structure behavior parameters

No.	Structure behavior parameters	Importance degree	State	Variation
b_1	Axial fixation model of axis	0.5	moderate	big
b_2	Radial fixation model of axis	0.9	higher	bigger
b_3	Structure design of axis	0.8	moderate	-
b_4	Layout of parts on axis	0.8	high	-
b_5	Axial fixation model of parts on axis	0.5	high	big
b_6	Radial fixation model of parts on axis	0.8	higher	big
b_7	Bearing style	0.7	high	big
b_8	Layout of bearing	0.7	high	big
b_9	Mating of axis and gear	0.8	high	big
b_{10}	Mating of axis and bearing	0.7	high	big
b_{11}	Lubricating of axis	0.5	high	moderate
b_{12}	Lubricating of bearing	0.7	high	moderate
b_{13}	Seal model of axle end	0.2	low	smaller
b_{14}	Surface roughness of axis	0.3	low	moderate
b_{15}	The method of reducing stress concentration	0.7	high	-
b_{16}	Processing of axis	0.4	high	-
b_{17}	Assemble process of axis	0.5	moderate	big

7. Conclusions

By the analysis of fuzzy mapping relationships between functional domain and physical domain, both the important and unimportant design parameters are to be identified. According to the fuzzy mapping relationships in different design stages, the property table of structure behavior parameters is established, and it can be used to guide the whole process of design. During the development process, the fuzzy mapping relationships and the property table can be changed based on the dynamic development environment to make the design information self-supplement, which can make the function of product perfect. Moreover, by the analysis of fuzzy mapping relationships, it is of avail to reduce the frequent modification, cut development cost, and shorten development time.

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References

1. J. S. Deng, X. J. Han and X. Zeng, *Theory, Method and Technology of Product Concept Design* (China Mechine Press, Beijing, 2002).
2. H. Vigain, N. Mats and T. Derrick, "Decision making software tools for product development based on axiomatic design theory", *Annals of the CIRP* **45** (1996) 135-139.
3. D. R. Wallace and N. P. Suh, "Information-based design for environmental problem solving", *Annals of the CIRP* **42** (1993) 175-180.

4. A. Kusiak and J. Wang, "Decomposition of the design process", *Transactions of ASME, Journal of Mechanical design* **115** (1993) 687-695.
5. A. Kusiak, J. Wang, D. W. He and C. X. Feng, "Structured approach for analysis of design processes", *IEEE Transactions on Components, Packaging, and Manufacturing Technology Part A* **18** (1995) 664-673.
6. A. Kusiak, E. Szczerbicki and K. Park, "Novel approach to decomposition of design specifications and search for solutions", *International Journal of Production Research* **29** (1991) 1391-1406.
7. C. X. Feng, P. G. Li and M. Liang, "Fuzzy mapping of requirement onto function in detail design", *Computer-Aided Design* **33** (2001) 425-437.
8. C. X. Feng, C. C. Huang, A. Kusiak and P. G. Li, "Representation of functions and features in detail design", *Computer-Aided Design* **28** (1996) 961-971.
9. C. X. Feng, D. E. Kroll and P. G. Li, "Fuzzy mapping of functions onto features in detail design", *International Journal of Production Research* **38** (2000) 2253-2272.
10. V. Catania, A. Puliafito and L. Vita, "Fuzzy approach to mapping problems", *Information Sciences* **95** (1996) 191-217.
11. Q. Q. Hu, "Manufacturing oriented fuzzy function-feature mapping", *Journal of China Textile University (English Edition)* **16** (1999) 25-30.
12. U. Belhe and A. Kusiak, "Modeling relationships among design activities", *Transactions of ASME, Journal of Mechanical Design* **118** (1996) 454-460.
13. C. Amaresh and T. P. Bligh, "A scheme for functional reasoning in conceptual design", *Design Studies* **22** (2001) 493-517.
14. H. Z. Huang, *Fuzzy Design* (China Mechine Press, Beijing, 2002).
15. D. Dubois and H. Prade, *Fuzzy Sets and Systems* (Academic Press, INC, New York, 1980).