

An interactive fuzzy multi-objective optimization method for engineering design

Hong-Zhong Huang^{a,*}, Ying-Kui Gu^b, Xiaoping Du^c

^a*School of Mechatronics Engineering, University of Electronic Science and Technology of China, Chengdu, Sichuan 610054, China*

^b*School of Mechanical & Electrical Engineering, Jiangxi University of Science and Technology, Ganzhou, Jiangxi 341000, China*

^c*Department of Mechanical and Aerospace Engineering, University of Missouri-Rolla, MO 65409, USA*

Received 12 October 2005; received in revised form 16 November 2005; accepted 10 December 2005

Available online 31 January 2006

Abstract

The coupling of performance functions due to common design variables and uncertainties in an engineering design process will result in difficulties in optimization design problems, such as poor collaboration among design objectives and poor resolution of design conflicts. To handle these problems, a fuzzy interactive multi-objective optimization model is developed based on Pareto solutions, where the metric function and some additional constraints are added to ensure the collaboration among design objectives. The trade-off matrix at the Pareto solutions was developed, and the method for selecting weighting coefficients of optimization objectives is also provided. The proposed method can generate a Pareto optimal set with the maximum satisfaction degree and the minimum distance from ideal solution. The favorable optimal solution can be then selected from the Pareto optimal set by analyzing the trade-off matrix and collaborative sensitivity. Two examples are presented to illustrate the proposed method.

© 2006 Elsevier Ltd. All rights reserved.

Keywords: Interactive optimization; Pareto solutions; Fuzzy optimization

1. Introduction

Engineering design is a process of formulating a plan for the satisfaction of human needs through a cycle of steps that include problem definition, conceptualization, embodiment, and detailing. Conflicts are ubiquitous in an engineering design process. For example, conflicts exist among requirements from groups of designers, manufacturers, suppliers, and clients, who have their own objectives and preferences. Engineers may focus on requirements in engineering aspects, such as strength, stiffness, and stability; administrators may focus on manufacturing cost and safety; clients may focus on ease of use, operation cost, and reliability. Many of these requirements conflict with each other. For instance, improving reliability will increase the cost. Therefore, conflicts always exist in design objectives in any engineering design process. It is important to resolve the objective conflicts in engineering design.

Conflict resolution is a complex and time-consuming process.

Multi-objective optimization is a design methodology that optimizes a collection of objective functions systematically and simultaneously. It has been increasingly used in engineering applications for design automation over multiple conflicting design objectives. One traditional method to ease the difficulties in multi-objective optimization problems is to choose only one objective and incorporate other objectives as constraints. The other traditional method is to simply combine all the objectives into a single objective function (Coello and Christiansen, 2000). Even though it is convenient to use these two methods, there are disadvantages associated with them.

- (1) The first approach limits the choices (design options) available to designers.
- (2) The second approach models the original problem in an inadequate manner, generating a solution that requires a further analysis to make sure it is reasonable to the designers.

*Corresponding author. Tel.: +86 28 8320 6916; fax: +86 28 8320 6916.
E-mail address: hzhuang@uestc.edu.cn (H.-Z. Huang).

- (3) Both of the methods are not inherently capable of design exploration.
- (4) Both of the methods require a priori selection of weights or targets for each of the objective functions. The upfront selection is extremely hard before the optimization process.
- (5) The designer's preferences are usually ignored to some extent.

To resolve the conflicts over design objectives collaboratively, in this work, a fuzzy interactive multi-objective optimization model is developed with adding a metric function and additional constraints. This model generates a Pareto optimal set first. A favorable optimal solution is then selected from the Pareto optimal set with several techniques such as analyzing the trade-off matrix, contacting collaborative sensitivity analysis, and adjusting the weighting coefficients of important degree of design objectives. The proposed method can generate the maximum satisfaction degree and the minimum distance between each of the objectives and its ideal solution. Moreover, the process of selecting the threshold of satisfaction degree and the weighting coefficients of objectives is a self-adjustment process and this adjustment improves the collaborative relationships among the objectives.

The paper is structured as follows. Section 2 briefly reviews of the related literature on fuzzy multi-objective optimization. Section 3 develops an interactive fuzzy multi-objective optimization model based on Shih's fuzzy structure optimization model. Section 4 presents two examples to illustrate the proposed method. Finally, conclusions are provided in Section 5.

2. Related literature review

Conventional optimization methods assume that all the design information (parameters and models) are precisely known, the constraints delimit a well defined set of feasible decisions, and the objectives are well defined to capture designers' intention. However, incomplete and uncertain input information is typical for practical problems of multi-objective optimization decision-making. This is mainly caused by fuzzy performance criteria, fuzzy ideas of decision-maker, and linguistic evaluations of quality, to mention but a few (Huang, 1997; Huang and Li, 2005, Huang et al., 2005(a–d); Gu and Huang, 2004). Fuzzy set theory enables one to model uncertainty or vagueness resulting from linguistic terms. Bellman and Zadeh (1970) inspired the development of fuzzy optimization by providing the aggregation operators, which combines the fuzzy goals and fuzzy decision space. After this work, there come out a great number of articles dealing with the fuzzy optimization problems. The collection of papers on fuzzy optimization edited by Slowinski (1998) and Delgado et al. (1994) gives the main stream of this topic.

Fuzzy set theory is widely used in solving mathematical programming problems. Zimmermann (1978) initiated the application of fuzzy theory to optimization by solving theoretical, fuzzy, linear programming problems. Zimmermann (1976) also proposed a max–min approach, which was used for solving fuzzy mathematical problems with fuzzy objectives and fuzzy constraints. Lai and Hwang (1994) proposed an augmented max–min approach, which is essentially an extension of Zimmermann's approach. Werner (1987a, b) proposed an interactive decision support system that aids in solving multiple objective programming problems subject to crisp and fuzzy constraints. One part of the system is an extension of a well-known fuzzy sets approach evaluating possible solutions by their degrees of membership to objectives and constraints. Delgado et al. (1990), Cadenas and Verdegay (2000) and Verdegay (1984) discussed fuzzy mathematical programming problems with fuzzy objective coefficients. In their approach the k th objective λ -constraint approach were used for solving fuzzy multi-objective problems with fuzzy objective coefficients. Chanas (1983) presented the possibility of the identification of a complete fuzzy decision in fuzzy linear programming by using the parametric programming technique. This parametric approach can analytically describe the set of solutions incorporating the whole range of possible values of the fuzzy decision and provides some information on other alternatives close to the optimal solution.

Fuzzy set theory is also widely used in solving multi-objective optimization for mechanical systems and structure design problems. Wang and Wang (1985) used a level-cuts approach to solve non-linear, structural problems with fuzzy constraints (and crisp objectives). Rao (1987 a–c) used explicit, continuous membership functions for fuzzy constraints and fuzzy objectives to optimize mechanical systems and structure design; membership functions for the objective function and for the constraints are aggregated into a single, standard optimization problem. Rao's method of λ -formulation yields a unique compromise solution with maximum overall satisfaction for fuzzy optimum structural design. Furthermore, he introduced the α -cut approach, which provides the results in a parametric form for multi-objective problems. Xu (1988) also transformed problems with fuzzy constraints into standard optimization problems with a slightly different format; the final solution is then determined with a bound-constrained optimization approach. Despite the significant amount of existing work in fuzzy optimization, there are few investigations on the use of fuzzy theory to determine feasible points of constrained problems.

The fuzzy decision-making and fuzzy logic are also useful tools to solve the multi-objective design problems. Shih and Chang (1995) presented a global criterion method by fuzzy logic to obtain solutions for multi-criteria crisp or fuzzy structural design, which is not only capable of acquiring the non-dominated solution, but also capable of achieving the highest degree of satisfactory design. Loetamonphong et al. (2002) studied the optimization

problems that have multiple objective functions subject to a set of fuzzy relation equations. Huang (1997) presented a fuzzy multi-objective optimization decision-making method, which can be used for the optimization decision-making on two or more objectives of system reliability.

Formulating a fuzzy optimization problem entails developing membership functions for each constraint and each objective. A relatively high value for a membership function of a constraint set indicates a near or definite membership in the set, i.e., a high likelihood of the constraint satisfaction. Therefore, the goal of a fuzzy optimization problem is to maximize all membership functions simultaneously. Most often, this is done using a formulation similar to the min–max formulation for multi-objective optimization (Baykasoglu and Sevim, 2003). In terms of fuzzy optimization, both of the objective functions and constraint functions are treated as modified constraints. Consequently, fuzzy optimization lends itself to multi-objective optimization where additional objective functions are modeled as constraints. However, in many fuzzy multi-objective optimization models, the conflicting degree among objectives and the designer’s preferences are neglected to some extent. How to model and resolve the conflicts is still an ongoing research topic.

In this work, our purpose is to develop an interactive multi-objective optimization model based on fuzzy theory to decrease the conflicting degree among design objectives and maximize the degree of constraint satisfaction.

3. Interactive fuzzy multi-objective optimization model

3.1. Pareto solution

A general multi-objective optimization problem is to find the design variable set X that optimizes a vector of objective functions $f(X) = \{f_1(X), f_2(X), \dots, f_n(X)\}$ over the feasible design space. The problem is modeled as follows:

$$\begin{aligned} &\text{Minimize } f(X) = \{f_1(X), f_2(X), \dots, f_n(X)\}, \\ &\text{Subject to } h_i(X) = 0, \quad i = 1, 2, \dots, I, \\ &\quad g_j(X) \leq 0, \quad j = 1, 2, \dots, J, \\ &\quad X_k^u \geq X_k \geq X_k^l, \quad k = 1, 2, \dots, K, \end{aligned} \tag{1}$$

where $f_1(X), f_2(X), \dots, f_n(X)$ are the individual objective functions. $h_i(X)$ and $g_j(X)$ are equality and inequality constraint functions, respectively. X_k^u and X_k^l are the lower and upper bounds of X_k , respectively.

The Pareto optimal solution is defined as follows (Tappeta and Renaud, 2001).

A vector of X^* is a Pareto optimal if there exists no feasible vector X which would decrease some objective function without causing a simultaneous increase in at least one objective function. Mathematically, the Pareto optimal solution is expressed as below.

A design vector X^* is a Pareto optimum if and only if, for any X and i ,

$$f_j(X) \leq f_j(X^*), \quad j = 1, 2, \dots, n, \quad j \neq i \Rightarrow f_i(X) \geq f_i(X^*). \tag{2}$$

In general, there exist a number of Pareto optimal solutions to a multi-objective optimization problem. Thus, the designer must select a compromise or satisfying solution from the Pareto optimal solution set according to his or her preference. It can be shown that if X' is a global optimal solution, then X' is also a Pareto optimal solution.

3.2. Finding the ideal value of each objective function

The optimization model for finding the ideal value of each objective is given by

$$\begin{aligned} &\text{Minimize } f_t(X), \quad t = 1, 2, \dots, n, \\ &\text{Subject to } h_i(X) = 0, \quad i = 1, 2, \dots, M, \\ &\quad g_j(X) \leq 0, \quad j = 1, 2, \dots, N, \\ &\quad X_k^u \geq X_k \geq X_k^l, \quad k = 1, 2, \dots, P, \end{aligned} \tag{3}$$

The solution to the above model is the ideal solution of each objective function, X'_t , and the objective function at the ideal solution is then given by

$$f_t^u = f_t(X'_t) \quad (t = 1, 2, \dots, n), \tag{4}$$

where f_t^u is the ideal value of i th objective function.

3.3. Establishing fuzzy interactive multi-objective optimization model

3.3.1. Metric function

Let X be the ideal solution, and f_i^u be the ideal value of i th objective function, then the metric function which evaluates X is defined as (Shih and Chang, 1995)

$$d(X) = \sqrt[\alpha]{\sum_{i=1}^n \left| \frac{f_i(X) - f_i^u}{f_i^u} \right|}, \tag{5}$$

where α can be selected from the universe $[1, +\infty]$. $\alpha = 2$ is usually used. Minimizing this metric function results in a commonly encountered min–max method (Shih and Chang, 1995), since for this metric the optimum X can be defined as

$$F(X) = \min_x \max_i \left| \frac{f_i(X) - f_i^u}{f_i^u} \right|. \tag{6}$$

The degree of importance of each objective criterion can be incorporated in metric function with the following additional constraints

$$\omega_i \left| \frac{f_i(X) - f_i^u}{f_i^u} \right| \leq \varepsilon, \tag{7}$$

and

$$\sum_{i=1}^n \omega_i = 1, \tag{8}$$

where ω_i represents the degree of importance of the i th objective criterion, and ε represents the allowable degree of deviation from the ideal solution for objectives (the ideal value is 0).

3.3.2. Establishing fuzzy multi-objective optimization model

The scenario of a multi-objective optimization problem itself is subjective and can be modeled by fuzzy decision-making due to the conflicting objectives and the nature of human decision on conflict resolution.

In fuzzy set theory, membership functions are established to characterize the fuzziness of fuzzy sets. The membership function values vary between zero and one. The elements in a fuzzy set with membership value 1 reflect that they are in the core of the fuzzy set. The membership function value is zero for the element outside the fuzzy set. The elements with membership function value between zero and one construct the boundary of the fuzzy set. In order to use fuzzy set theory to solve the optimization problems, the fuzzy constraints have to be formed first. These constraints originated from the given crisp constraints by relaxing the bounds. A corresponding membership function is established to describe the fuzziness of each constraint. In addition to fuzzy constraints, fuzzy objective functions are also needed. Each objective function is converted into a pseudo-goal. A membership function is associated with the pseudo-goal. The pseudo-goal has membership function value one if the design is located at the optimum from the single-objective optimization problem with the same constraints for the multi-objective design. It is obvious that solving the multi-objective optimization problem is essential to simultaneously make all membership function values of the pseudo-goals as large as possible.

The proposed procedure is summarized as follows.

(1) Finding the minimal feasible value and maximum feasible value of each objective function:

$$m_i = \min_{1 \leq l \leq n} f_i(X_l^*) = f_i(X_l^*), \tag{9}$$

$$M_i = \max_{1 \leq l \leq n} f_i(X_l^*), \tag{10}$$

where m_i and M_i are the minimum feasible value and maximum feasible value of i th objective function.

(2) Establishing the membership function of each fuzzy objective function: Most applications that involve fuzzy set theory tend to be independent of the specific shape of the membership functions. Various types of membership functions are used, such as a linear membership function, a tangent type of a membership function, an interval linear membership function, an exponential membership function, inverse tangent membership function, logistic type of membership function, and concave piecewise linear mem-

bership function. Example problems have suggested that varying the nature of the membership function does affect the final solution, but the differences between the various outcomes are not substantial (Chen, 2001).

The fuzzy objective stated by a designer can be quantified by eliciting a corresponding membership function using the following trapezoidal representation:

$$\mu_{f_i}(X) = \begin{cases} 1, & f_i(X) \leq m_i, \\ \frac{M_i - f_i(X)}{M_i - m_i}, & m_i < f_i(X) < M_i, \\ 0, & f_i(X) \geq M_i. \end{cases} \tag{11}$$

(3) Establishing the membership function of each fuzzy constraint function: In the traditional optimization, the design feasibility is considered as either satisfied or violated. For many engineering applications, the transition from infeasibility to feasibility is not obvious, because of not only the vague information in the design constraints, but also the factors that can affect the design scenario, such as designer’s knowledge, manufacture precision, and material properties. For this reason, the constraints are modeled in such a way that the transition from infeasible state to feasible state is smooth and gradual with subjectivity. For simplicity, a linear membership function is used to reflect the smooth transition. Other types of the membership function can also be used depending on the problems under consideration. The linear membership function is given by

$$\mu_{g_j}(X) = \begin{cases} 1, & g_j(X) \leq b_j, \\ \frac{[(b_j + d_j) - g_j(X)]}{d_j}, & b_j < g_j(X) < b_j + d_j, \\ 0, & g_j(X) \geq b_j + d_j, \end{cases} \tag{12}$$

where b_j and $b_j + d_j$ form an allowable fuzzy transition interval for the j th inequality constraint.

(4) Additional constraints: The ideal value of the degree of objective deviation in additional constraints is 0. Considering the collaborative relationship among objectives and their membership functions, the additional constraints are introduced by the following equality constraints (Shih and Chang, 1995):

$$\omega_i \left| \frac{f_i(X) - f_i^u}{f_i^u} \right| = \omega_j \left| \frac{f_j(X) - f_j^u}{f_j^u} \right|, \quad i, j = 1, 2, \dots, n, \quad i \neq j \tag{13}$$

and

$$\sum_{i=1}^n \omega_i = 1. \tag{14}$$

The goal of adding additional constraints into the multi-objective optimization model is to make collaboration among the objectives. By doing so, not only the degree of importance of each objective is considered, but also the

deviation between each objective and its ideal value can be minimized.

(5) Establishing fuzzy multi-objective optimization model: The fuzzy multi-objective optimization model can be developed as follows:

$$\begin{aligned}
 &\text{Maximize } \lambda \\
 &\text{Subject to } \lambda \leq \mu_{\tilde{f}_i}(X), \quad i = 1, 2, \dots, n, \\
 &\quad \lambda \leq \mu_{\tilde{h}_i}(X), \quad i = 1, 2, \dots, I, \\
 &\quad \lambda \leq \mu_{\tilde{g}_j}(X), \quad j = 1, 2, \dots, J, \\
 &\quad \omega_i \left| \frac{f_i(X) - f_i^u}{f_i^u} \right| = \omega_j \left| \frac{f_j(X) - f_j^u}{f_j^u} \right|, \quad (15) \\
 &\quad i, j = 1, 2, \dots, n, \quad i \neq j, \\
 &\quad \sum_{i=1}^n \omega_i = 1, \\
 &\quad 1 \geq \lambda \geq 0, \\
 &\quad X_k^u \geq X_k \geq X_k^l, \quad k = 1, 2, \dots, K.
 \end{aligned}$$

3.4. Selecting the weighting coefficients ω_i

Generally, the degrees of importance of individual objectives are different, which are decided by many factors, such as product usage, market orientation, designer’s preferences, and so on. The relative importance of objectives can be classified into the following two categories.

(1) Objective relative importance, which is determined by the objective requirements of product design, including the influence factors coming from the product itself, such as use, performance, and so on.

(2) Subjective relative importance, which is proposed by the designers based on their preferences and is decided by the subjective factors, such as designer’s preferences, product market orientation, etc.

The relative importance has a very important influence on modeling and solving multi-objective optimization problems.

The weighting coefficient ω_i can be used to represent the design degree of importance corresponding to the i th objective criterion. The selection of ω_i is highly subjective and correlated with other $\omega_j (i \neq j)$. Usually, ω_i is distributed in a small space. The normal-distribution-type membership function of ω_i , as shown in Fig. 1, is adopted

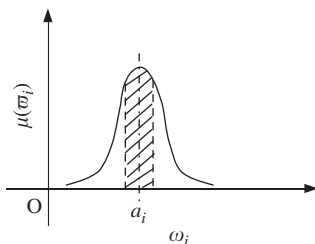


Fig. 1. Membership function of weighting coefficient representing the relative importance of the i th criterion.

and is given by

$$\begin{aligned}
 \mu(\omega_1) &= e^{-k(\omega_1 - a_1)^2}, \\
 \mu(\omega_2) &= e^{-k(\omega_2 - a_2)^2}, \\
 &\dots\dots \\
 \mu(\omega_{n-1}) &= e^{-k(\omega_{n-1} - a_{n-1})^2}, \\
 \mu(\omega_n) &= 1 - \sum_{i=1}^{n-1} \mu(\omega_i), \quad (16)
 \end{aligned}$$

where $k > 0$.

Given a threshold λ (λ can be obtained by fuzzy synthetic evaluation), a set of ω_i , which makes $\mu(\omega_i) \geq \lambda$ be satisfied, can be selected. The greater λ is, the stricter the selection of the ω_i is. On the contrary, the smaller λ is, the softer the selection is; a softer selection will benefit the interactive collaboration among the objectives.

The changing process of λ is a process of selecting appropriate ω set. The process is an adjustment process on the design parameters, and this adjustment can improve the collaborative relationships among the objectives.

It is noted that different optimization results can be obtained by selecting different λ from $[0, 1]$. There must exist one λ^* that can make the process reach or approximate the global optimum under the proposed model. The interactive optimization process continues while automatically selecting different λ and ω_i . Then the Pareto solutions can be obtained with different λ and ω_i on the base of the design requirements. To determine where to stop the search process, in each iterative calculating process, the trade-off matrix and the collaborative sensitivity at the Pareto solution should be analyzed. The iterative optimization process continues only if the Pareto sensitivity of current Pareto solution is satisfied.

3.5. Collaborative sensitivity analysis

The sensitivity analysis at a given Pareto point provides the variation in one objective given the variation in another objective. The purpose of collaborative sensitivity analysis is to minimize the degree of conflict among objectives, and the analysis results can be used as stopping criteria for the optimization process. It is noted from the discussion in Section 3.1 that the favorable optimal solution that is solved by the collaborative optimization model is a Pareto optimum solution. It is necessary to analyze the collaborative sensitivity (i.e., Pareto sensitivity $f_i(X)/f_j(X)$) at a given favorable optimum solution. That is to say, the change in one objective will result in the change in other objectives on the Pareto curve (or surface) along a given direction. To differentiate $f_i(X)$ with respect to $f_j(X)$ at X , one can conduct a Pareto sensitivity analysis along the feasible descent direction of the other objective functions.

$$df_i = d_{f_i}^T dX = \left[\frac{\partial f_i}{\partial x_1}, \frac{\partial f_i}{\partial x_2}, \dots, \frac{\partial f_i}{\partial x_m} \right]^T, \quad (17)$$

$$dX = [d_{f_i}^T d_{f_j}]^{-1} d_{f_i}^T d_{f_j} \tag{18}$$

$$\frac{df_i}{df_j} = d_{f_i}^T [d_{f_j}^T d_{f_j}]^{-1} d_{f_j} \tag{19}$$

This sensitivity information can be represented in a matrix form, called the trade-off matrix (Tappeta and Renaud, 2001; Tappeta et al., 2000).

$$T = \begin{bmatrix} 1 & \frac{df_2}{df_1} & \dots & \frac{df_n}{df_1} \\ \frac{df_1}{df_2} & 1 & \dots & \frac{df_n}{df_2} \\ \dots & \dots & \dots & \dots \\ \frac{df_1}{df_n} & \frac{df_2}{df_n} & \dots & 1 \end{bmatrix} \tag{20}$$

The trade-off matrix represents the fact that the objective functions are coupled with each other. The trade-off between any two objective functions exists only the corresponding off-diagonal element in the trade-off matrix is negative, and under this condition it is possible to improve the objectives. The *i*th row of trade-off matrix represents the trades-offs required for a possible improvement in the *i*th objective. The magnitude of the trade-off is given by the absolute value of the corresponding off-diagonal element. As illustrated in Fig. 2, when two objectives are minimized simultaneously, if $|df_i/df_j|$ is very small at a given Pareto optimum solution Q_1 , then it represents that f_i is satisfied, and the variation of f_j has smaller influence on f_i . At the given Pareto optimum solution Q_n , if $|df_i/df_j|$ is large, then it represents that f_i is not satisfied, and the change of f_j has a large influence on f_i . Therefore, based on the trade-off between $f_i(X)$ and $f_j(X)$, the objectives can be improved along the feasible descent direction further.

The flow chart of the proposed method is given in Fig. 3. As shown in Fig. 3, when the design preference or the threshold λ changes, so do the weighting coefficients ω . This will result in the adjustment of design variables at the same time. Because this adjustment is processed under the same design constraints, the change of one variable will result in the changes in the other variables. This is an interactive negotiation process among design parameters.

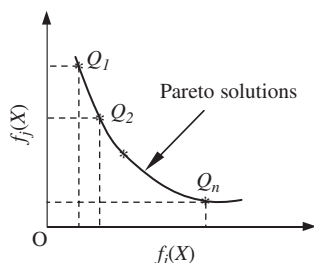


Fig. 2. Pareto solution in a problem where two objective criteria are minimized.

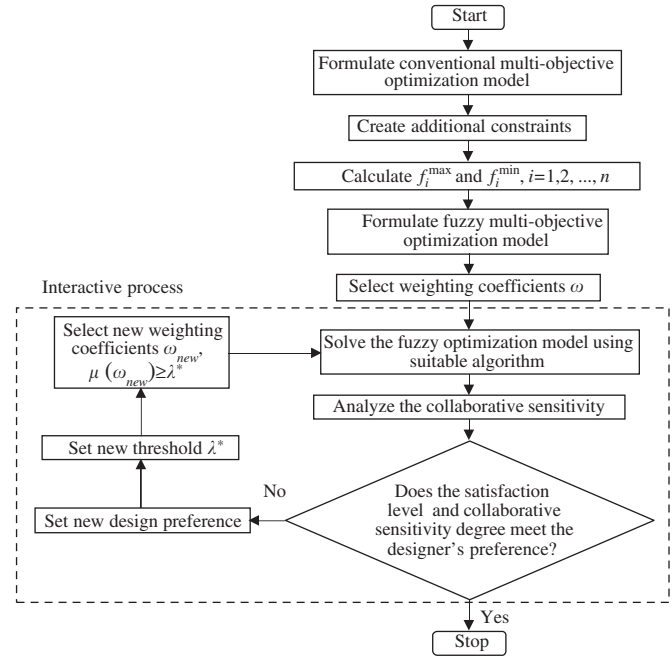


Fig. 3. Flow diagram of fuzzy interactive multi-objective optimization.

4. Examples

Two examples are used to demonstrate the proposed method in this section.

4.1. Example 1—Simply supported I-beam design

A simply supported I-beam is shown in Fig. 4 (Hajela and Shih, 1990). The objective is to select design variables $X(x_1, x_2, x_3, x_4)^T$ to minimize both the total cross-sectional area and its deflection at the midspan under the applied loads P and Q . The length of the beam is $L = 200$ cm, the forces are $P = 600$ kN and $Q = 50$ kN, the Young's modulus of elasticity is $E = 2 \times 10^4$ kN/cm², and the permissible bending stress of the beam material is $\sigma_b = 16$ kN/cm².

Two objective functions are given below. The cross-sectional area is $f_1(X) = 2x_2x_4 + x_3(x_1 - 2x_4)$, and the vertical deflection at the midspan is $f_2(X) = (PL^3/48EI)$, where

$$I = \frac{x_3(x_1 - 2x_4)^3 + 2x_2x_4[4x_4^2 + 3x_1(x_1 - 2x_4)]}{12}$$

The constraint functions are given below.

The stress constraint is expressed by

$$\frac{M_Y}{Z_Y} + \frac{M_Z}{Z_Z} \leq \sigma_b,$$

where M_Y, M_Z are the maximal bending moments in the Y and Z directions, respectively.

The geometric constraints are given by

$$10 \leq x_1 \leq 80,$$

$$10 \leq x_2 \leq 50,$$

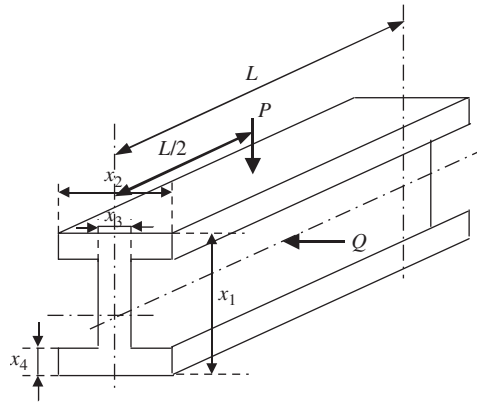


Fig. 4. Definition of design variables and loading for I-beam.

$$0.9 \leq x_3 \leq 5,$$

and

$$0.9 \leq x_4 \leq 5.$$

The multi-objective optimization problem is then formulated as

$$\begin{aligned} &\text{Minimize } (f_1, f_2), \\ &f_1(X) = 2x_2x_4 + x_3(x_1 - 2x_4), \\ &f_2(X) = \frac{PL^3}{48EI}, \\ &I = \frac{x_3(x_1 - 2x_4)^3 + 2x_2x_4[4x_4^2 + 3x_1(x_1 - 2x_4)]}{12}, \\ &\text{Subject to } \frac{180000x_1}{x_3(x_1 - 2x_4)^3 + 2x_2x_4(4x_4^2 + 3x_1(x_1 - 2x_4))} \\ &\quad + \frac{15000x_2}{(x_1 - 2x_4)x_3^3 + 2x_4x_3^3} \leq 16, \\ &10 \leq x_1 \leq 80, \\ &10 \leq x_2 \leq 50, \\ &0.9 \leq x_3 \leq 5, \\ &0.9 \leq x_4 \leq 5. \end{aligned} \tag{21}$$

The ideal values of the objectives are listed in Table 1.

The optimal results solved through the fuzzy interactive collaborative optimization model are listed in Table 2 for $\lambda = 1, 0.9, 0.8, 0.7, 0.6$.

After 5 optimization iterations, the weighting coefficients are away from the centers while still meet the design requirements. Though the off-diagonal elements in the trade-off matrix is negative, its absolute value (the magnitude of the trade-off) is very small ($4.4437e - 005$). Therefore, the solution meets the design requirements, and the process terminates.

The optimization results from Shih and Chang (1995) are also listed in Table 3 for comparison. From the results we see that only the deviation between objectives and their ideal values is taken into account, but the degree of importance corresponding to the objective criterion is

Table 1
The ideal value of sub-objective

	$(f_1(X), f_2(X))$	(x_1, x_2, x_3, x_4)
$(\min f_1, f_2)$	(127.4124, 0.0615)	(60.4765, 41.4464, 0.9000, 0.9000)
$(f_1, \min f_2)$	(850.0000, 0.0059)	(80.0000, 50.0000, 5.0000, 5.0000)
$(\max f_1, f_2)$	(850.0000, 0.0059)	(80.0000, 50.0000, 5.0000, 5.0000)
$(f_1, \max f_2)$	(528.1008, 0.3286)	(15.6202, 50.0000, 5.0000, 5.0000)

neglected. In our model, however, not only the degree of importance corresponding to the objective criterion and the trade-off among the objectives, but also the deviation between objectives and their ideal values, are considered. Therefore, the result solved by the fuzzy interactive optimization based on Pareto solution are more realistic to engineering applications.

4.2. Example 2—gear box design

A gear box (Kupapatz and Azarm, 2001) was originally formulated as a single-objective optimization problem. For the purpose of demonstration, the problem is adapted to a multi-objective optimization problem by converting two original constraints into objective functions. The optimization model is shown as follows:

$$\begin{aligned} &\text{Minimize } F(X) = \{f_1(X), f_2(X), f_3(X)\}, \\ &X = [x_1, x_2, x_3, x_4, x_5, x_6, x_7]^T, \\ &\text{Subject to } g_1(X) = 27x_1^{-1}x_2^{-2}x_3^{-3} - 1 \leq 0, \\ &g_2(X) = 397.5x_1^{-1}x_2^{-2}x_3^{-2} - 1 \leq 0, \\ &g_3(X) = 1.93x_2^{-1}x_3^{-1}x_4^3x_6^{-4} - 1 \leq 0, \\ &g_4(X) = 1.93x_2^{-1}x_3^{-1}x_5^3x_7^{-4} - 1 \leq 0, \\ &g_5(X) = x_2x_3 - 40 \leq 0, \\ &g_6(X) = x_1x_2^{-1} - 12 \leq 0, \\ &g_7(X) = 5 - x_1x_2^{-1} \leq 0, \\ &g_8(X) = 1.9 - x_4 + 1.5x_6 \leq 0, \\ &g_9(X) = 1.9 - x_5 + 1.5x_7 \leq 0, \\ &g_{10}(X) = A_1/B_1 - 1300 \leq 0, \\ &g_{11}(X) = A_2/B_2 - 850 \leq 0, \\ &2.6 \leq x_1 \leq 3.6, \\ &0.7 \leq x_2 \leq 0.8, \\ &17 \leq x_3 \leq 28, \\ &7.3 \leq x_4 \leq 8.3, \\ &7.3 \leq x_5 \leq 8.3, \\ &2.9 \leq x_6 \leq 3.9, \\ &5.0 \leq x_7 \leq 5.5, \end{aligned} \tag{22}$$

where

$$\begin{aligned} f_1(X) = &0.7854x_1x_2^2(10x_3^2/3 + 14.9334x_3 \\ &- 43.0934) - 1.508x_1(x_6^2 + x_7^2) \\ &+ 7.4777(x_6^3 + x_7^3) + 0.7854(x_4x_6^2 + x_5x_7^2), \end{aligned}$$

Table 2
The optimum results with different threshold and weighting coefficients

	λ	$\omega = (\omega_1, \omega_2)$	β	$X = (x_1, x_2, x_3, x_4)$	$f(X) = (f_1, f_2)$
1	1.0	(0.40, 0.60)	0.9718	(80.0000, 26.1303, 1.4637, 4.7086)	(349.3860, 0.0128)
2	0.9	(0.42, 0.58)	0.9725	(80.0000, 49.9860, 1.2242, 2.3464)	(326.7680, 0.0126)
3	0.8	(0.45, 0.55)	0.9723	(80.0000, 50.0000, 1.1312, 2.2856)	(313.8876, 0.0130)
4	0.7	(0.50, 0.50)	0.9728	(80.0000, 35.7683, 1.0355, 3.0966)	(297.9494, 0.0138)
5	0.6	(0.55, 0.45)	0.9739	(80.0000, 50.0000, 0.9000, 2.0820)	(276.4525, 0.0143)

Table 3
The optimization results from Hajela and Shih (1990)

	$\omega = (\omega_1, \omega_2)$	$X = (x_1, x_2, x_3, x_4)$	$f(X) = (f_1, f_2)$
1	(0.45, 0.55)	(79.99, 49.99, 0.90, 2.390)	(307.53, 0.0127)
2	(0.55, 0.45)	(80.00, 50.00, 0.90, 2.083)	(276.55, 0.0143)
3	(0.65, 0.35)	(79.99, 50.00, 0.90, 1.790)	(247.88, 0.0163)
4	(0.80, 0.20)	(80.00, 39.79, 0.90, 1.725)	(206.14, 0.0205)

Table 4
The physical meaning of decision variables and objective and constraint functions

x_1	Gear face width (cm)	$g_2(X)$	Contact stress of teeth
x_2	Teeth module (cm)	$g_3(X)$	Transverse displacement of shaft 1
x_3	Number of teeth of pinion	$g_4(X)$	Transverse displacement of shaft 2
x_4	Distance between bearing 1 (cm)	$g_5(X)$	Generated torque constraint
x_5	Distance between bearing 2 (cm)	$g_6(X)$	Generated torque constraint
x_6	Diameter of shaft 1 (cm)	$g_7(X)$	Generated torque constraint
x_7	Diameter of shaft 2 (cm)	$g_8(X)$	Generated torque constraint
$f_1(X)$	Volume of the gear box (cm ³)	$g_9(X)$	Generated torque constraint
$f_2(X)$	Stress of shaft 1	$g_{10}(X)$	Stress of shaft 1
$f_3(X)$	Stress of shaft 2	$g_{11}(X)$	Stress of shaft 2
$g_1(X)$	Bending stress of teeth		

Table 5
The optimum results with different threshold and weighting coefficients

	λ	$\omega = (\omega_1, \omega_2, \omega_3)$	β	$X = (x_1, x_2, x_3, x_4, x_5, x_6, x_7)$	$f(X) = (f_1, f_2, f_3)$
1	1	(0.30, 0.30, 0.40)	0.9587	(3.58, 0.70, 23, 7.36, 8.14, 3.45, 5.40)	(4361.3, 1004.5, 797.4)
2	0.9	(0.35, 0.38, 0.27)	0.9606	(3.52, 0.70, 24.8, 25, 8.20, 3.62, 5.37)	(4588.8, 870.0, 810.8)
3		(0.35, 0.35, 0.30)	0.9625	(3.60, 0.71, 20, 7.93, 7.96, 3.36, 5.41)	(3765.5, 1089.3, 793.0)
4	0.8	(0.40, 0.25, 0.35)	0.9618	(3.55, 0.71, 24, 7.68, 8.19, 3.79, 5.48)	(4821.6, 757.7, 762.9)
5		(0.40, 0.28, 0.32)	0.9622	(3.58, 0.71, 18, 8.24, 8.23, 3.61, 5.40)	(3425.0, 879.8, 797.6)
6	0.7	(0.45, 0.30, 0.25)	0.9632	(3.57, 0.70, 20, 8.17, 8.00, 3.53, 5.45)	(3762.0, 939.8, 775.7)
7		(0.45, 0.25, 0.30)	0.9601	(3.54, 0.70, 21, 7.96, 8.16, 3.69, 5.45)	(4001.6, 822.1, 775.7)
8	0.6	(0.50, 0.20, 0.30)	0.9706	(3.51, 0.70, 20, 7.89, 7.99, 3.89, 5.41)	(3812.9, 702.0, 793.0)

$$f_2(X) = A_1/B_1,$$

$$f_3(X) = A_2/B_2,$$

$$A_1 = \sqrt{(745x_4x_2^{-1}x_3^{-1}) + 1.69 \times 10^7},$$

$$B_1 = 0.1x_6^3,$$

$$A_2 = \sqrt{(745x_5x_2^{-1}x_3^{-1}) + 1.575 \times 10^8},$$

and

$$B_2 = 0.1x_7^3.$$

The physical meaning of design variables and objective and constraint functions are given in Table 4.

The optimal results obtained through fuzzy interactive collaborative optimization model are listed in Table 5, for $\lambda = 1, 0.9, 0.8, 0.7, 0.6$, respectively.

After 7 optimization iterations, the optimal solution with the minimal trade-off among the objectives is obtained. At this time, the corresponding off-diagonal elements are

negative; even though this suggests that the collaborative relationships among objectives could be improved further, the value of λ could not be loosened any more.

5. Conclusions

Product design usually involves a complicated multi-objective optimization process. Many efficient solutions can be obtained due to the influence of subjective and objective factors in a design process. The following conclusions are drawn based on preceding sections.

(1) A Pareto optimal set can be obtained from the proposed method. The changing process of λ is a process to select appropriate ω set. If λ decreases, the limitation of ω would be loosened. This will produce a set of Pareto solutions among which an optimal solution can be selected through analyzing the trade-off matrix and collaborative sensitivity.

(2) The proposed method can generate the maximum degree of satisfaction and the minimum distance between each of the objectives and their ideal solutions, both the design degree of importance corresponding to the objective criterion and the deviation between the objectives and their ideal values are taken into account.

(3) The process of selecting λ and ω is a self-adjustment process of the design variables, and this adjustment can improve the collaborative relationships among the objectives. The interactive collaboration among the objectives is realized through the collaborative sensitivity analysis at the Pareto solutions during the optimization process.

(4) Compared with the crisp optimization, the proposed interactive fuzzy multi-objective optimization method offers a greater threshold degree for interactive collaboration among the objectives by fuzzy processing of the objectives and constraints.

Acknowledgement

This research was partially supported by the National Natural Science Foundation of China under the Contract number 50175010, the National Excellent Doctoral Dissertation Special Foundation of China under the Contract number 200232, and the Excellent Young Teachers Program of the Ministry of Education of China under the Contract number 1766. Constructive suggestions and comments from the referees and editors are very much appreciated.

References

- Baykasoglu, A., Sevim, T., 2003. A review and classification of fuzzy approaches to multi objective optimization. International XII Turkish Symposium on Artificial Intelligence and Neural Networks-TAINN 2003.
- Bellman, R.E., Zadeh, L.A., 1970. Decision making in a fuzzy environment. *Management Science* 17, 141–164.
- Chanas, S., 1983. The use of parametric programming in fuzzy linear programming. *Fuzzy Sets and Systems* 11, 243–251.
- Cadenas, J.M., Verdegay, J.L., 2000. Using ranking functions in multi objective fuzzy linear programming. *Fuzzy Sets and Systems* 111, 47–53.
- Chen, L., 2001. Multiobjective design optimization based on satisfaction metrics. *Engineering Optimization* 33, 601–617.
- Coello, C.A., Christiansen, A.D., 2000. Multiobjective optimization of trusses using genetic algorithms. *Computers and Structures* 75 (6), 647–660.
- Delgado, M., Verdegay, J.L., Vila, M.A., 1990. Relating different approaches to solve linear programming problems with imprecise costs. *Fuzzy Sets and Systems* 37, 33–42.
- Delgado, M., Kacprzyk, J., Verdegay, J.L., Vila, M.A., 1994. *Fuzzy Optimization: Recent Advances*. Physica-Verlag, New York.
- Gu, Y.K., Huang, H.Z., 2004. Fuzzy mapping between physical domain and function domain in design process. *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems* 12 (1), 7–20.
- Hajela, P., Shih, C.J., 1990. Multiobjective optimum design in mixed integer and discrete design variable problems. *AIAA Journal* 28 (4), 670–675.
- Huang, H.Z., 1997. Fuzzy multi-objective optimization decision-making of reliability of series system. *Microelectronics Reliability* 37 (3), 447–449.
- Huang, H.Z., Li, H.B., 2005. Perturbation fuzzy finite element method of structural analysis based on variational principle. *Engineering Applications of Artificial Intelligence* 18 (1), 83–91.
- Huang, H.Z., Wu, W.D., Liu, C.S., 2005a. A coordination method for fuzzy multi-objective optimization of system reliability. *Journal of Intelligent and Fuzzy Systems* 16 (3), 213–220.
- Huang, H.Z., Tian, Z.G., Zuo, M.J., 2005b. Intelligent interactive multiobjective optimization method and its application to reliability optimization. *IIE Transactions* 37 (11), 983–993.
- Huang, H.Z., Li, Y.H., Xue, L.H., 2005c. A comprehensive evaluation model for assessments of grinding machining quality. *Key Engineering Materials* 291–292, 157–162.
- Huang, H.Z., Sun, Z.Q., Zuo, M.J., Tiao, Z.G., 2005d. Bayesian reliability assessment of gear lifetime under fuzzy environments. In: *Proceedings: Annual Reliability and Maintainability Symposium*, pp. 232–237.
- Kupapatz, A., Azarm, S., 2001. Immune network simulation with multiobjective genetic algorithms for multidisciplinary design optimization. *Engineering Optimization* 33 (2), 245–260.
- Lai, Y.J., Hwang, C.L., 1994. *Fuzzy Multiple Objective Decision Making, Methods and Applications*, Lecture Notes in Economics and Mathematical Systems. Springer, Berlin, Heidelberg.
- Loetamonphong, J., Fang, S.C., Young, R.E., 2002. Multi-objective optimization problems with fuzzy relation equation constraints. *Fuzzy Sets and Systems* 127, 141–164.
- Rao, S.S., 1987a. Multi-objective optimization of fuzzy structural systems. *International Journal for Numerical Methods in Engineering* 24, 1157–1171.
- Rao, S.S., 1987b. Description and optimum design of fuzzy mechanical systems. *ASME Journal of Mechanisms, Transmissions and Automation in Design* 109, 126–132.
- Rao, S.S., 1987c. Optimum design of structures in a fuzzy environment. *AIAA Journal* 25, 1633–1636.
- Shih, C.J., Chang, C.J., 1995. Pareto optimization of alternative global criterion method for fuzzy structural design. *Computers and Structures* 54 (3), 455–460.
- Slowinski, R., 1998. *Fuzzy Sets in Decision Analysis, Operations Research and Statistics*. Kluwer Academic Publishers, Boston.
- Tappeta, R.V., Renaud, J.E., 2001. Interactive multi-objective optimization design strategy for decision based design. *ASME Journal of Mechanical Design* 123 (6), 205–215.
- Tappeta, R.V., Renaud, J.E., Messac, A., 2000. Interactive physical programming: tradeoff analysis and decision making in multicriteria optimization. *AIAA Journal* 38 (5), 917–926.

- Verdegay, J.L., 1984. A dual approach to solve the fuzzy linear programming problem. *Fuzzy Sets and Systems* 14, 131–141.
- Wang, G.-Y., Wang, W.-Q., 1985. Fuzzy optimum design of structures. *Engineering Optimization* 8, 291–300.
- Werners, B., 1987a. Interactive multiple objective programming subject to flexible constraints. *European Journal of Operational Research* 31, 342–349.
- Werners, B., 1987b. An interactive fuzzy programming system. *Fuzzy Sets and Systems* 23, 131–147.
- Xu, C.W., 1988. Fuzzy optimum design of structures based on fuzzy decision-making. In: *Proceedings of the Seventh International Conference on Offshore Mechanics and Arctic Engineering*. American Society of Mechanical Engineers, New York, pp. 429–432.
- Zimmermann, H.J., 1976. Description and optimization of fuzzy systems. *International Journal of General Systems* 2, 209–215.
- Zimmermann, H.J., 1978. Fuzzy programming and linear programming with several objective functions. *Fuzzy Sets and Systems* 1, 45–55.