Mechanics Based Design of Structures and Machines: An International Journal

Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/lmbd20

A Polynomial Algorithm for Model Updating of Engineering Truss#

Chun Nam Wong [a], Jingqi Xiong [a], Hong-Zhong Huang [a] & Yao Jiang Zhao [b]

[a] School of Mechanical, Electronic and Industrial Engineering, University of Electronic Science and Technology of China, Chengdu, Sichuan, China

[b] Jinchuan Group Ltd., Jinchang, Gansu, China

Version of record first published: 26 Feb 2010.

To cite this article: Chun Nam Wong, Jingqi Xiong, Hong-Zhong Huang & Yao Jiang Zhao (2010): A Polynomial Algorithm for Model Updating of Engineering Truss# , Mechanics Based Design of Structures and Machines: An International Journal, 38:1, 1-24

To link to this article: http://dx.doi.org/10.1080/15397730903347491

Mechanics Based Design of Structures and Machines, 38: 1–24, 2010

ISSN: 1539-7734 print/1539-7742 online
DOI: 10.1080/15397730903347491

A POLYNOMIAL ALGORITHM FOR MODEL UPDATING OF ENGINEERING TRUSS#

Chun Nam Wong[1], Jingqi Xiong[1], Hong-Zhong Huang[1], and Yao Jiang Zhao[2]
[1]*School of Mechanical, Electronic and Industrial Engineering, University of Electronic Science and Technology of China, Chengdu, Sichuan, China*
[2]*Jinchuan Group Ltd., Jinchang, Gansu, China*

A polynomial algorithm is established to update structural variants of an engineering truss model. By expanding the bivariate to multivariate approach, eigenparameters are written as the higher order Lagrange factor functions of the structural variants. Using Taylor series expansion of these functions, higher order single- and multiple-variants system equations are constructed. Fixed-free finite element beam is established to analyze the convergences of first- to third-order algorithms under small to large percentage single and multiple modifications. The engineering truss model is constructed to validate the order analysis under different percentage modifications of the model elements. Both single and multiple modification cases are investigated using the model.

Keywords: Engineering truss; Model updating; Multivariate polynomial; Taylor series; Termination criterion.

INTRODUCTION

Model updating is a design process which modifies the system parameters of engineering model through matching the analytical responses with the actual responses. It is a test and analysis correlation procedure including (1) analytical model establishment, (2) test/simulated response acquisition, and (3) model updating algorithm. Initially, exact formulation, finite element model, or numerical computation approaches are used to establish the analytical model. Then different measurement techniques are employed to acquire the test responses. They include vibration mode, physical dimension, part temperature, thermoelastic measurement, ambient test, and static concentrated load (Bagchi, 2005; Bakir et al., 2007; Butkewithsch and Steffen, 2002; Humbert et al., 1999; Ip and Tse, 2001; Sinha et al., 2001; Steeneckers et al., 2007; Teughels and Roeck, 2004; Wahab et al., 1999;

Received November 7, 2008; Accepted July 29, 2009
#Communicated by I. Elishakoff.
Correspondence: Chun Nam Wong, School of Mechanical, Electronic and Industrial Engineering, University of Electronic Science & Technology of China, Chengdu, Sichuan 610054, P.R. China; E-mail: znhuang@uestc.edu.cn

Xi et al., 2000; Zhang, 1999). Finally, the system parameters are updated by the developed algorithm using the actual responses from engineering system.

Model updating algorithms can be classified as direct or iterative generally. In direct algorithms, the modification process is determined through direct mathematical operation between system parameters and system characteristic responses. For example in Xi's work, the finite element model was transformed to the space-fixed frame for the rotation of circular saw, where vibration mode shapes remained the same and natural frequencies were affected by the product of the rotation speed and order of the mode shape (Xi et al., 2000). Shape optimization was used for the desired configuration changes, while reliable model was used in the design of heavy truck side bumper by Butkewithsch and Steffen (2002). Ip and Tse (2001) developed inverse scheme for characterizing the dynamics flexural and shear moduli using Timoshenko beam model. Steeneckers et al. (2007) developed a model update method from output-only transmissibility measurements. The known modal matrix was decomposed by Zhang (1999) using singular value decomposition technique and the best updating solution was defined according to the best approximation theory. A matrix update method was used to update the beam and free shell elements of a three span continuous steel force deck bridge located in western Canada as proposed by Bagchi (2005).

On the other hand, some algorithms determine the modification value in an iterative approach. Humbert et al. (1999) used thermoelastic measurements to improve the finite element model of a thin plate bending using a variable metric Gauss–Newton method with a polynomial line search. Sinha et al. (2001) proposed a gradient-based updating method to detect spring support locations by updating the position parameters of the support through the optimization of an error criterion. An iteration sensitivity-based method to update a finite element model with ambient test was proposed by Teughels and Roeck (2004). In the works of Wahab et al. (1999) and Bakir et al. (2007), a sensitivity-based finite element model updating technique was used to update modification using static concentrated load. For computing the eigenparameter perturbations, Chen (2007) had established the derivatives of modes for distinct, repeated, or close eigenvalues. Perturbations of distinct, repeated, or close eigenvalues, complex modes of systems with real unsymmetric matrices, defective or near defective systems were derived. However in these formulations, only up to second-order perturbations and exact derivatives are computed.

Herein, the developed algorithm is iterative in nature, which generates an empirical function by interpolating the system responses. It is based on eigenvalue reanalysis to construct the polynomial function of the eigenparameters. Spath (1995) made use of the fundamental Lagrange polynomials (Issacson and Keller, 1973; Spath, 1995) to develop the Lagrange form of the bivariate polynomial interpolation. With the basic form of the Lagrange factor function of each separate variant, the eigenparameter function is generalized to its multivariate polynomial interpolation form. Also, different order derivative terms are computed using the constructed polynomial functions. Higher order polynomial equations developed in this work are established by relating these polynomial functions and derivative terms with the structural variants.

POLYNOMIAL EIGENPARAMETER FUNCTION AND ITS DERIVATIVE

In this model updating process, Taylor series expansion equation is utilized to relate the changes of the eigenparameters with the modifications of structural variants. By inverse computation, these modifications in the structural variants are estimated from an optimization algorithm. The updating process is repeated until a specific termination criterion is reached.

Multivariate Polynomial Eigenparameter Function

Let $\overline{G} = (G_1, G_2, \ldots, G_m)^T$ be the structural variant vector of the computational structure, where G_i denotes the stiffness value of the ith structural variant including Young's modulus, moment of inertia, or stiffness value. These correspond to the modification of material properties due to dimensional change or material selection in structural design. $\overline{G} = (G_1, G_2, \ldots, G_m)^T$ is the structural variant vector of the updated structure, while $\overline{G}_\delta = (G_{\delta 1}, G_{\delta 2}, \ldots, G_{\delta m})^T$ is the structural variant vector of the actual structure. The selected sets of n eigenparameter pairs corresponding to the updated and actual structures are denoted by $\overline{\Phi}$ and $\overline{\Phi}_\delta$ respectively, where $\overline{\Phi} = (\lambda^1, \bar{\phi}^1, \lambda^2, \bar{\phi}^2, \ldots, \lambda^n, \bar{\phi}^n)^T$ and $\overline{\Phi}_\delta = (\lambda_\delta^1, \bar{\phi}_\delta^1, \lambda_\delta^2, \bar{\phi}_\delta^2, \ldots, \lambda_\delta^n, \bar{\phi}_\delta^n)^T$ with λ^k and $\bar{\phi}^k$ $(k = 1, 2, \ldots, n)$ being the kth eigenvalue and eigenvector of updated structure, while λ_δ^k and $\bar{\phi}_\delta^k$ $(k = 1, 2, \ldots, n)$ being the kth eigenvalue and eigenvector of actual structure. These eigenparameter pairs corresponding to different combination of Ki prescribed values of G_i (denoted by $G_i^{\langle 1\rangle}, G_i^{\langle 2\rangle}, \ldots,$ and $G_i^{\langle Ki\rangle}$), are calculated autonomously using a finite element model. By denoting its structural variant vectors $(G_1^{\langle k1\rangle}, G_2^{\langle k2\rangle}, \ldots, G_m^{\langle km\rangle})^T$ as $\langle k1k2\cdots km\rangle$, its eigenparameter vector is denoted by $\overline{\Phi}_{\langle k1k2\cdots km\rangle}$ where each vector is partitioned in the form $\overline{\Phi}_{\langle k1k2\cdots km\rangle} = (\lambda^1_{\langle k1k2\cdots km\rangle}, \bar{\phi}^1_{\langle k1k2\cdots km\rangle}, \lambda^2_{\langle k1k2\cdots km\rangle}, \bar{\phi}^2_{\langle k1k2\cdots km\rangle}, \ldots, \lambda^n_{\langle k1k2\cdots km\rangle}, \bar{\phi}^n_{\langle k1k2\cdots km\rangle})^T$ $(j = 1, 2, \ldots, Ki)$. For the kth pair, $\lambda^k_{\langle k1k2\cdots km\rangle}$ $(k = 1, 2, \ldots, n)$ is the kth eigenvalue and $\bar{\phi}^k_{\langle k1k2\cdots km\rangle}$ $(k = 1, 2, \ldots, n)$ is the kth eigenvector. This procedure is called the structural reanalysis. Note that only the selected degrees of freedom (d.o.f) of the structure are used in the selected eigenvectors, which correspond to low stiffness-to-mass ratios (Allemang and Brown, 1982; Penny et al., 1994). Based on the $k1 \times k2 \times \cdots \times km$ ordered sets, $[G_{\langle k1k2\cdots km\rangle}, \overline{\Phi}_{\langle k1k2\cdots km\rangle}]$, one can interpolate m-variate polynomial function of $\overline{\Phi}$ in terms of $G_1, G_2, \ldots, G_m$ as

$$\overline{\Phi} = \sum_{k1=1}^{K1} \sum_{k2=1}^{K2} \cdots \sum_{km=1}^{Km} \overline{\Phi}_{\langle k1k2\cdots km\rangle} L_1^{\langle k1\rangle}(G_1^{\langle k1\rangle}) L_2^{\langle k2\rangle}(G_2^{\langle k2\rangle}) \cdots L_m^{\langle km\rangle}(G_m^{\langle km\rangle}), \tag{1}$$

where $\overline{\Phi} = (\lambda^1, \bar{\phi}^1, \lambda^2, \bar{\phi}^2, \ldots, \lambda^n, \bar{\phi}^n)^T$, and $L_i^{\langle ki\rangle}(G_i^{\langle ki\rangle})$ is the Lagrange factor function of the ith structural variant at the kith interpolated stiffness value, given by

$$L_i^{\langle ki\rangle}(G_i^{\langle ki\rangle}) = \prod_{\substack{k=1 \\ k\neq ki}}^{Ki} \left(\frac{G_i - G_i^{\langle k\rangle}}{G_i^{\langle ki\rangle} - G_i^{\langle k\rangle}} \right). \quad (ki = 1, 2, \ldots, Ki) \tag{2}$$

First-order derivative terms can be obtained by direct derivative on the corresponding Lagrange factor $L_i^{\langle ki\rangle}(G_i^{\langle ki\rangle})$ with respect to G_i. For single variant modification, the first-order eigenparameter derivative is

$$\frac{\partial\overline{\Phi}}{\partial G_1} = \sum_{k1=1}^{K1} \overline{\Phi}_{\langle k1\rangle} \frac{\partial L_1^{\langle k1\rangle}(G_1^{\langle k1\rangle})}{\partial G_1}, \tag{3}$$

where

$$\frac{\partial L_1^{\langle k1\rangle}(G_1^{\langle k1\rangle})}{\partial G_1} = \sum_{\substack{k1(1)=1 \\ k1(1)\neq k1}}^{K1} \frac{1}{G_i^{\langle k1\rangle} - G_1^{\langle k1(1)\rangle}} \prod_{\substack{kj=1 \\ kj\neq k1 \\ kj\neq k1(1)}}^{K1} \left[\frac{G_1 - G_1^{\langle kj\rangle}}{G_1^{\langle k1\rangle} - G_1^{\langle kj\rangle}}\right].$$

There is one summation series in this Lagrange factor derivative. Meanwhile, for multiple variants modification with $m > 1$, the first-order eigenparameter derivative can be expressed as

$$\frac{\partial\overline{\Phi}}{\partial G_i} = \underbrace{\sum_{k1=1}^{K1}\sum_{k2=1}^{K2}\cdots\sum_{km=1}^{Km}}_{m \text{ summation}} \overline{\Phi}_{\langle k1k2\cdots km\rangle} L_i^{\langle ki\rangle}(G_i^{\langle ki\rangle}) \cdots \frac{\partial L_i^{\langle ki\rangle}(G_i^{\langle ki\rangle})}{\partial G_i} \cdots L_m^{\langle km\rangle}(G_m^{\langle km\rangle}), \tag{4}$$

where

$$\frac{\partial L_i^{\langle ki\rangle}(G_i^{\langle ki\rangle})}{\partial G_i} = \sum_{\substack{k=1 \\ k\neq ki}}^{Ki} \frac{1}{G_i^{\langle ki\rangle} - G_i^{\langle k\rangle}} \prod_{\substack{kj=1 \\ kj\neq ki \\ kj\neq k}}^{Ki} \left[\frac{G_i - G_i^{\langle kj\rangle}}{G_i^{\langle ki\rangle} - G_i^{\langle kj\rangle}}\right].$$

On the other hand, the second-order derivative consists of two terms namely the repeated differential and the unrepeated differential. For the unrepeated term, it is given by the direct first-order derivative of the Lagrange factor according to the structural variant involved. Special care is given to the repeated terms where direct second-order derivative of the corresponding Lagrange factor is encountered. For single variant with $m = 1$, we have the simplified form of the second-order eigenparameter derivative

$$\frac{\partial^2\overline{\Phi}}{\partial G_1^2} = \sum_{k1=1}^{K1} \overline{\Phi}_{\langle k1\rangle} \frac{\partial^2 L_1^{\langle k1\rangle}(G_1^{\langle k1\rangle})}{\partial G_1^2}, \tag{5}$$

where

$$\frac{\partial^2 L^{\langle k1\rangle}(G_1^{\langle k1\rangle})}{\partial G_1^2} = \sum_{\substack{k1(1)=1 \\ k1(1)\neq k1}}^{K1} \frac{1}{G_1^{\langle k1\rangle} - G_1^{\langle k1(1)\rangle}} \sum_{\substack{k1(2)=1 \\ k1(2)\neq k1 \\ k1(2)\neq k1(1)}}^{K1} \frac{1}{G_1^{\langle k1\rangle} - G_1^{\langle k1(2)\rangle}} \prod_{\substack{kj=1 \\ kj\neq k1 \\ kj\neq k1(1) \\ kj\neq k1(2)}}^{K1} \left[\frac{G_1 - G_1^{\langle kj\rangle}}{G_1^{\langle k1\rangle} - G_1^{\langle kj\rangle}}\right].$$

Meanwhile, for multiple variants with $m > 1$, the second-order eigenparameter derivative is expressed as

$$\frac{\partial^2 \overline{\Phi}}{\partial G_i \partial G_j} = \begin{cases} \sum_{k1=1}^{K1} \sum_{k2=1}^{K2} \cdots \sum_{km=1}^{Km} \overline{\Phi}_{\langle k1k2\cdots km\rangle} L_1^{\langle k1\rangle}(G_1^{\langle k1\rangle}) & \text{for } i \neq j \\ \quad \cdots \dfrac{\partial L_i^{\langle ki\rangle}(G_i^{\langle ki\rangle})}{\partial G_i} \cdots \dfrac{\partial L_j^{\langle kj\rangle}(G_j^{\langle kj\rangle})}{\partial G_j} \cdots L_m^{\langle km\rangle}(G_m^{\langle km\rangle}) & \\ \sum_{k1=1}^{K1} \sum_{k2=1}^{K2} \cdots \sum_{km=1}^{Km} \overline{\Phi}_{\langle k1k2\cdots km\rangle} L_1^{\langle k1\rangle}(G_1^{\langle k1\rangle}) & \text{for } i = j \\ \quad \cdots \dfrac{\partial^2 L_i^{\langle ki\rangle}(G_i^{\langle ki\rangle})}{\partial G_i^2} \cdots L_m^{\langle km\rangle}(G_m^{\langle km\rangle}) & \end{cases}, \tag{6}$$

where

$$\frac{\partial^2 L_i^{\langle ki\rangle}(G_i^{\langle ki\rangle})}{\partial G_i^2} = \sum_{\substack{ki(1)=1 \\ ki(1)\neq ki}}^{Ki} \frac{1}{G_i^{\langle ki\rangle} - G_i^{\langle k\rangle}} \sum_{\substack{ki(2)=1 \\ ki(2)\neq ki \\ ki(2)\neq ki(1)}}^{Ki} \frac{1}{G_i^{\langle ki\rangle} - G_i^{\langle km\rangle}} \prod_{\substack{kj=1 \\ kj\neq ki \\ kj\neq ki(1) \\ kj\neq ki(2)}}^{Ki} \left[\frac{G_i - G_i^{\langle kj\rangle}}{G_i^{\langle ki\rangle} - G_i^{\langle kj\rangle}}\right].$$

Moreover, the pth-order derivative terms are constructed from different combinations of higher order repeated and unrepeated derivatives of the Lagrange factor. For single variant, the pth-order eigenparameter derivative is

$$\frac{\partial^p \overline{\Phi}}{\partial G_1^p} = \sum_{k1=1}^{K1} \overline{\Phi}_{\langle k1\rangle} \frac{\partial^p L_1^{\langle k1\rangle}(G_1^{\langle k1\rangle})}{\partial G_1^p}, \tag{7}$$

where

$$\frac{\partial^p L_1^{\langle k1\rangle}(G_1^{\langle k1\rangle})}{\partial G_1^p} = \sum_{\substack{k1(1)=1 \\ k1(1)\neq k1}}^{K1} \frac{1}{G_1^{\langle k1\rangle} - G_1^{\langle k\rangle}} \cdots \sum_{\substack{k1(p)=1 \\ k1(p)\neq k1 \\ k1(p)\neq k1(p-1) \\ \vdots \\ k1(p)\neq k1(1)}}^{K1} \frac{1}{G_1^{\langle k1\rangle} - G_1^{\langle k1(p)\rangle}} \times \prod_{\substack{kj=1 \\ kj\neq k1 \\ kj\neq k1(1) \\ \vdots \\ kj\neq ki(p)}}^{K1} \left[\frac{G_1 - G_1^{\langle kj\rangle}}{G_1^{\langle k1\rangle} - G_1^{\langle kj\rangle}}\right].$$

There are p summation series in this Lagrange factor function. Furthermore, for multiple variants with $m > 1$, there are two extreme cases in the pth

eigenparameter derivative namely, the structural variants are all different and all structural variants are the same, i.e.,

$$\underbrace{\frac{\partial^p \overline{\Phi}}{\partial G_i \partial G_j \cdots \partial G_t}}_{p \text{ terms}} = \begin{cases} \underbrace{\sum_{k1=1}^{K1} \sum_{k2=1}^{K2} \cdots \sum_{km=1}^{Km}}_{m \text{ summation}} \overline{\Phi}_{\langle k1k2\cdots km\rangle} L_1^{\langle k1\rangle}(G_1^{\langle k1\rangle}) \cdots \dfrac{\partial L_i^{\langle ki\rangle}(G_i^{\langle ki\rangle})}{\partial G_i} \\ \quad \cdots \dfrac{\partial L_j^{\langle kj\rangle}(G_j^{\langle kj\rangle})}{\partial G_j} \cdots \dfrac{\partial L_t^{\langle kt\rangle}(G_t^{\langle kt\rangle})}{\partial G_t} \cdots L_m^{\langle km\rangle}(G_m^{\langle km\rangle}) \quad \text{for } i \neq j \neq \cdots \neq t \\ \underbrace{\sum_{k1=1}^{K1} \sum_{k2=1}^{K2} \cdots \sum_{km=1}^{Km}}_{m \text{ summation}} \overline{\Phi}_{\langle k1k2\cdots km\rangle} L_1^{\langle k1\rangle}(G_1^{\langle k1\rangle}) \\ \quad \cdots \dfrac{\partial^p L_i^{\langle ki\rangle}(G_i^{\langle ki\rangle})}{\partial G_i^p} \cdots L_m^{\langle km\rangle}(G_m^{\langle km\rangle}) \quad \text{for } i = j = \cdots = t \end{cases} \tag{8}$$

where

$$\frac{\partial^p L_i^{\langle ki\rangle}(G_i^{\langle ki\rangle})}{\partial G_i^p} = \sum_{\substack{ki(1)=1 \\ ki(1)\neq ki}}^{Ki} \frac{1}{G_i^{\langle ki\rangle} - G_i^{\langle ki(1)\rangle}} \cdots \times \sum_{\substack{ki(p)=1 \\ ki(p)\neq ki \\ ki(p)\neq ki(p-1) \\ \vdots \\ ki(p)\neq ki(1)}}^{Ki} \frac{1}{G_i^{\langle ki\rangle} - G_i^{\langle ki(p)\rangle}} \prod_{\substack{kj=1 \\ kj\neq ki \\ kj\neq ki(1) \\ \vdots \\ kj\neq ki(p)}}^{Ki} \left[\frac{G_i^- G_i^{\langle kj\rangle}}{G_i^{\langle ki\rangle} - G_i^{\langle kj\rangle}}\right].$$

Note that these derivative terms process the symmetric property, such as that in the pth-order term

$$\frac{\partial^p \overline{\Phi}^k}{\partial G_i \partial G_j \cdots \partial G_t} = \frac{\partial^p \overline{\Phi}^k}{\partial G_t \partial G_j \cdots \partial G_i}. \tag{9}$$

After the establishment of eigenparameter functions and their derivatives, efforts are made to relate the changes of structural variants to the changes in these eigenparameters using the Taylor series expansion about the original structure.

Establishment of Polynomial System Equation

In general, one can use Taylor series expansion to obtain the change of the n eigenparameter pairs due to $\delta G_i = G_{\delta i} - G_i$ $(i = 1, 2, \ldots, m)$:

$$\overline{\Phi}_\delta - \overline{\Phi} = \sum_{i=1}^{m} \frac{\partial \overline{\Phi}}{\partial G_i}\delta G_i + \frac{1}{2!}\sum_{i=1}^{m}\sum_{j=1}^{m} \frac{\partial^2 \overline{\Phi}}{\partial G_i \partial G_j}\delta G_i \delta G_j + \cdots + \underbrace{\frac{1}{p!}\sum_{i=1}^{m}\sum_{j=1}^{m}\cdots\sum_{s=1}^{m}}_{p \text{ summations}} \frac{\partial^p \overline{\Phi}}{\partial G_i \partial G_j \cdots \partial G_t}\delta G_i \delta G_j \cdots \delta G_t + \overline{\varepsilon}_\Phi^p, \tag{10}$$

where $\overline{\varepsilon}_\Phi^p$ is the error vector in representing the selected eigenparameter pairs by polynomial function of structural variants of order p. The partial derivative terms in Eq. (10) with respect to different structural variants can be calculated from Eqs. (3)–(8).

Now the polynomial system equations are established. In what follows, the error term in Eq. (10) is neglected in order to obtain the polynomial system equations corresponding to the n selected eigenparameter pairs. The number of equations is larger than that of the unknowns δG_i (that is m) in order to improve the uniqueness of the obtained update. They are used in the first step of this algorithm to update the modification location. Left-hand side of Eq. (10) represents the differences of the eigenparameters between the actual and original structures, which are known. The structural variants in Eq. (10) can be determined using an optimization algorithm, such as quasi-Newton algorithm. When Eq. (10) is of first order, δG_i can also be determined by using the generalized inverse algorithm.

ITERATIVE ALGORITHM FOR SOLVING POLYNOMIAL SYSTEM EQUATION

Many of the existing experimental methods (known as design of experiments) work on the establishment of an input-output matrix and match it to the desired eigenparameters. For each structural variant, a number of nominal values are defined that represent the range for which the effect of that structural variant is desired to be known. The set-up is costly and tedious as thousands of experiments are involved. We give a moderate modification of $\pm 25\%$ as an example. For five structural variants with 10 trial increments of 5% original values, the total number of matrix experiments is 10^5. Thus the works demanded increase exponentially as the numbers of structural variants and trial runs increase. Nevertheless, the termination accuracy cannot be refined as the modification ranges are in fixed percentages.

This iterative algorithm is developed to update structural variants using the established polynomial system equations, bound the updated results, and apply the termination criterion to refine the updates in a single process. As the Taylor series equation contains the nonlinear coefficients, we use nonlinear optimization algorithm (Wong et al., 2004) that can iterate the interpolated stiffness values with these coefficients inversely. In the first update, the same equation as that in Eq. (10) is derived. From the known changes of the eigenparameters on the left-hand

side of Eq. (10), one can determine the changes of the structural variants using an optimization algorithm. They are denoted by $\delta\overline{G}^{(q)}$, where the number in the subscript denotes the number of update. The structural variants are modified by

$$\overline{G}^{(q+1)} = \overline{G}^{(q)} + \frac{\delta\overline{G}^{(q)}}{\|\delta\overline{G}^{(q)}\|}\eta^{(q)} \cdot G_h, \tag{11}$$

where $\|\delta\overline{G}^{(q)}\| = \sqrt{\delta\overline{G}^{(q)} \cdot \delta\overline{G}^{(q)}}$, G_h is assumed to be the original stiffness value for all the structural members, and $\eta^{(q)}$ is the modification scale factor of the qth update. In each update, Eq. (10) is modified by replacing $\overline{G}$ with $\overline{G}^{(q)}$, and G_i, G_j, ..., and G_s with $G_i^{(q)}$, $G_i^{(q)}$, ..., and $G_s^{(q)}$, respectively. From the calculated changes of the eigenparameters on the left-hand sides of the resulting polynomial system equations, one calculates inversely the changes of the structural variants $\delta\overline{G}^{(q)}$, and modifies the structural variants by using Eq. (11). This process is completed when the termination criterion is reached. On the other hand, changes of structural variants are bounded by the scale factors of a generic sequence. It allows the modification values to jump into the principal domain initially, but does not allow it to jump outside the principal domain afterward. Usually, in large percentage modifications, $\eta^{(q)}$ is set initially at a large percentage (e.g., 50%), which allows the updated solution to jump directly into the principal domain. Then it is reduced progressively so that the solution domain can be confined in the solution ranges.

TERMINATION INDICATOR AND CRITERION

Termination criterion is established to control the accuracy of iterative process. For the model updating of engineering truss model, the objective of model updating is met when the eigenparameters of the modified model are closest to that of the actual model. As a result, the norm of difference vector $\|\mathrm{d}\|$ is chosen as the termination indicator.

One of the first termination criterion is stated by Paskov (1995). The iteration process stops when either one of the following termination indicators drops below the criterion in the first time:

1. Norm of the eigenparameter difference vector first drops below nd ratio of the norm of normalized modified eigenparameter;

$$\|\mathrm{d}\|^{(q_{nd})} = \sum_{i=1}^{n}\left(\left\|\bar{\phi}^i_{q_{nd}} - \bar{\phi}^i_d\right\| + \left\|\frac{\lambda^i_{q_{nd}} - \lambda^i_d}{\lambda^i_d}\right\|\right) \le nd\left(\sum_{i=1}^{n}\left\|\bar{\phi}^i_d\right\| + n\right), \tag{12}$$

   where q_{nd} is the smallest update number for which the minimum allowed level is reached.
2. Maximum of absolute updated structural variant vector first drops below nd ratio of the maximum of absolute original structural variant vector,

$$\|\mathrm{d}\|^{(q_{nd})} = \max\left\{\mathrm{abs}(\overline{G}^{(q_{nd})})\right\} \le nd \cdot \max\left\{\mathrm{abs}(\overline{G}_h)\right\}, \tag{13}$$

where q_{nd} is the smallest update number for which the minimum allowed level is reached. Otherwise it stops when the number of updates q_{nd} exceeds the maximum preset value $n_{\max}$, i.e., $q_{nd} > n_{\max}$.

ORDER ANALYSIS ON FIXED-FREE BEAM MODEL

To investigate the applicability on different orders of the developed algorithm, various modification cases are introduced to a fixed-free beam model. The beam of length $L_t = 0.7\,\text{m}$, width $W = 0.0254\,\text{m}$, and thickness $H = 0.0031\,\text{m}$ has an area moment of inertia $I = \frac{1}{12}WH^3 = 6.3058 \times 10^{-11}\,\text{m}^4$ and a mass density $\rho = 2715\,\text{kg/m}^3$. Its finite element model shown in Fig. 1 is used to model its transverse vibration. The beam is divided into four typical elemental units with the length of each element being $l_e = \frac{L_t}{4}$, and there are 5 nodes. With V_i and θ_i denoting the translational and rotational displacements at node i ($i = 1, 2, \ldots, 5$), the displacement vector of the ith ($i = 1, 2, \ldots, 4$) element is $[V_i, \theta_i, V_{i+1}, \theta_{i+1}]^T$. Young's modulus is assumed to be constant over each beam element and that of the ith element is denoted by G_i. The Young's modulus of the original beam is $G_h = 69 \times 10^9\,\text{N/m}^2$. Hence $G_i^{(1)} = G_h$ for $i = 1, 2, \ldots, 4$. Small, medium, and large level of modifications, which correspond to the reduction in stiffness values of 20%, 50%, and 80% respectively, are simulated on the model. Mass matrix of the ith beam element is

$$M_i^e = \frac{\rho WHl_e}{420}\begin{bmatrix} 156 & 22l_e & 54 & -13l_e \\ 22l_e & 4l_e^2 & 13l_e & -3l_e^2 \\ 54 & 13l_e & 156 & -22l_e \\ -13l_e & -3l_e^2 & -22l_e & 4l_e^2 \end{bmatrix}, \tag{14}$$

and its stiffness matrix is

$$K_i^e = \frac{G_i I}{l_e^3}\begin{bmatrix} 12 & 6l_e & -12 & 6l_e \\ 6l_e & 4l_e^2 & -6l_e & 2l_e^2 \\ -12 & -6l_e & 12 & -6l_e \\ 6l_e & 2l_e^2 & -6l_e & 4l_e^2 \end{bmatrix}. \tag{15}$$

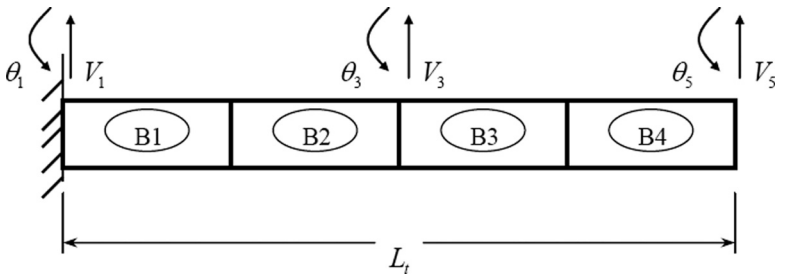

Figure 1 Finite element model of the four-element fixed-free beam.

Using the standard assembly process yields the 10×10 global mass and stiffness matrices. Constraining the translational and rotational displacements of the first node at the fixed-end to be zero yields the M and K matrices with dimension $N \times N$, where $N = 8$ is the d.o.f of the system. The displacement vector of the system, involving the displacements of the second through fifth node, is $[V_2, \theta_2, V_3, \theta_3, \ldots, V_5, \theta_5]^T$. The matrix $\frac{\partial K}{\partial G_i}$ $(i = 1, 2, \ldots, 4)$ can be obtained from K by setting $G_i = 1$ and $G_1 = \cdots = G_{i-1} = G_{i+1} = \cdots = G_N = 0$.

Without loss of generality, the first modification scale factor is set to $\eta^{(q1)}$ until $q1$th iteration. Efforts are made such that in most cases, $G^{(q1)}$ becomes saturated and oscillating in a stable range when the $q1$th update is reached. The second scale factor is set to $\eta^{(q2)}$, such that $G^{(q2)}$ becomes saturated at the $q2$th update. Finally the scale factor is set to $\eta^{(q3)}$, where the solution converges in an optimum path to the desired solution at $q3$th update. Therefore the generic sequence is set as

$$\eta^{(q)} = \left\{ \eta^{(q1)}, \eta^{(q1)}, \ldots, \underbrace{\eta^{(q1)}}_{q1}, \eta^{(q2)}, \eta^{(q2)}, \ldots, \underbrace{\eta^{(q2)}}_{q2}, \eta^{(q3)}, \eta^{(q3)}, \ldots, \underbrace{\eta^{(q3)}}_{q3} \right\}. \tag{16}$$

Multiple Small Percentage Structural Modification

For small percentage structural modification case, 10% modifications are introduced at the Young's moduli of elements B2, B3, and B4 respectively, $\overline{G}_\delta = (B1\ B2\ B3\ B4)^T = G_h(1.1\ 0.9\ 1\ 1.1)^T$. First-order algorithm is applied to assess this case, which terminates at $nd = 1e-8$ under criterion (2) of Eq. (13) or $n_{\max} = 100$. The first scale factor $\eta^{(q1)}$ is set to 0.1 with $q1 = 20$. In the second stage, $\eta^{(q2)}$ is set to

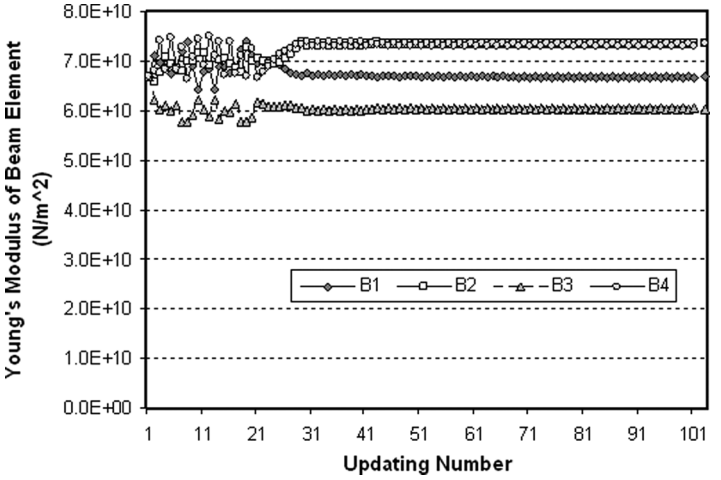

Figure 2 Case B_o1_s234: updating of fixed-free beam elements B2, B3, and B4 with ±10% modification by first-order algorithm.

0.02 with $q2 = 40$. For the third stage, $\eta^{(q3)}$ is set to 0.01 with $q3 = 60$. The generic sequence is set as

$$\eta^{(q)} = \left\{ 0.1, 0.1, \ldots, \underbrace{0.1}_{(20)}, 0.02, 0.02, \ldots, \underbrace{0.02}_{(40)}, 0.01, 0.01, \ldots, \underbrace{0.01}_{(60)} \right\}.$$

From the updating curves plotted in Fig. 2, element B3 drops to lower level, fluctuates chaotically until update 20, and remains at the solution level. All other elements interact with each other in this region, and they propagate gradually toward update 30. Afterward they remain at the solution levels and terminate at update 43. Then second-order algorithm is used with the results in Fig. 3. In this case, element B2 ramps up to the solution level until update 20. Elements B3 and B4 propagate gradually toward the solution levels. Then they remain flat at the solution levels. Element B1 fluctuates in large percentage, then propagates gradually toward the solution level. Then it remains at the solution level. Finally, third-order algorithm is applied to the case, and the updating curves are plotted in Fig. 4. Element B3 rises and remains flat until update 10, then it oscillates and drops to the solution level. Afterward it remains flat at the solution level. Element B1 rises to a peak and drops regularly toward the solution level. Then they propagate gradually toward the solution levels. Element B2 rises to a peak, drops, and rises again to the solution level in an oscillatory pattern until update 20. Then it continues to rise gradually to the solution level. For element B4 it rises to the peak level and drops in fluctuation pattern. And it rises gradually toward the solution level. From this analysis, one can observe that the first-order algorithm converges at update 43. The second-order algorithm converges at update 50. Meanwhile the third-order

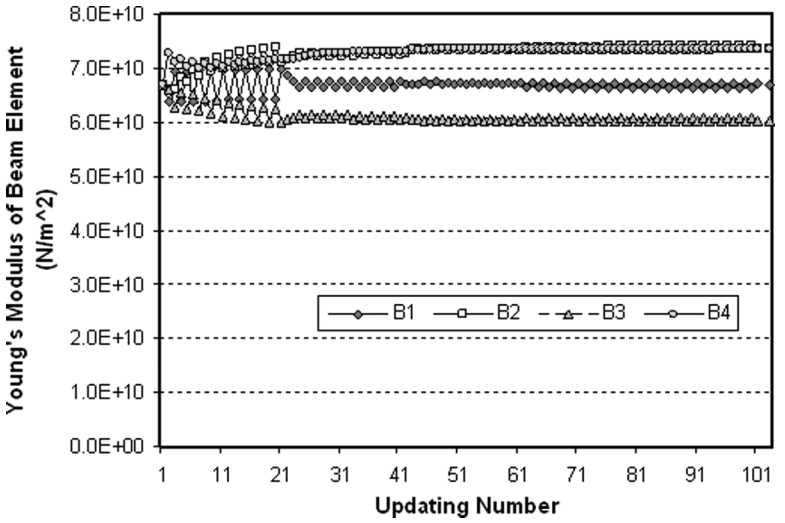

Figure 3 Case B_o2_s234: updating of fixed-free beam elements B2, B3, and B4 with ±10% modification by second-order algorithm.

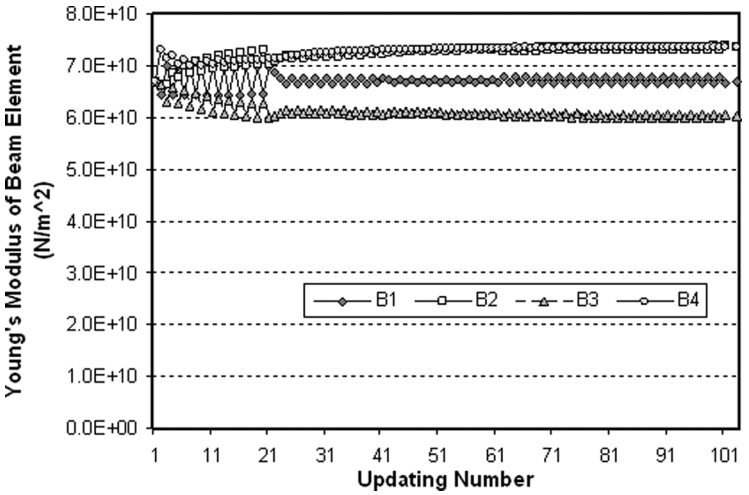

Figure 4 Case B_o3_s234: updating of fixed-free beam elements B2, B3, and B4 with ±10% modification by third-order algorithm.

algorithm converges at update 60. Therefore the first-order algorithm is the best for the small percentage modification.

Multiple Medium Percentage Structural Modification

For the medium percentage structural modification case, 25% modifications are introduced at the Young's moduli of elements B1 and B3 respectively, i.e.,

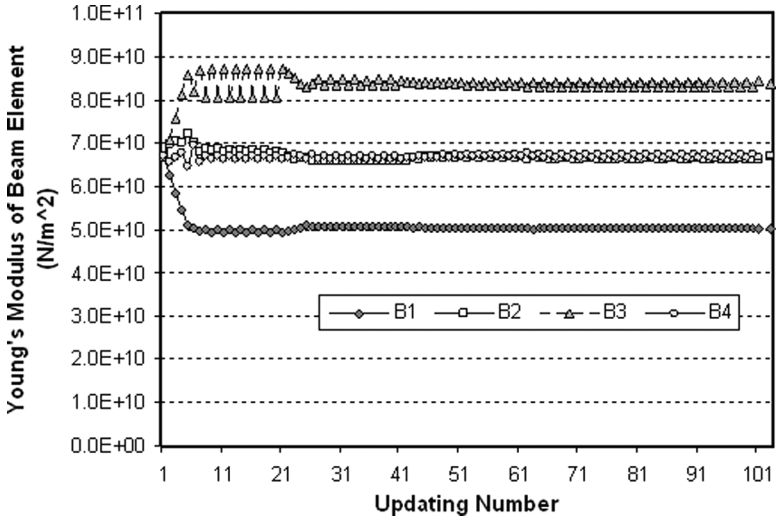

Figure 5 Case B_o1_m13: updating of fixed-free beam elements B1 and B3 with ±25% modification by first-order algorithm.

$\overline{G}_\delta = (B1\ B2\ B3\ B4)^T = G_h(0.75\ 1\ 1.25\ 1)^T$. For this structure, the scale factors are insignificant in bounding the updating ranges. Thus the generic sequence is set same as before,

$$\eta^{(q)} = \left\{ 0.1, 0.1, \ldots, \underbrace{0.1}_{(20)}, 0.02, 0.02, \ldots, \underbrace{0.02}_{(40)}, 0.01, 0.01, \ldots, \underbrace{0.01}_{(60)} \right\}.$$

Iteration process terminates at $nd = 1e-8$ under criterion (2) of Eq. (13) or $n_{\max} = 100$. Updating curves of using the first-order algorithm are plotted in Fig. 5. Element B3 rises to the solution level, while element B1 drops to the solution level in five updates. Elements B2 and B4, arrange chaotically to the solution levels in five updates. Then the four elements oscillate in the range until update 20. Afterward, they converge to the solution levels in four to five updates. In the subsequent updates, they just propagate around the solution levels and terminate with criterion (2) where $nd = 1e-8$. On the other hand, second-order algorithm is applied to this case (Fig. 6). Element B3 rises to the solution level around update 20, then it remains flat at the solution level. Elements B3 and B4 fluctuate oscillatory until update 20, then they remain flat at the solution level. Finally, third-order algorithm is used with the results plotted in Fig. 7. Element B1 drops rapidly to the lower level, then it propagates gradually around the solution level. Afterward, it remains flat at the solution level. Element B3 ramps up to the solution level until update 20, then it remains flat at the solution level. Elements B2 and B4 oscillate through the valley region, fluctuate, and decrease to the lower levels and terminate under the same criterion. From this analysis, one can observe that the third-order algorithm converges at update 42 and the first-order algorithm converges at update

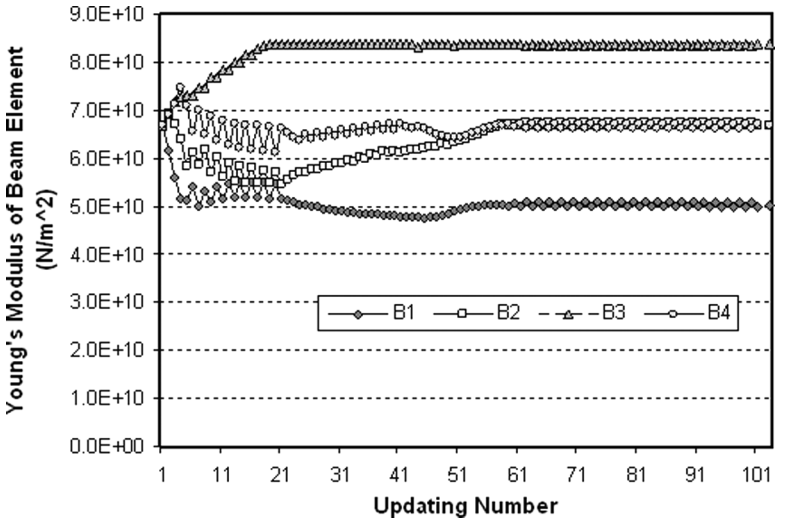

Figure 6 Case B_o2_m13: updating of fixed-free beam elements B1 and B3 with ±25% modification by second-order algorithm.

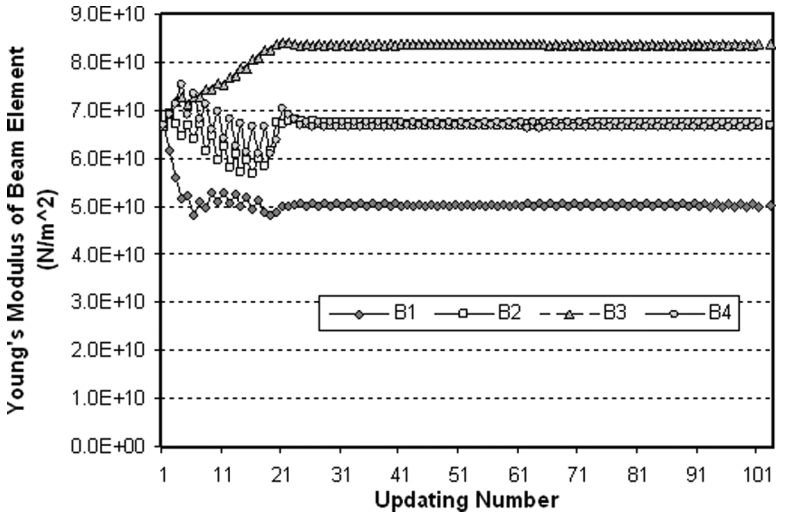

Figure 7 Case B_o3_m13: updating of fixed-free beam elements B1 and B3 with ±25% modification by third-order algorithm.

50. Meanwhile, the second-order algorithm converges at update 57. Therefore, the third-order algorithm is the best for the medium percentage modification.

Single Large Percentage Structural Modification

Assume that there is 50% modification at the second element B2 in the large percentage structural modification case with $\overline{G}_\delta = (B1\ B2\ B3\ B4)^T = G_h(1\ 1.5\ 1\ 1)^T$. Again, the generic sequence is set as

$$\eta^{(q)} = \left\{ 0.1, 0.1, \ldots, \underbrace{0.1}_{(20)}, 0.02, 0.02, \ldots, \underbrace{0.02}_{(40)}, 0.01, 0.01, \ldots, \underbrace{0.01}_{(60)} \right\}.$$

Consider first-order algorithm with updating curves as shown in Fig. 8. Element B2 rises to the solution level in five updates, then it propagates in specific percentage. Afterward it ramps up a little bit to the accurate level update 25, remains flat at solution level, and converges at update 32. For elements B1 and B3, they interact complementary in around ten updates. Afterward they interact and slightly propagate at the solution levels. For element B4, it remains flat at solution level initially, then drops to lower level in two updates. Then it ramps up and propagates to the solution level. For second-order algorithm in Fig. 9, element B2 rises gradually toward the solution level until update 30. Elements B1 and B4 rise and fall interactively with each other until update 30. Then they rise gradually toward the solution levels and converge at update 54. When the third-order algorithm is applied, the updating curves are plotted in Fig. 10. It is interesting to note that they follow the similar updating patterns as the second-order algorithm and converge at update 54. Reason behind that needs to be further investigated. Average percentage

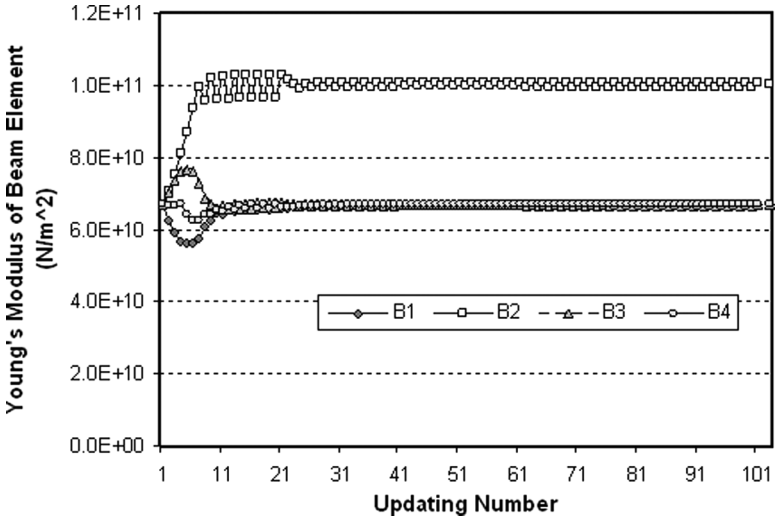

Figure 8 Case B_o1_l2: updating of fixed-free beam element B2 with +50% modification by first-order algorithm.

error deviated from second-order algorithm is 2.29e–2% with maximum of 1.041% and minimum of –2.23e–1%. From this analysis, we found that the first-order algorithm is the best for the large percentage modification.

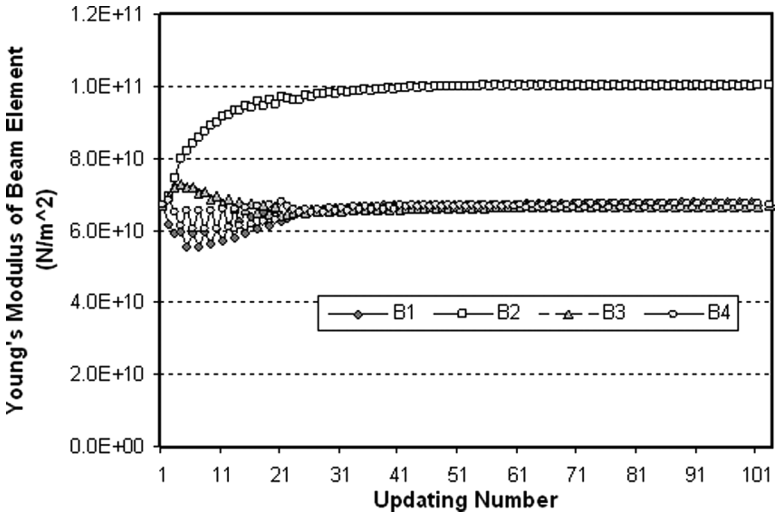

Figure 9 Case B_o2_l2: updating of fixed-free beam element B2 with +50% modification by second-order algorithm.

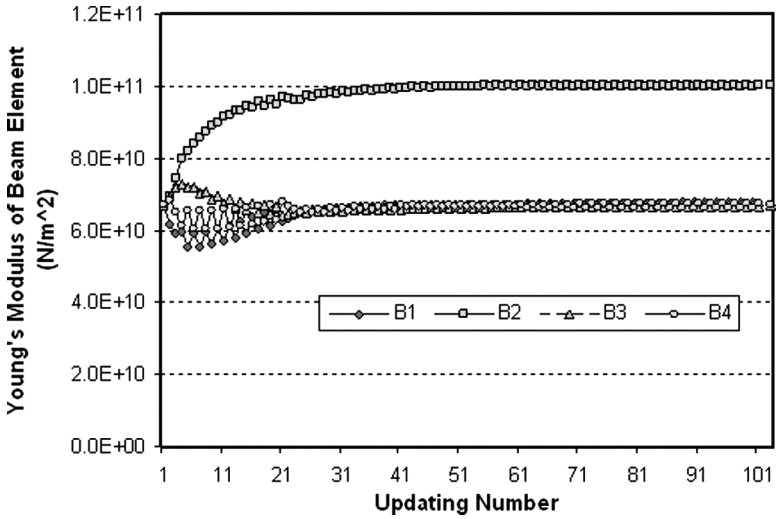

Figure 10 Case B_o3_l2: updating of fixed-free beam element B2 with +50% modification by third-order algorithm.

FINITE ELEMENT MODEL SIMULATION OF FOUR-BAY ENGINEERING TRUSS

In this study, different modification cases are simulated using the finite element model of a typical four-bay engineering truss. This truss is a structure with typical baywise unit of three-dimensional rectangular truss. It is fixed to the ground at the four foundation columns. In the structural reanalysis program, it calls in the stiffness matrix of each building unit to assemble the overall stiffness matrix. Both the fixed-free beam and this truss are composed of four elemental units, and similar boundary conditions are applied to these structures. Hence eigenvalue analysis of the beam model is analogue to that of the engineering truss, and order analysis of the beam can be applied to this structure. Design of its finite element model was mainly based on the following principles. It is physically alike to the actual truss composed of the same major elements listed in Table 1. Also, its d.o.fs are similar to the actual structure. Each of them can be detached without affecting the whole structure. The elements are assembled at the joints and nodes as the actual structure. Based on the aforesaid criteria, the model created is shown in Fig. 11. Dimensions

Table 1 Detail dimensions of engineering truss elements

Element	Nominal length (mm)	Cross-sectional dimensions (mm)
Column	609.6	Angle 38.1 × 38.1 × 6.35 (Thickness)
Diagonal-1	845.6	Bar 25.4 × 12.7
Diagonal-2	779.6	Bar 25.4 × 12.7
Horizontal-1	586	Bar 25.4 × 12.7
Horizontal-2	486	Bar 25.4 × 12.7

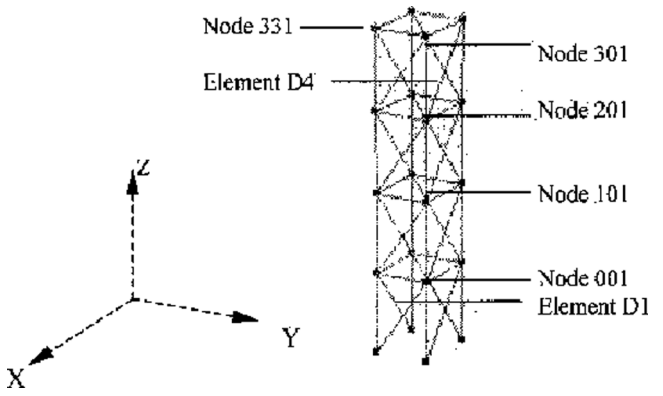

Figure 11 Finite element model of four-bay engineering truss.

of the elements are the same as the actual structure, while the nodes are arranged for the ease of numbering. The model is composed of aluminium truss elements which are assembled together by the constrained d.o.f. Boundary conditions at each node connected to the ground are modeled as grounded d.o.f.

An automated program using the aforementioned principles is generated in conjunction with structural reanalysis using a finite element code. The program reads in the eigenparameter database of the original analytical model and its interpolated models for each structural variant separately. Then it reconstructs the reduced-order modes according to the selected master d.o.f using the database. Through this process the right modes are extracted and arranged in order, while the phases of the eigenparameters are corrected. As the truss elements are connected directly to the nodal points, efforts are made to identify the modified elements by the change of the mode shapes at these nodal points. In the variation analysis of mode selection, profound linear relationships are indicated between the fundamental bending modes (in X and Y direction of Fig. 11) and the structural variants. This finding proved that these modes are highly sensitive to the corresponding p values and the higher order polynomials serve as suitable representations of the eigenparameter functions. From Table 2, one can observe that the most affected structural variants by the first X-bending mode and Y-bending mode are those of diagonal and column elements at first bay. Meanwhile, the most affected structural variants by the first torsional mode are those of diagonal and column elements at all bays. From this analysis, the fundamental modes give direct implications on

Table 2 Relationship between mode shapes and structural variants

Mode	Most affected element
First X-Bend	Bay 1: diagonal and column elements
First Y-Bend	Bay 1: diagonal and column elements
First Torsion	All bays: diagonal and column elements

the locations of modifications. This approach is particularly useful for updating a complicated large structure, which is composed of many detachable elements.

All these modes then went through the orthogonalization processes. As indicated by Lim (1990), the mass orthogonalized eigenvectors can be defined by the matrix orthogonalization procedure as

$$\bar{\phi}^k = \bar{\phi}^k \Big/ \sqrt{\bar{\phi}^{kT} M \bar{\phi}^k}. \tag{17}$$

In most engineering components, columns and horizontals are usually strong major elements taking most of the static design loads. Diagonals are weak minor elements instead to well position the major elements. When all diagonals at each individual bay are modified, i.e., $G_{\delta i} = Di$, the number of structural variants to be modified is four, i.e., $m = 4$. Now consider the number of independent equations. There are 16 equations from each bending modes and 32 equations from the torsional mode of the truss model. This sum up to 64 equations, i.e., $N = 64$. Therefore, this is an overconstrained system and its unique solution should be robustly obtained. Simulated cases of both single and multiple structural modifications are discussed below.

Multiple Small Percentage Structural Modification

According to the order analysis of fixed-free beam, second-order algorithm is the best to update small percentage modifications. For this multiple modifications case T_o1_s124, 10% modifications are introduced at the first and third bays at all the diagonals as the truss elements. Therefore, its modification vector is written as $\overline{G}_\delta = (D1\ D2\ D3\ D4)^T = G_h(1.1\ 0.9\ 1\ 1.1)^T$. For small structural modification updating, one can set the initial scale factor to a general level, e.g., 10% of the original structure's stiffness value, $\eta^{(q1)} = 0.1G_h$ with $q1 = 20$. In the second and third stages, $\eta^{(q2)}$ and $\eta^{(q3)}$ are set to 0.1 also. The generic sequence is set as

$$\eta^{(q)} = \left\{ 0.1, 0.1, \ldots, \underbrace{0.1}_{(20)}, 0.1, 0.1, \ldots, \underbrace{0.1}_{(40)}, 0.1, 0.1, \ldots, \underbrace{0.1}_{(60)} \right\}.$$

Termination criteria are set at convergence level $nd = 5e - 5$ in criterion (1) of equation (12) or $n_{\max} = 100$. By applying the iterative polynomial algorithm, the updating results are plotted in Fig. 12. Element B2 drops to the lower level initially. Then it oscillates with diminishing amplitude, which vanishes near the ultimate solution. Element D1 rises to a higher level and then oscillates with a diminishing amplitude, which vanishes near the ultimate solution. Then for element D4, it rises to the higher level then oscillates with an increasing amplitude until update 12. It oscillates with constant amplitude and arrives at the solution level. Element D3 remains constant in the updating process. Termination chart of the case is plotted in Fig. 13. It drops rapidly initially, and oscillates with decreasing amplitude. Afterward, it oscillates with constant amplitude until update 31 where the set termination criterion is attained.

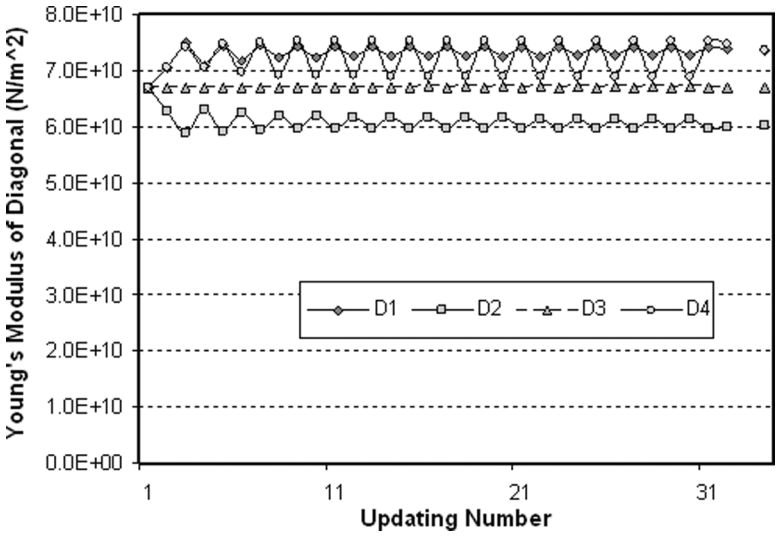

Figure 12 Case T_o1_s124: updating of engineering truss elements D1, D2, and D4 with ±10% modifications by first-order algorithm.

Multiple Medium Percentage Structural Modification

For the multiple structural modification case T_o2_m23, two sets of truss elements, i.e., all diagonals at the second and third bays, with medium percentage (25%) modifications are introduced. Its modification vector is given as $\overline{G}_\delta = G_h(1\ 0.75\ 1.25\ 1)^T$. The first-scale factor $\eta^{(q1)}$ is set to 0.5 with $q1 = 20$. In the second stage, $\eta^{(q2)}$ is set to 0.3 with $q2 = 40$. For the third stage, $\eta^{(q3)}$ is set to 0.1 with $q3 = 60$. The generic sequence is set as

$$\eta^{(q)} = \left\{ 0.5, 0.5, \ldots, \underbrace{0.5}_{(20)}, 0.3, 0.3, \ldots, \underbrace{0.3}_{(40)}, 0.1, 0.1, \ldots, \underbrace{0.1}_{(60)} \right\}.$$

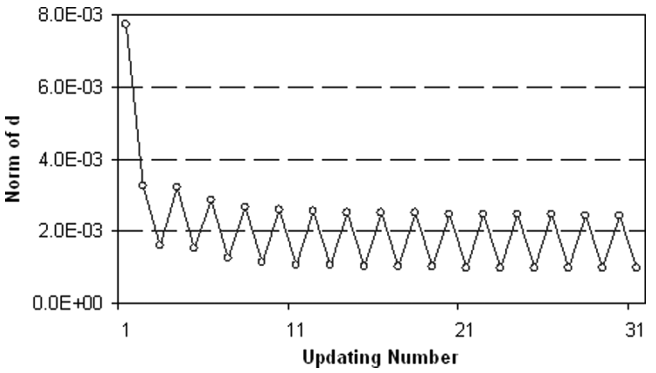

Figure 13 Case T_o1_s124: termination indicator of engineering truss elements D1, D2, and D4 with ±10% modifications by first-order algorithm.

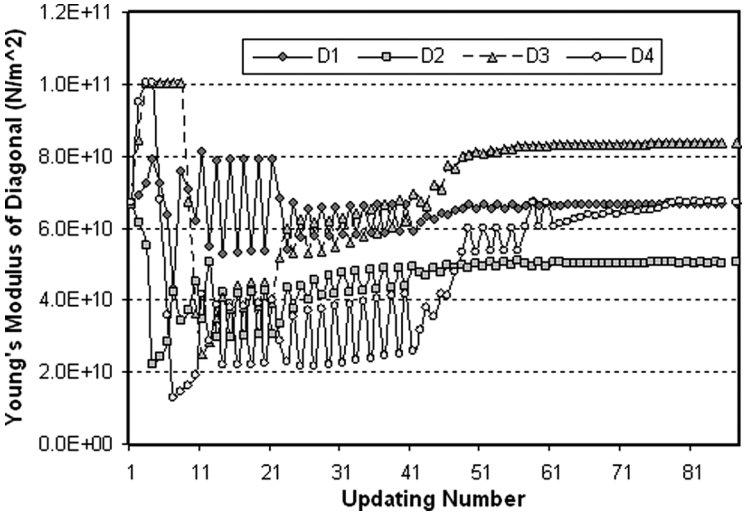

Figure 14 Case T_o2_m23: updating of engineering truss elements D2 and D3 with ±25% modifications by second-order algorithm.

Termination criteria are set as the small percentage modification case. Although third-order algorithm is suitable to update medium percentage modification as indicated by the order analysis, our analysis on this more complicated truss is limited to second-order at this stage. In the updating chart of Fig. 14, elements D1 to D4 fluctuate in large percentages in the first 10 updates. Then they oscillate in large percentage until update 20. Afterward they change patterns and oscillate until gradually toward the solution levels. The termination chart (as shown in Fig. 15) indicates a two-peak pattern during the convergence. For the first peak, it jumps from 6.95e–2 to 2.52e–1. Meanwhile for the second peak, it ramps from 1.43e–1

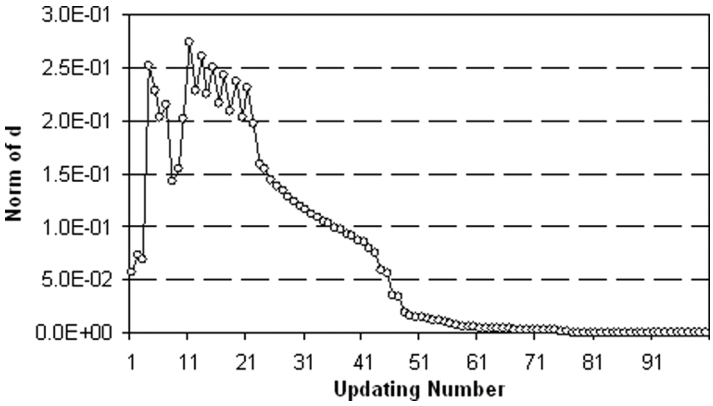

Figure 15 Case T_o2_m23: termination indicator of engineering truss elements D2 and D3 with ±25% modifications by second-order algorithm.

to 2.73e–1. Then it drops gradually to update 42 and drops rapidly until update 48. Afterward, it converges gradually toward the ultimate solution at update 85.

Single Large Percentage Structural Modification

From the order analysis, first-order algorithm is most appropriate to update large percentage modification. Large percentage modification case T_o1_l4 with $Di = 0.5G_h$ is generated with the modification vector $\overline{G}_\delta = G_h(1\ 1\ 1\ 1.5)^T$. For large percentage structural modification, one can set the initial scale factor to a medium percentage, e.g., 50% of the Young's modulus of the original structure with $\eta^{(q1)} = 0.5G_h$ with $q1 = 20$. In the second stage, $\eta^{(q2)}$ is set to 0.3 with $q2 = 40$. For the third stage, $\eta^{(q3)}$ is set to 0.1 with $q3 = 60$. The generic sequence is set as

$$\eta^{(q)} = \left\{ 0.5, 0.5, \ldots, \underbrace{0.5}_{(20)}, 0.3, 0.3, \ldots, \underbrace{0.3}_{(40)}, 0.1, 0.1, \ldots, \underbrace{0.1}_{(60)} \right\}.$$

The d norm (Fig. 16) oscillates in large percentage in the first stage until update 20. In the second stage, its oscillation amplitude diminishes until update 40. Meanwhile, in the third stage, its amplitude increases gradually until update 60. Finally, it decreases gradually until the termination criterion is reached at update 72. In the updating chart (Fig. 17), element D4 oscillates in a large range until update 20. Then it ramps to a higher level and oscillates in a smaller range until update 40. It drops to the solution level and oscillates in a smaller range until update 60. Afterward it rises gradually toward the solution level. Similar pattern can be observed for elements B1, B2, and B3, but with smaller ranges while they remain around the solution levels.

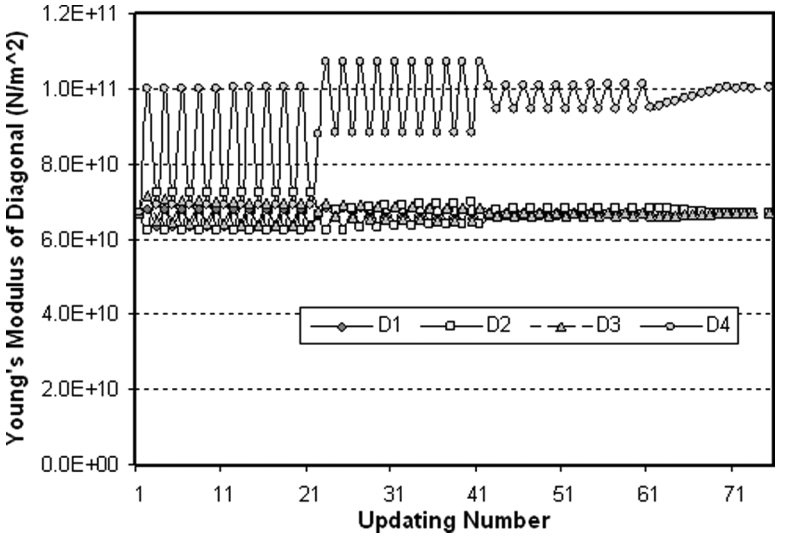

Figure 16 Case T_o1_l4: updating of engineering truss element D4 with +50% modification by first-order algorithm.

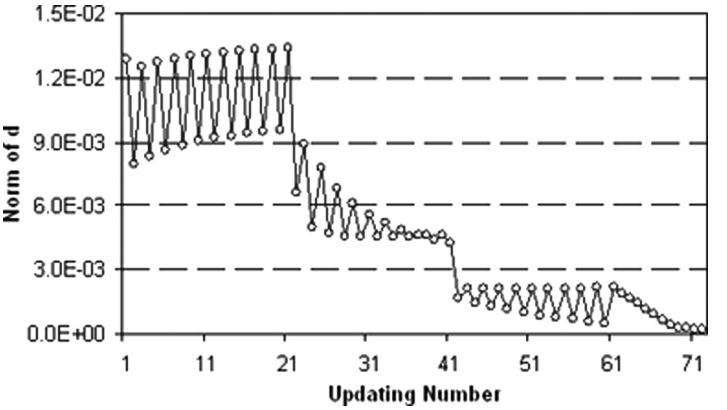

Figure 17 Case T_o1_l4: termination indicator of engineering truss element D4 with +50% modification by first-order algorithm.

CONCLUSION

Lagrange factor functions and Taylor series expansions are integrated to define the multivariate polynomial functions between actual eigenparameters and structural variants. Using the fixed-free beam model, the first-order algorithm is the best for the small percentage modification, the third-order algorithm is the best for the medium percentage modification, and the first-order algorithm is the best for the large percentage modification. For single modification cases on the engineering truss, the updating errors dropped to around one percent. For multiple modifications cases, the updating errors were rapidly converged within few percents. Development of third-order algorithm on the engineering truss is needed for further investigation. As the algorithms are robust at small selected d.o.f. to total d.o.f. ratios, they are validated to be effective in updating both single and multiple modifications of the engineering truss.

NOTATION

The following symbols are used in this paper:

G_i $(i = 1, 2, \ldots, m)$	ith structural variant of the updated structure;
$\overline{\Phi}_{\langle k1k2\ldots km\rangle}$	eigenparameter pairs corresponding to different combination of Ki prescribed values of G_i (denoted by $G_i^{\langle 1\rangle}$, $G_i^{\langle 2\rangle}$, $\cdots$ and $G_i^{\langle Ki\rangle}$);
$L_i^{\langle ki\rangle}(G_i^{\langle ki\rangle})$	Lagrange factor function of the mth structural variant at the kith interpolated stiffness value;
$\overline{G} = (G_1, G_2, \ldots, G_m)^T$	structural variant vector of the updated structure;
$\overline{G}_\delta = (G_{\delta 1}, G_{\delta 2}, \ldots, G_{\delta m})^T$	structural variant vector of the actual structure;
$\overline{\Phi}$	selected sets of eigenparameter pairs corresponding to the updated structure;
$\overline{\Phi}_\delta$	selected sets of eigenparameter pairs corresponding to the actual structure;

$\overline{\varepsilon}_{\Phi}^{p}$	error vector in representing the kth eigenparameter pair by polynomial function of structural variants of order p;
$\overline{G}^{(q)}$	structural variant vector at the qth number of updates; and
$\eta^{(q)}$	modification scale factor.

ACKNOWLEDGMENT

The authors greatly appreciate the support from the Jinchuan Group Ltd. under the project "Model updating of mining scraper design model and its health monitoring using sensor data" with grant number H04010801W080756. In addition, the first author would like to thank Prof. A.S. Khan of University of Maryland, Baltimore County, for his help in the finite element model and simulation aspects of this work.

REFERENCES

Allemang, R. J., Brown, D. L. (1982). *A Correlation Coefficient for Modal Vector Analysis*. Proceedings of the 1st International Modal Analysis Conference (Orlando, FL). Union College, Schenectady, NY, 110–116.

Bagchi, A. (2005). Updating the mathematical model of a structure using vibration data. *Journal of Vibration and Control* 11(12):1469–1486.

Bakir, P. G., Reynders, E., Roeck, G. D. (2007). Sensitivity-based finite element model updating using constrained optimization with a trust region algorithm. *Journal of Sound and Vibration* 305:211–225.

Butkewithsch, S., Steffen, V. (2002). Shape optimization model updating and empirical modeling applied to the design synthesis of a heavy truck side guard. *International Journal of Solids and Structures* 39:4747–4771.

Chen, S. H. (2007). *Matrix Perturbation Theory in Structural Dynamic Design*. Beijing: Science Press, No. 9787030186980.

Humbert, L., Thouverez, F., Jezequel, L. (1999). Finite element dynamic model updating using modal thermoelastic fields. *Journal of Sound and Vibration* 228(2):397–420.

Ip, K. H., Tse, P. C. (2001). Determination of dynamic flexural and shear moduli of thick composite beams using natural frequencies. *Journal of Composite Materials* 35(17):1553–1569.

Issacson, E., Keller, H. B. (1973). *Analyze Numerischer Verfahren*. Frankfurt: Verlag Harri Deutch.

Lim, T. W. (1990). Submatrix approach to stiffness matrix correction using modal test data. *AIAA Journal* 28(6):1123–1130.

Paskov, S. H. (1995). Termination criteria for linear problems. *Journal of Complexity* 11:105–137.

Penny, J. E. T., Friswell, M. I., Garvey, S. D. (1994). Automatic choice of measurement locations for dynamic testing. *AIAA Journal* 32(2):407–414.

Sinha, J. K., Mujumdar, P. M., Moorthy, R. I. K. (2001). Detection of spring support locations in elastic structure using a gradient-based finite element updating technique. *Journal of Sound and Vibration* 240(3):499–519.

Spath, H. (1995). *Two Dimensional Spline Interpolation Algorithms*. Wellerley, Massachusetts: A.K. Peters.

Spath, H. (1995). *One Dimensional Spline Interpolation Algorithms*. Boston: A.K. Peters.

Steeneckers, G., Devriendt, C., Guidlaume, P. (2007). On the use of transmissibility measurements for finite element model updating. *Journal of Sound and Vibration* 303:707–727.

Teughels, A., Roeck, G. D. (2004). Structural modification identification of the bridge Z24 by F model updating. *Journal of Sound and Vibration* 278:589–610.

Wahab, M. M. A., Roeck, G. D., Peeters, B. (1999). Parametrization of modification in reinforced concrete structures using model updating. *Journal of Sound and Vibration* 228(4):717–730.

Wong, C. N., Zhu, W. D., Xu, G. Y. (2004). On an iterative general-order perturbation method for multiple structural damage detection. *Journal of Sound and Vibration* 203(3):167–189.

Xi, F., Wang, X. G., Qin, Z. (2000). Modeling and analysis for active control of circular saw vibration. *Journal of Vibration and Control* 6:1225–1241.

Zhang, X. (1999). A best matrix approximation method for updating the analytical model. *Journal of Sound and Vibration* 223(5):759–774.