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New evaluation methods for conceptual design selection using computational intelligence techniques†

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Abstract

The conceptual design selection, which aims at choosing the best or most desirable design scheme among several candidates for the subsequent detailed design stage, oftentimes requires a set of tools to conduct design evaluation. Using computational intelligence techniques, such as fuzzy logic, neural network, genetic algorithm, and physical programming, several design evaluation methods are put forth in this paper to realize the conceptual design selection under different scenarios. Depending on whether an evaluation criterion can be quantified or not, the linear physical programming (LPP) model and the RAOGA-based fuzzy neural network (FNN) model can be utilized to evaluate design alternatives in conceptual design stage. Furthermore, on the basis of Vanegas and Labib's work, a multi-level conceptual design evaluation model based on the new fuzzy weighted average (NFWA) and the fuzzy compromise decision-making method is developed to solve the design evaluation problem consisting of many hierarchical criteria. The effectiveness of the proposed methods is demonstrated via several illustrative examples.

Keywords: Conceptual design selection; Design evaluation; Linear physical programming; Fuzzy logic; Neural network; Genetic algorithm; NFWA; Fuzzy compromise decision-making

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1. Introduction

At the conceptual design stage, several rough design alternatives may be generated based on the functional and performance requirements from customers. The purpose of conceptual design selection is to choose the best or most desirable design scheme among several candidates for the subsequent detailed design stage. As the conceptual design has a great influence on cost, robustness, reliability, manufacturability, and development time of final products, and the cost of changes of final products increases by ten times or greater when changes are made at the conceptual design stage. It is therefore crucial for designers to use effective tools to appropriately evaluate and choose the best design alternative.

The process of conceptual design is very complicated. Most of the time, information managed at this stage is incomplete, uncertain, and imprecise [1]. Several methods have been reported as of late to address these issues in evaluation of conceptual design. Malekly et al. [2] proposed a systematic decision process for selecting the best design idea by means of a novel integrated optimization-based methodology which integrates experts' knowledge and experiences via the fuzzy set method. Akay et al. [3] presented a new concept selection methodology that extends the fuzzy information axiom (FIA) approach to incorporate IT2FSs, namely interval-type-2 fuzzy information axiom. Since the vague set theory is superior to the fuzzy set theory in dealing with uncertain and imprecise judgments of decision makers (DMs) owing to its ability of supporting opposing evidences, Geng et al. [4] developed a new integrated design concept evaluation approach based on vague sets, in which a modified weighted least squares model (WSLM) based on vague sets was proposed to aggregate all individual judgments in group decision-making. The orders of alternatives were ranked according to the synthetic vague decision matrix. An analytic hierarchy process along with a simulation analysis method was proposed by Ayag [5] to evaluate conceptual design alternatives in a new product development environment. As customer requirements play a very important role in the evaluation and decision process of conceptual design schemes of products, an evaluation and decision method based on customer requirements was proposed in Ref. [6] using the fuzzy reasoning and the BP neural network.

In this paper, several design evaluation methods using computational intelligence techniques, such as the fuzzy logic, the neural network, the genetic algorithm, and the physical pro-

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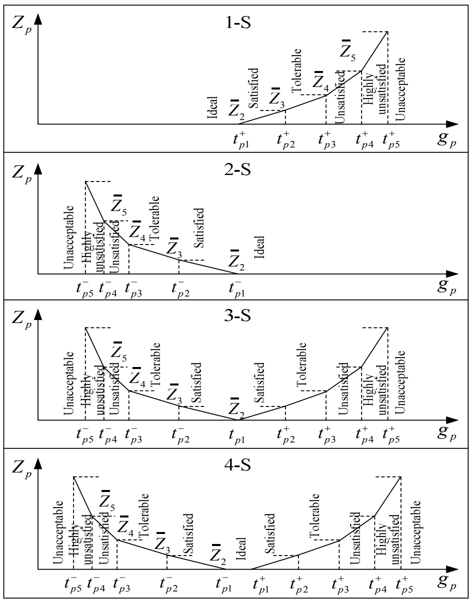

Fig. 1. Class functions of the *p*th design criterion.

gramming are developed. Depending on whether evaluation criteria can be quantified or not, the linear physical programming (LPP) model and the RAOGA-based fuzzy neural network (FNN) model are applied to evaluate design alternatives at the conceptual design stage. Furthermore, on the basis of Vanegas and Labib's work [1], a multi-level conceptual design evaluation model based on the NFWA and fuzzy compromise decision-making method is proposed to solve design evaluation problems consisting of many hierarchical criteria.

The rest of the paper is organized as follows. Section 2 gives an overview of the LPP model for evaluating design alternatives. The FNN model along with a RAOGA-based fuzzy learning algorithm is detailed in section 3. Section 4 introduces a multi-level conceptual design evaluation model based on the NFWA and fuzzy compromise decision-making method, and section 5 concludes this paper.

2. Design evaluation and decision-making based on LPP

The decision matrix is one of the most popular evaluation methods in engineering design. The main drawback of this method is that decision makers have to specify a set of weights, which are physically meaningless [7]. To overcome this shortcoming, the linear physical programming (LPP) model is introduced in this section to evaluate design alternatives when values of evaluation criteria can be quantified.

2.1 Linear physical programming

The LPP proposed by A. Messac, with the intent of substantially reducing computational cost of large scale problems, has been recognized as an effective way to solve multidisciplinary optimization problems and applied in many fields [8-15]. In the LPP, a decision maker expresses his or her preferences for each criterion by using four different classes, i.e. smaller is better (1-S), larger is better (2-S), value is better (3-S), and range is better (4-S) [16], as shown in Fig. 1.

The horizontal axis corresponds to the value of the *p*th criterion, i.e. g_p , whereas the value of class functions or preference functions, denoted as Z_p , is represented by the vertical axis. The parameters $t_{p1}^+,\cdots,t_{p5}^+$ are physically meaningful, and they are specified by decision makers to quantify the preference associated with the *p*th design criterion.

2.2 LPP-based design evaluation

In the LPP model, a weighted sum of deviations over all ranges (*s* = 2~5) and criteria *(p* = 1~*ns*) is defined as the aggregated objective function. The LPP-based model for concept evaluation and decision-making can be mathematically expressed as follows [8]

$$\begin{aligned} \min_{d_{ps}^-,d_{ps}^+,x} \ & J=\sum_{p=1}^{n_s}\sum_{s=2}^{5}(\tilde{w}_{ps}^- d_{ps}^- + \tilde{w}_{ps}^+ d_{ps}^+) \\ \text{s.t. } & g_p(x)-d_{ps}^+ \le t_{p,s-1}^+;\ d_{ps}^+\ge 0;\ g_p(x)\le t_{p5}^+ , \\ & g_p(x)+d_{ps}^- \ge t_{p,s-1}^-;\ d_{ps}^-\ge 0;\ g_p(x)\ge t_{p5}^- \\ & x_{\min}\le x\le x_{\max} \end{aligned} \tag{1}$$

where d_{pi}^- and d_{ps}^+ denote respectively the negative and positive deviations of the criterion value $g_p(x)$ from its target values $t_{p,s-1}^-$ and $t_{p,s-1}^+$.

To ensure the one vs. others criteria (OVO) rule, it is more beneficial to improve a worse criterion than a better criteria. Let $\bar{Z}_s = Z_s - Z_{s-1}$ ($2\le s\le 5$), one has $\bar{Z}_s=\beta(n_s-1)\bar{Z}_{s-1}$ ($3\le s\le 5,\ n_s>1,\ \beta>1$), where n_s denotes the number of soft criteria, and β is used as a convexity parameter. The length of the *s*th criterion is defined as $\bar{t}_{ps}^+ = t_{ps}^+ - t_{p(s-1)}^+$ and $\bar{t}_{ps}^- = t_{ps}^- - t_{p(s-1)}^-$ ($2\le s\le 5$). The slope value of the class function to the *i*th criterion takes the form $w_{ps}^+=\bar{Z}_s/\bar{t}_{ps}^+$, $w_{ps}^-=\bar{Z}_s/\bar{t}_{ps}^-$ ($2\le s\le 5$). These slopes change from range to range and from criterion to criterion. Let $\tilde{w}_{ps}^+ = w_{ps}^+ - w_{p(s-1)}^+$, $\tilde{w}_{ps}^- = w_{ps}^- - w_{p(s-1)}^-$, and $w_{p1}^- = w_{p1}^+ = 0$. Once the slopes are known, the convexity requirement can be verified by the relationship $\tilde{w}_{\min}=\min_{p,s}(\tilde{w}_{ps}^+,\tilde{w}_{ps}^-)>0$. When the convexity satisfies some certain demand, the increasing weights of $\tilde{w}_{ps}^-$ and $\tilde{w}_{ps}^+$ can be identified. One can see that weights are related to slopes of class functions. Based on given preference ranges of different criteria, the LPP weights algorithm can be used to increase weights $\tilde{w}_{ps}^-$ and $\tilde{w}_{ps}^+$.

2.3 Realization procedure

Based on preference structures for different criteria, the LPP weights algorithm can be used to calculate weights in the aggregate objective function. In a LPP model, for the purpose of design evaluation, the value of aggregate objective functions is treated as the total score and calculated for each design alternative. The lower value of total score, the better the design scheme is. The flowchart of the LPP-based design evaluation process is shown in Fig. 2.

2.4 Illustrative example

In this section, an automobile engine is used as an example to demonstrate and verify the effectiveness of the LPP-based design evaluation model. Three different automobile engine designs are shown in Table 1 [17]. Three criteria, i.e. oil consumption, unit power quality, and life, are considered in design evaluation. The LPP method is used to evaluate the design of the automobile engine.

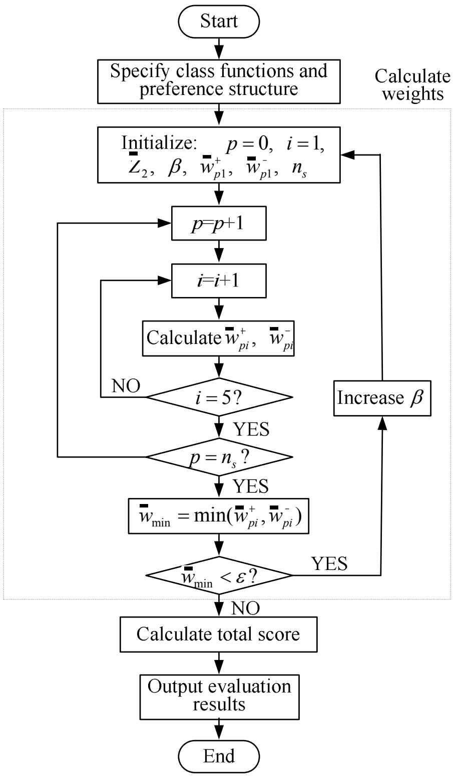

Fig. 2. The process of design evaluation based on the LPP.

Table 1. Design criteria values of all automobile engines.

	Design A	Design B	Design C
Oil consumption ($g/(kW\cdot h)$)	280	340	220
Unit power quality (kg/kW)	2.2	2.4	1.9
Life (km)	100×10^3	120×10^3	80×10^3

Table 2. Desirable ranges of each criterion.

Design criteria		Ideal	Satisfied	Tolerable	Unsatisfied	Highly unsatisfied
	Class	t_{p1}	t_{p2}	t_{p3}	t_{p4}	t_{p5}
Oil consumption	1-S	220	260	310	360	400
Unit power quality	1-S	1.7	2.1	2.6	3.1	3.5
Life	2-S	300×10^3	140×10^3	90×10^3	40×10^3	20×10^3

Table 2 shows the preferences of each criterion. In this example, the class chosen for oil consumption and unit power quality is Class 1-S, while the class of life is 2-S.

Once evaluation criteria are quantified, design evaluation can be carried out using the LPP-based model by specifying the class functions and preferences for different criteria. The process presented in the previous section is followed to calculate the total scores of three automobile engine designs. The associated total scores are shown in Table 3.

From Table 3, it can be easily seen that the rank of three design alternatives is Design C > Design A > Design B. Design C is the best one. The results are the same with those in Ref. [17]. This demonstrates the effectiveness of the LPP-based evaluation model.

3. RAOGA-based fuzzy neural network model for design evaluation

The LPP-based design evaluation model can be used to evaluate designs when evaluation criteria can be precisely quantified. In most cases, evaluation criteria are difficult to quantify, but expressed via linguistic variables such as “good”, “medium”, and “poor”, etc. The LPP-based evaluation model cannot provide the flexibility of handling evaluation problems with imprecise criteria. A fuzzy neural network (FNN) model is developed in this section to evaluate design alternatives when evaluation criteria can only be imprecisely described. In the proposed method, a feed-forward neural network-based fuzzy reasoning is used to evaluate concepts, and a RAOGA-based learning algorithm is adopted to seek the optimal fuzzy weights and thresholds.

3.1 Basic theory of fuzzy set

The fuzzy set introduced by Zadeh is a mathematical way to represent imprecision and vagueness. A fuzzy set is characterized by its membership function. For example, the fuzzy set $\tilde{A}$ can be described as

$$\tilde{A}=\{[x,\mu_{\tilde{A}}(x)],x\in X\} \tag{2}$$

Table 3. Evaluation result of each scheme.

	Design A	Design B	Design C
Total score	168.45	206.37	39.83
Rank	2	3	1

where $\mu_{\tilde{A}}(x)$ is the membership function of $\tilde{A}$. The α-cut of $\tilde{A}$, denoted as $\tilde{A}_\alpha$, is a crisp set consisting of all members of X whose membership degrees in $\tilde{A}$ are greater than or equal to α. This can formally be written as

$$\tilde{A}_\alpha = \{x \mid \mu_{\tilde{A}}(x) \geq \alpha\} . \quad (3)$$

Fuzzy sets that are defined on the set R of real numbers have a special significance in fuzzy set theory. Their membership functions have some quantitative meaning and may, under certain conditions, be viewed as representations of fuzzy numbers or fuzzy intervals. A fuzzy number is a special case of a fuzzy set. To qualify as a fuzzy number, a fuzzy set $\tilde{A}$ on R must satisfy at least the following three requirements: (1) $\tilde{A}$ is a normal fuzzy set; (2) The α-cuts of $\tilde{A}$ are closed intervals for every $\alpha \in [0,1]$; (3) The support of $\tilde{A}$, $\tilde{A}_0$, is a closed interval.

The arithmetic operation on fuzzy numbers at any α-cut level is based on the extension of the interval-valued arithmetic operations. Consider two fuzzy numbers, $\tilde{A}$ and $\tilde{B}$, and let $*$ denote any of interval-valued arithmetic operations (say addition, subtraction, multiplication, and division). Then, $\tilde{A} * \tilde{B}$ is defined in terms of its α-cuts, $(\tilde{A} * \tilde{B})_\alpha$, by the formula

$$(\tilde{A} * \tilde{B})_\alpha = \tilde{A}_\alpha * \tilde{B}_\alpha \quad (4)$$

for any $\alpha \in (0,1]$.

The triangular fuzzy number is commonly used in practice, and can be denoted by a triplet (l,m,u) indicating the lower limit of support, the mode, and the upper limit of support. The addition of two triangular fuzzy numbers yields a triangular fuzzy number as follows

$$(l_1,m_1,u_1) \oplus (l_2,m_2,u_2) = (l_1 + l_2, m_1 + m_2, u_1 + u_2) . \quad (5)$$

However, the multiplication of two triangular fuzzy numbers does not generally produce a triangular fuzzy number, but can be approximated by a triangular as follows

$$(l_1,m_1,u_1) \otimes (l_2,m_2,u_2) = (l_1 l_2, m_1 m_2, u_1 u_2) . \quad (6)$$

Repeating this approximation for a number of multiplications produces significant error. To avoid this problem, one can use a numerical method based on α-cuts or the extension principle directly. We further discuss it in the following sections.

3.2 Fuzzy neural network

Artificial neural network (ANN) is a new information processing technique which simulates biological neurons using computers and retains enough structure to work like a biological neural processing unit. The ANN has high computing precision, strong capability of self-learning and self-association. However, the traditional ANN has several disadvantages. For example, imprecise information cannot be manipulated, and a plenty of learning samples are required to achieve a desired high precision. Taking the advantage of the fuzzy logic in terms of dealing with imprecise information, the fuzzy neural network (FNN) which integrates the fuzzy logic and the artificial neural network and possesses the advantages of both fuzzy logic and ANN, and it is capable of learning, association, identification, self-adaptation, and processing fuzzy information, such as experts' knowledge. The FNN is the routine neural network, e.g. the feed forward neural network, Hopfield neural network, etc., incorporated with fuzzy inputs or/and fuzzy weights [18]. In this paper, we only focus on fuzzy weights and fuzzy inputs FNN where inputs and weights of the FNN are treated as fuzzy variables [19, 20]. As the FNN simulates the logic thinking of human brain and has a strong capability of approaching nonlinear function, it is able to cope with following issues oftentimes faced with in design evaluation: (1) Variables concerned are very complex. Not only variables themselves are uncertain, but also relationships between variables and evaluation results are fuzzy. (2) Subjectivity is inevitable in the process of determining the weights of ANN. The FNN is, therefore, applied to conduct conceptual design evaluation.

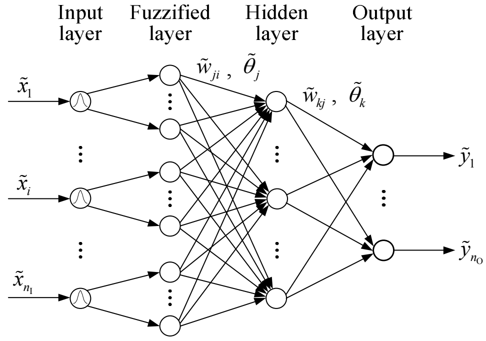

Fig. 3. FNN model of conceptual design evaluation.

3.3 RAOGA-based FNN model

3.3.1 Structure of evaluation model

The FNN model used in design evaluation is shown in Fig. 3, where n_I, n_H and n_O are numbers of neurons in the input layer, the hidden layer, and the output layer, respectively.

Input layer

The design alternatives generated in conceptual design phase are qualitative, but yet quantifiable. Suppose neurons of the input layer receive a vector $\tilde{X}_p = [\tilde{x}_{p1}, \tilde{x}_{p2}, \cdots, \tilde{x}_{pn_I}]$ of fuzzy inputs. The input of the ith neuron is defined as

$$\tilde{I}_{pi} = \tilde{x}_{pi} . \quad (7)$$

Fuzzified layer

Input information must be fuzzified at first. Put another way,

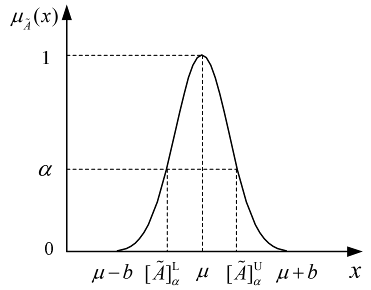

Fig. 4. The membership function of a Gaussian fuzzy number.

the value of every fuzzy input is calculated according to the corresponding membership function. In engineering practices, triangular or trapezoidal functions are usually used to depict fuzzy variables, because samples required by these functions are very simple. However Kuo and Xue [21] concluded that the assumption of triangular or trapezoidal function is not similar to the thinking of human being. In general, many variables in practical problems belong to the normal distribution. They suggested replacing triangular or trapezoidal function by the asymmetric Gaussian function. In this paper, to simplify the problem and for a clear illustration purpose, the symmetric normal function is used to represent membership function of fuzzy numbers. The membership function of a Gaussian fuzzy number is shown in Fig. 4.

The membership function $\mu_{\tilde{A}}(x)$ can be expressed as follows:

$$\mu_{\tilde{A}}(x)=\begin{cases}1 & x=\mu \\ \exp(-\dfrac{(x-\mu)^2}{2\sigma^2}) & x\neq\mu\end{cases}. \tag{8}$$

The Gaussian fuzzy number can be denoted with two-tuples (μ,σ), where μ is the center of a Gaussian function. According to the "3σ rules", we can let $\sigma=b/3$. As shown in Fig. 4, b is the half-span length of the Gaussian function along horizontal axis.

Hidden layer

Input information of the input layer can be evaluated through the hidden layer. The total input and output of the jth neuron of the hidden layer are defined as

$$\tilde{I}_{pj}=\sum_{i=1}^{n_{\rm I}}\tilde{w}_{ji}\cdot\tilde{x}_{pi}+\tilde{\theta}_j\ , \tag{9}$$

$$\tilde{O}_{pj}=f(\tilde{I}_{pj}) \tag{10}$$

where $\tilde{I}_{pj}$ is the fuzzy input of jth neuron of the hidden layer. $\tilde{w}_{ji}$ represents a fuzzy weight of a connection between ith neuron of the input layer and jth neuron of the hidden layer. $\tilde{\theta}_j$ is the fuzzy threshold of jth neuron of the hidden layer. $\tilde{O}_{pj}$ is the fuzzy output of jth neuron of the hidden layer.

Output layer

Similar to the hidden layer, the total input and output of neuron j of the output layer are defined as

$$\tilde{I}_{pk}=\sum_{j=1}^{n_{\rm H}}\tilde{w}_{kj}\cdot\tilde{O}_{pj}+\tilde{\theta}_k\ , \tag{11}$$

$$\tilde{O}_{pk}=f(\tilde{I}_{pk}) \tag{12}$$

where $\tilde{I}_{pk}$ is the fuzzy input of kth neuron of the hidden layer, $\tilde{w}_{kj}$ represents a fuzzy weight of a connection between jth neuron of the hidden layer and kth neuron of the output layer, $\tilde{\theta}_k$ is the fuzzy threshold of kth neuron of the output layer, $\tilde{O}_{pk}$ is the fuzzy output of kth neuron of the output layer, $f(\cdot)$ is a sigmoid nonlinear function. The detailed calculation of fuzzy numbers and fuzzy outputs can be found in Ref. [22].

3.3.2 The modified adaptive genetic algorithm

To train the proposed FNN via existing samples, appropriate optimization methods are needed. In this paper, we develop a modified adaptive genetic algorithm to seek the optimal settings of parameters in the FNN.

(I) Determination of fitness function

The success of GA-based optimization greatly depends on appropriately choosing the fitness function. Let $\tilde{Y}_p=(\tilde{y}_{p1},\tilde{y}_{p2},\cdots,\tilde{y}_{pn_{\rm O}})$ ($p=1,2,\cdots,m$) be the fuzzy output vector corresponding to the fuzzy input vector $\tilde{X}_p$. The error function for the α-cuts of the fuzzy output $\tilde{O}_{pk}$ from the kth output neuron and the corresponding fuzzy target $\tilde{y}_{pk}$ is defined as follows [22]:

$$E_{pk\alpha}=E_{pk\alpha}^{\rm L}+E_{pk\alpha}^{\rm U} \tag{13}$$

where

$$E_{pk\alpha}^{\rm L}=\alpha\cdot\frac{([\tilde{y}_{pk}]_\alpha^{\rm L}-[\tilde{O}_{pk}]_\alpha^{\rm L})^2}{2}, \tag{14}$$

$$E_{pk\alpha}^{\rm U}=\alpha\cdot\frac{([\tilde{y}_{pk}]_\alpha^{\rm U}-[\tilde{O}_{pk}]_\alpha^{\rm U})^2}{2} \tag{15}$$

where $E_{pk\alpha}^{\rm L}$ and $E_{pk\alpha}^{\rm U}$ can be viewed as the squared errors for the lower and the upper limits of the α-cuts, respectively. For the pth training sample, the error function for the α-cuts of the fuzzy output vector $\tilde{O}_p$ and the fuzzy target vector $\tilde{Y}_p$ is defined as

$$E_{p\alpha}=\sum_{k=1}^{n_{\rm O}}E_{pk\alpha}\ . \tag{16}$$

Thereby, the error function of all α-cuts of all fuzzy outputs and fuzzy targets is defined as

$$E=\sum_{p=1}^{m}\sum_{\alpha}E_{p\alpha}\ . \tag{17}$$

$$\cdots \underbrace{\underbrace{10010110}_{w_{ji1}}\ \underbrace{01111010}_{w_{ji2}}}_{\tilde{w}_{ji}} \cdots \underbrace{\underbrace{10101100}_{w_{kj1}}\ \underbrace{01100111}_{w_{kj2}}}_{\tilde{w}_{kj}} \cdots$$

Fig. 5. Binary encoding of fuzzy weights in the FNN.

The fitness function takes the value

$$\tilde{f} = \frac{1}{1+E}\,. \tag{18}$$

With the decrease of the squared error E, the fitness value approaches its maximum.

(II) Coding individual solution

To implement the GA to a specific optimization problem, coding individual solution is another important issue. In this paper, the binary encoding is used to construct chromosomes of fuzzy weights and thresholds. A single chromosome carries values of all fuzzy weights of the FNN in a binary format. Every weight is represented by a Gaussian fuzzy number. Encoding of fuzzy weights in RAOGA-based FNN evaluation model is shown in Fig. 5. The precision level of 8 bits per parameter of a fuzzy weight is used.

In Fig. 5 $\tilde{w}_{ji} = (w_{ji1}, w_{ji2})$ is a Gaussian fuzzy number of the connection weight between the ith neuron of the input layer and the jth neuron of the hidden layer. $\tilde{w}_{kj} = (w_{kj1}, w_{kj2})$ is the Gaussian fuzzy number of the connection weight between the jth neuron of the hidden layer and the kth neuron of the output layer.

(III) RAOGA

As an excellent searching technique with a strong global search capability, the GA can make the evolution process gradually converge to the optimal solution via selection, crossover, and mutation operations. Two typical deficiencies exist when the simple genetic algorithm (SGA) is used in practical applications. The first deficiency is the premature convergence at the early stage of an evolutionary process. The other is random search trend at the middle or final stage of revolutionary process because of the weakness of individual competition. The former case results in converging to a local optimal solution while the latter leads to lower convergence speed. In this research, a ranking-based adaptive evolutionary operator genetic algorithm (RAOGA) [23] is adopted to optimize fuzzy weights and thresholds of the FNN. In other words, solutions are ranked according to their fitness values first, and then the ratios of selection, crossover, and mutation are adaptively determined according to their ratings. Both the convergence speed and solution quality are improved greatly with RAOGA.

The evolutionary process of the RAOGA can be described via Markov chain. Let a state space be $S = B^{l*n}$, where l represents the binary length of every individual and n is the population size of the GA. Every element in the state space can be viewed as an integer of binary codes. Let $\pi_i(k)$ represent the kth binary segment of the ith element in state space, i.e. $\pi_i(k)$ is the kth individual in the population of state i.

In the RAOGA, the ranking of individuals is performed according to their fitness values. In the state i after ranking, the fitness value of left individual is greater than the right, i.e.

$$f_i(1) \geq f_i(2) \geq \cdots \geq f_i(k) \geq \cdots \geq f_i(n)\,. \tag{19}$$

The selection ratio of $\pi_i(k)$ in the evolutionary process is defined as

$$s(\pi_i(k)) = \frac{1}{n} + \alpha(t) \cdot \frac{n+1-2k}{n(n+1)} \qquad (k = 1, 2, \cdots, n) \tag{20}$$

where $\alpha(t)$ is the adaptive coefficient of selection operator, $0 < \alpha(t) < 1$. $\alpha(t)$ is a function of time, which can be a either continuous or piecewise discontinuous function. The larger the value of $\alpha(t)$, the larger the difference of selection ratios between adjacent individuals is in the state i. In the early stage of an evolutionary process, to avoid local premature convergence and keep the diversity of population, the value of $\alpha(t)$ should be small. In the middle or final stage of evolutionary process, the optimal fitness value of population has approached the optimal individual and the competition between individuals has weakened. To avoid random search, the value of $\alpha(t)$ should be large. In the process of practical applications, we usually let $\alpha(t)$ be the following finite piecewise function

$$\alpha(t) = \begin{cases} \alpha_1 & 0 \leq t < T_1 \\ \alpha_2 & T_1 \leq t < T_2 \\ \cdots & \cdots \\ \alpha_m & T_{m-1} \leq t < \infty \end{cases} \qquad (t, T_1, \cdots, T_{m-1} \in N)\,. \tag{21}$$

The action of crossover is to combine useful genetic information in a pair of individuals which are to be swapped to generate offspring. In the reproductive process of individuals, crossover can recombine gene modes and may generate offspring with an excellent performance. The crossover ratio of RAOGA is defined as

$$p_c(\pi_i(k)) = \lambda_c \cdot [1 - \exp(-\frac{\beta_c(k-1)}{n})] \quad (k = 1, 2, \cdots, n) \tag{22}$$

where λ_c and β_c are constants, $0 < \lambda_c \leq 1$.

Mutation simulates the change of gene mode caused by accidental factors in natural evolutionary environment. In general, mutation randomly picks one of the bits in a chromosome and flips it. The mutation ratio of the RAOGA is defined as

$$p_m(\pi_i(k)) = \lambda_m \cdot [1 - \exp(-\frac{\beta_m(k-1)}{n})] \; (k = 1, 2, \cdots, n) \tag{23}$$

where λ_m and β_m are constants, $0 < \lambda_m \le 1$.

In GA-based optimization, mutation can increase the diversity of population and prevent the algorithm from premature convergence. Excellent individuals generated through mutation can be kept during the evolutionary process. Inferior individuals are gradually eliminated during the process of population evolution.

In the RAOGA, the selection, crossover, and mutation are all adaptive genetic operators. Based on ratings of individual fitness values, they can guarantee a fast global convergence through adjusting ratios of genetic operators self-adaptively, keeping current optimal solutions and fastening the evolutionary speed of poor solutions.

Applying the RAOGA to train fuzzy weights and thresholds of FNN can avoid the use of differentiable information and simplify the computing process. The training of fuzzy thresholds is similar to that of fuzzy weights and does not be discussed here.

3.3.3 RAOGA-based learning mechanism of FNN

The learning process of the FNN is to construct a nonlinear mapping between fuzzy inputs and fuzzy outputs, and to make the FNN capable of association and judgment. The learning process of the RAOGA-based FNN is summarized as follows:

Step 1) Map the solution space into genetic search space represented by binary codes. Set values of λ_c , λ_m , β_c and β_m . Determine the population size, the evolutionary parameter $\alpha(t)$ and termination conditions of the RAOGA, and construct a fitness function.

Step 2) Initiate the fuzzy weights $\tilde{w}_{ji}(0)$, $\tilde{w}_{kj}(0)$, and fuzzy thresholds $\tilde{\theta}_j(0)$, $\tilde{\theta}_k(0)$.

Step 3) For each learning sample p ($p = 1,2,\cdots,m$), where p is the number of learning samples, execute the next step.

Step 4) Repeat the following procedures:

(1) Calculate α -cuts of the fuzzy output vector $\tilde{O}_p$ corresponding to the fuzzy input vector $\tilde{X}_p$;

(2) Calculate the total squared error E .

Step 5) Evaluate each individual in the population of the RAOGA.

Step 6) If termination conditions are met, go to Step 10.

Step 7) Rank individuals according to fitness values and calculate the selection ratio of each individual.

Step 8) Compute the crossover ratio and create new individuals by crossover. Replace poor parent individuals with newly generated ones.

Step 9) Compute the mutation ratio and mutate at randomly selected points. Return to Step 5.

Step 10) Stop the search process and get the optimal fuzzy weights and fuzzy thresholds of the FNN.

3.3.4 Fuzzy centroid method of design ranking

The fuzzy centroid method is one of the most widely used methods for ranking alternatives [24]. The calculating process of fuzzy centroid method is as follows.

Consider the following discrete fuzzy set

Table 4. Linguistic description of tank engines.

Evaluation criteria		Linguistic variables			
		Design 1	Design 2	Design 3	Design 4
c_1	Strengthening coefficient	Medium	Very good	Very poor	Very good
c_2	Volume power	Poor	Good	Very good	Medium
c_3	Oil consumption	Very good	Medium	Poor	Good

$$\tilde{A} = \{\frac{\mu_1}{x_1},\cdots,\frac{\mu_i}{x_i},\cdots,\frac{\mu_n}{x_n}\} \tag{24}$$

where $\mu_i = \mu_{\tilde{A}}(x_i)$ is the discretized membership function of x_i to fuzzy set $\widehat{A}$. The cenroid of fuzzy set $\widehat{A}$ can be calculated as follows

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i \mu_{\tilde{A}}(x_i)}{\sum_{i=1}^{n} \mu_{\tilde{A}}(x_i)} . \tag{25}$$

In the FNN evaluation model, the calculation of fuzzy inputs and outputs is based on α -cuts. The fuzzy outputs of output neurons can be viewed as the boundary values of fuzzy intervals corresponding to a series of α -cuts. When serial values of α are determined, the fuzzy output can be represented in the format of Eq. (24). Therefore, we can calculate fuzzy centroids of all design options according to Eq. (25) and rank them correspondingly.

3.4 Case study: design evaluation and selection for tank engines

The FNN is capable of associating and reasoning after learning fuzzy rules. In this section, the conceptual design evaluation of tank engines is used as an example to illustrate the capability of the proposed RAOGA-based FNN evaluation model. The values of each design scheme with respect to evaluation criteria are represented by linguistic variables tabulated in Table 4. Evaluation criteria considered in this study are strengthening coefficient, volume power, and oil consumption.

As shown in Fig. 6, Gaussian fuzzy numbers are defined to represent linguistic values of "very poor (VP)", "poor (P)", "medium (M)", "good (G)", and "very good (VG)".

For this specific design evaluation problem, the number of input neurons is three in the FNN, which are corresponding to the three evaluation criteria (strengthening coefficient, volume power, and oil consumption). The number of hidden neurons is eight. The number of output neurons is one, which is the evaluation value.

In the learning algorithm of the FNN, parameters are specified as follows:

(1) Values of α : $\alpha = 0.2, 0.4, 0.6, 0.8, 1.0$;

Table 5. Evaluation results of tank engine designs.

	$\alpha=0.2$		$\alpha=0.4$		$\alpha=0.6$		$\alpha=0.8$		$\alpha=1.0$		Centroid $\bar{x}$	Rating
	$[\tilde{O}]^{L}_{0.2}$	$[\tilde{O}]^{U}_{0.2}$	$[\tilde{O}]^{L}_{0.4}$	$[\tilde{O}]^{U}_{0.4}$	$[\tilde{O}]^{L}_{0.6}$	$[\tilde{O}]^{U}_{0.6}$	$[\tilde{O}]^{L}_{0.8}$	$[\tilde{O}]^{U}_{0.8}$	$[\tilde{O}]^{L}_{1.0}$	$[\tilde{O}]^{U}_{1.0}$		
Design 1	0.25	0.53	0.26	0.47	0.29	0.48	0.35	0.45	0.38	0.39	0.3867	4
Design 2	0.41	0.76	0.46	0.74	0.52	0.70	0.55	0.69	0.62	0.62	0.6130	1
Design 3	0.29	0.60	0.33	0.55	0.38	0.52	0.42	0.48	0.44	0.45	0.4467	3
Design 4	0.34	0.72	0.39	0.68	0.42	0.64	0.45	0.60	0.52	0.53	0.5283	2

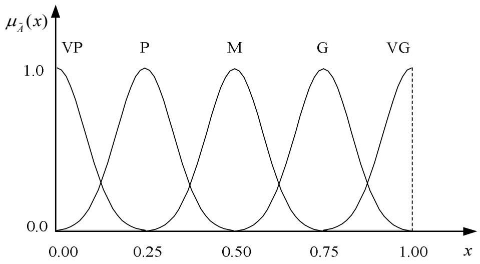

Fig. 6. Membership function of linguistic variables.

(2) The population size of the RAOGA is set to 50, $\lambda_c=1$, $\lambda_m=0.2$, $\beta_c=5$, $\beta_m=10$. $\alpha(t)$ is defined by the following piecewise function:

$$\alpha(t)=0.2+\frac{t}{400} \qquad (t\ge 300). \tag{26}$$

(3) The termination condition is that the maximum iterative number is 20000.

The four tank engine designs in Table 4 are evaluated by the trained FNN model. Table 5 is the output of the RAOGA-based FNN corresponding to a serial values of $[a_i, b_i]$, $i=1,2,\cdots,n$. From Table 5, it can be easily seen that Design 2 is the best scheme.

Applying the RAOGA to adjust fuzzy weights and thresholds of the FNN can avoid the use of differentiable information and simplify the calculating process. The RAOGA-based learning mechanism can accelerate the searching process and guarantee the optimization converge to the global optimal solution.

4. Multi-level evaluation model based on NFWA and fuzzy compromise decision-making

In the process of conceptual design evaluation, there are oftentimes many evaluation criteria with hierarchical relationships. The LPP-based design evaluation and the RAOGA-based fuzzy neural network model don't possess the flexibility to solve evaluation problems with hierarchical criteria. To overcome this issue, a multi-level conceptual design evaluation model based on the NFWA and the fuzzy compromise decision-making method is introduced in this section to address the problem when too many evaluation criteria and hierarchy of criteria exist. This multi-level evaluation model is constructed on the basis of Vanegas and Labib's work [1]. In a multi-level evaluation model, the group AHP based on the fuzzy Delphi method is used to identify the fuzzy number of weights of all criteria. The NFWA and the fuzzy compromise decision-making method are applied to calculate the overall desirability of each design alternative level by level [19].

4.1 New fuzzy-weighted average (NFWA) based on fuzzy Delphi AHP

4.1.1 Fuzzy Delphi AHP method

Identification of importance of all the evaluation criteria is a complicated task. The AHP method is mainly applied to deal with complicated decision-making problems systematically [26]. The relative importance of all criteria can be obtained by pair-wise comparison of any two criteria. In this research, a group of AHPs based on fuzzy Delphi method are utilized to determine fuzzy numbers of weights of all criteria [27]. When the fuzzy Delphi based AHP method is used to determine weights of criteria, there are several steps to be followed:

(1) Identify all the criteria of a problem and construct a hierarchy structure.

(2) Construct a pair-wise comparison matrix.

Pair-wise comparison starts with comparing the relative importance of two selected items. If *n* items are associated with *n* weights, the relative importance, a_{ij}, of the *i*th item in comparison to the *j*th item is obtained by

$$a_{ij}=\frac{w_i}{w_j}. \tag{27}$$

The pair-wise relative importance satisfies

$$\mathbf{Aw}=\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}\begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix}=n\begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix}. \tag{28}$$

Since an item is equally important with itself, the value of all diagonal elements a_{ii} is 1. Values of the elements in the upper triangle of the matrix are the reciprocal values of the elements in the lower triangle of this matrix, i.e. $a_{ij}=1/a_{ji}$.

In a process of identifying the relative importance of each criterion, the relative importance of one criterion to another

Table 6. Scales for pairwise comparison of criteria.

Relative importance a_{ij}	Comparison of the *i*th item and the *j*th item
1	The *i*th item is equally important with the *j*th item
3	The *i*th item is moderately more important than the *j*th item
5	The *i*th item is strongly more important than the *j*th item
7	The *i*th item is very strongly more important than the *j*th item
9	The *i*th item is extremely more important than the *j*th item
2,4,6,8	These are intermediate comparison values
Reciprocals	These are values for inverse comparisons a_{ji}

can be specified by a scale ranging from 1 to 9 as shown in Table 6.

(3) Calculate relative importance of all the criteria.

Calculating the maximum eigenvalues $\lambda_{\max}$ and the corresponding eigenvectors of the matrix $\mathbf{A}$. The standardized eigenvectors corresponding to the maximum eigenvalues are the weights.

(4) Check consistency.

In general, values of a_{ij} are estimated by experts' judgments. Estimation errors result in inconsistency of the data in matrix $\mathbf{A}$. A consistency index (CI) was introduced as a measure to evaluate the consistency deviation. CI can be computed by

$$\mathrm{CI} = (\lambda_{\max} - n)/(n-1). \tag{29}$$

With different values of *n*, different numerical values are generated, which are called random consistency index (RI).

$$\mathrm{RI} = 1.98(n-2)/n \tag{30}$$

The ratio of CI and RI for the same order matrices is called the consistency ratio (CR). A matrix with consistency ratio less than 0.1 is considered as good enough to calculate weights of items.

(5) Calculate fuzzy numbers of weights with the fuzzy Delphi method.

In this work, normal triangular fuzzy numbers are applied to represent fuzzy weights of criteria. Let $\tilde{w}_i$ be the fuzzy weight of a criterion c_i and $\tilde{w}_i = (c_i, a_i, b_i)$, one has:

$$\begin{aligned} & c_i = \min_j\{\tilde{w}_{ji}\},\ \ b_i = \max_j\{\tilde{w}_{ji}\} \\ & a_i = (\prod_{j=1}^{n} w_{ji})^{1/n},\ \ i = 1,2,\cdots,n. \end{aligned}, \tag{31}$$

where w_{ji} is the weight of c_i identified by the *j*th decision maker.

4.1.2 New fuzzy-weighted average (NFWA)

The fuzzy weighted average (FWA) is a combination of extended algebraic operations, and has been commonly used in design evaluation to calculate the overall desirability of a design alternative. As mentioned above, membership functions of fuzzy numbers can be approximated using a number of α-cuts, which are a set of *n* intervals $[a_i, b_i]$, $i = 1,2,\cdots,n$ in which $\mu(x) \ge \alpha_i$ for $a_i < x < b_i$. The FWA algorithm based on the extension principle when the function $y = f(x_1, x_2, \cdots, x_n)$ is

$$\tilde{y} = \frac{\sum_{i=1}^{n} \tilde{w}_i \cdot \tilde{x}_i}{\sum_{i-1}^{n} \tilde{w}_i} \tag{32}$$

where $\tilde{y}$, $\tilde{x}_i$ and $\tilde{w}_i$ are fuzzy numbers, and the algebraic operations performed are defined by Eq. (4).

Even though the FWA algorithms have been developed and implemented by several works, as denoted by Vanegas and Labib [1], the FWA may increase imprecision unnecessarily, and operations on fuzzy numbers, particularly for division, are difficult to be carried out. They also proposed a new fuzzy-weighted average (NFWA) to manipulate fuzzy numbers to obtain more meaningful results.

Let the fuzzy number $\tilde{w}_i$ represent the weight of the criterion c_i ($i = 1,2,...,n$, and n is the number of criteria), and the fuzzy number $\tilde{D}_i$ represent the performance of a design alternative with respect to this criterion. The α-cut of the overall desirability of an alternative $\tilde{D}_i$ is given by

$$D_\alpha = [\tilde{D}_{\alpha a}, \tilde{D}_{\alpha b}] \tag{33}$$

where

$$\tilde{D}_{\alpha a} = \min\left(\frac{\sum_{i=1}^{n} \tilde{D}_{i\alpha a} \cdot w_i}{\sum_{i=1}^{n} w_i}\right) \tag{34}$$

and

$$\tilde{D}_{\alpha b} = \max\left(\frac{\sum_{i=1}^{n} \tilde{D}_{i\alpha b} \cdot w_i}{\sum_{i=1}^{n} w_i}\right) \tag{35}$$

where $w_i \in [\tilde{w}_{i\alpha a}, \tilde{w}_{i\alpha b}]$ for all $i \in \{1,2,\cdots,n\}$ and $\alpha \in (0,1]$. $\tilde{D}_{\alpha a}$ and $\tilde{D}_{\alpha b}$ represent the lower and the upper limits, respectively, of the α-cut $\tilde{D}_\alpha$. $\tilde{D}_{i\alpha a}$ and $\tilde{D}_{i\alpha b}$ represent the lower and the upper limits, respectively, of the α-cut $\tilde{D}_{i\alpha}$, and $\tilde{w}_{i\alpha a}$ and $\tilde{w}_{i\alpha b}$ represent the lower and the upper limits, respectively, of the α-cut $\tilde{w}_{i\alpha}$.

In this paper, the evaluation and ranking steps for design alternatives by using the fuzzy Delphi AHP based NFWA are as follows.

Step 1) Construct the linguistic descriptions of the desirability and importance levels of all design candidates, and identify the fuzzy numbers to characterize linguistic values.

Step 2) Calculate the fuzzy numbers of weights of all criteria using the fuzzy Delphi AHP method.

Step 3) Calculate the overall desirability levels of all design alternatives using the NFWA method.

Step 4) Evaluate and rank all design candidates.

The NFWA is designed to perform each operation by taking into account initial imprecision so as to obtain meaningful results. The next section illustrates the utilization of the NFWA based on the fuzzy Delphi AHP through an example.

4.2 Multi-level design evaluation model based on NFWA and fuzzy compromise decision-making

4.2.1 Fuzzy compromise decision-making

The fuzzy compromise decision-making is constructed by referring to the fuzzy ideal solution, the fuzzy negative ideal solution, the fuzzy ideal and negative solutions together. The fuzzy ideal solution is composed of the maximum corresponding to each fuzzy criterion value. The fuzzy negative ideal solution consists of the minimum of each fuzzy criterion value. In fuzzy compromise decision-making method, the difference between fuzzy ideal solutions and fuzzy negative ideal solutions is measured by the Hamming distance. A small distance between the fuzzy ideal solution and the fuzzy negative ideal solution is favorable [29]. In this work, the fuzzy ideal and negative ideal solutions are both considered as datum. It is detailed as follows:

(1) Normalize the matrix of fuzzy criterion values.

(i) Normalize the fuzzy criterion values of the benefit type. $\tilde{x}_i (i = 1, 2, \cdots, m)$ is the fuzzy criterion values, let

$$(\cdot)_i^{\max} = \max_i \{(\cdot)_i\} . \quad (36)$$

If $\tilde{x}_i$ is a triangular fuzzy number, i.e. $\tilde{x}_i = (a_i, b_i, c_i)$, then the normalized fuzzy criterion value $\tilde{r}_i (i = 1, 2, \cdots, m)$ is

$$\tilde{r}_i = (\frac{a_i}{c_i^{\max}}, \frac{b_i}{b_i^{\max}}, \frac{c_i}{a_i^{\max}} \wedge 1) . \quad (37)$$

(ii) Normalize the fuzzy criterion values of the cost type. $\tilde{x}_i (i = 1, 2, \cdots, m)$ is the value of a fuzzy criterion, let

$$(\cdot)_i^{\min} = \min_i \{(\cdot)_i\} . \quad (38)$$

If $\tilde{x}_i$ is a triangular fuzzy number, then

$$\tilde{r}_i = (\frac{a_i^{\min}}{c_i}, \frac{b_i^{\min}}{b_i}, \frac{c_i^{\min}}{a_i} \wedge 1) . \quad (39)$$

(2) Weight the normalized matrix of fuzzy criterion values.

Let $\tilde{w}_j = (a; \alpha, \beta)$ and $\tilde{x}_{ij} = (c; \delta, \gamma)$ be *L-R* fuzzy numbers, values in the matrix of fuzzy criterion are

$$\tilde{r}_{ij} = \tilde{w}_j \tilde{x}_{ij}, \ \forall i, j \quad (40)$$

where $\tilde{r}_{ij}$ can be calculated by the following Bonissone approximate integration function

$$\tilde{r}_{ij} = (ac; a\gamma + c\alpha - \alpha\gamma, a\delta + c\beta - \beta\delta) . \quad (41)$$

(3) Determine the fuzzy ideal solution $\tilde{M}^+$.

$$\tilde{M}^+ = (\tilde{M}_1, \tilde{M}_2, \cdots, \tilde{M}_n) \quad (42)$$

where $\tilde{M}_j = \max\{\tilde{r}_{1j}, \tilde{r}_{2j}, \cdots, \tilde{r}_{mj}\} (j = 1, 2, \cdots, n)$ is the fuzzy maximum corresponding to the fuzzy weighted value of the criterion *j*, the membership function is

$$\mu_{\tilde{M}_j}(r) = \sup_{\substack{r = r_1 \vee r_2 \vee \cdots \vee r_m \\ (r_1, r_2, \cdots, r_m) \in R^m}} \min\{\mu_{\tilde{r}_{1j}}(r_1), \mu_{\tilde{r}_{2j}}(r_2), \cdots \mu_{\tilde{r}_{mj}}(r_m)\} . \quad (43)$$

(4) Determine the difference D_i^+ of scheme i and the fuzzy ideal solution $\tilde{M}^+$.

$$D_i^+ = \sqrt{\sum_{j=1}^{n} [d(\tilde{r}_{ijL}, \tilde{M}_{jL}) + d(\tilde{r}_{ijR}, \tilde{M}_{jR})]^2} \quad (44)$$

(5) Determine the fuzzy negative ideal solution $\tilde{M}^-$.

$$\tilde{M}^- = (\tilde{m}_1, \tilde{m}_2, \cdots, \tilde{m}_n) \quad (45)$$

where $\tilde{m}_j = \min\{r_{1j}, r_{2j}, \cdots, r_{mj}\} (j = 1, 2, \cdots, n)$ is the fuzzy minimum corresponding to the fuzzy weighted value of the criterion *j*, the membership function is

$$\mu_{\tilde{M}_j}(r) = \sup_{\substack{r = r_1 \wedge r_2 \wedge \cdots \wedge r_m \\ (r_1, r_2, \cdots, r_m) \in R^m}} \min\{\mu_{\tilde{r}_{1j}}(r_1), \mu_{\tilde{r}_{2j}}(r_2), \cdots \mu_{\tilde{r}_{mj}}(r_m)\} . \quad (46)$$

(6) Determine the difference D_i^- of scheme i and the fuzzy ideal solution $\tilde{M}^-$.

$$D_i^- = \sqrt{\sum_{j=1}^{n} [d(\tilde{r}_{ijL}, \tilde{m}_{jL}) + d(\tilde{r}_{ijR}, \tilde{m}_{jR})]^2} \quad (47)$$

(7) Determine the relative approach degree D_i of scheme i and the fuzzy ideal solution $\tilde{M}^+$.

$$D_i = \frac{D_i^-}{D_i^+ + D_i^-}, \ i = 1, 2, \cdots, m \quad (48)$$

(8) Decide the rank of designs according to the value of D_i. The greater value, the better.

4.2.2 Application of multi-level design evaluation model

When a hierarchy of evaluation criteria has more than two levels, the FAHP and the NFWA based design evaluation methods are applied to evaluate designs level-by-level recursively, starting from the bottom level up to the top level. When it reaches the top level, the fuzzy compromise decision-making is used to perform the last level evaluation. The ranks of design alternatives are determined according to the results of the fuzzy compromise decision-making.

4.3 Example

In this section, a boring fixture with three possible design schemes is used as an example to demonstrate the proposed multi-level design evaluation model.

Covering basis of design demands, main technical and economical characters in conceptual design of the fixture, the constructed evaluation criterion system is shown in Table 7. There are five linguistic terms of “very poor (VP)”, “poor (P)”, “medium (M)”, “good (G)”, and “very good (VG)” to describe the satisfaction of evaluation criteria. Fig. 7 provides the definition of triangular fuzzy numbers for evaluation criteria.

Triangular fuzzy numbers for evaluation criterion weights identified by fuzzy Delphi analytical hierarchy process are shown in Table 8.

(1) The first-level evaluation. Based on above definitions, the first-level evaluation is conducted using the NFWA method. Table 9 lists the results of the first-level evaluation.

(2) The second-level evaluation. The second-level evaluation is conducted based on the results of the first-level evalua-

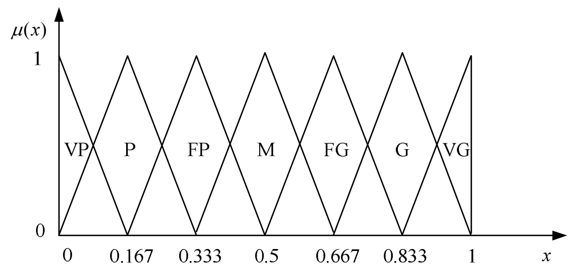

Fig. 7. Fuzzy numbers for capturing linguistic descriptions.

Table 7. Evaluation criteria system and design description of boring fixtures.

Evaluation criteria			Linguistic variables of evaluation criteria		
			S_1	S_2	S_3
Work reliability	Allocation reliability	Allocation datum is reasonable	Good	Good	Good
		Easy to implementation	Very good	Very good	Very good
	Clamping reliability	Good clamping quality	Very good	Very good	Good
		Good quality of anti-loosing	Good	Good	Very good
Simple to manufacture	Easy to manufacture	Little component types	Very good	Very good	Medium
		Complexity degree is low	Very good	Medium	Very poor
		Standardization degree is high	Medium	Very good	Very good
	Simple to assembly		Very good	Good	Very good
Usage quality	Simple to operation	Convenient to clamp	Very good	Very good	Medium
		Convenient to install	Very poor	Good	Good
	Convenient to maintenance		Very good	Very good	Very poor

Table 8. Triangular fuzzy numbers of all evaluation criteria weights of the boring fixture.

U_i	U_{ij}	u_{ijk}
(0.3938,0.3997,0.4028)	(0.4762,0.5004,0.5263)	(0.6875,0.6995,0.7143)
		(0.2857,0.3002,0.3125)
	(0.4737,0.4987,0.5238)	(0.5,0.5227,0.5454)
		(0.4546,0.4766,0.5)
(0.2856,0.2937,0.3023)	(0.5653,0.5932,0.6154)	(0.1927,0.2027,0.2054)
		(0.4943,0.5014,0.5106)
		(0.2967,0.2987,0.3003)
	(0.3846,0.4059,0.4347)	
(0.2949,0.3064,0.3127)	(0.6875,0.6965,0.7059)	(0.3846,0.4002,0.4166)
		(0.5834,0.5995,0.6154)
	(0.2941,0.3034,0.3125)	

Table 9. Evaluation results of the first level.

Evaluation criterion weights		Results of the first-level evaluation		
		S_1	S_2	S_3
(0.3938,0.3997,0.4028)	(0.4762,0.5004,0.5263)	(0.6148,0.7832,0.9523)	(0.6148,0.7832,0.9523)	(0.6148,0.7832,0.9523)
	(0.4737,0.4987,0.5238)	(0.5759,0.7462,0.9165)	(0.5759,0.7462,0.9165)	(0.5835,0.7538,0.9241)
(0.2856,0.2937,0.3023)	(0.5653,0.5932,0.6154)	(0.4492,0.6173,0.7844)	(0.4147,0.5835,0.7509)	(0.2978,0.4662,0.6343)
	(0.3846,0.4059,0.4347)	(0.5,0.667,0.833)	(0.667,0.833,1)	(0.5,0.667,0.833)
(0.2949,0.3064,0.3127)	(0.6875,0.6965,0.7059)	(0.2951,0.4667,0.6387)	(0.5974,0.7665,0.9358)	(0.5279,0.6997,0.8719)
	(0.2941,0.3034,0.3125)	(0.5,0.667,0.833)	(0.5,0.667,0.833)	(0.167,0.333,0.5)

Table 10. Evaluation results of the second level.

Evaluation criterion weights	Results of the second-level evaluation		
	S_1	S_2	S_3
(0.3938,0.3997,0.4028)	(0.5944,0.7647,0.9353)	(0.5944,0.7647,0.9353)	(0.5984,0.7685,0.9389)
(0.2856,0.2937,0.3023)	(0.4687,0.6375,0.8055)	(0.5117,0.6849,0.8592)	(0.3756,0.5478,0.7207)
(0.2949,0.3064,0.3127)	(0.3554,0.5275,0.6994)	(0.5670,0.7363,0.9183)	(0.4151,0.5884,0.7250)

Table 11. Evaluation results of the third level using the NFWA method.

	Triangular fuzzy number of total quality	m	σ	R	Rank
S_1	(0.4826,0.6546,0.8269)	0.6480	0.0049	0.8216	2
S_2	(0.5610,0.7326,0.9083)	0.7340	0.0050	0.8645	1
S_3	(0.4748,0.6485,0.8114)	0.6449	0.0047	0.8201	3

tion. Table 10 shows the results of the second-level evaluation.

(3) The third-level evaluation. After the evaluation of the first two levels, the evaluation of the third level is performed by the fuzzy compromise decision-making as follows:

The *L-R* fuzzy numbers of the decision-making matrix $\tilde{D}$ and the weight vector $\tilde{w}$ are

$$\tilde{D}=\begin{bmatrix}(0.7647;0.1703,0.1706) & (0.6375;0.1688,0.1680) & (0.5275;0.1721,0.1719)\\(0.7647;0.1703,0.1706) & (0.6849;0.1732,0.1743) & (0.7363;0.1693,0.1820)\\(0.7685;0.1701,0.1704) & (0.5478;0.1722,0.1729) & (0.5884;0.1733,0.1366)\end{bmatrix},$$

$$\tilde{w}=[(0.3997;0.0059,0.0031),(0.2937;0.0081,0.0086),(0.3064;0.0115,0.0063)].$$

Since the decision-making matrix and the weight vector have been normalized, no further normalization operation is needed. The fuzzy weighted decision-making matrix is

$$\tilde{V}=[\tilde{r}_{ij}]=\begin{bmatrix}(0.3057;0.0717,0.0700) & (0.1872;0.0531,0.0536) & (0.1616;0.0568,0.0550)\\(0.3057;0.0717,0.0700) & (0.2012;0.0553,0.0553) & (0.2256;0.0621,0.0554)\\(0.3072;0.0716,0.0698) & (0.1609;0.0538,0.0538) & (0.1803;0.0470,0.0557)\end{bmatrix}.$$

The fuzzy ideal solution $\tilde{M}^+$ and the fuzzy negative ideal solution $\tilde{M}^-$ are

$$\tilde{M}^+=[(0.3072;0.0716,0.0698),(0.2012;0.0553,0.0553),(0.2256;0.0621,0.0554)],$$
$$\tilde{M}^-=[(0.3057;0.0717,0.0700),(0.1609;0.0538,0.0538),(0.1616;0.0568,0.0550)],$$

respectively. The differences between each design and the corresponding fuzzy ideal solution or fuzzy negative solution are

$$D_1^+=0.1828,\ D_2^+=0.1153,\ D_3^+=0.2435,$$
$$D_1^-=0.0723,\ D_2^-=0.2137,\ D_3^-=0.0526,$$

respectively. The relative approach degrees between each design and fuzzy ideal solutions are respectively as follows

$$D_1=0.2834,\ D_2=0.6495,\ D_3=0.1776.$$

When the fuzzy compromise decision-making is used, the greater the relative approach degree, the better the design is. From this point of view, we can easily see that the rank of three boxing fixture designs is $S_2>S_1>S_3$. The third-level evaluation can also be done using the NFWA. Table 11 lists the results by using the NFWA.

Comparing evaluation results of fuzzy compromise decision-making and the NFWA, one can find that the ranks are identical. When the NFWA method is used, the differences of R between designs are not obvious. This may increase the difficulty to rank designs. In fuzzy compromise decision-making, the relative approach degrees between each design and the fuzzy ideal solution are calculated. This increases the ability to rank fixture designs and the decision is less likely to be affected by the error from approximation.

5. Conclusions

Design evaluation is a complex and challenging task. Several design evaluation methods under different conditions are proposed by using the LPP model and the RAOGA-based FNN model. They are further applied to evaluate design alternatives in the conceptual design. Furthermore, on the basis of Vanegas and Labib' work, a multi-level conceptual design evaluation model based on NFWA and fuzzy compromise decision-making method is developed when the evaluation problem has many hierarchical criteria. Several design evaluation examples have demonstrated and verified different proposed methods.

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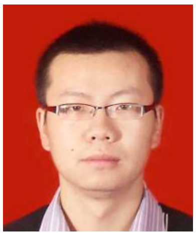

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