A Novel Viscosity-Based Model for Low Cycle Fatigue–Creep Life Prediction of High-Temperature Structures

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ABSTRACT: Damage evolution during low cycle fatigue, creep, and their interaction behavior is actually a ductility exhaustion process in response to cyclic and static creep. In this article, a novel viscosity-based model for low cycle fatigue–creep life prediction is presented in an attempt to condition viscosity-based approaches for general use in isothermal and thermo-mechanical loading. In this model, it was assumed that only plastic and creep strains caused by tensile stress lead to ductility consumption under stress-controlled loading. Moreover, with its simple expression, the mechanisms of the loading waveform, temperature, and mean stress effects are taken into account within a low cycle fatigue–creep regime. Predicted fatigue lives using the proposed model were found to be in good agreement with reported experimental data from literature. Compared with the generalized strain energy damage function method, the mean strain rate, Smith–Watson–Topper and Goswami's ductility models, the proposed model is widely applicable and more precise in the prediction of low cycle fatigue–creep life.

KEY WORDS: creep, high-temperature structure, life prediction, low cycle fatigue, viscosity.

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INTRODUCTION

CURRENTLY, A NUMBER of mechanical components are used at severe conditions such as high temperature or high pressure for their high efficiency. These components operating in aviation, power generating, petrochemical, and other industries are subjected to cyclically varying loads and often fail within a limited number of load cycles, e.g., below about 10^5 cycles (Zhang, 2010). The high reliability and safety of these components are largely due to a combination of improved materials, improved life prediction capabilities, and highly conservative design. Thus, a better understanding of the loading conditions to which today's high-temperature structures are subjected as well as developing better analytical tools of fatigue damage is needed. The motivation behind this is that if the predictive capability of fatigue damage at high temperature can be improved, then statistically, a more efficient design should be feasible.

The most common failure mode for these structures is low cycle fatigue (LCF) at high temperature which is an interactive mechanism of different processes such as time-independent plastic strain, time-dependent creep, and environmental corrosion, oxidation and the complex interaction between them. These damage mechanisms challenge fatigue design methods and may induce dangerously inaccurate life predictions. Thus, even though researchers have explored interactions between fatigue and time-dependent damage mechanisms for many years, studies on the complicated behaviors of materials at high temperature, the designs of such components considering effects of different time-dependent damage mechanisms, and the remaining life to failure are recognized to be essential nowadays.

Increasing attention has been paid to the study of fatigue and creep interaction in either isothermal or thermal–mechanical fatigue conditions. The deformation and failure mechanism of low cycle fatigue–creep (LCF–C) at high temperature are very complex. They may act independently or in combination depending on various test and material parameters, such as temperature, strain rate, hold time, and time-dependent damage processes such as creep, dynamic strain aging, and environmental attack. Some typical methods for LCF–C life prediction have been developed such as linear damage summation (Zhang, 2010), frequency modified Manson–Coffin equation (Coffin, 1976), frequency separation (FS) technique (Coffin, 1974), strain range partition (Manson et al., 1971), strain energy damage function (SEDF) model (Ostergren, 1967), and ductility exhaustion (DE) approach (Goswami, 1995, 1997). In practical application of these models, obtaining various parameters in these equations can be very difficult. Hence, highly precise methods of life prediction are needed to make the design effective.

Robustness of a life prediction method is a crucial key point; there is renewed interest in approaches based on strain energy (Lee et al., 2008; Payten et al., 2010; Zhu et al., 2011a, b). Recently, energy-based approaches have been used to predict damage for fatigue and fatigue–creep cycling based on the hysteresis loops area (Koh, 2002; Lee et al., 2008; Zhu et al., 2011a). In order to account for creep and mean strain or stress effects on the LCF life, a ductility-based model has been previously derived (Zhu et al., 2011a) and applied to a number of LCF tests on a GH4133 Superalloy. Using the mean strain rate (MSR) as the main factor associated with the fracture life, Fan et al. (2007) developed a MSR model based on the investigation of fatigue–creep interaction behavior in stress control mode for 1.25Cr0.5Mo steel. With the assumption that creep damage is measured by the absorbed internal energy density, Payten et al. (2010) put forward a strain energy density exhaustion approach, which is derived from considerations of mechanistic cavity growth. All these models have their own capabilities, shortcomings and have shown their validity on a limited number of alloys and loading conditions. However, the following problems still need to be carefully solved to accurately predict the LCF–C life: quantification of fatigue–creep interaction, description of temperature, loading waveform, mean stress effects, and appropriate damage accumulation rule.

The authors' previous work (Zhu and Huang, 2010) has clearly showed that it is possible: (1) to correlate the fatigue–creep damage and the life with a viscosity-based parameter E_p; (2) to reflect the effects of time-dependent damaging mechanisms on LCF–C life; (3) to identify the main influential factor of LCF–C life, the maximum stress and stress range at minimum stress $\sigma_{\min} \leq 0$, and mean stress at minimum stress $\sigma_{\min} > 0$. Further development and modifications to these issues are in progress which will make the estimation/prediction of LCF–C life *via* DE theory with high accuracy, simplicity, and wide application scope possible.

The objectives of this article lead to a novel viscosity method for LCF–C life prediction based on the correlation between failure/fatigue and energy. The proposed approach uses the fatigue–creep toughness as the main indicator of damage defining the dynamic viscosity with additional terms to account for the effects of time-dependent damaging mechanisms and mean stress. This model has to comply with industrial requirements for high prediction capability, ease of use, physical observations-based and simplicity. Following this, the prediction results of the proposed method are compared with the MSR (Fan et al., 2007), the generalized strain energy damage function (GSEDF) (Zhu and Huang, 2010), Smith–Watson–Topper (SWT) (Smith et al., 1970) and Goswami's ductility model (Goswami, 1997). A comparison between the prediction and the experimental results was conducted. It is showed that the proposed model is capable of providing accurate fatigue life prediction. The capabilities of this method are then further discussed.

PREVIOUS RESEARCH SCOPE

According to the interaction mechanisms of failure at high temperature, a good life prediction method must consider not only the effects of stress/strain level, loading history, impurity content, but also creep factors such as hold time, strain rate, and temperature. The LCF life is dependent on test parameters (Zhu et al., 2011c). Though several energy-based methods for predicting LCF life have been developed (Zhu and Huang, 2010), a GSEDF model with the capability to easily incorporate the effects of time-dependent damaging mechanisms on LCF life is essential for the future development of general prediction criterion addressing LCF–C at high temperature. In this study, a trapezoid load diagram was used to analyze the conditions of most alloys under high temperature, pressure, and cyclic loading. The load diagram of fatigue–creep interaction is plotted in Figure 1.

With regard to the stress cycle shown in Figure 1, T_{du}, T_{dl}, T', and T'' represent the tensile hold time, compressive hold time, tension-going time, and compression-going time, respectively, in one loading cycle when $\sigma_{\max} > 0$ and $\sigma_{\min} < 0$. Here, T_{dl} is the tensile hold-time when $\sigma_{\min} > 0$, T_0 and T the total time period, and the period time not including the hold time where $T = T' + T''$.

Considerable effort has been extended in defining suitable damage parameter which correlates the life to failure (Voyiadjis and Kattan, 2009).

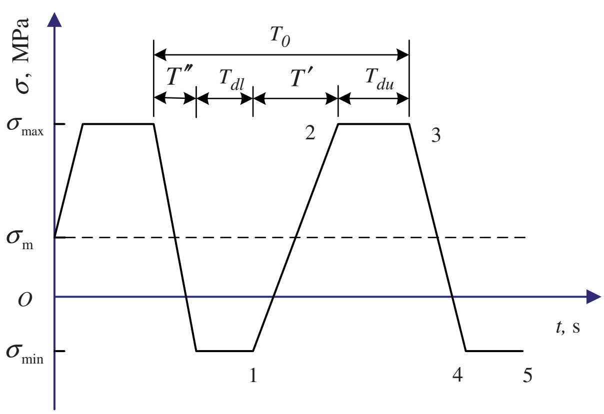

Figure 1. *Stress-time with trapezoidal loading waveform.*

Similar to the energy criterion proposed in the study of Stowell (1966), the energy parameter accumulated per cycle under fatigue–creep interaction can be described by the stress area under loading waveforms, and above the zero-stress line, as shown in Figure 2.

The parameter E_p per cycle with the shadows, as shown in Figure 2 can be calculated by the function given below

$$E_p = T_{du}\sigma_{\max} + (T_{dl} + T)\sigma_{\min} H(\sigma_{\min}) + \frac{T}{2} f(\sigma_{\max}, \sigma_{\min}) \tag{1}$$

and

$$f(\sigma_{\max}, \sigma_{\min}) = \begin{cases} \Delta\sigma, \sigma_{\min} > 0 \\ \frac{\sigma_{\max}^2}{\Delta\sigma}, \sigma_{\min} \leq 0 \end{cases} \tag{2}$$

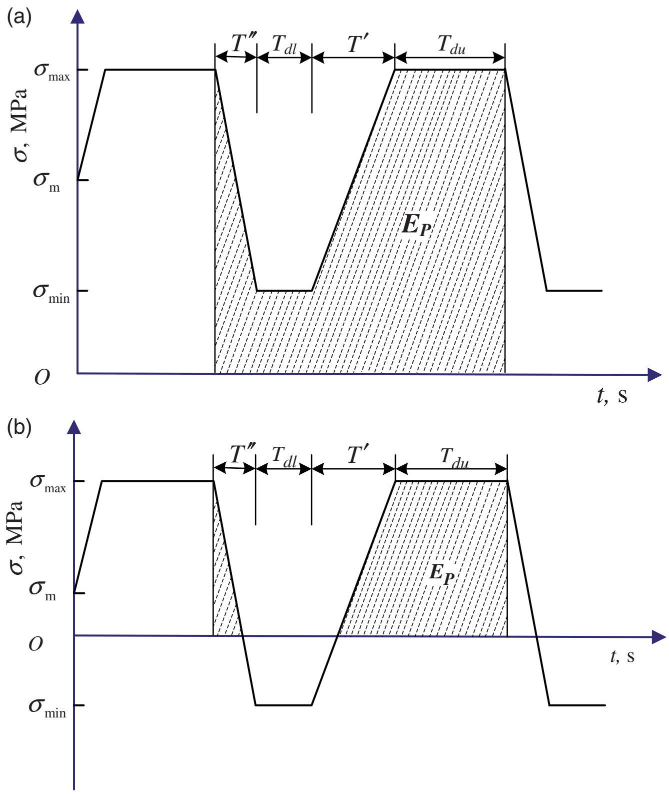

Figure 2. *The energy parameter E_p for different stress ratios: (a) $\sigma_{\min} > 0$ and (b) $\sigma_{\min} \leq 0$.*

where $H(\sigma_{\min})$ is the unit step function of $\sigma_{\min}$ and $f(\sigma_{\max}, \sigma_{\min})$ called the 'stress conversion function' determined by the maximum and minimum stresses and material properties. All materials are sensitive to the tensile stress and tensile dwell. As the dwell time in the tension direction is increased, the resulting life decreased. Compared with tensile dwell effects, the effects of compressive stress and compressive dwell on the fatigue–creep life are complex and vary by materials (Goswami and Hanninen, 2001a, b; Zhang, 2010). Using the assumption that only tensile stress can induce fatigue–creep damage (Chrzanowski, 1976), the function $H(\sigma_{\min})$ can be defined as follows

$$H(\sigma_{\min}) = \begin{cases} 1, \sigma_{\min} \geq 0 \\ 0, \sigma_{\min} < 0 \end{cases} \tag{3}$$

Power law index is established mainly based on the strain energy damage function model and analysis of large amounts of test data. In order to reduce the difference between the approximate and real strain energy absorbed during the damage process and get a higher precision, the strain energy and fatigue life can be written using the power law index

$$\Delta\varepsilon_{in}\left(E_p\right)^{\phi} N_f^{\alpha} = C \tag{4}$$

where $\Delta W_t = \Delta\varepsilon_{in}\left(E_p\right)^{\phi}$ is the GSEDF and ϕ the stress damage exponent related to environment conditions.

Considering that without hold-time in the loading waveform, $T_{du} = T_{dl} = 0$, the model reduces to an expression similar to the SEDF model when $\phi = \frac{1}{2}$. So, it is called a GSEDF model. The expression of LCF life without hold-time can be written as follows

$$N_f = \begin{cases} C\left(\Delta\varepsilon_{in}\sigma_{\max}\sqrt{T/2\Delta\sigma}\right)^{\beta}, \sigma_{\min} \leq 0 \\ C\left(\Delta\varepsilon_{in}\sqrt{T\sigma_{\mathrm{m}}}\right)^{\beta}, \sigma_{\min} > 0 \end{cases} \tag{5}$$

Under different loading waveforms, fatigue–creep life prediction can be determined using Equations (1)–(5). The comparison between the GSEDF, FS, and SEFS methods shows that the GSEDF model provides a higher precision of life prediction than the FS and SEFS methods (Zhu and Huang, 2010). In order to minimize the difference between the approximate and real ductility exhausted during the fatigue process, the accuracy of the proposed parameter E_p will be further investigated based on DE theory in the following section.

A NOVEL VISCOSITY-BASED LIFE PREDICTION MODEL FOR LCF–C

Fatigue is a damage accumulation process in which material property deteriorates continuously under cyclic loading. Experimental results have shown that the toughness of a material, a mechanical property parameter which combines both strength and plasticity of a material, is sensitive to the fatigue–creep damage process. A certain quantity of energy (material toughness), actually the ductility is gradually exhausted during the damage process of a material under LCF–C. Under cyclic loading, the continuous reduction of the material ductility indicates the progressive exhaustion of the ability to absorb energy, which is directly associated with the irreversible process of energy dissipation during fatigue failure. A value of energy dissipated in a material during one cycle of loading or during all the cycles up to the failure, is usually calculated from a history of the changes in cyclic strain and stress combined with the number of cycles. The more damage that is accumulated, the more ductility is exhausted. Once a critical toughness threshold is reached, fracture occurs. According to this relationship, the exhausted ductility can be used to indicate the damage accumulation under LCF–C loading. Several failure criteria and the corresponding life assessment methods have been developed based on this idea (Goswami, 1997; Fan et al., 2007; Zhu and Huang, 2010; Payten et al., 2010; Zhu et al., 2011b).

Under cyclic loading, the interactive behavior between stress and strain during deformation can be represented by hysteresis loops. Recent research suggests that a viscosity-based approach can be used to quantify this interaction (Goswami, 1997, 2004). In each cycle (points 1–5 in Figure 1), the parameter E_p was defined based on the loading waveform. The corresponding hysteresis loop is shown in Figure 3. In this article, the parameter E_p is associated with the stress, strain, and the material toughness based on the DE theory.

Ductility exhaustion theory assumes that, during tension, fatigue, and creep fracture processes, damage evolution usually can be associated with the continuous DE of the material. And failure occurs once accumulated strain reaches a critical ductility. Goswami (1995, 1997) developed a ductility model based on the assumption that deformation for LCF at high temperature can be represented in terms of viscous behavior. The dynamic viscosity should account for the strain range effects and can be presented based on the fundamental viscosity concept. Enlightened by this, dynamic viscosity υ_d is defined as (Goswami, 2004)

$$\upsilon_d = \Delta\sigma \cdot (\Delta\varepsilon_t / \dot{\varepsilon}) \quad (6)$$

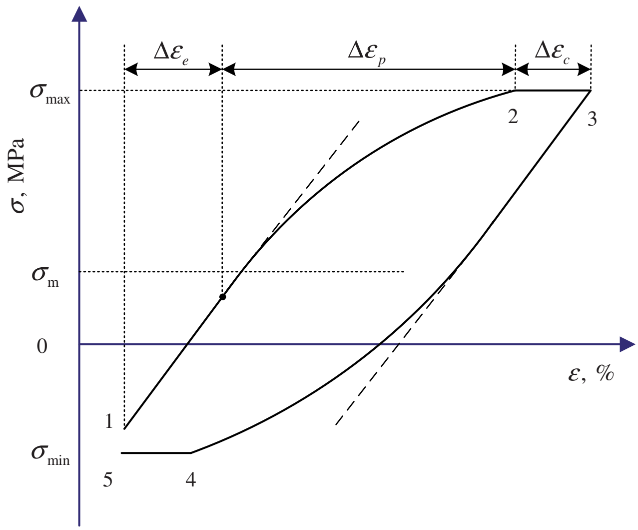

Figure 3. *Hysteresis loop under stress control with trapezoidal loading waveform.*

where υ_d, $\Delta\sigma$, $\Delta\varepsilon_t$, and $\dot{\varepsilon}$ are the dynamic viscosity, the stress range, the total strain range, and the strain rate, respectively.

The ability of a material to accommodate permanent deformation was defined in terms of material toughness. Failure criterion was defined in terms of dynamic viscosity, which was equal to the material toughness and could be expressed as

$$\sum f(\upsilon_d) = T_m \tag{7}$$

where T_m is the total energy dissipated by the material cumulatively up to failure, called the fatigue toughness. The material toughness is a product of ductility and cyclic strength (Goswami, 2004). Using the Edmund and White equation, ductility can be determined as follows

$$\text{Ductility} = \Delta\varepsilon_p N_f \tag{8}$$

Then, material toughness can be obtained by

$$T_m = \Delta\varepsilon_p \cdot N_f \cdot \sigma_{sat} \tag{9}$$

where σ_{sat} is the saturated tension stress at half-life ($N_f/2$).

Since creep damage is sensitive to the tensile hold time instead of compressive hold time (Zhang, 2010). The key factor leading to the failure of high-temperature structures in certain environments is the fatigue–creep interaction, which comes from thermally induced stresses and strains. Meanwhile, the effects of time-dependent test parameters, such as hold time, loading waveform and temperature, showed that the interactions between fatigue and creep are complex interactions with environmental factors. These interactions are very difficult to incorporate into a life prediction model. Hence, defining a suitable damage model to account for the interaction of fatigue and time-dependent damage caused by creep, which can account for temperature and mean stress effects, is considered to be very important when the results of LCF–C testing are applied to simulate thermal fatigue conditions.

According to the physical significance of the parameter E_p in Equation (1) and the dynamic viscosity υ_d in Equation (6), it should be noted that the latter is included in the former. And the essential difference between them is that the former includes the tensile elastic energy input which causes no damage compared with the latter. Based on the above description, the parameter E_p is actually a viscosity-based parameter. In order to reduce the difference between the approximate and real ductility exhausted during the fatigue process, a new definition of dynamic viscosity is presented using the parameter E_p and the tensile elastic energy input ΔW_{FL} per cycle which causes no damage

$$\upsilon_d = E_p - T_0 \cdot \Delta W_{FL} \tag{10}$$

where the tensile elastic energy input which causes no damage, ΔW_{FL}, was determined by

$$\Delta W_{FL} = \sigma_{\lim}^2 / 2E \tag{11}$$

where $\sigma_{\lim}$ is the fatigue limit of material.

Substituting Equations (1) and (11) into Equation (10) results in the following equation set

$$\upsilon_d = \begin{cases} T_{du}\sigma_{\max} + (T_{dl} + T)\sigma_{\min} + \frac{T}{2}\Delta\sigma - T_0\dfrac{\sigma_{\lim}^2}{2E}, \sigma_{\min} > 0 \\ T_{du}\sigma_{\max} + \frac{T}{2}\dfrac{\sigma_{\max}^2}{\Delta\sigma} - T_0\dfrac{\sigma_{\lim}^2}{2E}, \sigma_{\min} \leq 0 \end{cases} \tag{12}$$

For simplicity, it is assumed that the stress cycle period and fatigue life follow a power law index relationship. Thus, the dynamic viscosity during the fatigue process can be expressed as given below

$$\sum f(\upsilon_d) = k\left(E_p - T_0 \cdot \Delta W_{FL}\right)^{\gamma} \tag{13}$$

where k and γ are material constants related to the environment conditions, such as temperature.

Fatigue toughness which describes both strength and plasticity of a material is a more sensitive mechanical property parameter to the fatigue damage process than others. Considering that fatigue crack grows only in the tension stage, Ostergren (1967) proposed the strain energy damage function model (Viswanathan, 1995). This model assumes that only tensile inelastic strain energy can induce the crack opening and propagation. The strain energy damage function ΔW_{str} is approximately expressed by multiplication of the inelastic strain range $\Delta\varepsilon_{in}$ and the maximum tension stress $\sigma_{\max}$. The relationship between strain energy and fatigue life is expressed by the power exponent function, i.e.

$$\Delta W_{str} N_f^{\nu} = \Delta\varepsilon_{in}\sigma_{\max} N_f^{\nu} = C \tag{14}$$

where ΔW_{str} is the strain energy, $\Delta\varepsilon_{in}$ the inelastic strain range of the stable hysteresis loop or half-life which is replaced by the plastic strain range $\Delta\varepsilon_p$ under pure fatigue loading.

It is interesting to note that Equations (9) and (14) have the similar form even though they are derived from different theoretical backgrounds. Comparing two equations based on the Ostergren's model, the fatigue toughness T_m can be given as

$$T_m = \Delta\varepsilon_{in}\sigma_{\max} N_f^{\nu} \tag{15}$$

As discussed earlier, a new LCF–C life prediction equation is derived equating these two terms, the dynamic viscosity Equation (13) and fatigue toughness Equation (15), as shown in Equation (7). Therefore, a new life prediction equation is derived by rearranging the following equation

$$k\left(E_p - T_0 \cdot \Delta W_{FL}\right)^{\gamma} = \Delta\varepsilon_{in}\sigma_{\max} N_f^{\nu} \tag{16}$$

where k is a material parameter which balances the units of this equation.

Rearranging various terms in Equation (16), the number of cycles to failure can be calculated from the following viscosity-based approach

$$N_f = k\left(E_p - T_0 \cdot \Delta W_{FL}\right)^{p} (\Delta\varepsilon_{in}\sigma_{\max})^{q} \tag{17}$$

Based on Equation (17), it is worth noting that this empirical model correlates failure/fatigue with energy using DE theory, which includes the factors influencing fatigue and creep lives. It is very easy to use this equation to predict the LCF–C life at high temperatures using only three material parameters (k, p, and q) in Equation (17), which can also be fitted from test data.

The ratcheting behavior of materials is a complex phenomenon and depends on a number of factors, including mean stress σ_m, stress amplitude

σ_a, frequency (total time period for one cycle T_0), loading history and microstructural characteristics (Xia et al., 1996). It is worth noting that Equation (17) incorporates most of these factors. Similar to the parameter developed in the study of Xia et al. (1996) and the SWT parameter (Smith et al., 1970), the effect of ratcheting on the fatigue life was characterized by the inelastic strain range $\Delta\varepsilon_{in}$, which consists of plastic strain range $\Delta\varepsilon_p$ and creep strain range $\Delta\varepsilon_c$. The strains of points 1–5 in Figure 1 were obtained by an extensometer and the inelastic strain range per cycle is expressed as

$$\Delta\varepsilon_{in} = \varepsilon_f - \varepsilon_0 \tag{18}$$

where ε_0 and ε_f are the initial and final strains, respectively, in a cycle.

Moreover, these three methods considered the effect of mean stress on the predicted life through the maximum stress, where $\sigma_{\max} = \sigma_a + \sigma_{\mathrm{m}}$. For the proposed viscosity-based model, both mean stress and ratcheting effects have been taken into account in the fatigue damage process.

For LCF–C interaction with hold time at high temperature, it follows from the discussions above that the fatigue life predicted in terms of the fatigue DE actually represents the degradation of both plasticity and strength of a material. Compared with other methods (Goswami, 1995, 1997; Ye and Wang, 2001), the development of this new model considers not only the loading waveform effects on LCF–C life, but also the effects of mean stress. Additionally, it provides a new way to determine the real dynamic viscosity per cycle and extends the application of DE theory to stress-controlled LCF–C interaction conditions. For actual components, the prediction accuracy of the proposed model was evaluated and was verified by analyzes of LCF–C life prediction in the following section.

VALIDATION OF THE NEW LIFE PREDICTION MODEL FOR LCF–C

To verify the feasibility and prediction capability of the viscosity-based life prediction model for LCF–C at high temperature, the proposed model was evaluated using experimental results of 1.25Cr0.5Mo steel and turbine disk material GH4133 under different temperatures from published sources (Wang, 2006; Fan et al., 2007).

Material and Test Conditions

The applicability of the new life prediction model was evaluated using LCF–C test results from the study of Wang (2006) and Fan et al. (2007).

The materials used in these experiments are pearlitic heat resistant steel 1.25Cr0.5Mo and Ni-base superalloy GH4133. The tests for the 1.25Cr0.5Mo steel were conducted using a trapezoid waveform (stress control) with a hold period of 5 s duration at $\sigma_{\max}$ and $\sigma_{\min}$, respectively, where $T_{du} = T_{dl} = 5s$ and each cycle took 20 s ($T_0 = 20s$). The material was machined into cylindrical specimens with 10 mm diameter and 32 mm gauge length. Under different stress ratios and mean stresses σ_m, the fatigue–creep tests were performed under the following conditions: two temperatures, 540°C and 520°C, stress amplitudes ranging from 25 to 190 MPa and four defined maximum stresses 200, 210, 220, and 230 MPa. These stress controlled fatigue–creep interaction tests were conducted using an electro-hydraulic servo fatigue testing machine EHF-EG250-40 L equipped with a heating furnace. The combined test conditions and the number of tests are summarized in Table A1.

In order to verify the effects of strain rate and the loading waveform on the LCF life at high temperature, the proposed model will be evaluated using experimental results of GH4133. The heat treatment conditions of this alloy are as follows: austenitization (8 h at 1353.15 K, air-cooled) and tempering (16 h at 1023.15 K, air-cooled). The tests for the GH4133 were performed under axial total strain control with a triangular fully reversed waveform, using an axial extensometer placed on the specimen. Numerous tests were carried out with the following various conditions: mechanical strain range of 0.5–1.4% for isothermal LCF at temperature 500°C and 400°C under strain ratio $R_\varepsilon = -1$, respectively. For further inquiries regarding detailed mechanical properties of the materials, test procedures and specimen specifications refer to Wang (2006), Fan et al. (2007), Chen et al. (2007), and Beijing Institute of Aeronautical Materials (1996).

Results and Discussion

The damage induced in the material due to the loading described in the previous section is to be estimated using each of the following methods: MSR, GSEDF, SWT, Goswami's ductility model, and the current model stated in Equation (17). The response of the material at half-life ($N_f/2$) is considered to be stable and representative of the life. For 1.25Cr0.5Mo steel at 520°C and 540°C, the Young's modulus E was 1.77×10^5MPa. Combining the experimental results and the parameters listed in Table A1, for different maximum stresses and stress ratios, the fitted life prediction model for 1.25Cr0.5Mo steel at 540°C according to Equation (17) is given by

$$N_f = 4.89057 \times 10^{14} \left(E_p - 20 \cdot \Delta W_{FL}\right)^{-0.837803} (\Delta\varepsilon_{in}\sigma_{\max})^{-0.907999} \qquad (19)$$

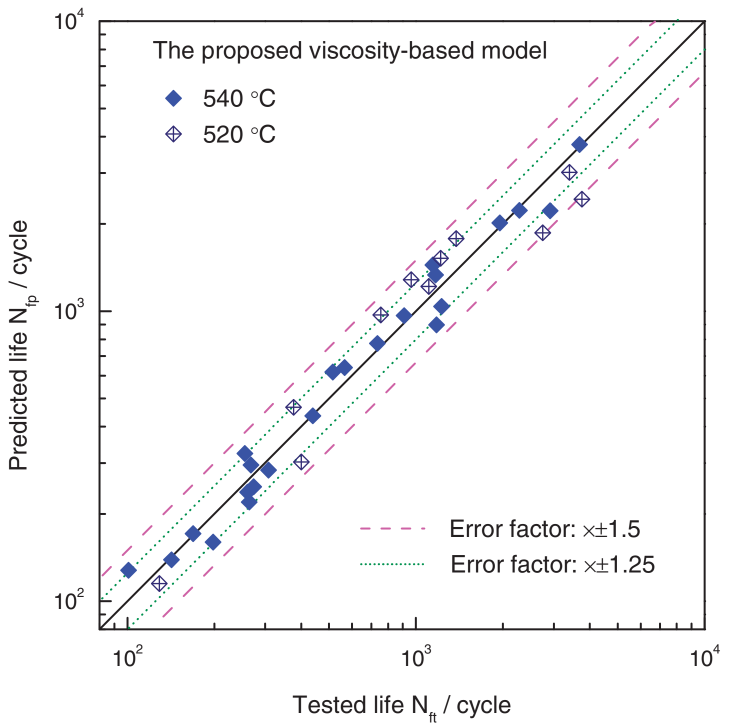

Figure 4. *Comparison between lives predicted by the new model and those tested for 1.25Cr0.5Mo steel.*

Similarly at 520°C, the fatigue life is approximately fitted as

$$N_f = 1.52045 \times 10^7 \left(E_p - 20 \cdot \Delta W_{FL}\right)^{-0.0101023} (\Delta\varepsilon_{in}\sigma_{\max})^{-0.938895} \tag{20}$$

For GH4133, the Young's modulus E was 1.992×10^5MPa. Under different total strain controls and strain ratios $R_\varepsilon = -1$, the fitted life prediction model for GH4133 at 500°C is given by

$$N_f = 4.08565 \times 10^{39} \left(E_p - 1.7138 \cdot \Delta W_{FL}\right)^{-4.0446} (\Delta\varepsilon_{in}\sigma_{\max})^{-0.268971} \tag{21}$$

Similarly at 400°C, the fatigue life prediction for GH4133 under strain ratio $R_\varepsilon = -1$ by Equation (17) is expressed as

$$N_f = 1.5365 \times 10^{34} \left(E_p - 1.7138 \cdot \Delta W_{FL}\right)^{-3.3111} (\Delta\varepsilon_{in}\sigma_{\max})^{-0.38978} \tag{22}$$

Good correlations between the experimental results and the theoretical predictions under different temperatures for these two materials are

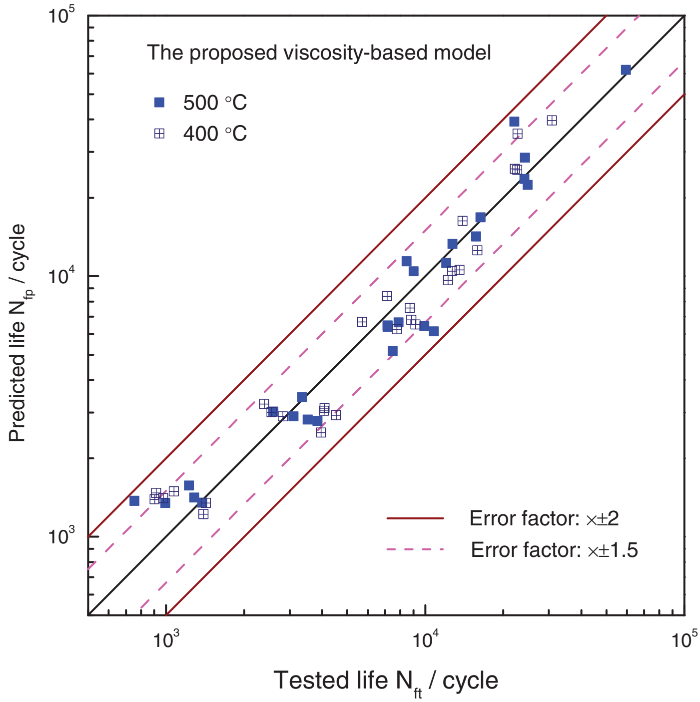

Figure 5. *Comparison between lives predicted by the new model and those tested for GH4133.*

observed, as shown in Figures 4 and 5. The dashed line in the graph corresponds to the ±1.5 factor indictors, the dot line for the ±1.25 factor indictors, and the solid line for the ±2 factor indictors. From Figures 4 and 5 and Table A1, all the predicted lives are within a factor of ±2, and about 33 out of 34 cyclic lives for 1.25Cr0.5Mo steel, 47 out of 55 cyclic lives for GH4133 are predicted within a factor of ±1.5. Obviously, the predicted results are in good agreement with the test ones.

To reflect the capability of the proposed model and evaluate its applicability under the effects of mean stress and creep, four other methods, the MSR (Fan et al., 2007), the GSEDF (Zhu and Huang, 2010), the Goswami's (1997) ductility model, and the SWT model (Smith et al., 1970), were employed for comparison purposes, respectively. The test data for 1.25Cr0.5Mo steel was assessed by three methods and is listed in Table A1. This table shows clearly that the life predictions using these

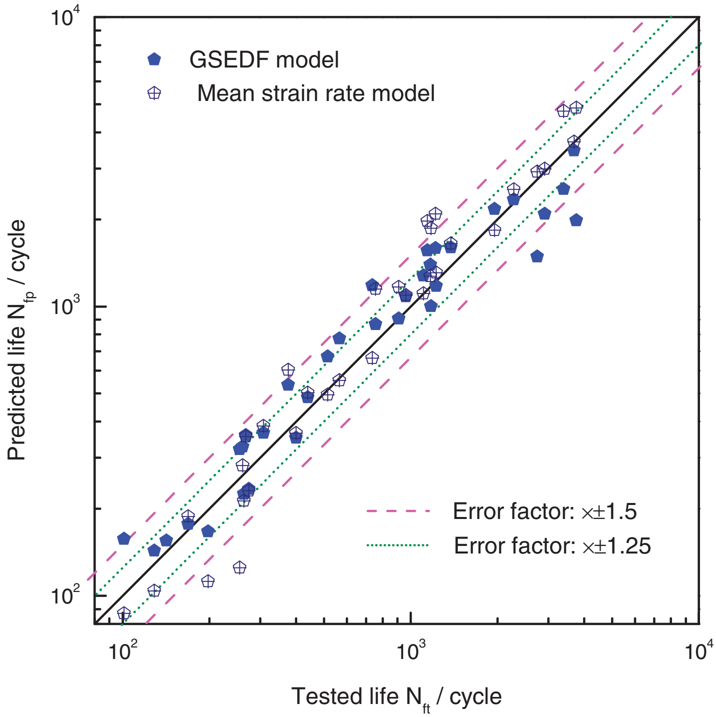

Figure 6. *Comparison between lives predicted by GSEDF, MSR methods and those tested for 1.25Cr0.5Mo steel.*

methods are in accordance with experimental results. The correlations between the experimental and the predicted lives by these four models for two materials are shown in Figures 6 and 7.

It is found that nearly all the predicted cyclic lives by the GSEDF, the MSR, and SWT models fall into a range within a scatter band of ±2, while about 30 out of 34, 26 out of 33, and 43 out of 55 cyclic lives predicted by the GSEDF, MSR, and the SWT models are within a factor of ±1.5, respectively. From Figure 7, the results show that only about 34 out of 55 cyclic lives are predicted within a factor of ±1.5 to the test ones by the Goswami's ductility model. Comparing the scatter band and the standard deviation of these methods, results indicate that the proposed model has a better predictability than others. The life prediction method using Equation (17) can predict LCF–C behavior well at a certain temperature, but whether it can be consolidated into one equation for a certain temperature interval or not will be assessed in the following section.

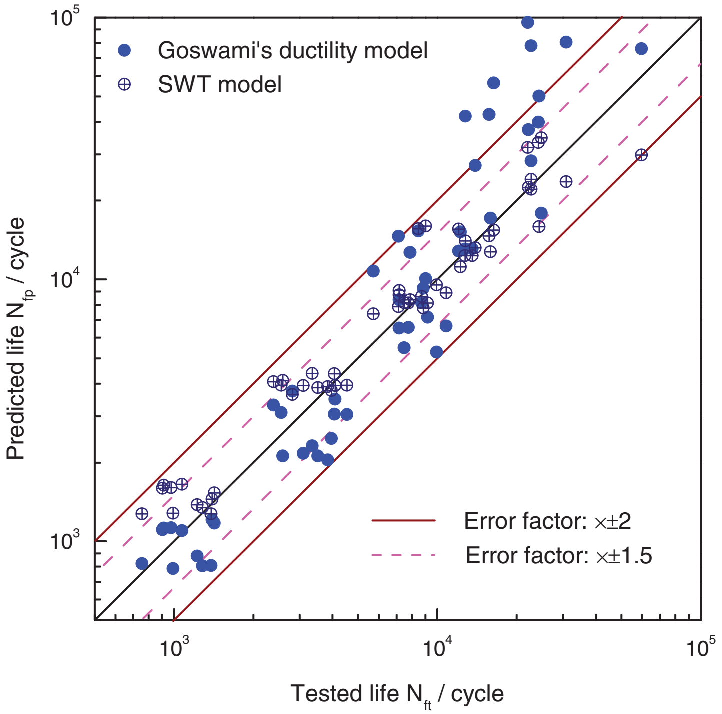

Figure 7. *Comparison between lives predicted by the Goswami's ductility model, SWT methods and those tested for GH4133.*

In practical engineering, the failure of high-temperature structures comes from thermally induced stresses and strains. Hence, developing a suitable life prediction model, which can account for temperature effects, is considered to be very important when the results of LCF testing are applied to the simulation of thermal fatigue conditions. If the effect of temperature on fatigue life can be properly described, life prediction at specific elevated temperatures is available in addition to the reference fatigue data normally obtained at room temperature. In general, thermal fatigue test is very difficult to perform and also takes a long time because temperature variation rate is much slow. Therefore, many researchers analyzed thermal fatigue with high-temperature isothermal fatigue data. Using Equation (17), the LCF–C life for 1.25Cr0.5Mo steel under different temperatures is predicted by the following equation

$$N_f = 4.07253 \times 10^{11}\left(E_p - 20 \cdot \Delta W_{FL}\right)^{-0.495795}(\Delta\varepsilon_{in}\sigma_{\max})^{-0.92377} \qquad (23)$$

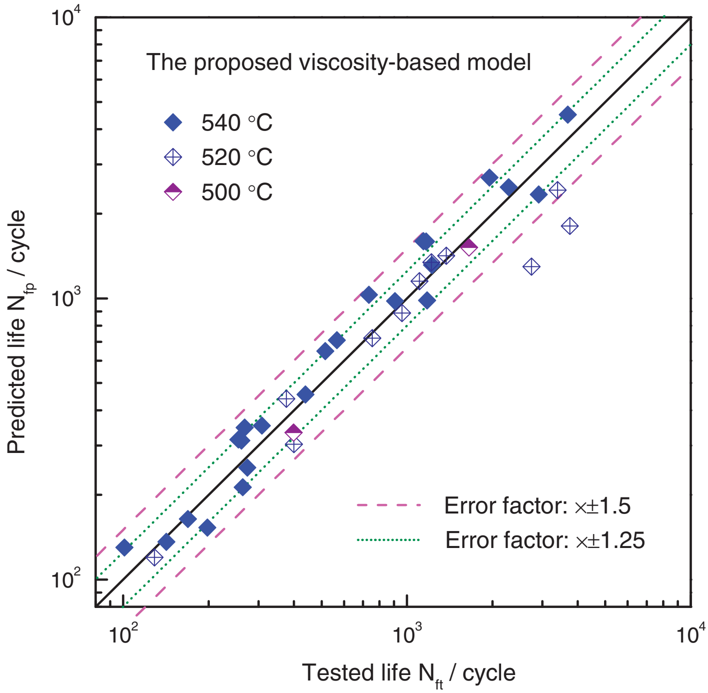

Figure 8. *Comparison between lives predicted by the new model and those tested for 1.25Cr0.5Mo steel under different temperatures.*

Similarly for GH4133 under different temperatures, the fatigue life is approximately fitted as

$$N_f = 4.99462 \times 10^{31}\left(E_p - 1.7138 \cdot \Delta W_{FL}\right)^{-3.0382}(\Delta\varepsilon_{in}\sigma_{\max})^{-0.370252} \quad (24)$$

Life obtained by Equations (23) and (24) at different temperatures are compared to the experimental life data, as shown in Figures 8 and 9, respectively.

The correlation of predicted and experimental life results is satisfactory for different temperature loading conditions. The fatigue life correction factor range is nearly equal to ±1.5 or better, with which 34 out of 36 cyclic lives for 1.25Cr0.5Mo steel and 46 out of 55 cyclic lives for GH4133 are predicted within a factor of ±1.5 by the proposed viscosity-based model. It should be noted that Equation (17) can be used to predict lives under thermal fatigue conditions. Through the DE theory, this model can transform the complex correlation between N_f and mechanisms of loading waveforms and mean stress into a rational relation. By which life

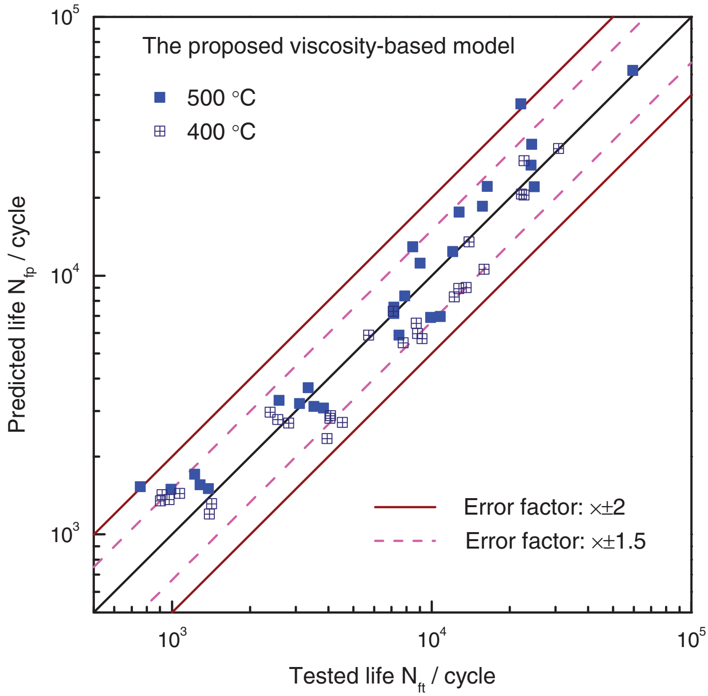

Figure 9. *Comparison between lives predicted by the new model and those tested for GH4133 under different temperatures.*

prediction for other conditions can easily be made. Furthermore, the proposed model enables the time dependencies of deformation characteristics to be described.

In summary, based on the same theoretical backgrounds and viscosity parameter E_p, the differences between the experimental and calculated LCF–C life by the proposed method and the GSEDF model are relatively small as both of them consider the effects of loading waveform and mean stress. Moreover, the new model incorporates the effect of temperature where the GSEDF model does not. Thus, it can be better used for life evaluation of high-temperature structures under thermal fatigue conditions. According to the theoretical derivation of the new model and the comparisons of the prediction results by these methods from Table A1 and Figures 4, 5, 8, and 9, it is obvious that the viscosity-based model proposed in this article has a better prediction capability than others.

According to the applicable conditions of the proposed model, it is valid for most metallic materials under uniaxial loading, such as carbon steels, cast irons, and alloy steels. Furthermore, it can reflect the fundamentals of fatigue–creep damage under stress or strain control. It has the advantages including few parameters, considering mean stress, temperature, and loading waveform effects, and has a higher life prediction precision when compared to other approaches. However, most engineering components and structures are subjected to complex loading conditions at which stress–strain cycles fluctuate with time. This leads to a cyclic creep–fatigue interaction. Thus, the application of this model under multiaxial loading, different hold times/ temperatures and materials need to be further evaluated.

CONCLUSIONS

Based on the DE theory and the GSEDF method, a new viscosity-based model has been proposed for LCF–C life prediction. The feasibility and validity of this proposed model was checked with the fatigue–creep interaction test data from literature. Some conclusions are drawn from the present investigation:

1. The viscosity-based parameter E_p correlates and describes the fatigue–creep damage with the mechanisms of loading waveform and mean stress. This leads to a great robust assessment method for high-temperature structures in practical engineering applications.
2. The proposed model is applicable for both the strain-controlled tests and stress-controlled tests under uniaxial loading. On the basis of DE theory, this model is able to describe the damaging processes during the interaction of LCF and creep as a dependence on loading parameters, σ_{max}, σ_{m}, σ_a, $\Delta\varepsilon_p$, strain rate and hold time within a certain temperature interval.
3. Compared with the GSEDF and the MSR methods, the proposed viscosity-based model is more suitable for predicting LCF–C life for high-temperature structures. It is worth noting that all the test data were within a factor of ±2 and nearly 97% of the test data for 1.25Cr0.5Mo steel, 85% of the test data for GH4133 were within a factor of ±1.5 of the predicted results. This strongly suggests that the proposed model is believed to be physically feasible and accepted as a general expression. Moreover, it has higher prediction accuracy and the capability to estimate thermal fatigue lifetime using calibration on isothermal fatigue tests only, leading to a great potential cost saving for future test programs given similar materials and loading conditions.
4. The main characteristics of this model compared with some existing methods for LCF–C life prediction based on energy (ductility) is that it takes the effects of creep, temperature, and mean stress on fatigue life into

account. Accordingly, it is better suited to describe the dynamic deterioration process of various high-temperature structural materials and hot section components even under thermal–mechanical fatigue.

NOMENCLATURE

C = Material constants representing the material energy absorption capacity
E = Young's modulus
N_f = Number of cycles to failure
N_{ft} = Number of tested life cycles
N_{fp} = Number of predicted life cycles
R_ε = Strain ratio
ΔW_{FL} = Strain energy density at the fatigue limit of the material
ΔW_{str} = Strain energy
T_m = Fatigue toughness
$\alpha, \beta, \gamma, \phi, v$ = Model parameters
$\dot{\varepsilon}$ = Strain rate
υ_d = Dynamic viscosity
$\Delta\varepsilon_c$ = Creep strain range
$\Delta\varepsilon_t$ = Total strain range
$\Delta\varepsilon_e$ = Elastic strain range
$\Delta\varepsilon_{in}$ = Inelastic strain range
$\Delta\varepsilon_p$ = Plastic strain range
σ_a = Stress amplitude
$\sigma_{\lim}$ = Fatigue limit
$\sigma_{\max}$ = Maximum stress
$\sigma_{\min}$ = Minimum stress
σ_m = Mean stress
σ_{sat} = Saturated stress range
$\Delta\sigma$ = Stress range

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APPENDIX

***Table A1.* Experimental parameters and life predictions of new method, the GSEDF model, and the MSR method for 1.25Cr0.5Mo steel.**

T (°C)	Maximum stress of tests (MPa)	Stress range, $\Delta\sigma(\sigma_{min}-\sigma_{max})$ (MPa)	Inelastic strain range of half-life, $\Delta\varepsilon_{in}$ (%)	Cyclic life tested, N_{ft} (cycle)	N_{fp} predicted by the GSEDF model (Zhu and Huang, 2010) (cycle)	N_{fp} predicted by the MSR model (Fan et al., 2007) (cycle)	N_{fp} predicted by the new method (cycle)
540	200 MPa	50 (150–200)	0.002700095	1952	2173	1832	2017
		150 (50–200)	0.001855671	3688	3458	3705	3760
		300 (−100 to 200)	0.012053573	908	908	1166	966
		250 (−50 to 200)	0.004487762	2914	2086	2990	2223
		350 (−150 to 200)	0.085710149	169	177	188	171
		350 (−150 to 200)	0.092444568	198	167	112	160
		350 (−150 to 200)	0.06484573	264	225	213	220
		350 (−150 to 200)	0.042463609	255	321	125	323
	210 MPa	210 (0–210)	0.010071726	1177	1002	1864	897
		210 (0–210)	0.005951911	1143	1563	1975	1445
		210 (0–210)	0.003694547	2280	2340	2539	2228
		260 (−50 to 210)	0.024450503	439	485	503	436
		260 (−50 to 210)	0.016635129	515	672	495	618
		160 (50–210)	0.005328134	1169	1392	1272	1335
		110 (100–210)	0.005969052	1227	1175	1309	1040
		360 (−150 to 210)	0.097583211	142	155	–	139
		60 (150 to 210)	0.007175124	735	1183	663	775

(continued)

Table A1. Continued

T (°C)	Maximum stress of tests (MPa)	Stress range, $\Delta\sigma(\sigma_{min}-\sigma_{max})$ (MPa)	Inelastic strain range of half-life, $\Delta\varepsilon_{in}$ (%)	Cyclic life tested, N_{ft} (cycle)	N_{fp} predicted by the GSEDF model (Zhu and Huang, 2010) (cycle)	N_{fp} predicted by the MSR model (Fan et al., 2007) (cycle)	N_{fp} predicted by the new method (cycle)
		310 (−100 to 210)	0.048775574	274	234	231	248
	220 MPa	220 (0–220)	0.013327132	566	775	555	640
		170 (50–220)	0.025915479	268	359	355	295
		320 (−100 to 220)	0.091740718	101	157	87	128
		120 (100–220)	0.023230701	308	366	386	283
		70 (150–220)	0.024522025	261	328	282	238
520	220 MPa	370 (−150 to 220)	0.103501	129	143	104	115
		320 (−100 to 220)	0.036789	400	352	365	302
		270 (−50 to 220)	0.008326	1106	1278	1107	1218
		120 (100–220)	0.003962	3753	1985	4855	2435
		170 (50–220)	0.003161	3397	2542	4736	3015
		70 (150–220)	0.005256	2745	1486	2919	1864
	230 MPa	280 (−50 to 230)	0.022058	376	536	603	467
		230 (0–230)	0.006258	1218	1590	2088	1525
		180 (50–230)	0.005293	1377	1600	1648	1781
		130 (100–230)	0.010084	755	867	1149	971
		80 (150–230)	0.007469	961	1083	1093	1285
		380 (−150 to 230)	–	–	–	–	–