Original Article

Concurrent Engineering: Research and Applications
21(4) 286–295

DOI: 10.1177/1063293X13499358
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A Pareto set coordination method for analytical target cascading

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Abstract

In the general analytical target cascading method, a weighted-sum formulation is commonly employed to coordinate the inconsistency between design points and assigned targets at each level while minimizing the cost. The determination of weighting coefficients is problem dependent. Improper selections of the weighting coefficients may result in incorrect solutions. To avoid using the weighting coefficients, a genetic algorithm optimization method is developed for the hierarchical design problem by using the Pareto set coordination method. The Pareto sets are obtained from the optimal solutions at each level while each subsystem chooses one solution based on the detailed information. Instead of setting point-valued targets and weighting coefficients, Pareto sets are computed and updated at multiple levels until targets are satisfied. Therefore, the genetic algorithm optimizer with Pareto set coordination for analytical target cascading can avoid choosing weighting coefficients. By doing so, the proposed method explores completed feasible solutions at each level and improves the convergence process. The results for the proposed method and the weighted-sum analytical target cascading are compared to illustrate the performance of the proposed method.

Keywords

Analytical target cascading, weighted-sum, genetic algorithm, Pareto set

Introduction

Design of an engineering system is often a challenging task due to its complexity. Originally, an engineering system design problem minimizes the cost subject to multiple disciplinary constraints, but it may be unpractical to solve the entire problem in one optimization formulation. One of the most important optimization algorithms, analytical target cascading (ATC) method (Allison, 2004; Kim et al., 2002, 2003a), has been developed and widely used to solve complex engineering design problems (Kim et al., 2003b; Kokkolaras et al., 2002, 2004; Li et al., 2008b). In the ATC method, the all-in-one (AIO) optimization formulation is decomposed into a hierarchical multilevel structure with one system level and multiple subsystem levels. For the system or each subsystem design problem, a multi-objective optimization problem is formulated. The purpose of using the multi-objective optimization model is to minimize the cost and meanwhile to reduce the inconsistency between different subsystems.

The deviations between the subsystem variables and the assigned targets (Balling and Sobieszczanski-Sobieski, 1995, 1996) have been considered as discrepancy functions. The discrepancy functions were then penalized in the objective functions (Demiguel and Murray, 2000; Gu et al., 2006) to find the optimal solution and meanwhile diminish the inconsistency during the optimization process. The ATC method considers the weighted discrepancy functions of design variables (Allison et al., 2005; Michelena et al., 2003) in the formulation of multi-objective optimization models. The weighting coefficients are then used to coordinate the targets and response in the solution process.

In the solution process of the general ATC, the system level assigns design targets to subsystems. In subsystems, the optimal solutions are identified by minimizing deviations between the subsystem variables

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and the targets subjected to the local constraints. The local optimal solutions are then updated back to the system level as design responses. The weighted discrepancy functions by considering the deviation of targets and the response points are minimized during the optimization processes of ATC. The selection of the weighting coefficients is vital for decreasing the discrepancy functions and coordinating the inconsistency between design points and assigned targets at each level during the optimization process. In other words, the correct optimal solutions can be obtained only when weighting coefficients are properly selected. Many approaches (Kim et al., 2006; Li et al., 2008a; Michalek and Papalambros, 2005a, 2005b; Tosserams et al., 2006) have been developed for the determination of proper weighting coefficients. Michalek and Papalambros (2005a) proposed a weighting coefficient updating method using the Karush–Kuhn–Tucker (KKT) first-order necessary conditions combined with user-specified inconsistency tolerances. Tosserams et al. (2006) developed an augmented Lagrangian coordination method with the alternating direction method of Lagrangian multipliers. Kim et al.(2006) formulated a Lagrangian dual coordination method to update the weighting coefficients. Li et al. (2008a) provided a diagonal quadratic approximation method by linearizing the cross terms of the discrepancy functions. Methods presented in Tosserams et al. (2006), Kim et al. (2006), and Li et al. (2008a) took the discrepancy functions as equality constraints and used different penalty terms to coordinate those equality constraints. The convergence rate of these methods becomes slow when the discrepancy tends to be small, and the penalty terms might oscillate during the iteration process.

Genetic algorithm (GA) can search different regions of a solution space to make it possible to find a diverse set of solutions for difficult optimization problems with nonconvex, discontinuous, and multi-modal objective functions with the selection, crossover, and mutation operations (Gen and Cheng, 2000; Viennet et al., 1996), and GA has been widely used to solve multi-objective optimization problems (Deb et al., 2002; Yousefi and Yusuff, 2013; Zhang et al., 2013). For an engineer or designer, reducing the design space to only the Pareto set allows the engineer to focus on important trade-offs without considering the full range of all possible parameters. Also, Pareto set can provide maximal information for the decision makers in the multi-objective optimization design process. Therefore, Pareto set solution is the pursuit for decision maker in the complicated engineering problems (Bradner and Davis, 2013; Kubica and Woźniak, 2012; Pardalos et al., 2012).

In this work, a GA optimization method with Pareto set coordination is formulated for the hierarchical design problem. The Pareto set coordination process is introduced to avoid setting point-valued targets and weighting coefficients to the subsystems. During the optimization process, Pareto sets using GA optimizer are obtained at each level and propagated to provide detailed performance ability of each subsystem. The multi-objective optimization problem is solved at the system level first. From the system-level optimization model, Pareto sets are obtained. The Pareto sets are then propagated to subsystems as targets. At the subsystem level, each subsystem formulates a multi-objective optimization problem based on the targets and Pareto solutions from system level and uses GA optimizer to solve this problem, and the optimal solution is sent back to the system level as responses. Based on the responses from the subsystem level, a new multi-objective design model is formulated at the system level. After that, solutions from the new design model are used to update the Pareto sets, which are later used as the new targets for the subsystems. The iteration continues until all the subsystems achieve their targets.

The remainder of this article is given as follows. In section "General ATC," the general ATC is briefly reviewed. In section "Pareto set coordination method," the Pareto set coordination method is introduced using GA optimizer, and the solution process is presented with a numerical example. Following that, in section "Reliability-based design optimization of a cylindrical gear reducer," a speed reducer design optimization problem is studied to demonstrate the performance of the proposed method.

General ATC

A generalized AIO model for complex engineering problem is given in equation (1). In this model, a large-scale system design is formulated and solved with fully integrated multidisciplinary analysis. This framework is usually not practical. To reduce the complexity, it may be demanding to decompose an engineering system into multiple manageable subsystems hierarchically or non-hierarchically. ATC is developed for the design optimization of hierarchical multilevel systems. Therefore, by using the ATC method, the model presented in equation (1) is decomposed into n subsystems, and each subsystem is decomposed into some components. The multilevel structure is shown in Figure 1

$$\begin{aligned}
&\min: f_{sys}\\
&\text{s.t. } \mathbf{g}_{sys}(\mathbf{x}, \mathbf{z}) \leq 0\\
&\quad\; \mathbf{h}_{sys}(\mathbf{x}, \mathbf{z}) = 0\\
&\quad\; \mathbf{g}_{sub,i}(x_i, \mathbf{y}_{sub,i}, \mathbf{u}_i) \leq 0\\
&\quad\; \mathbf{h}_{sub,i}(x_i, \mathbf{y}_{sub,i}, \mathbf{u}_i) = 0\\
&\quad\; f_{sys} = f_{sys}(\mathbf{x}, \mathbf{z})\\
&\quad\; i = 1, 2, \ldots, n
\end{aligned} \tag{1}$$

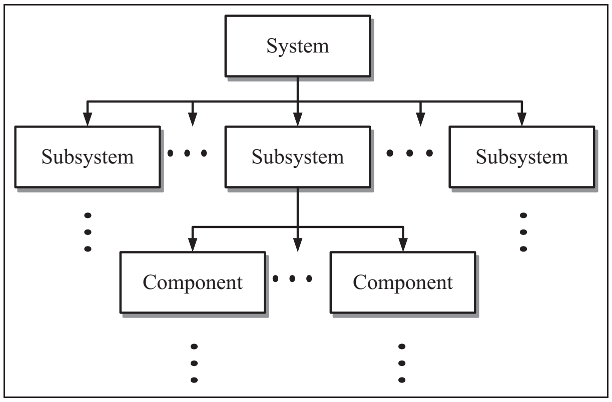

Figure 1. The hierarchically decomposed structure.

Form Figure 1, we can see that ATC is a multidisciplinary optimization (MDO) method for multilevel hierarchically decomposed engineering system. A two-level structure is presented for illustration. The top level is the system level and the lower level is the subsystem level. The design objective at the system level is to minimize the system performance and the deviation of the subsystem response with targets. The objective function in subsystem level is to achieve the targets assigned by system as far as possible. Both the design optimization models of system level and subsystem level are multi-objective. In the general ATC method, a weighted-sum formulation is usually used to coordinate the inconsistency between design points and assigned targets at each level while minimizing the cost. The design models at system level and subsystem level are given in equations (2) and (3), respectively

$$\begin{aligned} \min: & f_{sys} + \sum_{i=1}^{n} w_{x,i}(x_{sub,i} - x_{sub,i}^{sub})^2 \\ & + \sum_{i=1}^{n} w_{y,i}(\mathbf{y}_{sub,i} - \mathbf{y}_{sub,i}^{sub})^2 \\ \text{s.t. } & \mathbf{g}_{sys}(\mathbf{x}, \mathbf{z}) \leq 0 \\ & \mathbf{h}_{sys}(\mathbf{x}, \mathbf{z}) = 0 \\ & f_{sys} = f_{sys}(\mathbf{x}, \mathbf{z}) \\ & i = 1, 2, \ldots, n \end{aligned} \tag{2}$$

$$\begin{aligned} \min: & w_{x,i}(x_{sub,i} - x_{sub,i}^{sys})^2 + \mathbf{w}_{y,i}(\mathbf{y}_{sub,i} - \mathbf{y}_{sub,i}^{sys})^2 \\ \text{s.t. } & \mathbf{g}_{sub,i}(x_{sub,i}, \mathbf{y}_{sub,i}, \mathbf{u}_i) \leq 0 \\ & \mathbf{h}_{sub,i}(x_{sub,i}, \mathbf{y}_{sub,i}, \mathbf{u}_i) = 0 \end{aligned} \tag{3}$$

In the weighted-sum formulation, the weighting coefficients are determined for each objective function. However, the selection of the weighting coefficients is problem dependent, and improper selections of the weighting coefficients may lead to, unfortunately, slow convergence, oscillation, or even incorrect solutions. Pareto set can explore all the feasible solutions in design space and provide better understanding to the performance of system. In the hierarchical interactive structure, Pareto set makes more information be propagated between system level and subsystem level, which also avoids setting weighting coefficients. Therefore, the Pareto set coordination procedure with the help of GA is provided in section "Pareto set coordination method."

Pareto set coordination method

Given a multi-objective optimization problem

$$\begin{aligned} \min: & \ \mathbf{f}(\mathbf{x}) = [f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_n(\mathbf{x})] \\ \text{s.t.} & \ \mathbf{h}(\mathbf{x}) = 0, \mathbf{g}(\mathbf{x}) = 0 \end{aligned} \tag{4}$$

the feasible domain D is defined as

$$D = \{\mathbf{x} | \mathbf{h}(\mathbf{x}) = 0, \mathbf{g}(\mathbf{x}) = 0\} \tag{5}$$

A point $\mathbf{x}_0$ in the feasible domain D is a Pareto optimal if and only if there is no another $\mathbf{x}$ in D such that $f_i(\mathbf{x}) \leq f_i(\mathbf{x}_0)$ for all $i = \{1, 2, \ldots, n\}$ and $f_i(\mathbf{x}) < f_i(\mathbf{x}_0)$ for at least one i.

An important task for multi-objective optimization is to identify Pareto set points. The question is how to judge a point in the Pareto set. To solve this problem, the fitness function is employed as

$$G(\mathbf{x}_i) = \left[1 - \max_{i \neq j}\left(\min\left(f_1^i - f_1^j, f_2^i - f_2^j, \ldots, f_n^i - f_n^j\right)\right)\right] \tag{6}$$

The objectives in equation (6) should be scaled to a range [0 1], given in equation (7)

$$f_i(\mathbf{x}) = \frac{f_i(\mathbf{x}) - f_{i,\min}(\mathbf{x})}{f_{i,\max}(\mathbf{x}) - f_{i,\min}(\mathbf{x})} \tag{7}$$

Pareto set at system level

For a design problem at system level, to coordinate two subsystems and one common variable, the objective function is given by

$$\begin{aligned} F_{sys} = f_{sys} & + w_1(x_1 - x_1^{sub})^2 + w_2(x_2 - x_2^{sub})^2 \\ & + w_3(y_1 - y_1^{sub})^2 + w_4(y_1 - y_2^{sub})^2 \end{aligned} \tag{8}$$

We assume that there are m solutions at the system level, and the deviation function is defined as $d_{sub,i}^{sys,j} = (x_i - x_{sub,i}^{sub})^2, j = 1, 2, \ldots, m$, the Pareto set at the system level of the kth iteration is denoted as matrix $\mathbf{d}^{sys,k}$

$$\mathbf{d}^{sys,k} = \begin{bmatrix} d_{sub,1}^{sys,1} & d_{sub,2}^{sys,1} & d_{y1}^{sys,1} & d_{y2}^{sys,1} \\ d_{sub,1}^{sys,2} & d_{sub,2}^{sys,2} & d_{y1}^{sys,2} & d_{y2}^{sys,2} \\ \vdots & \vdots & \vdots & \vdots \\ d_{sub,1}^{sys,m} & d_{sub,2}^{sys,m} & d_{y1}^{sys,m} & d_{y2}^{sys,m} \end{bmatrix}$$

Pareto set at subsystem level

For design problems at the subsystem level, the objective function is

$$F_{sub} = w_i(x_i - x_i^{sys})^2 + \mathbf{w}_y(\mathbf{y}_i - \mathbf{y}_i^{sys})^2 \tag{9}$$

If there are l solution at the ith subsystem level, and the deviation function is defined as $d_{sub,i}^{sub,j} = (x_i - x_{sub,i}^{sys})^2$, $j = 1, 2, \ldots, l$, the Pareto set in the ith subsystem of the kth iteration is denoted as matrix $\mathbf{d}_{sub,i}^{sub,k}$

$$\mathbf{d}_{sub,i}^{sub,k} = \begin{bmatrix} d_{sub,i}^{sub,1} & \mathbf{d}_{y,i}^{sub,1} \\ d_{sub,i}^{sub,2} & \mathbf{d}_{y,i}^{sub,2} \\ \vdots & \vdots \\ d_{sub,i}^{sub,l} & \mathbf{d}_{y,i}^{sub,l} \end{bmatrix}$$

Solution process

The multi-objective optimization models at the system level and subsystem levels are presented in equations (10) and (11)

$$\begin{aligned} &\min: f_{sys} \\ &\min: (x_{sub,i} - x_{sub,i}^{sub})^2 \\ &\min: (\mathbf{y}_{sub,i} - \mathbf{y}_{sub,i}^{sub})^2 \\ &\text{s.t. } \mathbf{g}_{sys}(\mathbf{x}, \mathbf{z}) \le 0 \\ &\qquad \mathbf{h}_{sys}(\mathbf{x}, \mathbf{z}) = 0 \\ &\qquad f_{sys} = f_{sys}(\mathbf{x}, \mathbf{z}) \\ &\qquad i = 1, 2, \ldots, n \end{aligned} \tag{10}$$

$$\begin{aligned} &\min: (x_{sub,i} - x_{sub,i}^{sys})^2 \\ &\min: (\mathbf{y}_{sub,i} - \mathbf{y}_{sub,i}^{sys})^2 \\ &\text{s.t. } \mathbf{g}_{sub,i}(x_{sub,i}, \mathbf{y}_{sub,i}, \mathbf{u}_i) \le 0 \\ &\qquad \mathbf{h}_{sub,i}(x_{sub,i}, \mathbf{y}_{sub,i}, \mathbf{u}_i) = 0 \end{aligned} \tag{11}$$

Numerical procedure

Figure 2 shows the numerical procedure of the proposed method.

The procedure of Pareto set pursuing in multilevel multi-objective design optimization problem is illustrated as follows. A well-known ATC problem is taken from the literature (Kim et al., 2006) to facilitate the explanation of the procedures. The AIO formulation of this geometric problem is expressed by equation (12)

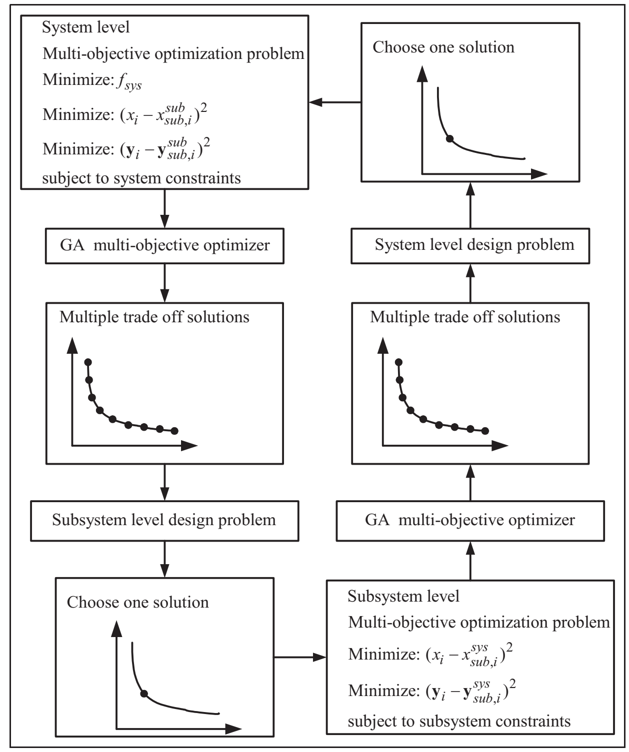

Figure 2. Solution process in system and subsystem. GA: genetic algorithm.

$$\begin{aligned} &\min: x_1^2 + x_2^2 \\ &\text{s.t. } g_1 = (x_3^{-2} + x_4^2) \times x_5^{-2} - 1 \le 0 \\ &\qquad g_2 = (x_6^{-2} + x_5^2) \times x_7^{-2} - 1 \le 0 \\ &\qquad g_3 = (x_8^2 + x_9^{-2}) \times x_{11}^{-2} - 1 \le 0 \\ &\qquad g_4 = (x_{10}^2 + x_8^{-2}) \times x_{11}^{-2} - 1 \le 0 \\ &\qquad g_5 = (x_{11}^2 + x_{12}^{-2}) \times x_{13}^{-2} - 1 \le 0 \\ &\qquad g_6 = (x_{11}^2 + x_{12}^2) \times x_{14}^{-2} - 1 \le 0 \\ &\qquad h_1 = x_1 - (x_3^2 + x_4^{-2} + x_5^2)^{1/2} = 0 \\ &\qquad h_2 = x_2 - (x_6^2 + x_7^2 + x_5^2)^{1/2} = 0 \\ &\qquad h_3 = x_3 - (x_8^2 + x_9^{-2} + x_{10}^{-2} + x_{11}^2)^{1/2} = 0 \\ &\qquad h_4 = x_6 - (x_{12}^2 + x_{13}^2 + x_{14}^2 + x_{11}^2)^{1/2} = 0 \\ &\qquad x_1, x_2, \ldots, x_{14} \ge 0 \end{aligned} \tag{12}$$

The geometric problem is decomposed into two-level structure, with constraints g_1, g_2, h_1, and h_2 in the system level; constraints g_3, g_4, and h_3 in subsystem 1; while constraints g_5, g_6, and h_4 in subsystem 2, which are shown in equations (13), (14), and (15), respectively

$$\begin{aligned}\min:\ & [x_1^2 + x_2^2, (x_3 - x_3^{sub})^2, (x_6 - x_6^{sub})^2 \\ & (x_{11} - x_{11}^{sub,1})^2, (x_{11} - x_{11}^{sub,2})^2] \\ \text{s.t. } & g_1 = (x_3^{-2} + x_4^2) \times x_5^{-2} - 1 \le 0 \\ & g_2 = (x_6^{-2} + x_5^2) \times x_7^{-2} - 1 \le 0 \\ & h_1 = x_1 - (x_3^2 + x_4^{-2} + x_5^2)^{1/2} = 0 \\ & h_2 = x_2 - (x_6^2 + x_7^2 + x_5^2)^{1/2} = 0 \\ & x_1, x_2, \ldots, x_7, x_{11} \ge 0\end{aligned} \tag{13}$$

$$\begin{aligned}\min:\ & [(x_3 - x_3^{sys})^2, (x_{11} - x_{11}^{sys})^2] \\ \text{s.t. } & g_3 = (x_8^2 + x_9^{-2}) \times x_{11}^{-2} - 1 \le 0 \\ & g_4 = (x_{10}^2 + x_8^{-2}) \times x_{11}^{-2} - 1 \le 0 \\ & h_3 = x_3 - (x_8^2 + x_9^{-2} + x_{10}^{-2} + x_{11}^2)^{1/2} = 0 \\ & x_3, x_8, x_9, x_{10}, x_{11} \ge 0\end{aligned} \tag{14}$$

$$\begin{aligned}\min:\ & [(x_6 - x_3^{sys})^2, (x_{11} - x_{11}^{sys})^2] \\ \text{s.t. } & g_5 = (x_{11}^2 + x_{12}^{-2}) \times x_{13}^{-2} - 1 \le 0 \\ & g_6 = (x_{11}^2 + x_{12}^2) \times x_{14}^{-2} - 1 \le 0 \\ & h_4 = x_6 - (x_{12}^2 + x_{13}^2 + x_{14}^2 + x_{11}^2)^{1/2} = 0 \\ & x_6, x_{11}, x_{12}, x_{13}, x_{14} \ge 0\end{aligned} \tag{15}$$

Step 1. Initial random sampling in the system level design problem

First, generate 50 sampling points in system level satisfying the constraints and calculate the fitness for each sampling point. Second, crossover and mutation operations are used to the sampling points to get the optimal design points and assign the optimal design points as targets to subsystems. The optimal solution set is shown in Figure 3.

From Figure 3, we can find that system's performance is not monotonic with the change of deviation with the response from subsystems. Based on the optimal solution set, designer can choose one point as target set to subsystems.

Step 2. Optimization design in subsystem level

Based on the targets from system level, generate sampling points satisfying constraints in subsystems and calculate the fitness function value of subsystems. Then, using crossover and mutation operations, the optimal design points in subsystems can be obtained and propagated to system level as response. The solution set of subsystems are presented in Figure 4.

Step 3. Repeat Step 1 and Step 2 until the deviation between targets and response can be accepted.

Reliability-based design optimization of a cylindrical gear reducer

The design optimization of a two-stage helical cylindrical gear reducer is employed to demonstrate the proposed method. The design objective is to minimize the speed reducer weight under the satisfaction of

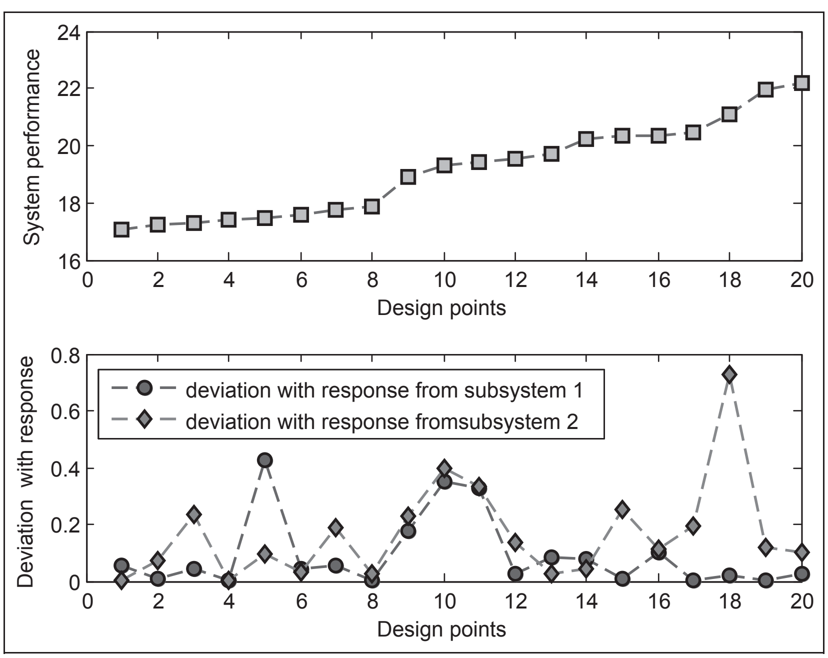

Figure 3. The optimal solution set in system level.

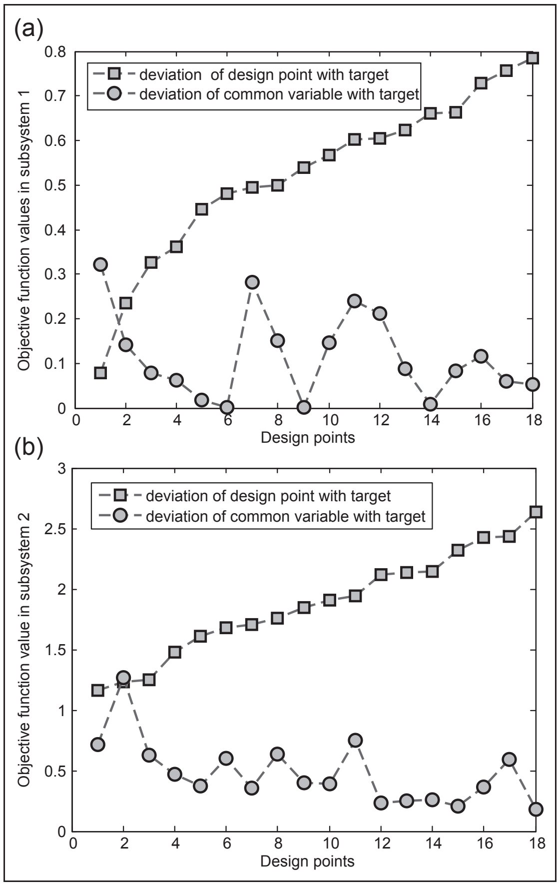

Figure 4. Optimal solution set in subsystems.

reliability constraints. The design variables are teeth module, number of pinion teeth, face width, and helix angle both in high-speed level and low-speed level. The transmission ratio is also taken as design variable. The information of design variables is listed in Table 1.

In the target cascading process of the proposed method, the sensitivity of the system to each subsystem can also be calculated. The sensitivity provides information for the reliability allocation and system reliability design. The Pareto set targets are propagated though the multilevel system, which provides more choices and degree of flexibility to designers.

1. Weight of the speed reducer

$$\min: f(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9) = \frac{\pi}{4} \times x_7 \left[\left(\frac{x_3 x_1}{\cos x_5} \right)^2 + \left(\frac{x_3 x_1 x_9}{\cos x_5} \right)^2 \right] + \frac{\pi}{4} \times x_8 \left[\left(\frac{x_4 x_2}{\cos x_6} \right)^2 + \left(\frac{x_4 x_2 i}{\cos x_6 \times x_9} \right)^2 \right] \tag{16}$$

Table 1. The design variables of the speed reducer.

Design variables	Description	
	Physical meaning	Sign
z_1	Number of pinion teeth in high-speed level	x_1
z_3	Number of pinion teeth in low-speed level	x_2
mn_{12}	Teeth module in high-speed level	x_3
mn_{34}	Teeth module in low-speed level	x_4
β_{12}	Helix angle in high-speed level	x_5
β_{34}	Helix angle in low-speed level	x_6
b_1 (mm)	Face width in high-speed level	x_7
b_2 (mm)	Face width in low-speed level	x_8
i_{12}	Transmission ratio	x_9

where i is the total transmission ratio.

2. Design constraints

$$\begin{aligned}
&g_1 = 17 - x_1 \le 0, g_2 = 17 - x_2 \le 0\\
&g_3 = 7.5° \times \frac{\pi}{180°} - x_5 \le 0\\
&g_4 = x_5 - 14.16° \times \frac{\pi}{180°} \le 0\\
&g_5 = 7.5° \times \frac{\pi}{180°} - x_6 \le 0\\
&g_6 = x_6 - 14.16° \times \frac{\pi}{180°} \le 0\\
&g_7 = 1.18i - x_{12}^2 \le 0, g_8 = x_{12}^2 - 1.62i \le 0\\
&g_9 = 0.98 - \phi_{d12} \le 0, g_{10} = \phi_{d12} - 1.77 \le 0\\
&g_{11} = 0.7 - \phi_{d34} \le 0, g_{12} = \phi_{d34} - 1.15 \le 0\\
&g_{13} = 2 - x_3 \le 0, g_{14} = 2 - x_4 \le 0\\
&g_{15} = 1 - \varepsilon_{\beta 12} \le 0, g_{16} = 1 - \varepsilon_{\beta 34} \le 0\\
&g_{17} = d_2 + \frac{(2 \times 1)x_3}{\cos x_5} + 20 - (d_3 + d_4) \le 0\\
&g_{18} = 2.30274 - u_{RH2} \le 0\\
&g_{19} = 2.30274 - u_{RH4} \le 0\\
&g_{20} = 2.30274 - u_{RF1} \le 0\\
&g_{21} = 2.30274 - u_{RF2} \le 0\\
&g_{22} = 2.30274 - u_{RF3} \le 0\\
&g_{23} = 2.30274 - u_{RF4} \le 0
\end{aligned} \tag{17}$$

where $u_{RH} = \frac{\overline{\sigma}_{HS} - \overline{\sigma}_H}{(s_{\sigma_{HS}}^2 + s_{\sigma_H}^2)^{1/2}}$ and $u_{RF} = \frac{\overline{\sigma}_{FS} - \overline{\sigma}_F}{(s_{\sigma_{FS}}^2 + s_{\sigma_F}^2)^{1/2}}$.

herein, $\overline{\sigma}_H$ and $\overline{\sigma}_{HS}$ denote the mean value of contact fatigue stress and contact fatigue strength, respectively; S_{σ_H} and $S_{\sigma_{HS}}$ denote the standard deviation of contact fatigue stress and contact fatigue strength, respectively; $\overline{\sigma}_F$ and $\overline{\sigma}_{FS}$ denote the mean value of bending stress and bending strength, respectively; and S_{σ_F} and $S_{\sigma_{FS}}$

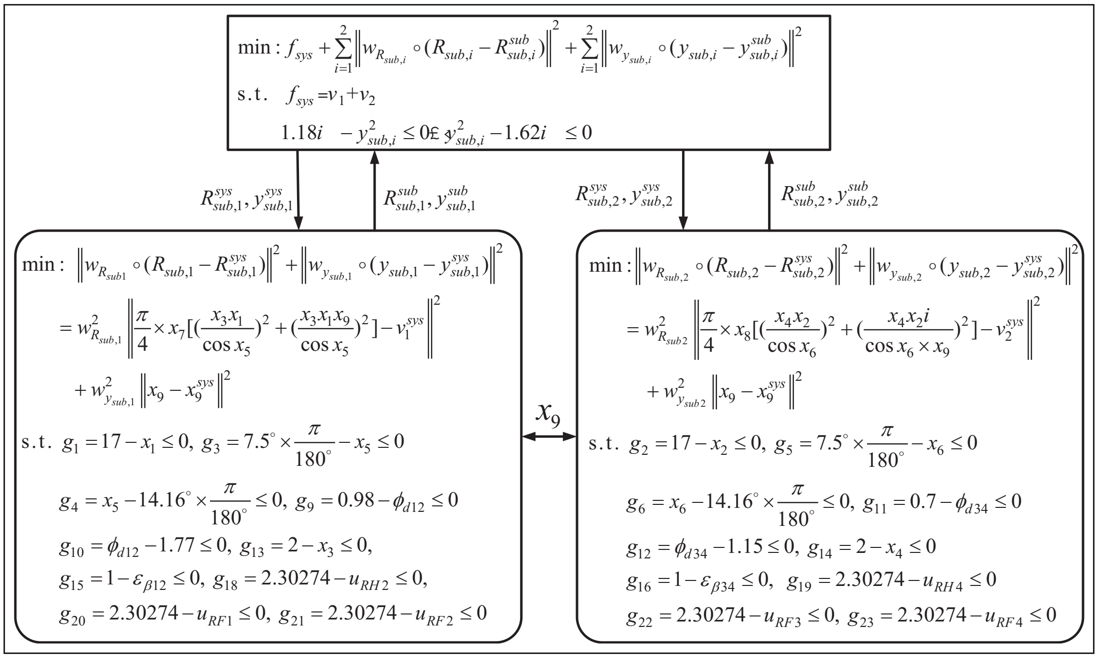

Figure 5. The hierarchically decomposed structure for reducer design.

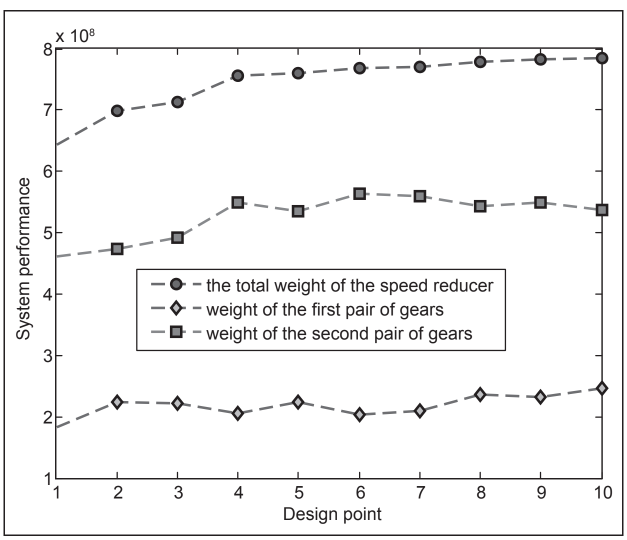

Figure 6. System's performance with different design points.

denote the standard deviation of bending stress and bending strength, respectively.

The design optimization model is decomposed into two hierarchical structures as shown in Figure 5. The system design problem is considered as the top level and two subsystems as the lower level. The transmission ratio is the common variable of the two subsystems.

Transmission ratio is a common variable between the two subsystems, and different transmission ratio will give different design point as presented in Figure 6. One optimal solution from Pareto set using Pareto set coordination with GA optimizer is shown in Table 2, and the solutions from AIO and general ATC with different weights are also provided in Table 2 for comparison.

The results illustrate that the general ATC method with different weights can lead to different solutions, and improper weighting coefficient may need large number of function evaluations (FE) to converge. Pareto set coordination method with GA optimizer only needs 4676 times for FE and can provide more detailed information such as the optimal results with different weighting coefficients for each design problem in the hierarchical structure, which is better for the information propagation.

Conclusion

Design optimization model of a complex engineering system is usually decomposed into hierarchical structures with one system level and multiple subsystem levels. The main task in the decomposed design optimization algorithm is to minimize the cost and diminish the discrepancy between subsystems simultaneously. In order to maintain the consistency between subsystems, the most commonly used ATC method formulates the multi-objective model in terms of the weighted discrepancy functions. The choice of the weighting coefficients of ATC method is problem dependent, and improper selections of the weighting coefficients may lead to incorrect solutions.

Table 2. Optimal design solution of reducer design.

Design variables	Optimization method				
	ATC[a]	ATC[b]	ATC[c]	AIO	PS-ATC
z_1	17	16	16	16	16
z_3	16	17	17	16	16
mn_{12}	10	10	10	10	10
mn_{34}	20	20	20	20	20
β_{12}	12.59	13.65	14.15	14.16	14.16
β_{34}	14.16	14.16	14.16	14.16	14.16
b_1 (mm)	185	190	186	162	165
b_2 (mm)	270	280	280	260	260
i_{12}	7.02	7.12	7.096	7.125	7.125
Objective	6.824×10^8	7.2853×10^8	7.3089×10^8	6.634×10^8	6.634×10^8
FE	77,807	29,554	306,573	–	4676

FE: function evaluations; ATC: analytical target cascading; AIO: all-in-one; PS-ATC: Pareto Set - ATC.
[a]$w = 1000$.
[b]$w = 2^k$.
[c]$w = 5^k$.

In this article, Pareto set coordination method with GA optimizer is proposed to solve the multi-objective design optimization problem. The method avoids setting weighting coefficients for each objective, which is required in the general ATC method. Pareto sets are obtained from the optimal solutions at each level, and each level can choose one solution from the Pareto set as targets. The Pareto set from each subsystem can provide more information to the system and let the system better understand the performance of a subsystem. The example of gear reducer design confirms that the Pareto set coordination method can efficiently and accurately find the optimal solutions.

Declaration of conflicting interests

The authors declare that there is no conflict of interest.

Funding

This research was partially supported by the National Natural Science Foundation of China under the contract number 51075061, the Research Fund for the Doctoral Program of Higher Education of China (New Faculty) under the contract number 20100185120029, and the Fundamental Research Funds for the Central Universities under the contract numbers ZYGX2010J093 and ZYGX2012J098.

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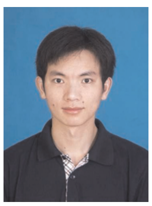

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Appendix I

Notation

f_{sys}	cost function in the system level
$\mathbf{g}_{sub,i}$	inequality constraints in the ith subsystem
$\mathbf{h}_{sub,i}$	equality constraints in the ith subsystem
n	number of subsystems
$\mathbf{w}_x, \mathbf{w}_y$	weighting coefficients in analytical target cascading
$\mathbf{x}_i$	variables in the ith subsystem coupling with one level below
$\mathbf{y}_i$	common variables in the ith subsystem coupling with other subsystem at the same level
$\mathbf{z}_i$	local variables in the system level

Superscript

k	design point in the kth iteration
sub	subsystem level
sys	system level

Subscript

sub	subsystem level
sys	system level