

# An improved Corten-Dolan's model based on damage and stress state effects<sup>†</sup>

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## Abstract

The value of exponent  $d$  in Corten-Dolan's model is generally considered to be a constant. Nonetheless, the results predicted on the basis of this statement deviate significantly from the real values. In consideration of the effects of damage and stress state on fatigue life prediction, Corten-Dolan's model is improved by redefining the exponent  $d$  used in the traditional model. The improved model performs better than the traditional one with respect to the demonstration of a fatigue failure mechanism. Predictions of fatigue life on the basis of investigations into three metallic specimens indicate that the errors caused by the improved model are significantly smaller than those induced by the traditional model. Meanwhile, predictions derived according to the improved model fall into a narrower dispersion zone than those made as per Miner's rule and the traditional model. This finding suggests that the proposed model improves the life prediction accuracy of the other two models. The predictions obtained using the improved Corten-Dolan's model differ slightly from those derived according to a model proposed in previous literature; a few life predictions obtained on the basis of the former are more accurate than those derived according to the latter. Therefore, the improved model proposed in this paper is proven to be rational and reliable given the proven validity of the existing model. Therefore, the improved model can be feasibly and credibly applied to damage accumulation and fatigue life prediction to some extent.

*Keywords:* Corten-Dolan's model; Fatigue; Life prediction; Damage accumulation; Damage state; Stress state

## 1. Introduction

Fatigue is a main failure mode of structures used in the fields of aviation, mining, automobile, and marine equipment, among others. Fatigue damage is a process by which material properties deteriorate continuously under the action of cyclic loading stress. Thus, fatigue damage is related to external loads. That is, damage variables are influenced by stress or strain measurements [1-3]. The process of fatigue fracture is very complicated; once fatigue damage accumulates to a certain critical value, fatigue failure occurs. Thus the accumulation of fatigue damage is a key factor in predicting the lifespan of materials subjected to fatigue loading. An appropriate method must be designed to predict fatigue life and to assess fatigue damage accumulation accurately.

Numerous models have been developed since the proposition of fatigue damage accumulation theory. These models can be classified into two types: linear and nonlinear. An example of a linear damage accumulation model is Palmgren-Miner rule, or Miner's rule for short. This rule was proposed to predict the fatigue life of materials on the basis of the following assumptions [4-6]:

(1) A material absorbs the same amount of net work over each loading cycle under the arbitrary constant amplitude loading of fatigue. Thus, the rate of fatigue damage accumulation remains constant in each loading cycle.

(2) Loading stress that is less than the fatigue limit of a material cannot generate any form of fatigue damage. Fatigue damage is initialized and accumulated only when the material is subjected to a loading stress that is equal to or higher than its fatigue limit.

(3) The amounts of net work absorbed by a material are independent of one another under different loading stress levels. In other words, fatigue damage accumulation is unrelated to the loading sequences and the interactions among various loadings.

On the basis of the assumptions above, Miner [7] predicted fatigue damage accumulation under variable amplitude loading using the equation:

$$D = \sum_{i=1}^k \frac{n_i}{N_i}, \quad (1)$$

where  $D$  is the total fatigue damage.  $n_i$  and  $N_i$  are the number of loading cycles and fatigue life at the  $i$ -th loading

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$\sigma_i$ , respectively. The subscript  $i$  denotes the loading sequence number.

Fatigue failure occurs when cumulative fatigue damage  $D$  reaches a certain critical value. Miner's rule supposes that this critical value is equal to unity, which indicates that materials absorb the same amount of net work under constant or variable amplitude loading. The major advantage of Miner's rule is that this method is uncomplicated and is suitable for predicting the lifespans of many structures. However, problems are still encountered under the basic assumptions of Miner's rule. For instance, researchers have proven that low-amplitude loading, including the loading that is lower than the fatigue limit of the material, contributes to fatigue damage in variable amplitude loading [8-11] because high-amplitude loading can induce cracks. However, cracks can also be propagated by low amplitude loading. Miner's rule assumes that loading sequences and load interaction do not affect fatigue damage accumulation. By contrast, these two factors contribute significantly to fatigue life prediction [4, 12]. Thus, predictions made using Miner's rule are not always satisfactory; in some cases, they are considerably scattered and inaccurate.

Nonlinear accumulation theories and their corresponding models were developed to overcome the problems of Miner's rule and in consideration of the characteristics of fatigue damage. These models include damage curve-based models [12, 13], models for continuum damage mechanics [6, 14], models that consider load interactions [13, 15], energy-based damage methods [16-18], models based on physical property degradation [19, 20], ductility exhaustion methods [21, 22], and thermodynamic entropy-based theories [23-25].

Among these linear and nonlinear models, Miner's rule and Corten-Dolan's model are widely used to predict fatigue damage [26, 27]. The contribution of low-amplitude loading and the effects of load interactions are considered in Corten-Dolan's model; thus, many researchers utilize and recommend this model in fatigue life prediction [28-34]. However, the predictions made using Corten-Dolan's model cannot always satisfy engineering requirements [15, 35]. Therefore, this model must be improved.

At present, many new findings have been reported in relation to Corten-Dolan's model. Huang et al. [35] imported a fatigue strength degradation coefficient into the original model in the form of an exponent, verified the accuracy of this modification, and applied the modified model in the reliability analysis of fatigue life based on strength degradation theory. To investigate the effect of small loads on fatigue property, Zhou et al. [36] introduced a strengthening factor into Corten-Dolan's model. These researchers reported that prediction accuracy is enhanced in the modified model.

Nonetheless, the prediction performance of Corten-Dolan's model depends strongly on the accuracy of the critical exponent  $d$  used in the model. If the critical parameter cannot be calculated accurately, then the prediction made using the model is highly erroneous. Although Corten and Dolan suggested that this exponent is a constant and can be determined

through program tests under two-stress level loading, the process of obtaining the optimal exponent remains questionable. Chen [31] pointed out that exponent  $d$  is not a simple material constant. Moreover, this exponent is related not only to material property but also to the levels of loading spectrum. Thus, a fatigue test of two-level loading must be performed with different parameters. Moreover, Chen conducted block fatigue tests of two-stress level loading and proposed a method to determine appropriate values for exponent  $d$ . However, researchers realized that the program tests under two-stress level loading blocks differ considerably from those under random loading spectrums. The value of  $d$  is not always constant; rather, it decreases with an increase in loading stress. Thus, improved methods have been established as well. Feng [37] calculated the value of the exponent by applying active load spectra directly. This process improved estimation accuracy. Marciniak et al. [38] used Corten-Dolan's model to predict fatigue life under non-proportional loading and determined that predictions were overestimated as a result of the uncertainty of  $d$ . Zhu et al. [15] proposed a practical method to determine  $d$  by defining this variable as a function of loading stress amplitude to facilitate the consideration of load interaction effects.

According to our literature review, the method of determining exponent  $d$  is a key problem in the traditional Corten-Dolan's model. In line with this research direction, the current study investigates this issue further. Damage and stress state directly affect the accuracy of fatigue life predictions; therefore, these effects should be taken into account in Corten-Dolan's model. Otherwise, the predicted lifespans may differ significantly from the actual ones. Therefore, the traditional Corten-Dolan's model is improved by redefining exponent  $d$  to describe the effects of damage and stress state. The accuracy and reliability of the improved model are verified by experimental data. Furthermore, the prediction performance of this model is compared with that under Miner's rule, traditional Corten-Dolan's model, and an existing model. The results demonstrate that the improved model proposed in this paper can be used to assess cumulative damage and fatigue life.

## 2. Corten-Dolan damage accumulation theory

According to Corten-Dolan accumulation theory [39], the fatigue damage  $D$  following constant amplitude loading for  $n$  cycles is calculated through

$$D = mrn^a, \quad (2)$$

where  $a$  is a constant;  $m$  is the number of damage nuclei generated from the loading;  $r$  and  $n$  denote the damage coefficient and number of loading cycles, respectively.

Critical fatigue damage is computed with

$$D_c = m_1 r_1 N_1^{a_1}, \quad (3)$$

where  $D_c$  is the critical fatigue damage;  $a_1$  is a constant;  $m_1$  is the number of damage nuclei;  $r_1$  is damage coefficient; and  $N_1$  is the fatigue life that corresponds to the constant loading (Constant amplitude loading) or the maximum cyclic loading among the loading series applied to a specimen (Variable amplitude loading).

Subsequently, fatigue life is derived under variable amplitude loading. First, we discuss a case under the two-stress level loading block.

According to the statements reported in Ref. [40], the development of fatigue damage under two-stress level loading block is plotted by a thick solid line in Fig. 1. This development involves the alternating action of  $\sigma_1$  and  $\sigma_2$  ( $\sigma_1 > \sigma_2$ ). In the loading process, the sum of damage increments should be equal to the damage incurred when fatigue failure occurs under single constant amplitude loading, such as  $\sigma_1$ . In accordance with this theory, Corten and Dolan simplified parameters under the assumption  $a_1 = a_2 = a$  and combined the experimental results of cold-drawing wire specimens. Consequently, estimated life  $N$  under the two-stress level loading block can be obtained as follows:

$$N = \frac{N_1}{\alpha + (1 - \alpha) \left( \frac{\sigma_2}{\sigma_1} \right)^d}, \tag{4}$$

where  $\alpha$  is the ratio of the cyclic number of loading stress  $\sigma_1$  to that of the entire loading block. Eq. (4) illustrates Corten-Dolan's damage accumulation theory under a two-stress level loading block.

Under  $k$  levels of variable amplitude loading, the fatigue cumulative damage is given as follows:

$$D = \sum_{i=1}^k m_i r_i n_i^{a_i} = m_1 r_1 N_1^{a_1}, \tag{5}$$

where  $n_i$  is the number of loading cycles under  $\sigma_i$ .

Once fatigue nuclei are generated, they do not disappear but increase in number during the latter process of fatigue loading. Therefore,  $m_i = m_1$ . On the basis of the assumption  $a_i = a_1 = a$ , the ratio of experimental life and model prediction can be obtained by:

$$\frac{\sum n_p}{N_f} = \sum_{i=1}^k \frac{n_i}{N_1} (r_i/r_1)^{1/a} = 1. \tag{6}$$

The expression  $(r_i/r_1)^{1/a}$  is related to loading ratio  $(r_i/r_1)^{1/a} = (\sigma_i/\sigma_{max})^d$ . Thus, Eq. (6) is transformed into the following equation:

$$\frac{\sum n_p}{N_f} = \sum_{i=1}^k \frac{n_i}{N_1} (\sigma_i/\sigma_{max})^d = 1, \tag{7}$$

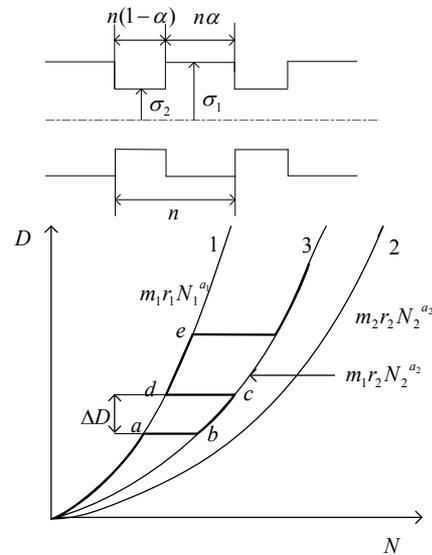


Fig. 1. Corten—Dolan damage-cycle curve under a two-stress level loading block.

where  $d$  is a material parameter whose value is determined through testing.

The expression above is an assumed criterion for fatigue failure analysis that has been widely used in fatigue life prediction [15, 40, 41].  $\sum n_p/N_f$  in Eq. (7) represents the ratio of practical fatigue life to the total number of cyclic predictions made when fatigue failure occurs. Thus, fatigue life is given as follows under a multi-level loading block condition:

$$N_f = \frac{N_1}{\sum_i \alpha_i (\sigma_i/\sigma_{max})^d}, \tag{8}$$

where  $N_f$  is the total cyclic number under multi-level loading blocks in the event of fatigue failure.  $N_1$  is the fatigue life of materials under maximum cyclic loading stress. Subscripts  $p$  and  $i$  represent the sequence number of the loading block and cyclic loading stress, respectively. Thus,  $\sum n_i = n_p$ . In addition,  $\sigma_{max}$  indicates the maximum stress of multi-level loading blocks;  $\alpha_i$  is the percentage of cyclic stress  $\sigma_i$ ; and  $d$  represents a material parameter derived from the experimental data.

Eq. (8) shows that parameter  $d$  is a key factor in enhancing the accuracy of fatigue life predictions if the value of this parameter can be determined appropriately [31]. On the basis of fatigue experimental data, Corten and Dolan [39] recommended that the value of the exponent parameter be determined as follows:

$$d = \begin{cases} 4.8 & \text{for high-strength steel,} \\ 5.8 & \text{for others.} \end{cases} \tag{9}$$

### 3. Improved Corten-Dolan's model based on damage and stress state effects

As shown in Eqs. (7)–(9), the effects of load interaction and low-amplitude loading are considered in Corten-Dolan's model, thus indicating a major advantage over Miner's rule. However, fatigue failure is a complicated process that is strongly associated with fatigue damage and applied loading stress [42]. In other words, fatigue damage accumulation and fatigue life are closely related to the damage and stress state effects of the specimens. Consequently, an improved Corten-Dolan's model is proposed in this paper to improve the accuracy of fatigue life prediction on the basis of the aforementioned viewpoints.

As per Eqs. (7) and (8), exponent  $d$  in Corten—Dolan's model is a unique material parameter. Corten and Dolan [39] suggest that this parameter is considered an empirical value as in Eq. (9). This value is fixed regardless of loading condition. However, researchers point out that the value of  $d$  cannot be constant for a given material. In reality, this value changes significantly with the load spectra [4, 15, 20, 31]. Moreover, fatigue failure is a dynamic process, as per the result of Ref. [42]. This failure is generated through the action of loading stress under a certain damage state. If the application of fatigue loads is stopped, damage is unchanged. Hence, specimen failure depends on the existing condition of damage and stress. If the damage is minor, then failure is induced only when the load is adequately great; if the damage is significant, then minimal stress initiates fatigue fracture. Therefore, fatigue failure not only depends on load spectra, but also on existing damage. On the basis of the research results and reviews above, the following assumption can be reasonably made: if parameter  $d$  can consider the effects of damage and stress state simultaneously, then fatigue failure can be effectively described by Corten-Dolan's model. Therefore, a novel method is introduced to determine the appropriate value of  $d$  in this model.

At present, relevant models can be used as references even though the relationship between the effects of damage and stress state and cumulative fatigue damage has not been ascertained. A nonlinear model was proposed in Ref. [20] on the basis of a two-parameter fatigue criterion. This criterion advocated that fatigue failure is influenced by the combined effect of the two parameters, including the degree of damage and stress conditions. This influence is established by defining an exponential function of the cyclic and loading ratios. Therefore, the form of the nonlinear model in Ref. [20] is employed in the current study. The value of exponent  $d$  in Corten-Dolan's model increases as fatigue loading decreases [4]. The value of  $d$  is assumed to be a function of cycle ratio ( $n_i/N_i$ ) and loading ratio ( $\sigma_i/\sigma_{max}$ ), as shown in Eq. (10).

$$d = \exp \left[ \left( \frac{n_i}{N_i} \right)^{\sigma_i/\sigma_{max}} \right] + \gamma. \quad (10)$$

Substituting Eq. (10) into Eq. (8) can modify Corten-Dolan's model as follows:

$$N = \frac{N_{max}}{\sum_i \alpha_i \left( \sigma_i/\sigma_{max} \right)^{\exp((n_i/N_i)\sigma_i/\sigma_{max}) + \gamma}}, \quad (11)$$

where  $N_{max}$  is the fatigue life under  $\sigma_{max}$ ,  $N_i$  is the cyclic number under the action of  $\sigma_i$  until fatigue failure and  $\gamma$  is a material constant that can be derived from experimental data and the criterion of fatigue failure. If experimental data are limited, then the corresponding equations based on the failure criterion can be obtained. The value of  $\gamma$  is determined through fitting analysis.

Eq. (11) is the formula for the improved Corten-Dolan's model proposed in this paper. In this model, damage state effects are considered by applying the ratio of the load cycle number to fatigue life under the  $i$ -th loading stress  $\sigma_i$ . Stress state effects can be observed from the ratio of  $i$ -th loading stress  $\sigma_i$  to the maximum load spectrum  $\sigma_{max}$ . Eq. (10) shows that these two ratios are important in determining the value of exponent  $d$ . For example, the value of  $d$  decreases with an increase in loading stress  $\sigma_i$  under a certain damage state because the expression  $n_i/N_i$  is less than or equal to unity. Then, fatigue life  $N$  is influenced by the variation in  $d$ . The tendency of the value of  $d$  to change with the applied load spectrum conforms strongly to the proposition in Ref. [4].

Substituting Eq. (10) into Eq. (7) yields the second expression of the ratio of the experimental life to the prediction of Corten—Dolan's model:

$$\frac{\sum n_p}{N_f} = \sum_{i=1}^k \frac{n_i}{N_{max}} \left( \sigma_i/\sigma_{max} \right)^{\exp((n_i/N_i)\sigma_i/\sigma_{max}) + \gamma}. \quad (12)$$

Corten-Dolan's model is thus modified using the proposed method to determine the value of exponent  $d$ , as in Eqs. (10) and (11). The expression of  $d$  considers the effects of both damage and stress state through ratios  $n_i/N_i$  and  $\sigma_i/\sigma_{max}$ . These effects are reflected in the fatigue life prediction model.

## 4. Experiments and analyses

### 4.1 Case study 1

The accuracy of the improved Corten-Dolan's model that considers the effects of damage and stress state in materials is evaluated in this section. Three sets of experimental data on normalized 45 steel, normalized 16 Mn steel, and hot-rolled 16Mn steel were used to predict fatigue life. Tests were conducted under two-stress level loading. Seven, five, and eight different loading modes were implemented for the normalized

Table 1. Experimental data and the predictions of Miner’s rule, Corten-Dolan’s model and the improved model for normalized 45 steel [3, 43].

Loading mode	$\sigma_1$ /MPa	$\sigma_2$ /MPa	$n_1$	$\frac{n_1}{N_{f1}}$	$n_2$	$\frac{n_2}{N_{f2}}$	$\left(\frac{\sum n_p}{N}\right)_M$	$\left(\frac{\sum n_p}{N}\right)_T$	$\left(\frac{\sum n_p}{N}\right)'$
Mode I	331.46	284.4	500	0.0100	423,700	0.8474	0.8574	3.4964	0.6776
Mode II	331.46	284.4	12,500	0.2500	250,400	0.5008	0.7508	2.3104	0.6854
Mode III	331.46	284.4	25,000	0.5000	168,300	0.3366	0.8366	1.8849	0.8043
Mode IV	331.46	284.4	37,500	0.7500	64,500	0.1290	0.8790	1.2807	0.8720
Mode V	284.4	331.46	125,000	0.2500	37,900	0.7580	1.0080	1.7866	0.9884
Mode VI	284.4	331.46	250,000	0.5000	38,900	0.7780	1.2780	2.8351	1.2128
Mode VII	284.4	331.46	375,000	0.7500	43,400	0.8680	1.6180	3.9537	1.4769

Table 2. Experimental data and the predictions of Miner’s rule, Corten-Dolan’s model and the improved model for normalized 16 Mn steel [3, 43].

Loading mode	$\sigma_1$ /MPa	$\sigma_2$ /MPa	$n_1$	$\frac{n_1}{N_{f1}}$	$n_2$	$\frac{n_2}{N_{f2}}$	$\left(\frac{\sum n_p}{N}\right)_M$	$\left(\frac{\sum n_p}{N}\right)_T$	$\left(\frac{\sum n_p}{N}\right)'$
Mode I	562.9	392.3	2	0.0005	73,600	0.9352	0.9357	2.2850	0.8825
Mode II	562.9	392.3	200	0.0504	59,400	0.7548	0.8052	1.8941	0.8500
Mode III	562.9	392.3	1,000	0.2520	56,300	0.7154	0.9674	1.9995	1.0286
Mode IV	562.9	392.3	1,700	0.4284	47,600	0.6048	1.0332	1.9059	1.1303
Mode V	562.9	392.3	2,450	0.6174	22,900	0.2910	0.9084	1.3282	1.0213

Table 3. Experimental data and the predictions of Miner’s rule, Corten-Dolan’s model and the improved model for hot-rolled 16 Mn steel [41, 44].

Loading mode	$\sigma_1$ /MPa	$\sigma_2$ /MPa	$n_1$	$\frac{n_1}{N_{f1}}$	$n_2$	$\frac{n_2}{N_{f2}}$	$\left(\frac{\sum n_p}{N}\right)_M$	$\left(\frac{\sum n_p}{N}\right)_T$	$\left(\frac{\sum n_p}{N}\right)'$
Mode I	394	345	9,350	0.1000	269,500	0.6701	0.7701	1.4342	0.6373
Mode II	394	345	19,700	0.2107	236,100	0.5870	0.7977	1.3795	0.6908
Mode III	394	345	39,400	0.4213	209,500	0.5209	0.9422	1.4585	0.8538
Mode IV	394	345	46,750	0.5000	159,300	0.3961	0.8961	1.2886	0.8377
Mode V	394	345	56,100	0.6000	74,000	0.1840	0.7840	0.9663	0.7633
Mode VI	345	394	181,000	0.4500	82,867	0.8863	1.3363	1.7824	1.2657
Mode VII	345	394	197,100	0.4900	80,970	0.8660	1.3560	1.8418	1.2757
Mode VIII	345	394	233,300	0.5800	59,570	0.6371	1.2171	1.7921	1.1123

45, normalized 16 Mn, and hot-rolled 16 Mn steel, respectively. The loading mechanisms were roughly similar, and the major difference among these three sets of experimental data is the concrete embodiment in loading amplitude value and cyclic number ratio. Two-stress level loading for normalized 45 steel displayed amplitude values of 331.46 and 284.4 MPa. Furthermore, the loading cycles were chosen such that the cyclic numbers of first-level loading stress to the fatigue life of the corresponding loading ratios reached 0.0100, 0.2500, 0.5000, 0.7500, 0.2500, 0.5000, and 0.7500. Second-level loading stress was then applied until failure. Given normalized 16Mn steel, the amplitude values increased to 562.9 and 392.3 MPa. Moreover, the ratios of loading cycle number were 0.0005, 0.0504, 0.2520, 0.4284, and 0.6174. The amplitude

values of loading stress for hot-rolled 16 Mn steel were 394 and 345 MPa, and second-level loading stress was initiated when the cyclic-ratios of first-level loading stress reached 0.1000, 0.2107, 0.4213, 0.5000, 0.6000, 0.4500, 0.4900, and 0.5800. The experimental results and parameters for fatigue life prediction are listed in Tables 1-3. The experimental conditions are not detailed in this paper; rather, they are presented in other studies [3, 41, 43, 44].

Material constant  $\gamma$  can be determined through fitting analysis using Eq. (10) and by combining the failure criterion and the experimental data in Tables 1-3. The calculated values of exponent  $d_i$  for normalized 45 steel were 16.5951, 15.9515, 15.6952, 15.4023, 15.5698, 15.9502, and 16.3983, as per the loading modes listed in Table 1.  $d_j$  values for nor-

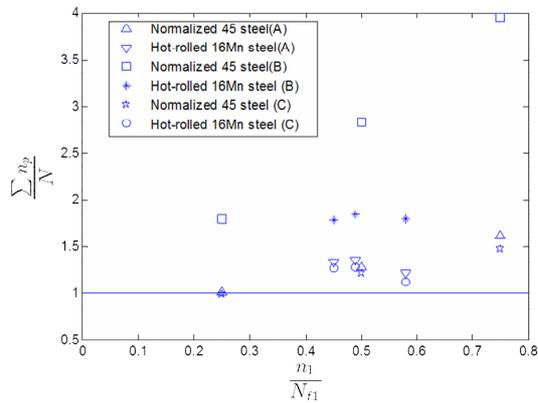


Fig. 2. Comparison of fatigue lives under low to high loading conditions as obtained using: (A) Miner’s rule; (B) Corten—Dolan’s model; (C) improved Corten—Dolan’s model.

malized 16Mn steel were computed to be 8.4358, 8.1138, 8.0463, 7.8616, and 7.3657 in the same manner given the loading modes in Table 2. Subscripts  $l = I, II \dots VII$  and  $j = I, II \dots V$  represented the sequence numbers of the loading modes for normalized 45 steel and 16Mn steel, respectively. Similarly, the  $d$  values for hot-rolled 16Mn steel could be calculated in sequence as 12.6491, 12.4991, 12.3860, 12.1863, 11.8817, 12.2705, 12.3350, and 12.4869 in accordance with Table 3. However, the value of  $d$  should be a constant according to the suggestion of Corten and Dolan [39]; thus, the value of  $d$  was set at 5.8 for normalized 45 steel and 16 Mn steel given that the two materials are not composed of high-strength steel. This setting is reflected in Eq. (9). A comparative analysis of the values of  $d$  as calculated from the proposed and the traditional methods reveals that the results of the two methods differ. In fact, the error is double and even triple the maximum.

As per the aforementioned analysis, the calculated values of exponent  $d$  using the traditional method varies significantly from those obtained with the proposed method. These differences inevitably affect the accuracy of fatigue life prediction. Tables 1-3 list the ratios of experimental life and predicted results as computed according to Miner’s rule, traditional Corten—Dolan’s model, and the improved model. The values are under the columns  $(\sum n_p/N)_M$ ,  $(\sum n_p/N)_T$  and  $(\sum n_p/N)$ . As per the final two columns, the differences in the values of exponent  $d$  induced considerable scatter in fatigue life predictions. A comparison of the predicted life-spans with experimental ones reveals that the errors associated with the traditional Corten-Dolan’s model are significantly greater than those observed when the improved model is used to process the experimental data. Miner’s rule and the improved Corten-Dolan’s model can generate approximately similar predictions; nonetheless, the prediction accuracy of the latter is occasionally higher than that of the former, especially given normalized 45 steel and hot-rolled 16 Mn steel under

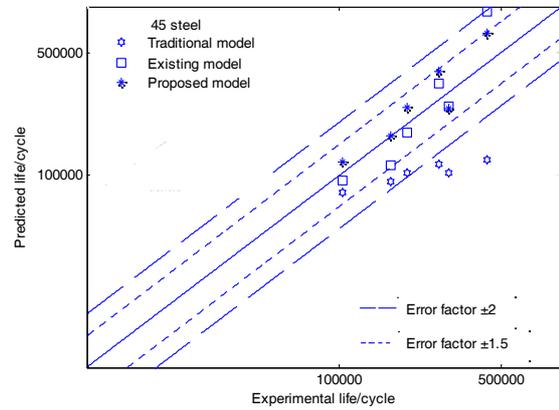


Fig. 3. Comparisons of experimental and fatigue lives for normalized 45 steel as assessed using the three different models.

low to high loading conditions. This effectiveness is clearly demonstrated in Fig. 2. The results predicted by Miner’s rule and the traditional model are dispersed over a wide area, especially those by the latter. The traditional model displays double and even triple the errors of those observed under Miner’s rule, as observed in Tables 1-3. A narrow scattering zone was also generated by the improved model when relative errors were all lower than or equal to 50%. Thus, the improved Corten-Dolan’s model, which considers the effects of damage and stress state in predicting the value of  $d$ , can predict fatigue life more accurately than the traditional model and Miner’s rule can.

4.2 Case study 2

To verify the prediction performance of the improved Corten-Dolan’s model further, the experimental data listed in Tables 1 and 2 were applied to an existing model [15]. Load interaction effects were taken into account in the existing model based on Corten-Dolan’s model, as expressed by Eqs. (13) and (14).

$$N = \frac{N_1}{\sum_{i=1}^k \alpha_i (\sigma_i/\sigma_1)^\mu \frac{\sigma_1^\lambda \delta_f^{1-\lambda}}{\sigma_i}}, \tag{13}$$

$$\frac{\sum n_p}{N} = \sum_{i=1}^k \frac{n_i}{N_1} (\sigma_i/\sigma_1)^\mu \frac{\sigma_1^\lambda \delta_f^{1-\lambda}}{\sigma_i}, \tag{14}$$

where  $\mu$  is a material constant;  $\lambda$  ( $0 < \lambda < 1$ ) represents the load interaction factor; and  $\delta_f$  is the initial static strength of a specimen.

Fatigue life can be predicted differently using this model. Therefore, the existing model is proven to predict fatigue life effectively [15].

A comparison of experimental and fatigue lives as assessed

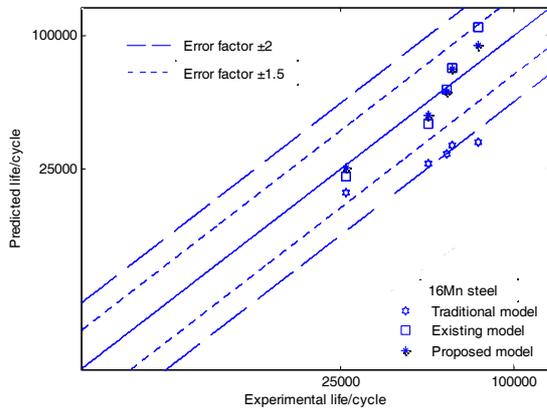


Fig. 4. Comparisons of experimental and fatigue lives for normalized 16 Mn steel as assessed using the three different models.

using the three models are shown in Figs. 3 and 4. The solid lines in the two figures indicate that the predicted results are equal to the experimental data, whereas the broken and dotted lines represent the error factors (Error factor = experimental life/predicted life) of  $\pm 2$  and  $\pm 1.5$ , respectively. The hexagrams, rectangles, and stars denote the predicted values obtained by the traditional Corten-Dolan's model, the existing model [15], and the improved model, respectively. All of the predictions by the improved model and the existing model are more consistent with the experimental data than those obtained with the traditional Corten-Dolan's model. In addition, most of the results derived from the improved model and existing model are approximately similar. All of the fatigue lives predicted by the existing model are within the factor of  $\pm 2$ , whereas all predictions by the improved model can fall into a factor of  $\pm 1.5$ . Thus, the existing model and the improved model display similar accuracy. Nonetheless, some of the prediction errors generated by the latter are smaller than those generated by the former. Given this condition, fatigue life can be predicted using the existing model presented in Ref. [15] such that engineering requirements are satisfied. Therefore, the improved Corten-Dolan's model that considers the damage and stress state effects of materials is effective and reliable.

## 5. Conclusions and discussion

This study examines the influence of damage and stress state on fatigue life prediction. This factor is not considered in the traditional Corten-Dolan's model. The major contributions of this research and the conclusions are summarized as follows.

(1) The value of exponent  $d$  is a unique and critical parameter in Corten-Dolan's model. As per the results presented above, the value of  $d$  significantly affects the accuracy of fatigue life prediction [15, 31, 38]. A novel method for determining the value of  $d$  accurately is thus proposed in this paper on the basis of previous research and in consideration of

the effects of damage and stress state.

(2) The traditional model is modified by substituting exponent  $d$  in the form of a function of cycle ratio ( $n_i/N_i$ ) and loading ratio ( $\sigma_i/\sigma_{max}$ ) into Corten-Dolan's model. The improved model exhibits an enhanced perfection performance when the damage and stress state effects of materials are considered.

(3) As per the comparisons made in this study, both Miner's rule and the improved model proposed in this paper can predict fatigue life more accurately than the traditional Corten-Dolan's model for normalized 45 steel, normalized 16 Mn steel, and hot-rolled 16 Mn steel. Nonetheless, the accuracy of the improved model is equal to or higher than that of Miner's rule. The errors incurred by the improved model are all within 50%; however, those observed by using Miner's rule are not.

(4) The existing model [15] differs slightly from the improved model with respect to most of the predicted results, even the improved model can occasionally generate more even predictions than the existing model can [15]. Overall, the predictions made by the existing model can satisfy engineering requirements. Therefore, the improved model presented in this paper is effective and reliable.

To predict fatigue life and investigate fatigue mechanism, a future work aims to study the effects of damage and stress state on the fatigue life of specimens in the cases of multi-level and random loading.

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