Introduction

Specifically, most of the mechanical components and/or engineering structures are often experienced variable amplitude cyclic loads during their service operation, the fatigue damage is introduced and the residual life of these structures will also be decreased with the possibly accumulated damage. Once the accumulative fatigue damage exceeds the critical failure threshold, the accident occurs. Thus, fatigue is one of the main failure reasons for these mechanical components and it is important to predict the reliability and useful life of these components. However, it is challenging to characterize the fatigue damage in a meaningful and reliable manner because fatigue damage is a process of the irreversible as well as stochastic. Therefore, to predict the residual life and reliability of these components effectively, there has been considerable interest in developing an accurate method to evaluate the fatigue damage accumulation.

Until now, based on the fatigue damage, various theories and models have been developed to predict the fatigue life of mechanical components and/or engineering structures. Note that fatigue damage accumulation theory can be generally classified into two categories: linear damage accumulation theory, which is also called the Palmgreen-Miner rule (just Miner's rule for short) (Miner, 1945); and nonlinear damage accumulation theories. Though the Miner's rule has been widely used in engineering, it has some drawbacks, for example, it not only neglects the effects of loading sequence and load interaction, but also the damage contribution induced by those stresses below the fatigue limit are ignored (Schijve, 2001). Accordingly, Miner's rule often results in the disagreement between the predicted and experimental value. Based on the characteristics of fatigue damage above described, some researchers have done some improvements works to the Miner's rule (Li et al., 2012; Rathod et al., 2011; Svensson, 2002; Zhu et al., 2011). Recently, Zhu et al. (2011) proposed a physics-based linear damage rule which considers the strengthening and damaging of low amplitude loads under different sequences. Rathod et al. (2011) developed a method employing the probabilistic modeling of fatigue damage accumulation under multi-level stress loading conditions, which combines the Miner's rule with the probabilistic S–N curves. Li et al. (2012) used the probability encoding method to predict the component life. However, due to its intrinsic deficiencies, no matter which version is used, life prediction using this rule is often unsatisfactory (Fatemi and Yang, 1998).

In order to remedy the shortcomings of Miner's rule, lots of researchers have done the works about the nonlinear damage accumulation theories, and there are extensive literatures concerning nonlinear damage models: continuum damage mechanics models (Besson, 2010; Dattoma et al., 2006; Yuan et al., 2013); energy-based damage methods (Jahed et al., 2007; Kreiser et al., 2007; Scott-Emuakpor et al., 2008); damage curve approaches (Manson and Halford, 1981); damage theories based on thermodynamic entropy (Risitano and Risitano, 2010; Naderi et al., 2010); damage rule considered the load interactions (Chen et al., 2011; Corten and Dolan, 1956; Li et al., 2001; Skorupa, 1999); damage theories based on physical property degradation (Cheng and Plumtree, 1998; Ye and Wang, 2001; Zhu et al., 2013). Though a number of works have been done in nonlinear damage accumulation, there are still some issues for nonlinear damage accumulation to be further researched especially in considering the effects of load interactions and the strength degradation of materials.

In this paper, an effort was made to get a better life prediction capability and applicability of the proposed nonlinear fatigue damage model which considers the effects of residual strength degradation. The remainder of this paper is organized as follows. ''The residual strength degradation'' section describes the strength degradation phenomenon and puts forward a residual strength degradation model. Then a nonlinear fatigue damage accumulation and a modified nonlinear fatigue damage model based on Manson–Halford model and considering the residual strength degradation are presented in ''Nonlinear fatigue damage accumulation modeling'', ''A modified nonlinear fatigue damage based on Manson–Halford model'' and ''A modified nonlinear fatigue damage accumulation model considering the residual strength degradation'' sections, respectively. The validation of the proposed model is processed in ''Validation of the residual strength degradation model and the proposed nonlinear fatigue damage accumulation model'' section. Finally, ''Conclusions'' section summarizes the paper and some conclusions are drawn.

The residual strength degradation

For the issue of fatigue failure, it is a damage accumulation process, and material property deteriorates continuously under cyclic loading. That is, on the one hand, the strength of a component degenerates gradually as depicted in Figure 1; on the other hand, its residual life will be reduced with the increasing working time. Thus, the evolution of fatigue damage will inevitably lead to the change of the internal microstructure of the material. According to the stress–strength interference (SSI) model, when the residual strength of the material is less than the loading stress, the failure occurs. And the limit state function $G(n)$ for fatigue reliability analysis can be written as

$$G(n) = g(\delta) - f(\sigma) \tag{1}$$

where $g(\delta)$ represents the residual strength and $f(\sigma)$ is the loading stress; when $G(n) > 0$, the material is at the safe domain, while $G(n) < 0$ the material is at the failure domain, when $G(n) = 0$ the material is at the boundary which is between the safe domain and the failure domain.

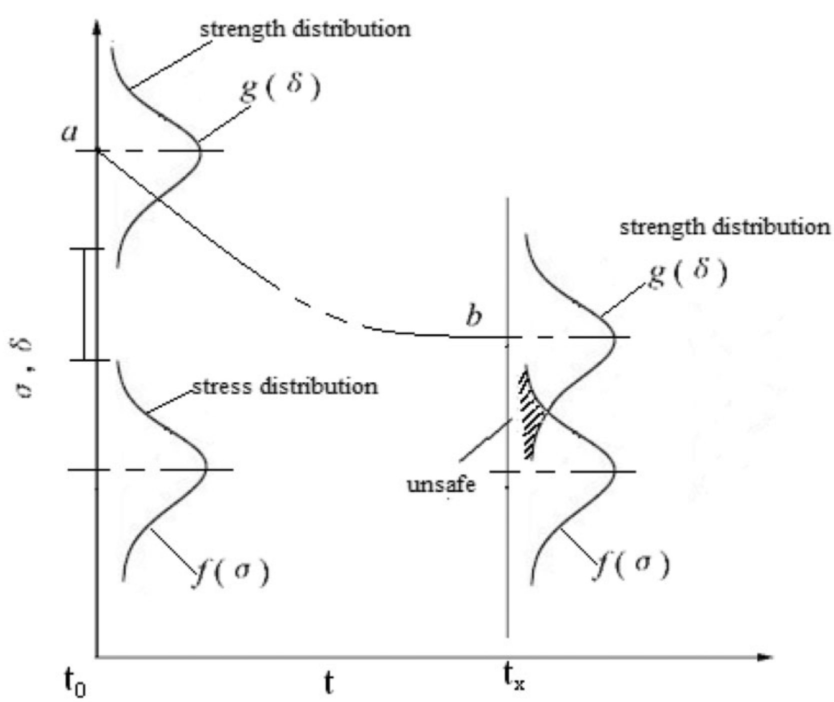

Figure 1. Dynamic variation of the stress and strength of materials.

It is well known that in order to investigate the relationship of the damage accumulation and the strength degradation, many studies have been done on this subject. Such as Chou and Croman (1978, 1979) used similar rate type differential equations to predict residual strength under a single stress level. In term of the *S–N* curve prediction methodology (Yang, 1977, 1978) developed several strength degradation models. Diao et al. (1999) predicted the residual strength under complex stress states and developed a generalized residual material property degradation model. Schaff and Davidson (1997a, 1997b) focus on the strength-based model for predicting the residual strength and life of composite structures subjected to constant amplitude and two-stress level loading conditions. Broutman and Sahu (1972) put forward a simple linear model. More detail comments on these models can be found in Philippidis and Passipoularidis (2007).

In this section, it should be emphasized that the residual strength degradation model was to be developed, and with an assumption of that the static strength degradation of material under constant amplitude loading can be calculated as (Lu et al., 1997)

$$\frac{\mathrm{d}\delta_R(n)}{\mathrm{d}n} = -\sigma^c \delta_R^{-b}(n) \tag{2}$$

where $\delta_R(n)$ is the residual strength of the material, n is the number of loading cycles at a given stress σ, c and b are dimensionless parameters which are related to the environment conditions, such as temperature, loading waveforms and loading frequency. If these parameters are determined at some given conditions, c and b become material constants. In addition, the residual strength $\delta_R(n)$ should satisfy the following initial and final boundary conditions

$$\delta_R(0) = \delta_{(0)} \quad \delta_R(N) = \sigma \tag{3}$$

where $\delta_{(0)}$ is the initial static strength of the material, N is the number of cycles to failure for the maximum applied stress and fatigue failure occurs when the residual strength reduces to the maximum stress level.

According to the theory of thermodynamics, fatigue damage is a process of the irreversible energy dissipation of the material. Hence, for the strength degradation model, the residual strength $\delta_R(n)$ should be a monotone decreasing function. And from equation (2), the proposed strength degradation model should meet the irreversible condition the reason is that $\frac{\mathrm{d}\delta_R(n)}{dn} < 0$. It is differentiated with respect to n after the two sides of equation (2), we can get

$$\frac{d^2(\delta_R(n))}{dn^2} = -\sigma^{2c} \delta_R^{-1-2b}(n) \tag{4}$$

From the result of equation (4), we can see that it is a convex function since $\frac{d^2(\delta_R(n))}{dn^2} < 0$. Thus, the proposed strength degradation model is reasonable in theory.

Then integrating equation (2) and combining with equation (3), it can easily yield the following equation for the residual strength degradation under the applied cyclic stress

$$\delta_R^{1+b}(n) = \delta_{(0)}^{1+b} - (1+b)\sigma^c n \tag{5}$$

When $n = N$, equation (5) can be rewritten as

$$\sigma^{1+b} = \delta_{(0)}^{1+b} - (1+b)\sigma^c N \tag{6}$$

The experimental methods to determine the value of parameters in equation (6) will take lots of cost and time. Therefore, an easy way to calculate the relative coefficients is needed, and from equation (5), it is clear that

$$(1+b)\sigma^c N = \delta_{(0)}^{1+b} - \sigma^{1+b} \tag{7}$$

Generally, the loading stress is less than the initial static strength of a material, thus $\sigma^{1+b} \ll \delta_{(0)}^{1+b}$, and the effect of σ^{1+b} can be ignored, equation (7) becomes

$$\sigma^c N = \frac{\delta_{(0)}^{1+b}}{1+b} \tag{8}$$

Equation (8) is the S–N curve of the material, where c and b are undetermined coefficients, if we assume that $\log(\sigma^c N) = \log\left(\frac{\delta_{(0)}^{1+b}}{1+b}\right) \Rightarrow \log(N) = B_1 + B_2\log(\sigma)$, then employing the least square method based on the available data of σ and N, the coefficients c and b can be obtained.

Similarly, for the multi-stress level loading condition, if fatigue damage is caused by the k level stress amplitude blocks, and the residual strength degradation of material after applied n_i cycles at σ_i can be obtained by

$$\delta_{R_i}^{1+b}(n_i) = \delta_{R_{i-1}}^{1+b}(n_{i-1}) - (1+b)\sigma_i^c n_i \tag{9}$$

Based on these concepts and assumptions stated above, an expression for estimating the residual strength degradation after k level stress can be developed as

$$\delta_{R_k}^{1+b}(n_k) = \delta_{(0)}^{1+b} - (1+b)\sum_{i=1}^{k} \sigma_i^c n_i \tag{10}$$

It should be noted that equation (10) indicates the ability of a material to withstand fatigue loading is decreasing with the variable amplitude loading stress, that is to say, the residual strength degraded continually with the working time. Therefore, to calculate the residual strength degradation, a residual strength degradation coefficient is introduced as

$$A_i = \frac{\delta_{R_i}(n_i)}{\delta_{R_{i-1}}(n_{i-1})} \tag{11}$$

where A_i represents the residual strength degradation coefficient of a material at any given stress σ_i, and $\delta_{R_i}(n_i)$ is the residual strength after n_i cycles at σ_i. Note that the initial state of loading cycles is zero and with the assumption of no initial damage, hence, the residual strength degradation coefficient for the initial state is $A_0 = 1$. When the damage is caused by the k level stress, combining equations (9), (10) and (11), the residual strength degradation of a material under k level stress can be obtained as

$$\begin{aligned}\delta_{R_k}(n_k) &= A_0 \times A_1 \times A_2 \times \cdots \times A_k \times \delta_{(0)} \\ &= \frac{\delta_{R_1}(n_1)}{\delta_{R_0}(n_0)} \times \frac{\delta_{R_2}(n_2)}{\delta_{R_1}(n_1)} \times \frac{\delta_{R_3}(n_3)}{\delta_{R_2}(n_2)} \times \cdots \times \frac{\delta_{R_i}(n_i)}{\delta_{R_{i-1}}(n_{i-1})} \times \delta_{(0)} \\ &= \frac{\delta_{R_i}(n_i)}{\delta_{R_0}(n_0)} \times \delta_{(0)}\end{aligned} \tag{12}$$

Assuming that $A_{(k)} = A_0 \times A_1 \times A_2 \times \cdots \times A_k$, and it can be obtained from the recursion formulas as follows

$$A_{(k)} = \left[1 - \frac{(1+b)\sum_{i=1}^{k} \sigma_i^c n_i}{\delta_{(0)}^{1+b}}\right]^{\frac{1}{1+b}} \tag{13}$$

Substituting equation (13) into equation (12) leads to

$$\delta_{R_k}(n_k) = \left[1 - \frac{(1+b)\sum_{i=1}^{k} \sigma_i^c n_i}{\delta_{(0)}^{1+b}}\right]^{\frac{1}{1+b}} \times \delta_{(0)} \tag{14}$$

Nonlinear fatigue damage accumulation modeling

As well known to us, fatigue damage accumulation is an irreversible process, and it can be treated as a measurement of degradation in resistance of materials. Furthermore, with the deterioration of the materials which is described aforementioned, the ultimate failures will occur if the degradation decreases to the failure critical threshold. And the degradation process can be depicted graphically as shown in Figure 2 which is implemented by Wang and Coit (2007) to explain why the variability of a degradation measurement increases monotonically with working time.

In Figure 2, D_0 is the initial damage and D_C is the failure critical threshold which varies appreciably among different components in practice. In terms of the physics of fatigue failure, the accumulative fatigue damage $D(n_i)$ at applied loading cycles n_i can be expressed as

$$D(n_i) = D_0 + kn_i^a \tag{15}$$

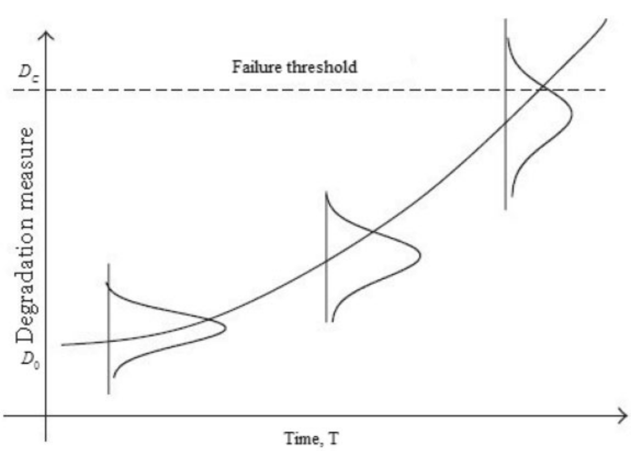

Figure 2. The degrading path.

where k is the rate of damage accumulation associated with cyclic loading n_i, which depends not only on the initial damage D_0, but also on the amplitude stress σ, and it is often determined by imposing a failure criterion. a is a yet-to-be determined damage accumulation parameter. According to the boundary conditions and failure criterion $D(0) = D_0$ and $D\left(N_f\right) = D_C$, and substituting these conditions into equation (15), we can get

$$k = \frac{D_C - D_0}{N_f^a} \tag{16}$$

Then substituting equation (16) into equation (15) leads to

$$D(n_i) = D_0 + (D_C - D_0)\left(\frac{n_i}{N_f}\right)^a \tag{17}$$

We assume that there is no initial damage and the damage failure occurs when $D_C = 1$. Thus, equation (17) can be rewritten as

$$D(n_i) = \left(\frac{n_i}{N_f}\right)^a \tag{18}$$

It is worth noting that equation (18) becomes the Miner's rule and Manson–Halford model when $a = 1$ and $a = \left(\frac{N_{f1}}{N_{f2}}\right)^{0.4}$, respectively. For the Manson–Halford model, some of the works have been done in this area. For example, Costa et al. (2012) used the Manson–Halford fatigue damage model to research the fatigue behavior of AA6082 friction stir welds under variable loadings; in Zhao et al.'s (2014) work, it has shown that the modified Manson–Halford model has a good ability to predict the multiaxial fatigue life of 50CrVA spring steel under proportional loading; based on the modified Manson–Halford model, Shukaev and Panasovs'kyi (2011) developed a method of fatigue life prediction for metal alloys under multiaxial low-cycle block deformation conditions.

A modified nonlinear fatigue damage based on Manson–Halford model

Lots of experimental data under completely reversed loading condition often indicate that the load sequence has a strong influence on the fatigue damage. In fact, under high–low loading sequences the accumulative damage $D < 1$ and under low–high loading sequences, the accumulative damage $D > 1$, which is shown in Figure 3.

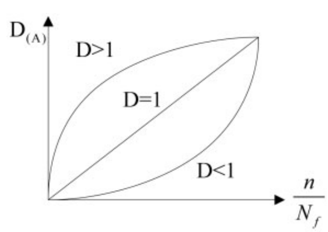

Figure 3. Different types of fatigue damage accumulation curves.

In this section, a nonlinear fatigue damage accumulation model based on the Manson–Halford model is presented with the damage curve approach. For a two-stress level loading, assuming a specimen is firstly loaded at stress σ_1 for n_1 cycles, accordingly the damage increment is from zero to x point along with the damage curve 1; and then the second segment loading stress σ_2 is applied for n_2 cycles up to failure, at the same time, the accumulative damage increases from y point to the z point along with the damage curve 2. The above-mentioned process can be described as shown in Figure 4.

In Figure 4, curve ox illustrates the fatigue damage for a material subjected to a constant amplitude loading at stress level σ_1 and the curve oyz illustrates the fatigue damage for a material experienced the constant amplitude loading at stress level σ_2. Specifically, both of these curves assume that the material at the begins has no initial damage. In addition, point x represents the accumulative damage after n_1 cycles at stress σ_1; point y represents the location of equivalent damage to point x under the second stress level σ_2, which is terms of the equivalence of fatigue damage. Applying this idea to the two-stress level loading condition, the beginning of the second segment corresponds to the point y, and the accumulative damage for the second segment should be along with path yz. Note that the curve oyz exhibits the nonlinear fatigue damage, and to achieve the requisite shift from point x to point y, it is possible to introduce an effective equivalent number of cycles n_{eff} under the stress level σ_2. And which is equivalent to the same amount of damage as caused by n_1 cycles at σ_1. Therefore, according to Manson–Halford model, the effective equivalent number of cycles can be determined from equation (19) which can be written as

$$\frac{n_{eff}}{N_{f2}} = \left(\frac{n_1}{N_{f1}}\right)^{\left(\frac{N_{f1}}{N_{f2}}\right)^{0.4}} \tag{19}$$

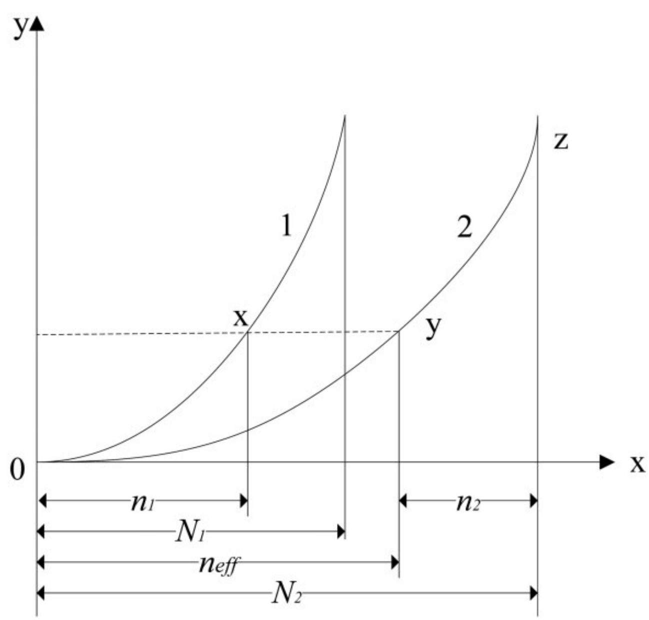

Figure 4. Fatigue damage process under two-stress level loading.

So the total damage after $n_{eff} + n_2$ cycles at σ_2 becomes

$$\frac{n_{eff}}{N_{f2}} + \frac{n_2}{N_{f2}} = \left(\frac{n_1}{N_{f1}}\right)^{\left(\frac{N_{f1}}{N_{f2}}\right)^{0.4}} + \frac{n_2}{N_{f2}} \tag{20}$$

It should be pointed that the exponent a of Manson–Halford model has a great impact on the accurately estimated the fatigue damage. Additionally, when estimating the fatigue damage we should consider not only the effects of load interactions but also the load sequences. However, in equation (20), only the effect of load sequences has been taken into account but the influence of loading interaction has been ignored. Hence, some works should be done to improve the accuracy of the Manson–Halford model. For example, by considering the physical laws of fatigue damage, Xu et al. (2012) suggested that the exponent parameter a of Manson–Halford model should be modified, which can consider both the load sequence and the effects of the load interaction at the same time, and the modified is as follows

$$0 < a = \left(\frac{N_{f1}}{N_{f2}}\right)^{0.4\min\left\{\frac{\sigma_1}{\sigma_2},\frac{\sigma_2}{\sigma_1}\right\}} < 1 \tag{21}$$

Then the damage accumulation model under two-stress level loading can be described as

$$\left(\frac{n_1}{N_{f1}}\right)^{\left(\frac{N_{f1}}{N_{f2}}\right)^{0.4\min\left\{\frac{\sigma_1}{\sigma_2},\frac{\sigma_2}{\sigma_1}\right\}}} + \frac{n_2}{N_{f2}} = 1 \tag{22}$$

For high–low loading condition, $0 < \frac{N_{f1}}{N_{f2}} < 1$ and $0 < a = \left(\frac{N_{f1}}{N_{f2}}\right)^{0.4\min\left\{\frac{\sigma_1}{\sigma_2},\frac{\sigma_2}{\sigma_1}\right\}} < 1$, thus the damage caused by stress level σ_2 follows

$$\frac{n_2}{N_{f2}} = 1 - \left(\frac{n_1}{N_{f1}}\right)^{\left(\frac{N_{f1}}{N_{f2}}\right)^{0.4\min\left\{\frac{\sigma_1}{\sigma_2},\frac{\sigma_2}{\sigma_1}\right\}}} < 1 - \frac{n_1}{N_{f1}} \tag{23}$$

Hence, the fatigue accumulative damage for high–low loading condition can be described as

$$\frac{n_1}{N_{f1}} + \frac{n_2}{N_{f2}} = \frac{n_1}{N_{f1}} + 1 - \left(\frac{n_1}{N_{f1}}\right)^{\left(\frac{N_{f1}}{N_{f2}}\right)^{0.4\min\left\{\frac{\sigma_1}{\sigma_2},\frac{\sigma_2}{\sigma_1}\right\}}} < \frac{n_1}{N_{f1}} + 1 - \frac{n_1}{N_{f1}} = 1 \tag{24}$$

Therefore, the accumulative damage for high–low loading condition is less than the unit. In the same way, the accumulative damage for low–high loading condition can be proven to be more than the unit. For the same two-stress level loading, there is no loading interaction effect, and $a = 1$. Equation (22) can be reduced to the Miner's rule

$$\frac{n_1}{N_{f1}} + \frac{n_2}{N_{f2}} = 1 \tag{25}$$

Similarly, using the equivalent damage characteristic, it is easy to get the fatigue damage accumulative formula under the multi-stress level loading condition in the sequential calculation manner, and which can be described as:

$$\left[\left[\left[\left(\frac{n_1}{N_{f1}}\right)^{\left(\frac{N_{f1}}{N_{f2}}\right)^{0.4\min\left\{\frac{\sigma_1}{\sigma_2},\frac{\sigma_2}{\sigma_1}\right\}}}+\frac{n_2}{N_{f2}}\right]^{\left(\frac{N_{f2}}{N_{f3}}\right)^{0.4\min\left\{\frac{\sigma_2}{\sigma_3},\frac{\sigma_3}{\sigma_2}\right\}}}+\frac{n_3}{N_{f3}}\right]^{\left(\frac{N_{f3}}{N_{f4}}\right)^{0.4\min\left\{\frac{\sigma_3}{\sigma_4},\frac{\sigma_4}{\sigma_3}\right\}}}+\cdots+\frac{n_{i-1}}{N_{f(i-1)}}\right]^{\left(\frac{N_{f(i-1)}}{N_{fi}}\right)^{0.4\min\left\{\frac{\sigma_{(i-1)}}{\sigma_i},\frac{\sigma_i}{\sigma_{(i-1)}}\right\}}} + \frac{n_i}{N_{fi}}=1 \tag{26}$$

where n_1, n_2, $n_3, \ldots, n_{i-1}, n_i$ are the numbers under $\sigma_1, \sigma_2, \sigma_3, \ldots, \sigma_{i-1}, \sigma_i$, and $N_{f1}, N_{f2}, N_{f3}, \ldots, N_{f(i-1)}, N_{fi}$ are the fatigue life under the different loading stresses, respectively. When $N_{f1} = N_{f2} = N_{f3} = \cdots = N_{f(i-1)} = N_{fi}$, equation (26) becomes the famous Miner's rule which is widely used because of its simplicity. However, it neglects the load sequence, etc. And it is generally known fact that the order of different loadings has significant influences on the fatigue behavior of materials. Equation (26) can consider not only the effects of load sequence, but also the load interaction, which is a modified nonlinear fatigue damage accumulation model based on Manson–Halford model.

A modified nonlinear fatigue damage accumulation model considering the residual strength degradation

Defining fatigue damage of materials is not a simple problem, since the evaluation of fatigue damage is usually complex, and it is influenced remarkably by many factors, such as the effects of load sequences, load interaction and the residual strength degradation. However, as noted in equation (26) which is a modified Manson–Halford model without considering the influence of strength degradation. So, in order to accurately calculate the fatigue damage, it is necessary to introduce the residual strength degradation into the proposed modified nonlinear model.

Based on the fatigue accumulation irreversible aforementioned, the residual strength of material is monotonous decreasing function. Furthermore, the fatigue damage created by the large loading for one cycle is larger than that created damage by the small loading for one cycle. In other words, the larger the loading stress is, the more the strength degradation is. Consequently, different loading sequence will result in different strength degradation, which leads to the different fatigue damage. In addition, the failure of fatigue is the residual strength degradation coefficient decreasing process which is from the initial value 1 to the final value 0. Furthermore, the effect of strength degradation is small at the first stage, and is deteriorating seriously along with the working time, Hence, there is a need to introduce the residual strength degradation function

$$\gamma = \exp\left[\alpha \cdot \left(\frac{1}{A} - 1\right)\right] \tag{27}$$

where α is the material coefficient which is related to the material properties and can be obtained from fatigue experimental data, A is the residual strength degradation coefficient which reflects the relationship between the strength degradation and the fatigue accumulation. Therefore, the residual strength degradation function, which exhibits the relationship among, fatigue damage under constant amplitude loads, residual strength degradation and the interaction of different stress loading, are acquired, and it is significant to introduce the factor γ into the modified fatigue damage accumulation model.

Therefore, using these concepts and the assumptions given above, the corresponding value of the damage induced by the n_i applied cycles at σ_i can be deduced by the following formula as

$$D(n_i) = \exp\left[\alpha \cdot \left(\frac{1}{A_{(i)}} - 1\right)\right]\left(\frac{n_i}{N_{fi}}\right)^{\left(\frac{N_{f(i-1)}}{N_{fi}}\right)^{0.4\min\left\{\frac{\sigma_{(i-1)}}{\sigma_i}, \frac{\sigma_i}{\sigma_{(i-1)}}\right\}}} \tag{28}$$

Equation (28) can be used to estimate the fatigue accumulative damage for components or materials under the multilevel loading, where $D(n_i)$ is the fatigue accumulation value which considers the loading sequence, the effects of load interaction and the residual strength degradation simultaneously, n_i and N_{fi} are the applied cycles and the number of cycles to failure under the application of σ_i respectively, and $A_{(i)}$ is the residual strength degradation coefficient which can be obtained by equation (11). It should be noted that there are no residual strength degradation effects if the residual strength degradation coefficient is equal to zero.

Validation of the residual strength degradation model and the proposed nonlinear fatigue damage accumulation model

In this section, the validity and accuracy of the residual strength degradation model and the proposed modified nonlinear fatigue damage accumulation model considering the residual strength degradation are evaluated, respectively, which are through a comparison of predicted and observed fatigue life distributions for metallic specimens subjected to two-stress level loadings.

Validation of the residual strength degradation model

The studied case in this subsection aims to illustrate the mathematical modeling and deduction of the residual strength degradation in ''The residual strength degradation'' section. Experimental data of the normalized 45 steel under the constant amplitude stress loading are used and the details of test conditions can be found in Lu et al. (1997). For these tests, the loading stress amplitude is $\sigma_1 = 395.02\,\text{MPa}$ and $\sigma_2 = 321.37\,\text{MPa}$, respectively. Based on the pre- and post-test analyses of S–N curve, the material parameter was obtained as $a = 8.114$ and $b = 6.358$. The initial static strength is $\delta_0 = 598.2\,\text{MPa}$. Finally, the residual strength degradation with the cycle ratio for

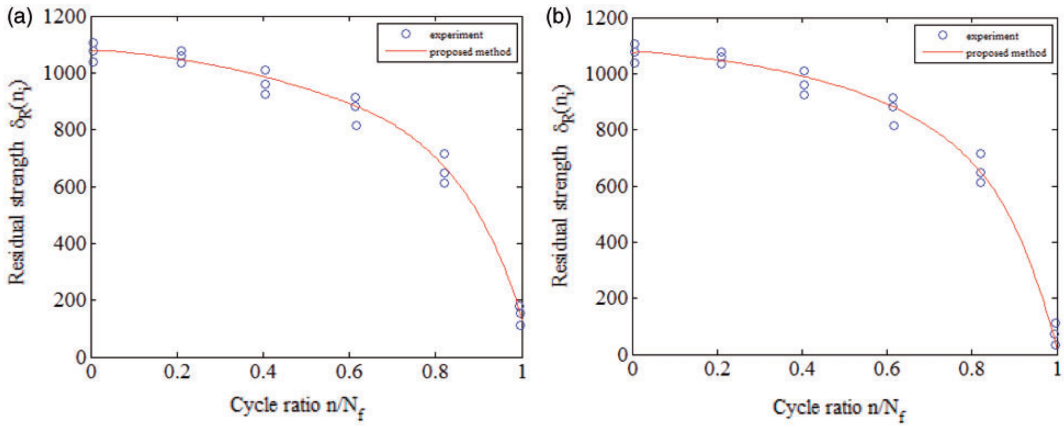

Figure 5. Residual strength degradation vs. cycle ratio for 45 steel under different stress loadings. (a) $\sigma_1 = 395.02$ MPa and (b) $\sigma_2 = 321.37$ MPa.

45 steel under different stress loadings σ_1 and σ_2 are presented in Figure 5, respectively. From the results, we can see that the predicted results are in good agreement with the experimental data.

Validation of the proposed nonlinear fatigue damage accumulation model considering the residual strength degradation

In order to verify the descriptive ability of equation (27), two categories of experimental data were used to verify the proposed model, details of test conditions are reported in Fang et al. (2006) and Shang and Yao (1999). For 30CrMnSiA, the material constants $a = 19.32$, $b = 15.59$ and $\alpha = 0.425$; the two-stress level loadings are $\sigma_1 = 836$ MPa and $\sigma_2 = 732$ MPa, and their cycles to failure are $N_{f1} = 7204$ and $N_{f2} = 55762$, respectively; the high-low load spectrum was $836 - 732$ MPa and low-high load spectrum was $732 - 836$ MPa. For normalized 16Mn steel, material constants $a = 8.27$, $b = 8.19$ and $\alpha = 0.676$; the two-stress level loadings are $\sigma_1 = 562.9$ MPa and $\sigma_2 = 392.3$ MPa, and their cycles to failure are $N_{f1} = 3968$ and $N_{f2} = 78723$, respectively; the high-low load spectrum was $562.9 - 392.3$ MPa and low-high load spectrum was $372.65 - 392.3$ MPa. Further, to reflect the capability of the new model, the test data are also assessed by Miner's rule and Hashin's rule. The results of 30CrMnSiA and normalized 16Mn steel between experiment and prediction are listed in Tables 1 and 2, respectively.

From Tables 1 and 2, we can see the comparison results of the Miner's rule, Hashin's rule and the proposed model for the 30CrMnSiA and normalized 16Mn steel under the two-stress level loading, which indicate that a good agreement is achieved by the proposed model and it has better life prediction capabilities than the conventional model because most of the relative errors by the proposed model are all in the 50%, and it is generally that if the relative error of the prediction and the experiment is at the range of $\frac{1}{2} - 2$ it will satisfy the need of the engineering applications. Meanwhile, it is clear that the fatigue accumulative damage predicted by the proposed model exceeds unity for low-high loading, and the fatigue damage accumulation value is less than unity and for high-low loading conditions, which demonstrates the effect of residual strength degradation and the loading interaction. Through the tests mentioned above, it can be concluded the proposed model has a good practicability and is reasonable because it considers the residual strength

Table 1. Experiment and prediction comparison of Miner's rule, Hashin's rule and proposed model for the 30CrMnSiA under the two-stress level loading.

Stress (MPa)	Loading sequence	n_1	$\frac{n_1}{N_{f1}}$	n_2	Miner $\frac{n_2}{N_{f2}}$	Experiment $\frac{n_2}{N_{f2}}$	Hashin's rule	Proposed model
836–732	High-Low	1800	0.208	32,450	0.792	0.582	0.732	0.9536
	High-Low	3000	0.417	16,002	0.583	0.287	0.755	0.9682
	High-Low	5000	0.694	6969	0.306	0.125	0.852	0.9568
732–836	Low-high	15,000	0.269	6501	0.731	0.903	1.108	0.9026
	Low-high	25,000	0.448	5400	0.552	0.750	1.129	1.0607
	Low-high	35,000	0.628	4428	0.372	0.615	1.255	1.1072
	Low-high	45,000	0.807	3254	0.193	0.425	1.172	1.0482

Table 2. Experiment and prediction comparison of Miner's rule, Hashin's rule and proposed model for the normalized 16Mn steel under the two-stress level loading.

Stress (MPa)	Loading sequence	n_1	$\frac{n_1}{N_{f1}}$	n_2	Miner $\frac{n_2}{N_{f2}}$	Experiment $\frac{n_2}{N_{f2}}$	Hashin's rule damage	Proposed model
562.9–392.3	High-low	200	0.0504	59,400	0.9496	0.7548	0.8356	0.9062
	High-low	1000	0.2520	56,300	0.7480	0.7154	0.9123	0.9837
	High-low	1700	0.4284	47,600	0.6716	0.6048	0.9482	0.9979
	High-low	2450	0.6174	22,900	0.3826	0.2910	0.9261	0.9721
	Low-High	64,400	0.240	62,800	0.760	0.798	1.145	1.0794
372.65–392.3	Low-High	116,000	0.433	62,900	0.567	0.799	1.276	0.9876
	Low-High	150,000	0.560	23,300	0.440	0.455	1.206	1.0056

degradation when calculate the fatigue accumulation damage which is different from the conventional model.

The application of the proposed model for reliability prediction

Based on the SSI model, the component is safe or reliable when the residual strength is greater than the loading stress, and the reliability is equal to all the sum of the probability that the loading stress is less than the residual strength, that is

$$R(\delta_R) = P(\sigma < \delta_R) = P[(\delta_R - \sigma) > 0] \tag{29}$$

Assuming that the residual strength follows lognormal distribution, then the reliability of a component can be derived in terms of the following equation

$$\ln \delta_R(n) = \frac{1}{1+b} \ln\left(\delta_0^{1+b} - (1+b)\sigma^c n\right) \sim N\left(\mu, \sigma_0^2\right) \tag{30}$$

Table 3. The reliability analysis of a component under constant amplitude loading.

Number	Life (cycle 10^3)	$R(n)$ (experiment)	$R(n)$ (proposed model)
1	462	0.939	0.950
2	697	0.859	0.831
3	707	0.763	0.771
4	757	0.675	0.696
5	931	0.588	0.583
6	994	0.512	0.521
7	1044	0.412	0.426
8	1323	0.325	0.317
9	1410	0.237	0.241
10	1867	0.149	0.151
11	2456	0.061	0.052

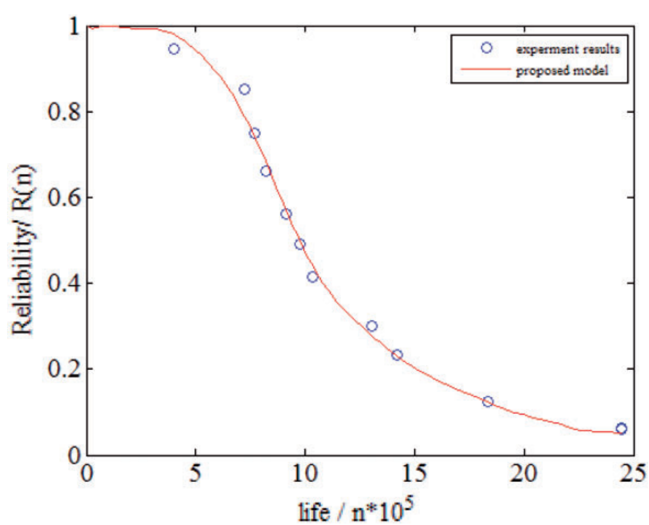

Figure 6. Fatigue reliability analysis of a component under constant amplitude loading.

where μ is the mean and σ_0 the standard deviation of the residual strength, and the value of them can be obtained from the experiment data through doing experiment under different cycles ratio n_i at the maximum stress $\sigma_{\max}$. Then according to the available value fitting equation (19), the mean and the standard deviation can be determined.

In this section, take the reliability analysis of the component under constant amplitude as an example, and employ the data of the LY-12cz (Guo and Yao, 2003). The expectation and variance of the residual strength are $\mu = 5.877$ and $\sigma_0 = 0.215$, respectively. Therefore, the results of fatigue reliability obtained from equation (29) are shown in Table 3 and Figure 6. From the results, we can see it clearly that the reliability almost keeps one because no enough degradation to cause the component failure, after that, with the continuous degradation, the system reliability begins to drop, until the 2.5×10^6 cycles the reliability for the component nearly drops down to zero which will be at the failure domain. At the same time, it is clear that there are good agreements between the prediction results by the proposed method and the experimental data.

Conclusions

In this paper, a modified nonlinear fatigue damage accumulation model considering the residual strength degradation is developed. The study is essential for the proper characterization of damage accumulation for the components during their service. The main conclusions can be summarized as follows: first, a general method for modeling residual strength degradation is developed to analyze the strength degradation and fatigue reliability. Further, this model's coefficients can be obtained by fitting the *S–N* curve. Second, based on the Manson–Halford theory, combining with the proposed residual strength degradation model, a new nonlinear fatigue damage accumulation model is presented, which considers the effects of load interaction, loading history and strength degradation in materials. It shows a better characterization of fatigue damage evolution over the conventional model. Third, experimental data from the literature are used to validate and verify the proposed model through comparing with the prediction results. Through all the comparative analyses, the proposed model has better prediction accuracy than the conventional model. Meanwhile, according to the applicable conditions of the Manson–Halford model, it is valid for most metallic materials, such as carbon steels, cast irons, and alloy steels, which meet the demands of practicability of the proposed model in actual engineering. Therefore, it should be pointed out that the new method presented in this paper can be better used for fatigue damage accumulation and reliability analysis. And it is worth considering in later research for applying the proposed nonlinear fatigue damage cumulative accumulation model considering the strength degradation to the multi-stress level loadings.

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Conflict of interest

None declared.

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