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Fatigue life prediction under variable amplitude loading using a non-linear damage accumulation model

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Abstract

Most of engineering components in service are usually subjected to variable cyclic loading. It is important to predict fatigue life and deal with the issue about fatigue damage accumulation for these components. One of the largest difficulties in fatigue failure analysis is to find a representative 'damage criterion' which can be easily connected with the Wöhler curve taken as the known material data. The most commonly used model is the Miner's rule which ignores the loading history effect, under the same loading conditions the experimental results are higher than the Miner expectations for low-to-high load sequence and are lower than the Miner expectations for high-to-low load sequence. The fatigue driving stress that causes fatigue damage is presented to predict residual fatigue life under variable amplitude loading. It increases with loading cycles until equals the fatigue strength when fracture occurs. By determining the equivalent number of cycles that yields the same fatigue driving stress as the previous loads, the remaining life can be predicted. The proposed damage criterion is connected cycle by cycle to the Wöhler curve and the experimental results are in a good agreement with the model predictions.

Keywords

Fatigue, damage criterion, load sequence, fatigue driving stress, life prediction

Introduction

Fatigue failure is one of the most common failure modes of mechanical components. Reliable lifetime prediction and fatigue damage analysis are particularly important in the design, optimization and safety assessments of these components (Ince and Glinka, 2014). Fatigue damage

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increases with the applied cycles in a cumulative manner which results to fracture. As the damage accumulates, the remaining lifetime under further loading becomes much more limited. Fatigue damage model plays a key role in the life prediction of mechanical components. Compared to the fatigue damage accumulation under constant amplitude loading, it is much more intractable to deal with fatigue damage subjected to variable amplitude loading. Therefore, it is important to formulate an efficient method to evaluate the fatigue damage accumulation and predict fatigue life and reliability (Cunha et al., 2014).

Cumulative fatigue damage issue is an old, but not yet resolved issue. Comprehensive reviews on the cumulative fatigue damage and life prediction methods have been conducted in Fatemi and Yang (1998) and Cui (2002). Cumulative fatigue damage theories can be classified into two categories: (1) linear damage cumulative theories and (2) non-linear damage cumulative theories. Miner's rule is the epitome of linear damage accumulation approach and receives extensive usage in engineering due to its simplicity (Luo et al., 2014; Manson and Halford, 1981; Zhu et al., 2012a). The main drawback of Miner's rule is that the damage accumulation process does not relate with the load conditions, the load sequences, the interaction between various loads, as well as the damage induced by stresses below the fatigue limit (Aid et al., 2011; Manson and Halford, 1981). So a lot of work (Aid et al., 2011; Dai et al., 2013; Luo et al., 2014; Zhu et al., 2012a, 2012b) has been carried out to get better and more exact life estimates under variable amplitude loading conditions. Many fatigue damage accumulation models have been proposed and a majority of these models are based on non-linear accumulation laws (Chaboche and Lesne, 1988; Ince and Glinka, 2014; Manson and Halford, 1981; Richart and Newmark, 1948). These methods can be classified into the following categories (Zhu et al., 2012b): damage curve based approaches (Dowling, 1972; Manson and Halford, 1981), continuum damage mechanics models (Chaboche, 1988; Cheng and Plumtree, 1998; Lemaitre and Plumtree, 1979; Mesmacque et al., 2005; Shang and Yao, 1998), models considered load interaction effects (Dai et al., 2013; Zhu et al., 2012b, 2013), energy based damage methods (Kim et al., 2013; Meneghetti, 2007; Valluri, 1961), physical property degradation based models (Bao and Shang, 2001; Bui-Quoc et al., 1971; Lagoda et al., 2005), ductility exhaustion based methods (Pavlou, 2002; Xie, 1992), and thermodynamic entropy based damage theories (Naderi et al., 2010; Macha et al., 2006; Marco and Starkey, 1954).

In this paper, a new non-linear damage accumulation rule is proposed to improve the deficiencies inherent in the linear damage accumulation rule and still maintains its simplicity in its application. The main advantage of this model is the ease of use which requests only the Wöhler curve.

Fatigue damage models

Linear damage cumulative rule

According to the published fatigue damage accumulation theories, the linear damage accumulation rule (LDR), also known as Miner's rule, is probably the most commonly used in engineering. The Miner's rule was proposed based on the following assumptions (Buxbaum, 1987; Miner, 1945):

(1) The rate of damage accumulation remains constant over each loading cycle.
(2) Damage occurs and accumulates only when the stress is higher than the fatigue limit.
(3) The failure of component is assumed to occur when cumulative damage reaches the unity.

According to the above assumptions, for a load spectrum that includes stress level, $S_1, S_2, \ldots, S_i$, the corresponding numbers of load cycles at these stress level are $n_1, n_2, \ldots, n_i$, respectively, N_{f1},

$N_{f2}, \ldots, N_{fi}$ represent the cycles to failure under each stress level. Fatigue life prediction under constant amplitude block loading can be predicted according to the Miner's rule:

$$D = \sum_{i=1}^{k} \frac{n_i}{N_{fi}} \tag{1}$$

However, most metallic materials exhibit much more complex behaviors than the ones modeled by the linear damage rule. It has been verified that metallic materials exhibit highly non-linear fatigue damage evolution with load dependency (Miller and Zachariah, 1977; Ye, 1996). The linear damage rule often gives inaccurate prediction results due to the following shortcomings (Aid et al., 2012; Buxbaum, 1987). It is not sensitive to the load sequence using the linear accumulation rule for fatigue damage analysis. The influence of load sequence on fatigue life is ignored. As it is well known from Miller and Zachariah (1977) and Lee et al. (2005), Miner's rule leads to optimistic results under high-to-low load sequence and to pessimistic results under low-to-high load sequence. The assessment error resulting from the Miner's rule is detrimental to the structure design and the safe use (Aid et al., 2012; Lee et al., 2005). According to this, a lot of work (Aid et al., 2012; Bao and Shang, 2001; Chaboche and Lesne, 1988; Lagoda et al., 2005; Shang and Yao, 1998) has been carried out to get better and more exact fatigue life estimates under variable amplitude loading conditions.

Non-linear fatigue damage accumulation model

Many researchers have tried to modify Miner's rule, but due to its intrinsic deficiencies, no matter which version is used, life prediction using this rule is often unsatisfactory (Shang and Yao, 1999; Krouse, 1967). Richart and Newmark (1948) introduced the concept of the damage curve to overcome the deficiencies associated with the linear damage rule. Based on this concept and the results of load sequence experiments, Marco and Starky (1954) developed a non-linear load dependent damage theory firstly, represented by a power law:

$$D = \sum_{i=1}^{k} \left[\frac{n_i}{N_{fi}}\right]^{C_i} \tag{2}$$

where C_i is a material parameter related to ith loading level. This rule allows to correctly taking into account the effects of loading sequences. It is shown that the Miner's sum $D = \sum_{i=1}^{k} \frac{n_i}{N_{fi}} > 1$ for low-to-high load sequences and $D = \sum_{i=1}^{k} \frac{n_i}{N_{fi}} < 1$ for high-to-low load sequences. Experience showed that only in some cases and for some materials, this law has shown good agreement with experimental results; moreover, the involved coefficients C_i have to be calculated for different loading conditions which limit their use in engineering applications (Chaboche and Lesne, 1988; Richart and Newmark, 1948; Shang and Yao, 1999; Zhu et al., 2011). As pointed out by Van Paepegem and Degrieck (2002), this conclusion cannot be applied to all materials in the existing experimental data in the literature.

According to the non-linearity of equation (2), fatigue damage under service loading needs to be computed in a cycle-by-cycle manner, which still requires a large amount of computational efforts. This disadvantage can be circumvented by approximating the non-linear function using double linear functions (Krouse, 1967). In each stage, a linear damage accumulation rule is applied.

For two-block loading condition, the double linear damage model is easy to implement. For the multi-block loading or spectrum loading condition, the determination of the model parameters becomes complicated (Krouse, 1967; Marco and Starkey, 1954; Shang and Yao, 1999; Gatts, 1961).

Several more complex fatigue damage accumulation functions have been proposed to improve the accuracy. Manson and Halford (1981) proposed a double damage curve approach, which combines the accurate parts of both the double linear damage approach and the damage curve approach. A more recent approach for fatigue damage accumulation is to use a non-linear continuum damage mechanics model (Aid et al., 2011; Dai et al., 2013; Luo et al., 2014; Zhu et al., 2012a). Despite the different proposed damage functions, the basic idea is to calculate the fatigue damage in an evolutionary manner using a scalar damage variable. The main difference lies in the number and characteristics of the parameters used in these models, the requirements for additional experiments, and their applicability (Zhu et al., 2012b).

A new non-linear fatigue damage accumulation model

From the brief discussion on fatigue damage models mentioned above, LDR is a simple rule, which often leads to inaccurate life prediction. Although most of non-linear fatigue damage models improve the deficiencies of LDR by considering load interaction and load sequence effects, which require detailed information such as material parameters, crack geometry, crack growth laws and other mechanisms. However, in engineering applications, the information is not fully available (Chaboche and Lesne, 1988; Zhu et al., 2011, 2013). In addition, non-linear fatigue damage models are usually computationally expensive.

Recently, a new approach using fatigue driving stress has been developed in Kwofie and Rahbar (2013, 2011). The fatigue driving stress is a function of the applied cyclic stress, numbers of loading cycles, and the numbers of cycles to failure. It increases with loading cycles until the fatigue strength is reached when fracture occurs. By determining the equivalent number of cycles that yields the same fatigue driving stress as the previous loads, the remaining life can be predicted. Many forms of fatigue damage equation have been derived based on Kwofie and Rahbar's work (2013). At the same time, new and modified theories were continuously developed in different categories, all of them are similar in form and the main differences lie in the number and the characteristics of parameters used in the model and the requirement for additional experiments to determine them. In this paper, the model suggested does not require too many material properties and it takes into account the loading history effects. This model is connected cycle by cycle based on $S-N$ curve.

Due to the large diversity of $S-N$ curves for different materials, it is difficult to establish a unified expression of a $S-N$ curve. It is generally accepted that $S-N$ representation can be done using a power function expression, as follows (Buxbaum, 1987):

$$N_f S^m = C \tag{3}$$

where C is the fatigue strength constant and m represents the slope of $S-N$ curve associated with material property, specimen configuration, loading mode, etc. and determined from experiments. Equation (3) shows the power function relationship between stress amplitude S and life N_f at a specific stress ratio R or mean stress S_m. Fatigue life N_f varies with fatigue stress S, where for higher S one observes less N_f while for lower S, one observes larger N_f. The fatigue driving stress S_D that causes fatigue damage is presented to predict residual fatigue life under variable amplitude loading condition. It increases with loading cycles until equals the fatigue strength constant C when fracture occurs (Richart and Newmark, 1948). This fatigue driving stress S_D is a function of applied cyclic stress S, number of loading cycles n, and the number of cycles to failure N_f, which due to applied cyclic stress S, may be expressed as a function of number of loading cycles, n.

$$S_D = N_f^{\beta} S^m \tag{4}$$

where $\beta = \frac{n}{N_f}$ is the expended life-fraction of load S, as the numbers of loading cycles n increases from zero to N_f, S_D increases from S to C. Thus, at $n = N_f$ we have

$$\begin{cases} \beta = \dfrac{n}{N_f} = 1 \\ C = N_f^{\beta} S^m \end{cases} \tag{5}$$

Two-level fatigue loading

To make the discussion easier for fatigue damage accumulation under stationary loading, we will consider a fatigue problem under a repeated two-block loading first.

Consider a two-step high-to-low load sequence loading as shown in Figure 1, where n_1 denotes the initial applied loading cycles with a higher stress S_1 and n_2 represents the remaining cycles to eventual fatigue failure with a lower stress S_2. The $S - N$ curve is used to obtain the fatigue lives N_{f1} and N_{f2}, for each loading stress as shown in Figure 1 (Lemaitre and Plumtree, 1979).

Non-linear damage curves under two-level fatigue loading are shown schematically in Figure 2. Each of these curves represents a different loading condition that leads to different cycles to failure. At each load level or life level, the relation between the fatigue driving stress (S_{D_1}) and the applied cycles or the life-fraction follows equation (4). If a life-fraction $\frac{n_1}{N_{f1}}$ is first applied along the curves representing the life N_{f1} to point A, the fatigue driving stress (S_{D_1}) increasing process will be represented by the life curve N_{f1} from zero to point A. If at this point a new loading stress with a life of N_{f2} is introduced and this loading is applied, the fatigue driving stress (S_{D_1}) will proceed from

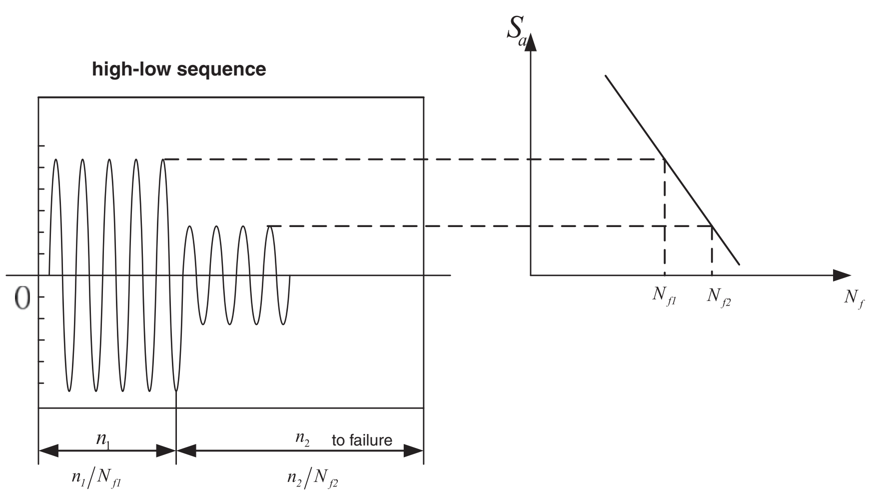

Figure 1. A block of two-step high–low sequence loading.

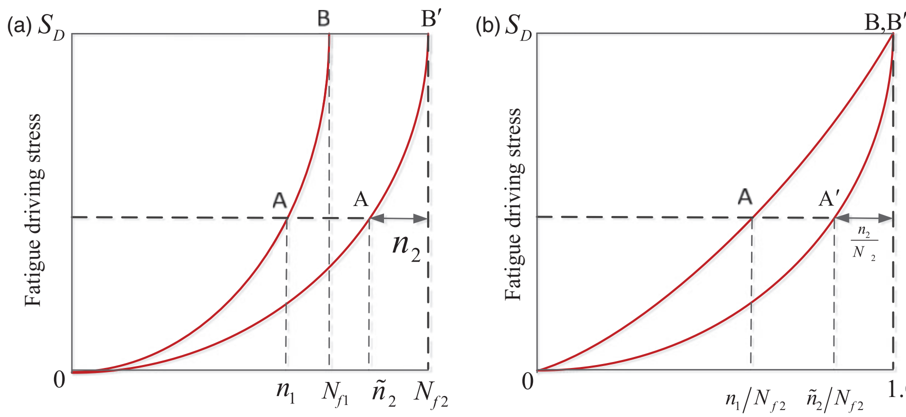

Figure 2. Non-linear damage accumulation (high-to-low load sequence). (a) N_f, cycles and (b) $\frac{n_1}{N_f}$, cycle fraction.

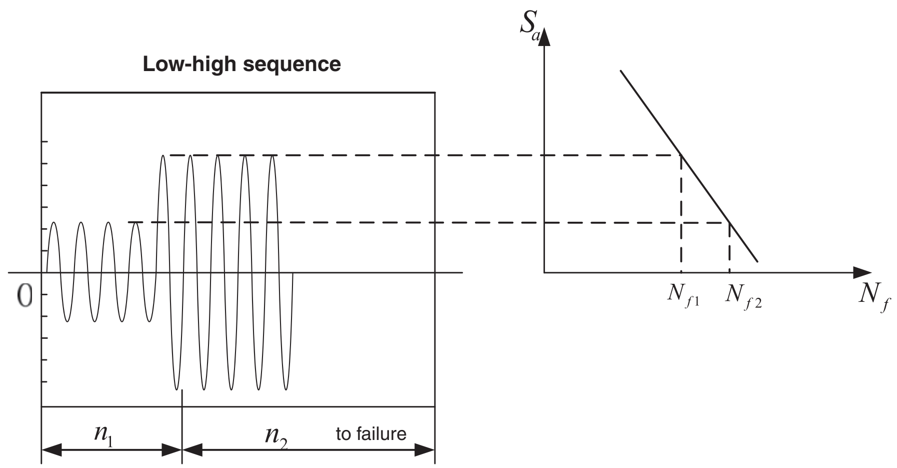

Figure 3. A two-step block loading under low-to-high load sequence.

point A to point A′ from the same fatigue driving stress (Cheng and Plumtree, 1998; Dowling, 1972; Lemaitre and Plumtree, 1979; Mesmacque et al., 2005).

If the load corresponding to the life-fraction $\frac{n_2}{N_{f2}}$ is applied from point A′ to point B′ at the life level N_{f2}, failure occurs when $S_{D_{1-2}} = C$ is reached at point B′. From Figure 2, it is clear that if a higher loading stress with a lower life along OA is first applied and followed by the lower load magnitude with a higher life along A′B′, the sum of life-fraction will be smaller than the unity. Thus, the estimated fatigue life depends on the loading sequence.

Similarly, consider a two-step low-to-high load sequence as shown in Figure 3. Let S_1 be the initial loading stress under n_1 cycles, followed by loading stress S_2 also applied n_2 cycles to eventual fatigue failure, and N_{f1},N_{f2} represent the cycles to failure under S_1 and S_2, respectively. According to Figure 4, a lower loading stress is applied first along OA and is followed by the higher loading stress along A′B, the summation of life-fractions is greater than the unity because the cycle ratio AA′ is calculated for twice. Thus, the estimated fatigue life depends on the loading sequence (Miner, 1945).

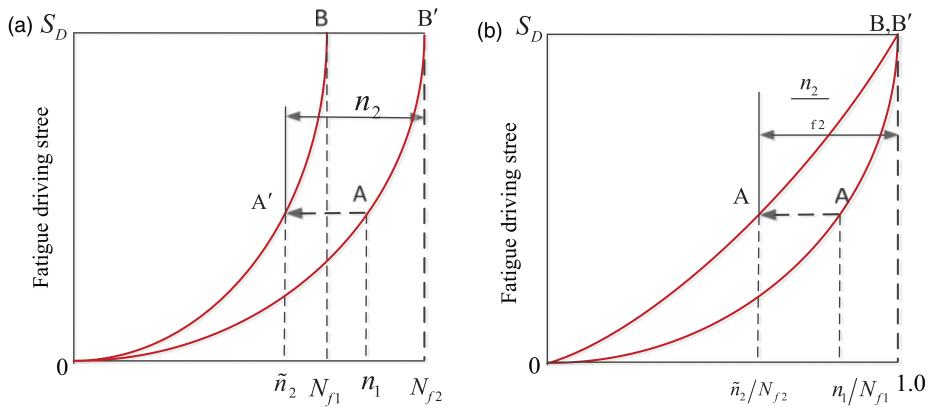

Figure 4. Non-linear damage accumulation (low-to-high load sequence). (a) N_f, cycles and (b) $\frac{n}{N_f}$, cycle fraction.

Fatigue driving stress at point A and point A′ are equal under two level loading as shown in Figure 4. According to equation (4), at point A, the fatigue driving stress (S_{D_1}) can be obtained by

$$S_{D1}= S_1^m N_{f1}^{\beta_1} \tag{6}$$

Let $\tilde{n}_2$ be the equivalent cycles under loading stress S_2 which yields to the same fatigue driving stress as the previous loading stress S_1 (i.e. $\tilde{S}_{D_2} = S_{D_1}$). Then the equivalent driving stress $\tilde{S}_{D_2}$ of loading stress S_2 is given as

$$\tilde{S}_{D_2} = S_2^m N_{f2}^{\tilde{\beta}_2} \tag{7}$$

Since $\tilde{S}_{D_2} = S_{D_1}$, equations (6) and (7) must be equaled, according to the $S-N$ curve, that is

$$S_1^m N_{f1}^{\beta_1} = S_2^m N_{f2}^{\tilde{\beta}_2} \tag{8}$$

$$S_{D_1} = \tilde{S}_{D_2} = S_1^m N_{f1}^{\frac{n_1}{N_{f1}}} = S_2^m N_{f2}^{\frac{\tilde{n}_2}{N_{f2}}} \tag{9}$$

Equation (9) implies that the driving stress that would be reached by loading stress S_2 at life-fraction of $\tilde{\beta}_2$ is equivalent to that reached by loading stress S_1 at life-fraction of β_1. Therefore, if n_2 is the subsequent number of applied cycles at loading stress S_2 after S_1, then the final driving stress due to both loads $S_{D_{1-2}}$ would be equivalent to that reached by S_2 when it was applied alone from zero life-fraction to total life-fraction of $\tilde{\beta}_2+\beta_2$. That is

$$S_{D_{1-2}} = S_2^m \cdot N_{f2}^{\tilde{\beta}_1+\beta_2} = S_2^m \cdot N_{f2}^{\frac{(\tilde{n}_2+n_2)}{N_{f2}}} \tag{10}$$

Combining equation (9) and equation (10) leads to

$$S_{D_{1-2}} = S_2^m \cdot N_{f2}^{\frac{(\tilde{n}_2+n_2)}{N_{f2}}} = S_2^m \cdot N_{f2}^{(\tilde{\beta}_2+\beta_2)} = S_1^m \cdot N_{f1}^{\beta_1} \cdot N_{f2}^{\beta_2} \tag{11}$$

When the combined driving stress $S_{D_{1-2}}$ reaches the critical value of C, we have

$$S_1^m \cdot N_{f1}^{\beta_1} \cdot N_{f2}^{\beta_2} = C \tag{12}$$

By dividing both sides of equation (12) by S_1 and noting from equation (3) that $\frac{C}{N_f} = S^m$, we obtain

$$N_{f1}^{\beta_1} \cdot N_{f2}^{\beta_2} = \frac{C}{S_1^m} = N_{f1} \tag{13}$$

Taking the natural log of both sides of equation (13) leads to

$$\beta_1 \ln N_{f1} + \beta_2 \ln N_{f2} = \ln N_{f1} \tag{14}$$

Dividing equation (14) by $\ln N_{f1}$, a new damage model is defined as

$$\beta_1 \frac{\ln N_{f1}}{\ln N_{f1}} + \beta_2 \frac{\ln N_{f2}}{\ln N_{f1}} = 1 \tag{15}$$

According to equation (15), we have

$$\beta_1 + \beta_2 \frac{\ln N_{f2}}{\ln N_{f1}} = 1 \tag{16}$$

Then,

$$\beta_2 \doteq (1 - \beta_1) \frac{\ln N_{f1}}{\ln N_{f2}} \tag{17}$$

Therefore, the sum of the life-fractions under two level loading yields

$$\beta_1 + \beta_2 = \beta_1 + (1 - \beta_1) \frac{\ln N_{f1}}{\ln N_{f2}} \tag{18}$$

According to equation (18), for high-to-low load sequence $S_1 > S_2$, $N_{f1} < N_{f2}$, resulting in $(1-\beta_1)\frac{\ln N_{f1}}{\ln N_{f2}} < (1-\beta_1)$ and hence, $\beta_1+\beta_2 < 1$. For low-to-high load sequence, $S_1 < S_2$, $N_{f1} > N_{f2}$, so that $(1-\beta_1)\frac{\ln N_{f1}}{\ln N_{f2}} > (1-\beta_1)$, hence $\beta_1 + \beta_2 > 1$.

Multi-level fatigue loading

Consider a three-step sequence loading as shown in Figure 5. Each of these curves represents a different loading condition that leads to a different cycle to failure.

In Figure 5, n_1 denotes the initial applied loading cycles at S_1, n_2 is the applied loading cycles at S_2, and n_3 the remaining cycles to eventual fatigue failure at S_3 (Kwofie and Rahbar, 2013; Lemaitre and Plumtree, 1979; Mesmacque and Garcia, 2005). The $S - N$ curve is used to obtain fatigue lives N_{f1}, N_{f2}, and N_{f3}, for each loading stress. Similarly, for a three-level fatigue loading, the third loading stress S_3 would commence at a driving stress where the second loading stress S_2 ends, that is

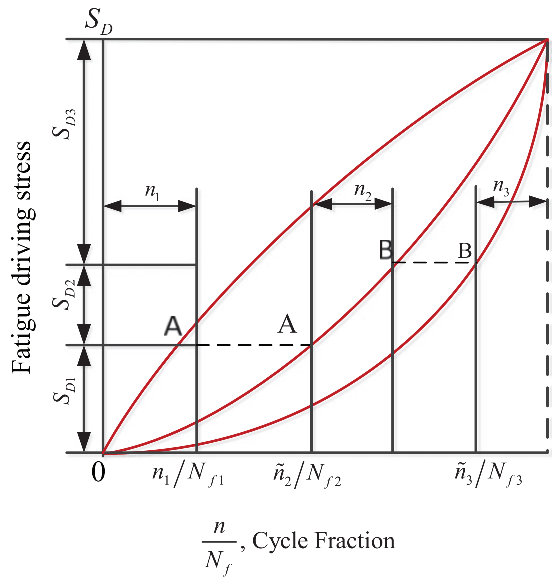

Figure 5. A block of three-level fatigue loading.

at driving stress equal $S_{D_{1-2}}$ of equation (11). Note that $\tilde{\beta}_3$ for which loading stress S_3 yields equivalent driving stress $\tilde{S}_{D_3}$ equals $S_{D_{1-2}}$. Thus the equivalent driving stress $\tilde{S}_{D_3}$ can be expressed as

$$\tilde{S}_{D_3} = S_3^m N_{f3}^{\tilde{\beta}_3} = S_{D_{1-2}} = S_1^m \cdot N_{f1}^{\beta_1} \cdot N_{f2}^{\beta_2} \tag{19}$$

If loading stress S_3 is applied for n_3 cycles, subsequent to loading stresses S_1 and S_2, then the final driving stress reached by all three loads $S_{D_{1-3}}$, would also be equivalent to that due to loading stress S_3 alone if it was applied from zero life-fraction to total life-fraction of $\tilde{\beta}_3 + \beta_3$. Therefore we have

$$S_{D_{1-3}} = S_3^m \cdot N_{f3}^{(\tilde{\beta}_3+\beta_3)} = S_3^m \cdot N_{f3}^{\tilde{\beta}_3} \cdot N_{f3}^{\beta_3} \tag{20}$$

Combining equation (19) and equation (20) leads to

$$S_{D_{1-3}} = S_1^m \cdot N_{f1}^{\beta_1} \cdot N_{f2}^{\beta_2} \cdot N_{f3}^{\beta_3} \tag{21}$$

It can be deduced from equation (21) that during cyclic loading the initial applied stress is amplified by a factor of $N_{fn}^{\beta_n}$ that depends on the applied loading stress and the number of loading cycles. Therefore for multi-level loading stress $S_1, S_2, S_3, \ldots, S_n$ with loading cycles $n_1, n_2, n_3, \ldots, n_n$ and failure lives $N_{f1}, N_{f2}, N_{f3}, \ldots, N_{fn}$, the cumulative damage caused would be due to fatigue driving stress $S_{D_{1-n}}$ by

$$S_{D_{1-n}} = S_1^m \cdot N_{f1}^{\beta_1} \cdot N_{f2}^{\beta_2} \cdot N_{f3}^{\beta_3} \cdots N_{fn}^{\beta_n} \tag{22}$$

Through dividing both sides of equation (22) by S_1 and noting from equation (3) that $\frac{C}{N} = S^m$, we obtain

$$N_{f1}^{\beta_1} \cdot N_{f2}^{\beta_2} \cdot N_{f3}^{\beta_3} \cdots N_{fn}^{\beta_n} = \frac{C}{S_1^m} = N_{f1} \tag{23}$$

Taking the natural logarithm of both sides of equation (23) leads to

$$\beta_1 \ln N_{f1} + \beta_2 N_{f2} + \beta_3 N_{f3} + \cdots + \beta_n N_{fn} = \ln N_{f1} \tag{24}$$

Rearranging equation (24) by $\ln N_{f1}$ leads to

$$\beta_1 \frac{\ln N_{f1}}{\ln N_{f1}} + \beta_2 \frac{\ln N_{f2}}{\ln N_{f1}} + \beta_3 \frac{\ln N_{f3}}{\ln N_{f1}} + \cdots + \beta_n \frac{\ln N_{fn}}{\ln N_{f1}} = 1 \tag{25}$$

In general, damage models are used to assess fatigue damage caused by previous loadings and predict the residual life of mechanical components. Almost invariably, all damage models can be expressed as a function of the life-fraction β, which is between zero and unity (Dowling, 1972). A new damage criterion is proposed according to equation (25), each item in the left-hand side of equation (25) defines the damage D_i due to applied loading stress S_i as

$$D_i = \beta_i \frac{\ln N_{fi}}{\ln N_{f1}} \tag{26}$$

where $\beta_i = \frac{n_i}{N_{fi}}$ is the expended life-fraction at S_i, N_{fi} is the failure life of S_i, and S_1 is the initial applied loading stress. The latter condition demands that $S_1 > \sigma_e$, where σ_e is the material's endurance limit. Therefore the cumulative damage D due to applied variable loads can be expressed as

$$D = \sum_{i=1}^{n} \beta_i \frac{\ln N_{fi}}{\ln N_{f1}} \tag{27}$$

Complete fracture would occur when the cumulative damage reaches unity (i.e. when $D = 1$), it yields

$$D = \sum_{i=1}^{n} \beta_i \frac{\ln N_{fi}}{\ln N_{f1}} = 1 \tag{28}$$

Equation (28) can be used for fatigue damage estimate under constant and variable amplitude loading conditions, where D is the fatigue accumulation value which considers loading sequence effects. For constant amplitude loading, $S_i = S_1$ and $\frac{\ln N_{fi}}{\ln N_{f1}} = 1$, thus equation (28) reduces to equation (29), which is the same as the Miner's rule (Zhu et al., 2013). Thus, the Miner's rule is a particular case of the presented damage model for constant amplitude loading.

$$\sum_{i=1}^{n} \beta_i = \sum_{i=1}^{n} \frac{n_i}{N_f} = 1 \tag{29}$$

In general it is required to determine the remaining life under variable cyclic loading after previous different loads have been applied. Thus, the remaining life-fraction β_i under variable cyclic loading can be estimated from equation (29) as

$$\beta_i = \frac{(1 - \beta_1)\ln(N_{f1}) - \beta_2 \ln(N_{f1}) - \beta_3 \ln(N_{f1}) - \cdots - \beta_{i-1} \ln(N_{f1})}{\ln(N_{fn})} \tag{30}$$

The remaining life n_i under variable cyclic loading can be estimated from equation (30) as

$$n_i = \frac{(1-\beta_1)\ln(N_{f1}) - \beta_2 \ln(N_{f1}) - \beta_3 \ln(N_{f1}) - \cdots - \beta_{i-1} \ln(N_{f1})}{\ln(N_{fn})}(N_{fn}) \tag{31}$$

The main advantage of the proposed model is removed inherent deficiencies in the linear accumulation rule. This is used to predict fatigue life under different loading conditions and the loading

Table 1. Life predictions by the Miner's rule, new model for 16Mn steel under two-level fatigue loading.

		Experimental				Miner's rule		New model	
Load level (MPa)	Load sequence	n_1	β_1	n_2 (result life)	β_2	Result	β_2	Result	β_2
562.9–392.3	Low–high	38,900	0.1450	75,500	0.9590	67,312	0.8550	69,492	0.870
		64,400	0.2400	70,690	0.8980	59,827	0.7600	63,792	0.801
		116,000	0.4330	62,900	0.7990	44,634	0.5670	52,978	0.676
372.65–392.3	High–low	200	0.0504	59,400	0.7548	74,730	0.9496	60,785	0.772
		1000	0.2520	48,430	0.6154	58,865	0.7480	57,341	0.728
		1700	0.4284	57,600	0.5411	44,983	0.5716	50,867	0.519

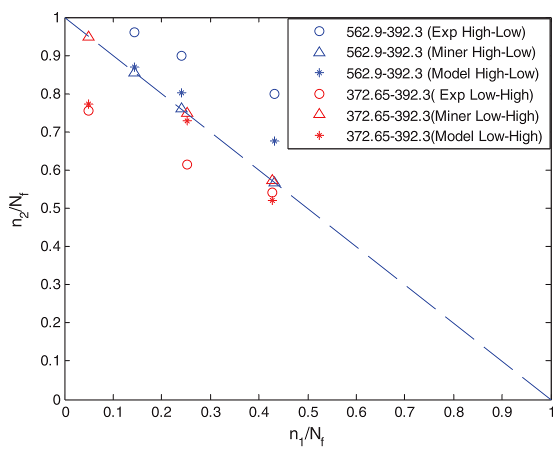

Figure 6. Comparison of experimental results and prediction by the proposed model.

Table 2. Number of cycles to failure at different stress amplitudes.

	Block number						
	1	2	3	4	5	6	7
Stress amplitude	2086	2000	1655	1103	965	900	827
Fatigue life	891	1160	3809	48,645	112,257	174,400	296,594

history effects are taken into account in this model. Moreover, the proposed model is much simpler to use than other non-linear models.

Validation of the proposed model

Two-level fatigue loading condition

In this section, the prediction results by the proposed model are compared with experimental data available under two-level fatigue loading in the literature.

According to equation (16) and equation (17), the damage accumulation model under two-level fatigue loading can be described as

$$\beta_2 = (1 - \beta_1)\frac{\ln N_{f1}}{\ln N_{f2}} \tag{32}$$

The remaining life n_i under two-level fatigue loading can be estimated as

$$n_2 = (1 - \beta_1)\frac{\ln N_{f1}}{\ln N_{f2}} N_{f2} \tag{33}$$

where n_1, n_2 indicate the cycle numbers at S_1 and S_2, N_{f1} and N_{f2} represent the fatigue failure life at the corresponding load level, respectively.

Table 3. Steel 300CVM life predictions under two-level fatigue loading (low-to-high load sequence).

S_1 − 1165, S_2 − 2000 MPa					S_1 − 1103, S_2 − 2000 MPa				
	Experiments		Model			Experiments		Model	
β_1	β_2	$\sum\beta_i$	β_2	$\sum\beta_i$	β_1	β_2	$\sum\beta_i$	β_2	$\sum\beta_i$
0.196	0.806	1.002	0.939	1.135	0.13	1.008	1.138	0.915	1.046
0.396	0.727	1.123	0.706	1.101	0.267	0.8160	1.083	0.779	1.046
0.586	0.493	1.079	0.483	1.069	0.533	0.898	1.431	0.712	1.245
0.786	0.226	1.012	0.250	1.036	1.07	0.024	1.093	0	1.07
S_1 − 827, S_2 − 2000 MPa					S_1 − 900, S_2 − 2086 MPa				
0.196	0.796	0.992	0.813	1.009	0.196	0.850	1.109	0.927	1.123
0.241	0.830	1.071	0.796	1.037	0.339	0.889	1.228	0.849	1.188
0.401	0.764	1.165	0.643	1.044	0.508	0.874	1.383	0.821	1.329
0.812	0.847	1.659	0.435	1.247	0.678	0.794	1.471	0.571	1.249
	Experiments					Model			
β_1	β_2		$\sum\beta_i$			β_2		$\sum\beta_i$	
0.241	0.776		1.017			0.832		1.073	
0.419	0.857		1.276			0.8409		1.259	
0.839	0.776		1.615			0.323		1.17	

Results from Shang and Yao. The test material is normalized 16Mn steel and tests were carried out under two-level fatigue loading. The high-to-low and low-to-high load spectrum were 372.65–392.3 MPa and 562.9–392.3 MPa, respectively. The tests were performed on a PQ1-6 rotating bending testing machine in stress-control mode under fully reversed loading conditions (Shang and Yao, 1998, 1999). The ultimate tensile strength was $\delta_b = 570.7$ MPaand the fatigue limit of smooth specimen under rotating bending loading was δ_{-1}=280.8 MPa. The details of test conditions can be found in (Shang and Yao, 1998; 1999).

A comparison of experimental results, Miner, and new model predicted life is given in Table 1.

By comparing the experimental data with the predicted results of Miner and new model, it is obvious that most prediction results are close to the experimental data as shown in Figure 6.

Results from Krouse and Moore. Krouse (1967) got the following results for maraging steel with material properties: yield stress δ_s=2098 MPa, and ultimate stress $\delta_b = 2590$ MPa. Numbers of cycles to failure at different stress amplitudes are given in Table 2.

Table 4. Steel 300CVM life predictions under two-level fatigue loading (high-to-low load sequence).

$S_1 - 2000$, $S_2 - 1165$ MPa					$S_1 - 2000$, $S_2 - 1103$ MPa				
	Experiments		Model			Experiments		Model	
β_1	β_2	$\sum \beta_i$	β_2	$\sum \beta_i$	β_1	β_2	$\sum \beta_i$	β_2	$\sum \beta_i$
0.16	0.696	0.856	0.718	0.878	0.151	0.58	0.731	0.552	0.706
0.312	0.508	0.82	0.588	0.900	0.16	0.383	0.543	0.549	0.709
0.482	0.375	0.857	0.443	0.925	0.32	0.383	0.703	0.447	0.764
0.642	0.258	0.9	0.306	0.940	0.508	0.214	0.722	0.321	0.829
0.794	0.25	1.044	0.176	0.970	0.633	0.196	0.829	0.240	0.873
$S_1 - 2000$, $S_2 - 724$ MPa					$S_1 - 2000$, $S_2 - 1379$ MPa				
0.071	0.723	0.794	0.697	0.768	0.071	0.803	0.874	0.832	0.903
0.16	0.187	0.347	0.342	0.502	0.16	0.741	0.901	0.7510	0.911
0.348	0.089	0.437	0.103	0.451	0.294	0.455	0.749	0.631	0.925
0.464	0.08	0.544	0.126	0.59	0.482	0.366	0.848	0.474	0.956
0.625	0.026	0.651	0.037	0.662	0.462	0.294	0.936	0.393	0.855
					0.785	0.25	1.035	0.176	0.961

	Experiments		Model	
β_1	β_2	$\sum \beta_i$	β_2	$\sum \beta_i$
0.16	0.517	0.677	0.635	0.795
0.16	0.482	0.642	0.635	0.795
0.32	0.258	0.578	0.362	0.682
0.473	0.142	0.615	0.281	0.754
0.642	0.089	0.731	0.190	0.832
0.803	0.062	0.865	0.105	0.908

A comparison of experimental results under low-to-high load sequence and new model prediction life is given in Table 3.

A comparison of experimental results under high-to-low load sequence and new model prediction life is given in Table 4.

The comparisons between the results obtained from the model and the experimental data are shown in Figure 7. The estimated results from the model are in good agreement with the experimental results. For low-to-high loading sequence, $\beta_1 + \beta_2 > 1$. For high-to-low loading sequence, $\beta_1 + \beta_2 < 1$.

Multi-level fatigue loading condition

In order to verify the descriptive ability of equation (27) under multi-level fatigue loading, the experimental data in Chaboche and Lesne (1988), four cyclic stress levels were considered and three different load sequences were applied. The results of aluminum alloys 6082 T6 for cumulative damage under block loading are given in Table 5. The experimental data under the block loading are given in Table 6.

According to equation (31), the remaining life n_i under four-level fatigue loading can be estimated as:

$$n_4 = \frac{\left[(1-\beta_1)\ln(N_{f1}) - \beta_2\ln(N_{f1}) - \beta_3\ln(N_{f1})\right]N_{f4}}{\ln(N_{f4})} \tag{34}$$

The experimental and predicted results under four-level block loading, for increasing, decreasing and random blocks for Al-6082-T6 alloy are shown in Table 6. Once again the proposed model and experimental results predict $\sum\beta_i > 1$ for low-to-high loading sequence and $\sum\beta_i < 1$ for

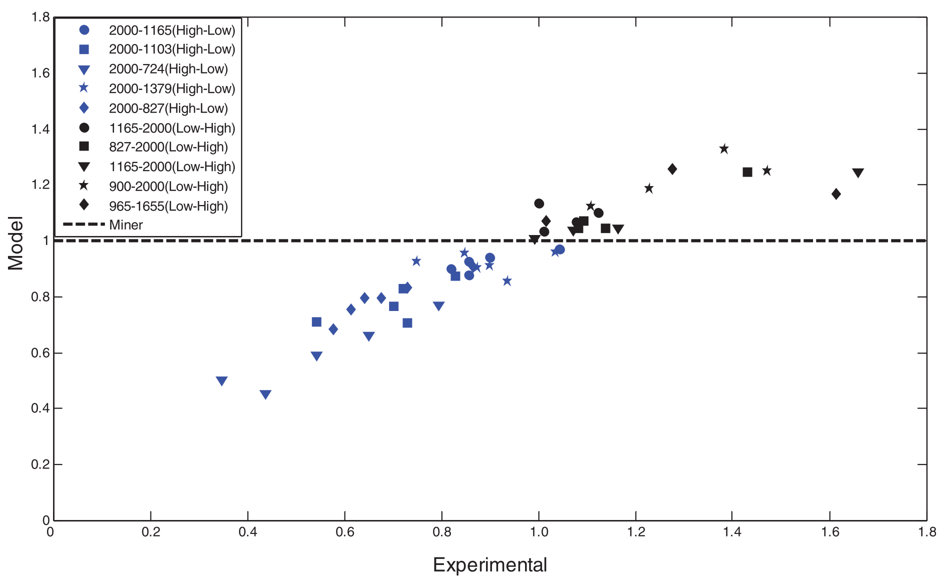

Figure 7. Comparison of experimental results and prediction by the proposed model.

high-to-low load sequence, which can be depicted as shown in Figure 8. It should be pointed out that the proposed model can be better used for fatigue life evaluation under multi-level load spectrums.

The proposed fatigue damage model improves the deficiencies in LDR by considering load interaction and sequence effects. Miner's rule has the simplest form and is easiest to be used for calculation. But Miner's rule does not account for the loading history, so the estimated results deviate from the experimental results. Although Marco and Starky model improve the deficiencies in LDR by considering load interaction and sequence effects, which are usually computationally expensive. Moreover, the proposed model is much simpler to use than other non-linear models.

According to the applicable conditions of the proposed model, it is valid for most metallic materials under uniaxial loading, such as carbon steels, cast irons, and alloy steels. Even though the proposed uniaxial fatigue damage cumulative model is verified by some experimental data, and good results are obtained, there is further need to verify the proposed model for amount of materials. In addition, the model proposed in this paper only suits the ductile material. For the brittle material, the further study

Table 5. Number of cycles to failure at different stress amplitudes.

Block number				
	1	2	3	4
S_{max}(MPa)	240	260	280	305
N_f	394,765	180,660	87,612	38,000

Table 6. Aluminum alloys 6082 life predictions under multi-level fatigue loading condition.

				Low-to-high load sequence							
01 load	240(n_1)	260(n_2)	280(n_3)	305							
				Experiments		Miner rule		Marco and Starky model		Proposed model	
				$\sum \beta_i$	(n_4)	$\sum \beta_i$	(n_4)	$\sum \beta_i$	(n_4)	$\sum \beta_i$	(n_4)
Cycles	103,000	26,258	19,427	1.1585	16,800	1	14,140	1.3142	19,140	1.2145	18,250
				High-to-low load sequence							
02 load	305(n_1)	280(n_2)	260(n_3)	240							
				Experiments		Miner rule		Marco and Starky model		Proposed model	
				$\sum \beta_i$	(n_4)	$\sum \beta_i$	(n_4)	$\sum \beta_i$	(n_4)	$\sum \beta_i$	(n_4)
Cycles	10,950	19,427	26,258	0.8271	52,500	1	136,050	0.745	35,420	0.8911	40,038
				Random load sequence							
03 load	280(n_1)	305(n_2)	260(n_3)	240							
				Experiments		Miner rule		Marco and Starky model		Proposed model	
				$\sum \beta_i$	(n_4)	$\sum \beta_i$	(n_4)	$\sum \beta_i$	(n_4)	$\sum \beta_i$	(n_4)
Cycles	19,427	10,950	26,258	0.8023	43,400	1	136,050	1.278	98,700	0.962	112,457

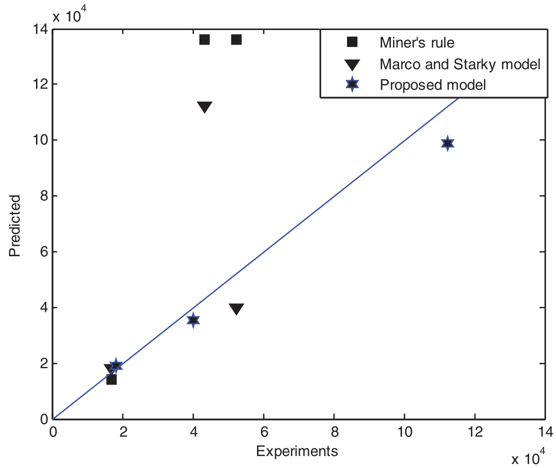

Figure 8. Comparison of experimental results and prediction by the proposed model.

also is needed. Besides, the application of this model to fatigue damage analysis under multiaxial cyclic loading (including non-proportional loading) will be further evaluated.

However, model predictions for high-to-low load sequence and low-to-high load sequence show lesser deviations from experimental data than that under random load sequence. Since the model in this paper is based on deterministic method, whereas in practice, the fatigue progress under random loading is uncertain in nature. Thus, further research on the load interaction and sequence effects on fatigue life under random load spectrums is desirable.

Conclusion

The proposed model in this paper improves the inherent deficiencies in the linear damage accumulation rule and still maintains its simplicity in its application. The main advantage of this model is the ease of use which request only the Wöhler curve (S–N). Moreover, it does not require too many material property parameters and it takes into account the loading history effects. The model correctly assesses the fatigue life under different loading conditions. Comparing with the experimental data, the prediction results by the proposed model are in a good agreement. The method gives more accurate and simple predictions for the engineer.

Conflict of interest

None declared.

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