

# A Practical Method for Determining the Corten–Dolan Exponent and Its Application to Fatigue Life Prediction

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**Abstract.** Based on the derivation and calculation of the Corten–Dolan exponent  $d$ , a practical method of determining its value is proposed. This exponent depends not only upon the materials, but also upon the load spectrums. Therefore its value is obtained by a function which decreases with increasing stress amplitude. This exponent was investigated through analysis of fatigue damage evolution to determine its parameters. The proposed method has been effectively proved by experimental data from literature. Utilization of the modified Corten–Dolan’s model significantly improves its life prediction capability when compared to the conventional model where that exponent was assumed to be constant.

**Keywords.** fatigue, Corten–Dolan’s model, nonlinear fatigue damage accumulation, life prediction.

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## Nomenclature

$a$	Damage propagation exponent
$d$	Corten–Dolan exponent
$D$	Total fatigue damage
$m$	Number of damaged nuclei
$r$	Coefficient of damage propagation rate depending on stress level
$N_1$	Number of cycles to failure at $\sigma_1$
$N_g$	Total number of cycles to failure
$n_i$	Number of cycles that incurred at $\sigma_i$
$n_r$	Total number of cycles in each repeated block

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$\sigma_1$	Maximum stress
$\sigma_i$	The $i$ th level stress amplitude, $i = 1, 2, \dots, k$
$\sigma_b$	Ultimate tensile strength
$\alpha$	Fraction of $n_r$ cycles that incurred at $\sigma_1$
$\alpha_i$	Proportions of the stresses at $\sigma_i$ , $\alpha_i = n_i/N_g$
$\mu$	Material constant
$\delta_f$	Initial static strength
$\lambda$	Load-interaction factor

## 1 Introduction

Reliable lifetime prediction and fatigue damage analysis are particularly important in the design, optimization and safety assessments of engineering structures. Fatigue damage increases with applied cycles in a cumulative manner which results in fracture. As the possible damage accumulates, the remaining lifetime under future loads becomes more limited. In order to estimate the remaining life, an assessment of the accumulated damage is required. The common purpose of damage accumulation analysis is to predict how far into the loading sequence a material can remain coherent before suffering catastrophic failure under a given complete loading spectrum.

Until now a large number of theories and models have been proposed to predict the fatigue life of engineering structures. In general, cumulative fatigue damage theories can be classified into two categories: (1) linear damage cumulative and (2) nonlinear damage cumulative theories. Comprehensive reviews of the cumulative fatigue damage and life prediction theories have been conducted by Fatemi et al. [1] and Cui [2]. The linear damage rule, also called Palmgren–Miner rule (just Miner rule for short), is commonly used in analyzing cumulative fatigue damage. However, Miner’s rule is widely acknowledged to be inadequate. This is partially based upon its empirical nature and it may result in evident errors due to the effects of loading sequence and load-interaction. Accordingly, a new physics-based linear damage rule was developed to consider the strengthening and damaging of low amplitude loads under different sequences using fuzzy sets theory [3, 4]. Fatigue damage assumes that material properties are continuously deteriorating under cyclic loading and the damage variable is dependent on the cyclic stress-strain relationship [5]. Based on these characteristics of fatigue damage, some

nonlinear cumulative damage theories are presented and can be categorized into six groups:

- (1) Damage theories based on the physical properties degradation of materials [5–7];
- (2) Damage curve approaches [8];
- (3) Continuum damage mechanics (CDM) approaches [6, 9, 10];
- (4) Energy based damage theories [11–14];
- (5) Theories accounting for load interaction effects [15–18];
- (6) Thermodynamic entropy based damage theories [19, 20].

Among these methods, Miner's rule and the Corten–Dolan's model have been widely used in engineering [21]. Comparing with other theories, the Corten–Dolan's model has a higher precision and wider application for life prediction [21, 22]. However, the Corten–Dolan's model sometimes gives prediction results with great error due to the method of determining the exponent  $d$  value. Currently, the determination of the  $d$  value is often empirical or semi-empirical, which needs more experimental tests and lacks a theoretical basis [21, 23]. It was suggested that if  $d$  was determined experimentally from two-stress level tests, the same value of  $d$  could be used to predict the life of a specimen in a multi-level test. Marsh [24, 25], however, found that the value of  $d$  obtained from two-stress level tests could not always be used to predict the life in more complicated block tests and in fact would lead to unsafe predictions in some conditions. Nevertheless Marsh concluded that the concept was useful and provided an appropriate value of  $d$ , which depended on the type of cumulative damage test being performed. Even though the Corten–Dolan's model could provide better predictions than other theories, these predictions were not necessarily safe. It is because experience in determining the  $d$  value is required to use this model and considerable caution is necessary in its application to extreme stress histories. Further improvements to the method of determining the exponent  $d$  in Corten–Dolan's model is needed.

In general, the value of  $d$  is given by two-stress level tests or semi-empirical. The objective of the present paper is to determine the Corten–Dolan exponent  $d$  value by a function of loading stresses instead of being constant. In this paper, first the Corten–Dolan's model capabilities is introduced in Section 2. In Section 3, derivation and calculation of the exponent  $d$  are given. Then, by introducing the effects of maximum stress and load interaction into the value of  $d$ , a nonlinear cumulative fatigue damage model is proposed on the basis of Corten–Dolan's model in Section 4. In Section 5, the proposed model is verified using two sets of experimental data. The predicted lives were then compared with actual test data and a good agreement between

them was observed. Finally in Section 6, conclusions are presented.

## 2 Corten–Dolan Cumulative Damage Theory

Fatigue is a damage accumulation process in which material property deteriorates continuously under loading. For fatigue damage, the following assumptions are well supported by data [25]:

- (1) A nucleation period (possibly a small number of cycles) is required to initiate permanent fatigue damage.
- (2) The number of nuclei (submicroscopic voids) that form throughout the member grows with the increase in the stress level.
- (3) Damage for a given stress amplitude propagates at an increasing rate with increased numbers of cycles.
- (4) The rate of damage per cycle grows with increasing stresses.
- (5) The total damage that constitutes failure in a given member is a constant for all possible stress histories that could be applied.
- (6) Damage continues to be propagating at stress levels that are lower than the minimum stress required to initiate damage.

According to the above assumptions, a nonlinear accumulative damage model was proposed by Corten and Dolan [15] to account for load interaction effects, which are derived from the microcosmic damage model of fatigue. The damage formula is expressed in terms of the applied cycles by a power-law relationship

$$D = mrN^a, \quad (1)$$

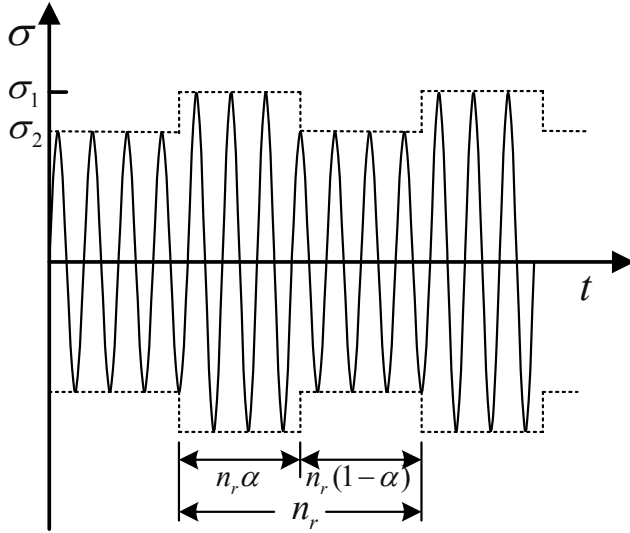
where  $D$  is the damage,  $N$  is the applied cycles and  $r$  and  $a$  are functions of stress level.

By a process of summing damage increments, equating the total damage to the damage at failure in a constant amplitude test at  $\sigma_1$ , and incorporating some simplifying assumptions based on numerous two-stress level block tests as shown in Fig. 1, the theory predicts the life  $N$  is given by

$$N_g = \frac{N_1}{\alpha + (1 - \alpha) (\sigma_2/\sigma_1)^d}. \quad (2)$$

Similarly, using these concepts and the assumptions stated earlier, Corten and Dolan developed an expression for estimating the number of cycles to failure for repeated blocks of many different stress levels as

$$N_g = \frac{N_1}{\sum_{i=1}^k \alpha_i (\sigma_i/\sigma_1)^d}. \quad (3)$$



**Figure 1.** A simple two-stress level load spectrum used in Corten–Dolan's model.

For comparison with Miner's rule, it is convenient to express Eq. (3) as the following:

$$\left(\frac{n_1}{N_1}\right) + \left(\frac{n_2}{N_1}\right) \left(\frac{\sigma_2}{\sigma_1}\right)^d + \left(\frac{n_3}{N_1}\right) \left(\frac{\sigma_3}{\sigma_1}\right)^d + \dots + \left(\frac{n_i}{N_1}\right) \left(\frac{\sigma_i}{\sigma_1}\right)^d = 1 \quad (4)$$

or simply,

$$\frac{\sum n_r}{N_g} = \sum_{i=1}^k \frac{n_i}{N_1} \left(\frac{\sigma_i}{\sigma_1}\right)^d. \quad (5)$$

Failure of the component is assumed to occur when  $\sum n_r/N_g = 1$ . According to the Corten–Dolan's model, an experimental value of  $d$  must be obtained for each material of interest. The parameter  $d$  is interpreted as the inverse slope of a hypothetical  $S/N$  curve (whose finite-life portion is assumed to be linear on log-log axes), which allows for the interaction between high and low stress levels. Corten and Dolan's two-stress level data on steels gave a mean value of  $d \approx 6.57$  (with a range from 6.2 to 6.9), which is compared to a value of the inverse slope of the constant stress amplitude data on log-log axes of about 7.5 [25]. The value of  $d$  can be determined experimentally from tests and by comparing with the value of the inverse slope of two-stress level  $S-N$  curve, such as mean value for Al 7075-T6 of  $d = 6.0$  [26].

According to Corten–Dolan's model, the exponent  $d$  is assumed to be constant. However, recent studies [22, 23] have shown that the Corten–Dolan exponent  $d$  is not simply a material constant which depends not only upon the materials. But it also depends upon the levels of load spectrum while its value can be obtained by a function of maximum

stress. Results indicated that the effects of mean stress on the  $d$  value are not evident [23]. Compared with the condition of variable amplitude loading, the maximum stress has primary influence on the exponent  $d$ . The exponent  $d$  is smaller at a higher maximum stress than that at a lower one. To apply the Corten–Dolan's model more accurately and account for the fatigue damage evolution, it is important to determine the value of exponent  $d$  within the Corten–Dolan's model effectively.

### 3 Derivation and Calculation of the Exponent $d$

Under two-stress level block tests, after substituting test data into Eq. (2) which can be rewritten as:

$$d = \frac{\log [N_g (1 - \alpha) / (N_1 - N_g \alpha)]}{\log (\sigma_1 / \sigma_2)}. \quad (6)$$

Note that it is easy to obtain the value of  $d$  directly from Eq. (6). But according to the calculation of  $d$ , it is difficult to characterize the physical meaning of exponent  $d$  [23]. Thus, Eq. (6) is rewritten as

$$d = \frac{\log \left[ \frac{N_g (1 - \alpha) + N_g \alpha (\sigma_1 / \sigma_2)^d}{N_g \alpha + N_g (1 - \alpha) (\sigma_2 / \sigma_1)^d} \right]}{\log (\sigma_1 / \sigma_2)}, \quad (7)$$

where  $N_g \alpha$  and  $N_g (1 - \alpha)$  are the cycles incurred at the higher stress level  $\sigma_1$  and the second stress level  $\sigma_2$ .  $N_g \alpha (\sigma_1 / \sigma_2)^d$  and  $N_g (1 - \alpha) (\sigma_2 / \sigma_1)^d$  correspond to the equivalent cycles of the first segment  $N_g \alpha$  conversion to the same damage produced at  $\sigma_2$  and the second segment  $N_g (1 - \alpha)$  conversion to the same damage produced at  $\sigma_1$ , respectively.

Thus the physical meaning of  $N_g (1 - \alpha) + N_g \alpha (\sigma_1 / \sigma_2)^d$  is the conversion of the total number of cycles  $N_g$  under two-stress level tests into the equivalent cycles to failure at  $\sigma_2$ . In a similar way,  $N_g \alpha + N_g (1 - \alpha) (\sigma_2 / \sigma_1)^d$  is the equivalent cycles to failure at  $\sigma_1$  for  $N_g$ . Now let

$$N'_2 = N_g (1 - \alpha) + N_g \alpha (\sigma_1 / \sigma_2)^d, \quad (8)$$

$$N'_1 = N_g \alpha + N_g (1 - \alpha) (\sigma_2 / \sigma_1)^d. \quad (9)$$

This allows Eq. (7) to be rewritten as:

$$d = \frac{\log [N'_2 / N'_1]}{\log (\sigma_1 / \sigma_2)}. \quad (10)$$

As noted in Eq. (10) the physical meaning of exponent  $d$  is interpreted as the inverse slope of a two-stress level vs. number of cycles plot (also called  $S-N$  curve) on log-log axes. For the two-stress level vs. number of cycles plot,  $N'_1$

and  $N'_2$  are the equivalent cycles at  $\sigma_1$  and  $\sigma_2$  under two-stress level tests.

Combining Eqs. (2) and (9) gives

$$N'_1 = N_1. \quad (11)$$

The equivalent cycles to failure at  $\sigma_1$  for  $N_g$  under the two-stress level tests is equal to the fatigue life at the constant amplitude  $\sigma_1$ , this then gives

$$d = \frac{\log [N'_2/N'_1]}{\log (\sigma_1/\sigma_2)} = \frac{\log [N'_2/N_1]}{\log (\sigma_1/\sigma_2)}. \quad (12)$$

The prediction of fatigue life via Miner rule in Eq. (13) is based on the  $S$ - $N$  curve under constant amplitude cyclic loading as in Eq. (14),

$$\sum_{i=1}^k \frac{n_i}{N_i} = 1, \quad (13)$$

$$N_1 \sigma_1^\beta = N_2 \sigma_2^\beta = \dots = N_i \sigma_i^\beta, \quad (14)$$

where  $\beta$  is the slope of the original  $S$ - $N$  curve, and Eq. (14) can be rewritten as

$$N_i = N_1 (\sigma_1/\sigma_i)^\beta. \quad (15)$$

Substituting Eq. (15) into Eq. (13), and integrating gives

$$\sum_{i=1}^k \frac{n_i}{N_i} = \sum_{i=1}^k \frac{n_i}{N_1 (\sigma_1/\sigma_i)^\beta} = N_g \sum_{i=1}^k \frac{\alpha_i (\sigma_i/\sigma_1)^\beta}{N_1} = 1. \quad (16)$$

This then gives

$$N_g = \frac{N_1}{\sum_{i=1}^k \alpha_i (\sigma_i/\sigma_1)^\beta}. \quad (17)$$

When the prediction of fatigue life is based on the  $S$ - $N$  curve under two-stress level cyclic loading (Eq. (12)),  $N'_2 = N_1 (\sigma_1/\sigma_2)^d$ , similarly,

$$N'_i = N_1 (\sigma_1/\sigma_i)^d. \quad (18)$$

Substituting Eq. (18) into Miner's rule in Eq. (13) gives:

$$N_g = \frac{N_1}{\sum_{i=1}^k \alpha_i (\sigma_i/\sigma_1)^d}. \quad (19)$$

After comparing Eq. (19) with Eq. (17), it can be easily noted that Eq. (19) is the Corten–Dolan's model. The Corten–Dolan's model modifies the slope of  $S$ - $N$  curve. According to Eqs. (16)–(19), the difference between theories of Miner and Corten–Dolan is that the former predicts

fatigue life based on  $S$ - $N$  curve under constant amplitude cyclic loading. But the latter is based on  $S$ - $N$  curve under two-stress level cyclic loading. The exponent  $\beta$  in Eq. (17) was determined by the inverse slope in the  $S$ - $N$  curve under constant amplitude cyclic loading. It corresponds to the exponent  $d$  in Eq. (19) and is determined by the inverse slope in the  $S$ - $N$  curve under two-stress level cyclic loading. Combining with the expressions for exponent  $d$  in Eq. (12) and  $\beta$  in Eq. (14), the difference between them is the parameters  $N'_2$  and  $N_2$ .  $N_2$  is the fatigue life at constant amplitude  $\sigma_2$ , but  $N'_2$  is the equivalent fatigue life at  $\sigma_2$  under two-stress level cyclic loading. Therefore, the value of exponent  $d$  reflects the impacts of maximum stress on the follow-up stresses and the load interaction between high and low stress levels. In order to explain the above-described behavior, a new method for determining the exponent  $d$  is proposed in the next section.

#### 4 A New Method for Determining the Exponent $d$

As the Corten–Dolan exponent  $d$  has a great impact on the estimated life, the value of  $d$  should be accurately determined in its application for life prediction. The experimental results in [22,23] have shown that the value of  $d$  changed significantly for different specimens of the same material and for different load spectrums. Hence, it is inappropriate to view the exponent  $d$  as a material constant in Corten–Dolan's model.

In this paper, the exponent  $d$  is defined as a function that decreases with the increasing loading stress amplitude. Details of the exponent  $d$  have been discussed based on the deduction of fatigue damage evolution and the principles of determining the  $d$  value.

The fatigue life of most materials has been observed to decrease more rapidly when the loading sequence is repeatedly changed after only few loading cycles [27]. It was classified as a "load interaction effect" [17]. It is assumed that the load interaction effect is a result of the damage that occurs during those "transition cycles" between different constant amplitude segments, where the magnitude of the mean stress changes. These interactions, which are highly dependent upon the loading sequence, make the prediction of fatigue life under variable amplitude loadings more complex than that under constant amplitude loading. This effect is accounted for in the proposed model through a "load-interaction factor".

Additionally, considering the following assumptions from tests:

- (1) The exponent  $d$  is a material parameter which is related with the material properties;
- (2) To a great extent, the value of  $d$  depends upon the levels of load spectrum. And very roughly, the value of



exponent  $d$  is inversely proportional to the stress amplitude of loading  $\sigma_{\max i}$ ;

(3) The mean stresses have little effect on the exponent  $d$ .

Using some experimental results and simplification [22, 23], a simple form of  $d$  to account for the load interaction effect can be derived as:

$$d(\sigma_i) = \mu \left( \frac{\sigma_1}{\sigma_i} \right)^\lambda \left( \frac{\delta_f}{\sigma_i} \right)^{1-\lambda} \quad (20)$$

where  $\lambda$  is the load-interaction factor which accounts for the loss in life that may result from sequencing effects, where  $0 < \lambda < 1$ . Constant amplitude loading corresponds to  $\lambda = 0$ . The effects of material properties on the fatigue life is characterized by the parameter  $\nu = \frac{\delta_f}{\sigma_i}$ ,  $\nu$  is the normalized peak stress based on the initial static strength  $\delta_f$  of the specimen. The effects of loading history on the fatigue life is therefore characterized by the parameter  $\gamma = \frac{\sigma_1}{\sigma_i}$ ,  $\gamma$  is the normalized peak stress based on the maximum stress  $\sigma_1$ .

Fatigue is a damage accumulation process in which material properties (such as strength) are continuously deteriorated under cyclic loading. According to residual strength degradation of material during fatigue, the effects of material properties on fatigue life are characterized by the initial static strength  $\delta_f$ , which is a measure of static strength to withstand fatigue loading at the initial state of material and often used in residual strength degradation models [28].

Considering the load interaction effects, rearranging terms in Eq. (20) leads to a new method for determining the value of exponent  $d$  as follows

$$d(\sigma_i) = \mu \frac{\sigma_1^\lambda \delta_f^{1-\lambda}}{\sigma_i} \quad (21)$$

By substituting Eq. (21) into Eq. (3), fatigue life expressed by Corten–Dolan’s model can be rewritten as shown in the following equation, in a form that defined the exponent  $d$  as a function

$$N_g = \frac{N_1}{\sum_{i=1}^k \alpha_i \left( \sigma_i / \sigma_1 \right)^\mu \frac{\sigma_1^\lambda \delta_f^{1-\lambda}}{\sigma_i}} \quad (22)$$

where the initial static strength  $\delta_f$  is experimentally determined. For metals, it can be obtained approximately by [29]

$$\delta_f \approx \sigma_b + 350 \text{ MPa} \quad (23)$$

It should be noted that the value of  $d$  varies significantly under different load spectrums. The parameter  $d$  within Corten–Dolan’s model is a variable instead of being constant, thus it can be called the dynamic Corten–Dolan’s model.

Recent studies [16–18, 30] show that the load interaction effect is a significant contributor to strength and life

loss for low-high and high-low two-stress level tests with various load block sizes. The load-interaction factor,  $\lambda$ , is a nondimensional value determined by comparing fatigue life data from small block and large block two-stress level fatigue tests with predicted results. According to Corten–Dolan’s 2<sup>nd</sup> and 4<sup>th</sup> key assumptions, a single value of  $\lambda$  is proportional to the damage extent of a material caused by the maximum stress  $\sigma_1$ , which can be given approximately by

$$\lambda \propto \frac{n_1}{N_1} \quad (24)$$

For simplicity, the value of  $\lambda$  can be obtained by  $\lambda = n_1/N_1$  as an estimate in engineering. For constant amplitude loading, there is no load interaction effect. Then  $\lambda = 0$  and the obtained result of  $d$  has no physical meanings. Eq. (22) is reduced to Miner’s rule

$$D = \frac{n}{N} = 1 \quad (25)$$

Using Eq. (5), the ratio of tested life to the life predicted by the dynamic Corten–Dolan’s model is as follows

$$\left( \frac{\sum n_r}{N_g} \right)^* = \sum_{i=1}^k \frac{n_i}{N_1} \left( \frac{\sigma_i}{\sigma_1} \right)^\mu \frac{\sigma_1^\lambda \delta_f^{1-\lambda}}{\sigma_i} \quad (26)$$

By combination of Eqs. (22) and (24), both the effects of load interaction and sequencing on the fatigue life can be characterized by dynamic value of  $d$ . To a certain extent, it considers the mechanism of load interaction effect on the fatigue life.

### 5 Fatigue Life Prediction Using Dynamic Corten–Dolan’s Model

The validity and accuracy of the proposed method for determination of Corten–Dolan exponent  $d$  is evaluated through a comparison of predicted and observed fatigue life distributions for metallic specimens subjected to various two-stress level uniaxial fatigue loadings [31].

In order to verify the descriptive ability of Eq. (22), two categories of experimental data from smooth specimens of normalized 45 steel and 16Mn steel were used to verify the dynamic Corten–Dolan’s model. The details of test conditions are reported in [31–33].

For normalized 45 steel, the high-low load spectrum was 331.463–284.4MPa and low-high load spectrum was 284.4–331.463MPa. For normalized 16Mn steel, the high-low load spectrum was 562.9–372.65MPa and low-high load spectrum was 372.65–392.3MPa. The tests were performed on a PQ1-6 rotating bending testing machine in stress-control mode under fully reversed loading conditions. The mechanical properties of 45 Steel and 16Mn

Steel are as follows. For normalized 45 steel, the ultimate tensile strength  $\sigma_b = 598.2$  MPa and the fatigue limit of smooth specimen under rotating bending loading  $\sigma_{-1} = 262.8$  MPa. For 16Mn steel, the ultimate tensile strength  $\sigma_b = 570.7$  MPa and the fatigue limit of smooth specimen under rotating bending loading was  $\sigma_{-1} = 280.8$  MPa. Test parameters and results are presented in Tables 1 and 2.

When using the proposed method in Eq. (21), material constant  $\mu$  was determined from limited amount of experimental data and the load-interaction factor  $\lambda$  was obtained by Eq. (24). According to the experimental data in Table 1 and mechanical properties of 45 Steel, the value of  $d$  in Eq. (21) for 45 steel is given by

$$d(\sigma_i)_{45 \text{ steel}} = 5.6186 \frac{\sigma_1^\lambda (9.482 \times 10^8)^{1-\lambda}}{\sigma_i} \quad (27)$$

For different two-stress level tests and load sequences, the fitted life prediction model for 45 steel is as follows

$$\left(\frac{\sum n_r}{N_g}\right)_{45 \text{ steel}}^* = \sum_{i=1}^k \frac{n_i}{N_1} \left(\frac{\sigma_i}{\sigma_1}\right)^{5.6186 \frac{\sigma_1^\lambda (9.482 \times 10^8)^{1-\lambda}}{\sigma_i}} \quad (28)$$

Similarly, the value of exponent  $d$  in Eq. (21) for 16Mn steel can be expressed as

$$d(\sigma_i)_{16\text{Mn}} = 3.8970 \frac{\sigma_1^\lambda (9.207 \times 10^8)^{1-\lambda}}{\sigma_i} \quad (29)$$

The fitted life prediction model for 16Mn steel can be written as follows

$$\left(\frac{\sum n_r}{N_g}\right)_{16\text{Mn}}^* = \sum_{i=1}^k \frac{n_i}{N_1} \left(\frac{\sigma_i}{\sigma_1}\right)^{3.8970 \frac{\sigma_1^\lambda (9.207 \times 10^8)^{1-\lambda}}{\sigma_i}} \quad (30)$$

Finally, fatigue life prediction results were calculated from Eqs. (27–30). Comparisons between experimental results and predicted lives for 45 and 16Mn steels are shown in Figs. 2–3. The broken line in the graphs represents a  $\pm 2$  factor indicator. From these tests, all of the cyclic lives are predicted within a factor of  $\pm 2$  by the new model. 12 out of 15 cyclic lives are predicted within a factor of  $\pm 1.5$  and the predicted results are in good agreement with the observed ones.

To reflect the capability of the new model, the test data are also assessed by the Miner's rule and conventional Corten–Dolan's model. As shown by the results, an adequate agreement is achieved and the fatigue lives predicted by the proposed method are better than those by the Corten–Dolan model. Two evaluating parameters of life assessments: scatter band and standard deviation were used, as shown in Figs. 2 and 3. The results show that all of the predicted cyclic lives by the new model are within a factor

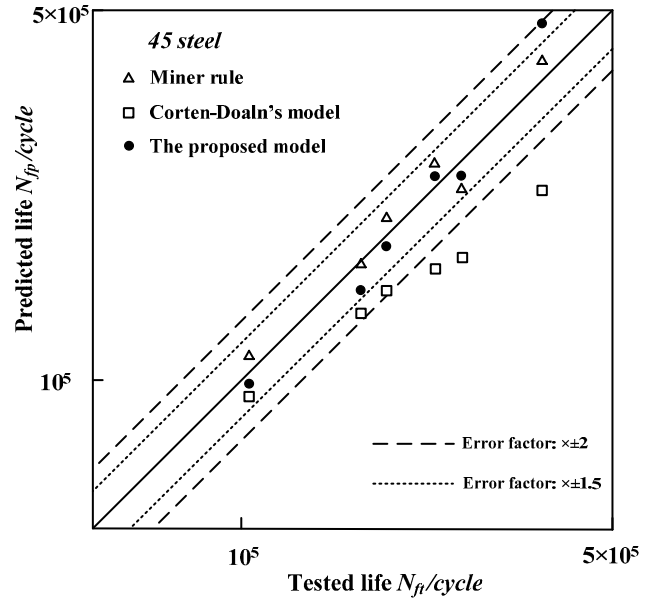


Figure 2. Comparison between lives predicted by the proposed model, Miner rule, Corten–Dolan's model and lives tested for normalized 45 steel.

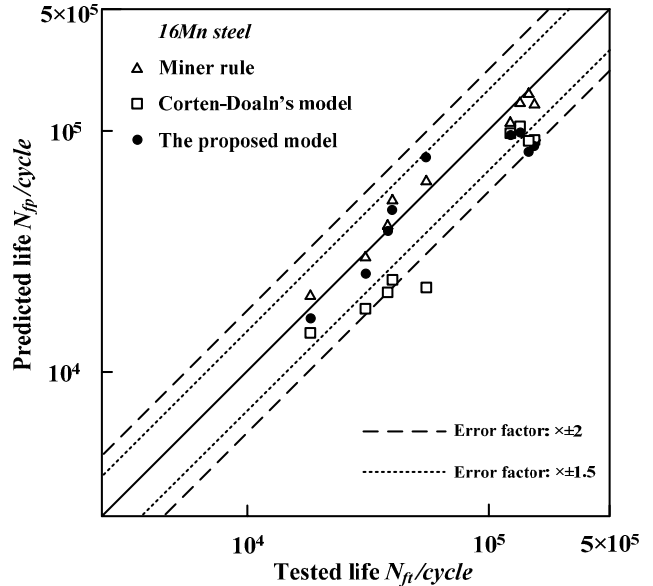


Figure 3. Comparison between lives predicted by the proposed model, Miner rule, Corten–Dolan's model and lives tested for normalized 16Mn steel.

of  $\pm 2$  to the test ones and nearly 80% of the test data was predicted within a factor of  $\pm 1.5$ . This was better than that 26.7% of test data prediction by the conventional Corten–Dolan's model. It was found that the cumulative damage calculated by the new model is less than 1 for high-low load spectrums and greater than 1 for low-high load spectrums,

Two-stress level tests/(MPa)	Load sequences	$n_1$	$\frac{n_1}{N_{f1}}$	$n_2$	Miner rule D	Corten-Dolan's model	$\frac{\sum n_r}{N_g}$	Dynamic Corten-Dolan's model $\left(\frac{\sum n_r}{N_g}\right)^*$
331.46~284.4	High-low	500	0.010	423700	0.8574	3.4962		0.5060
		12500	0.250	250400	0.7508	2.3103		0.8018
		25000	0.500	168300	0.8366	1.8848		1.1174
		37500	0.750	64500	0.8790	1.2807		1.1001
284.4~331.46	Low-high	125000	0.250	37900	1.0080	1.7858		1.4440
		250000	0.500	38900	1.2780	2.8337		1.1874

**Table 1.** Experimental parameters and life predictions by the new model, Miner rule, Corten–Dolan’s model for normalized 45 steel under two-stress level loading.

Two-stress level tests/(MPa)	Load sequences	$n_1$	$\frac{n_1}{N_{f1}}$	$n_2$	Miner rule D	Corten-Dolan's model	$\frac{\sum n_r}{N_g}$	Dynamic Corten-Dolan's model $\left(\frac{\sum n_r}{N_g}\right)^*$
562.9~392.3	High-low	2	0.0005	73600	0.9357	2.2850		0.6835
		200	0.0504	59400	0.8052	1.8942		0.8476
		1000	0.2520	56300	0.9674	1.9995		1.0193
		1700	0.4284	47600	1.0332	1.9059		1.2552
		2450	0.6174	22900	0.9084	1.3282		1.1218
372.65~392.3	Low-high	38900	0.1450	75500	1.1040	1.3258		1.3636
		64400	0.2400	62800	1.0380	1.4052		1.4678
		116000	0.4330	62900	1.2320	1.8928		2.0055
		150000	0.5600	23300	1.0150	1.7103		2.0152

**Table 2.** Experimental parameters and life predictions by the new model, Miner rule, Corten–Dolan’s model for normalized 16Mn steel under two-stress level loading.

which reflects the coaxing effects on fatigue life to a certain extent.

From Tables 1 and 2, Miner’s rule gives a better life prediction for simple loading tests than Corten–Dolan model. But it neglects the damage induced by stresses below the fatigue limit and load interaction effects for complex load spectrums. Meanwhile, the comparison between the Miner’s rule and the Corten–Dolan model was made by Singh [34] through two variable-load tests. Compared with Corten–Dolan’s model, the test results conflict with the behavior predicted by the Miner’s rule be-

cause it neglects the damage induced by low-amplitude loads. Comparing these three methods, results indicate that the proposed model and Miner’s rule have better life prediction capabilities than the conventional Corten–Dolan’s model.

It should be pointed out that the new method presented in this paper can be better used for fatigue life evaluation under multilevel load spectrums. In addition, the proposed model is suitable for ductile materials. The proposed model was verified by some experimental data and good results were obtained. But there is further need to verify them for

different materials under multilevel or random load spectrums.

## 6 Conclusions

The aim of this research was to present a new method for determining the Corten–Dolan exponent  $d$  and predict fatigue life by accounting for load interaction effects. The main achievements can be summarized as follows:

(1) A method has been developed to solve the problem of load interaction effects on fatigue life, which can be characterized by defining the exponent  $d$  as a function of load spectrum instead of being constant.

(2) By using the new method of determining the exponent  $d$  and considering the effects of load interaction, a modified Corten–Dolan's model was derived. It showed a better characterization of fatigue damage evolution over the conventional model.

(3) Both Miner's rule and proposed model yield more satisfactory life prediction results for 45 and 16Mn steels than the conventional Corten–Dolan's model. Moreover, it should be pointed out that Miner's rule gives a better life prediction under simple loading conditions than the others. Under complex load spectrums, however, it ignores the effects of load interaction on fatigue life.

Further research of the load interaction and sequencing effects on fatigue life under random load spectrums is desirable, leading to better fatigue life prediction and remaining life assessments.

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