Hindawi Publishing Corporation
Advances in Mechanical Engineering
Volume 2014, Article ID 790620, 12 pages
http://dx.doi.org/10.1155/2014/790620

Research Article

Bounded Target Cascading in Hierarchical Design Optimization

Xiaoling Zhang, Debiao Meng, Ruan-Jian Yang, Zhonglai Wang, and Hong-Zhong Huang

School of Mechanical, Electronic, and Industrial Engineering, University of Electronic Science and Technology of China, No. 2006, Xiyuan Avenue, West Hi-Tech Zone, Chengdu, Sichuan 611731, China

Correspondence should be addressed to Hong-Zhong Huang; hzhuang@uestc.edu.cn

Received 16 December 2013; Accepted 19 May 2014; Published 22 June 2014

Academic Editor: Zhenling Liu

For large scale systems, as a hierarchical multilevel decomposed design optimization method, analytical target cascading coordinates the inconsistency between the assigned targets and response in each level by a weighted-sum formulation. To avoid the problems associated with the weighting coefficients, single objective functions in the hierarchical design optimization are formulated by a bounded target cascading method in this paper. In the BTC method, a single objective optimization problem is formulated in the system level, and two kinds of coordination constraints are added: one is bound constraint for the design points based on the response from each subsystem level and the other is linear equality constraint for the common variables based on their sensitivities with respect to each subsystem. In each subsystem level, the deviation with target for design point is minimized in the objective function, and the common variables are constrained by target bounds. Therefore, in the BTC method, the targets are coordinated based on the optimization iteration information in the hierarchical design problem and the performance of the subsystems, and BTC method will converge to the global optimum efficiently. Finally, comparisons of the results from BTC method and the weighted-sum analytical target cascading method are presented and discussed.

1. Introduction

The multidisciplinary design optimization (MDO) problems have become very important in most engineering designs. The key issue for MDO problems is how to coordinate coupling disciplines, and various MDO frameworks have been developed based on their problem formulations and decomposition strategies. Martins and Lambe [1] provided a general introduction for the various developed approaches and summarized the merit of different MDO frameworks. Tosserams et al. [2] discussed the general characteristics of nested formulations and alternated formulations for MDO problems. Originally, the MDO problem minimizes the cost function subject to multiple disciplinary constraints but it may be unpractical to solve the entire problem in one optimization formulation; bilevel optimization formulations have received considerable attention to solve MDO problems, such as collaborative optimization (CO), which has been widely applied to the design of aerospace systems, and the analytical and computational aspects of CO were presented [3]. For large scale engineering problems, as a method to propagate system targets and coordinate common variables of subsystems through a hierarchical multilevel system formulation, analytical target cascading (ATC) method [4–6] has been developed and widely utilized to solve multidisciplinary engineering problems [7–10]. In the ATC method, the all-in-one (AIO) optimization formulation is decomposed into a hierarchical structure with one system level and multiple subsystems. In order to find the optimal solution of the original MDO problem, it is essential to minimize the cost function and reduce the inconsistency between the subsystems simultaneously in ATC.

The cumulative constraint violations in the subsystems, also known as the discrepancy functions, were firstly formulated and minimized to maintain the consistency in the MDO problems [11]. Besides, the differences between the subsystem variables have been considered as the discrepancy functions [12, 13]. The discrepancy functions were then penalized in the objective functions for finding the optimal solution and diminishing the inconsistency during the optimization processes [14, 15]. The ATC method firstly considered the weighted discrepancy functions of design variables in multiobjective formulations and chose weighting coefficients to coordinate the optimal solutions of the original MDO problems [16–19].

In the processes of solving the Analytical Target Cascading, the system level assigned its optimal solution to the subsystems as the design target points. In the subsystems, the optimal solutions were found in terms of minimizing the differences between the subsystem variables and the targets subject to the local constraints. The local optimal solutions were then updated back to the system level as the design responses. The weighted discrepancy functions in terms of the targets and the response points were diminished during the optimization processes of ATC. However, the selection of the weighting coefficients is crucial for decreasing the discrepancy functions to zero and coordinating the inconsistency between design points and assigned targets in each level during the optimization processes. In other words, the proper optimal solutions can only be obtained with an appropriate choice of weighting coefficients. Many approaches have been studied to determine the proper selection of weights [18–22]. Michalek and Papalambros [18] proposed a weights updating method using the KKT first order necessary conditions combined user-specified inconsistency tolerances. Tosserams et al. [20] developed an augmented Lagrangian coordination method with the alternating direction method of multipliers. Kim et al. [21] formulated a Lagrangian dual coordination method to update the weighting coefficients. Li et al. [22] provided a diagonal quadratic approximation method by linearizing the cross terms of the discrepancy functions. The above methods took the discrepancy functions as equality constraints and used different penalty terms to coordinate those equality constraints. The convergence rate of these methods becomes more slowly as the discrepancy becoming smaller, and the penalty terms might lead to oscillate during the iteration process.

In this paper, single objective functions in the hierarchical design optimization are formulated to avoid the problems associated with the weighting coefficients by a new bounded target cascading (BTC) method. Instead of assigning the point targets to the subsystems in ATC, the bounded targets are introduced in the new method. The target bounds are obtained from the optimal solutions in each level and utilized to reduce the variable bounds during the optimization processes. Furthermore, the bounds of responses are also determined to avoid the weighted discrepancy functions of the responses. If the common variables exist, they are coordinated based on their sensitivities with respect to design variables. With the aids of the target bounds, the optimal solutions can be determined in the reduced variable bounds while the consistency between the variables and the targets is maintained.

In the next section, the well-known MDO algorithm and the analytical target cascading method are firstly reviewed. In Section 3, the bounded target cascading method is proposed to solve MDO problems to avoid the problems associated with the weighting coefficients. Lastly, some numerical examples are solved by the BTC method and the comparisons of the results from the proposed method and the ATC method are also presented and discussed in Section 4.

2. MDO and Analytical Target Cascading

The original formulation of the MDO problem is given by

$$\begin{aligned}
\text{min:}\quad & f\left(\mathbf{x}^{m=1}, \mathbf{z}^{m=1}\right) \\
\text{w.r.t.}\quad & X_i^m, \mathbf{x}_i^m, \mathbf{Y}_i^m, \mathbf{y}_i^m, \mathbf{z}_i^m \\
\text{s.t.}\quad & \mathbf{g}_i^m\left(X_i^m, \mathbf{x}_i^m, \mathbf{Y}_i^m, \mathbf{z}_i^m\right) \le \mathbf{0} \\
& \mathbf{h}_i^m\left(X_i^m, \mathbf{x}_i^m, \mathbf{Y}_i^m, \mathbf{z}_i^m\right) = \mathbf{0} \\
& \underline{X}_i^m \le X_i^m \le \overline{X}_i^m \\
& \underline{\mathbf{Y}}_i^m \le \mathbf{Y}_i^m \le \overline{\mathbf{Y}}_i^m \\
& \underline{\mathbf{x}}_i^m \le \mathbf{x}_i^m \le \overline{\mathbf{x}}_i^m \\
& \underline{\mathbf{y}}_i^m \le \mathbf{y}_i^m \le \overline{\mathbf{y}}_i^m \\
& \underline{\mathbf{z}}_i^m \le \mathbf{z}_i^m \le \overline{\mathbf{z}}_i^m,
\end{aligned} \tag{1}$$

where f is the cost function; $\mathbf{x}_i^m$ stands for the ith subsystem variables in the mth level coupling with one level below; $\mathbf{z}_i^m$ is the local variable; and $m = 1$ represents the variables in the system level. In the ith subsystem, X_i^m is the variable coupling with one level above; $\mathbf{y}_i$ and $\mathbf{Y}_i$ denote the common variables from one level below and one level above, respectively; $\mathbf{g}_i^m$ and $\mathbf{h}_i^m$ are the inequality and equality constraints, respectively. The all-in-one formulation in (1) is often unpractical in engineering problems; therefore, it is decomposed into hierarchical optimization structures and solved by the MDO algorithms. The main goal of MDO methods is to minimize the cost function while the discrepancy between subsystems is diminished simultaneously.

Kim et al. [4] developed analytical target cascading (ATC) method to decompose the MDO problem into hierarchical structures with a system level and several subsystems. The system level is given as

$$\begin{aligned}
\text{min:}\quad & f\left(\mathbf{x}^{m,k}, \mathbf{z}\right) + \left\|\mathbf{w}_x^{m,k} \cdot \left(\mathbf{x}^{m,k} - \mathbf{x}^{m+1,k-1}\right)\right\|^2 \\
& + \left\|\mathbf{w}_y^{m,k} \cdot \left(\mathbf{y}^{m,k} - \mathbf{y}^{m+1,k-1}\right)\right\|^2 \\
\text{w.r.t.}\quad & \mathbf{x}^{m,k}, \mathbf{y}^{m,k}, \mathbf{z}^{m,k} \\
\text{s.t.}\quad & \mathbf{g}^m\left(\mathbf{x}^{m,k}, \mathbf{z}^{m,k}\right) \le \mathbf{0} \\
& \mathbf{h}^m\left(\mathbf{x}^{m,k}, \mathbf{z}^{m,k}\right) = \mathbf{0} \\
& \underline{\mathbf{x}}^{m,k} \le \mathbf{x}^{m,k} \le \overline{\mathbf{x}}^{m,k} \\
& \underline{\mathbf{y}}^{m,k} \le \mathbf{y}^{m,k} \le \overline{\mathbf{y}}^{m,k} \\
& \underline{\mathbf{z}}^{m,k} \le \mathbf{z}^{m,k} \le \overline{\mathbf{z}}^{m,k},
\end{aligned} \tag{2}$$

where $\mathbf{w}_x^{m,k}$ and $\mathbf{w}_y^{m,k}$ are the weighting coefficients. When the cost function is minimized, the weighted discrepancy terms are diminished simultaneously to maintain the consistency in the ith subsystem. Furthermore, a general formulation for each subsystem is defined as

$$\begin{aligned}
\text{min:}\quad & w_{X,i}^{m,k}\left\|X_i^{m,k}-X_i^{m-1,k}\right\|^2 \\
&+\left\|\mathbf{w}_{x,i}^{m,k}\cdot\left(\mathbf{x}_i^{m,k}-\mathbf{x}_i^{m+1,k-1}\right)\right\|^2 \\
&+\left\|\mathbf{w}_{Y,i}^{m,k}\cdot\left(\mathbf{Y}_i^{m,k}-\mathbf{Y}_i^{m-1,k}\right)\right\|^2 \\
&+\left\|\mathbf{w}_{y,i}^{m,k}\cdot\left(\mathbf{y}_i^{m,k}-\mathbf{y}_i^{m+1,k-1}\right)\right\|^2 \\
\text{w.r.t.}\quad & X_i^{m,k},\mathbf{x}_i^{m,k},\mathbf{Y}_i^{m,k},\mathbf{y}_i^{m,k},\mathbf{z}_i^{m,k} \\
\text{s.t.}\quad & \mathbf{g}_i^m\left(X_i^{m,k},\mathbf{x}_i^{m,k},\mathbf{Y}_i^{m,k},\mathbf{z}_i^{m,k}\right)\le\mathbf{0} \\
& \mathbf{h}_i^m X_i^{m,k},\mathbf{x}_i^{m,k},\mathbf{Y}_i^{m,k},\mathbf{z}_i^{m,k}=\mathbf{0} \\
& \underline{X}_i^{m,k}\le X_i^{m,k}\le\overline{X}_i^{m,k} \\
& \underline{\mathbf{x}}_i^{m,k}\le\mathbf{x}_i^{m,k}\le\overline{\mathbf{x}}_i^{m,k} \\
& \underline{\mathbf{Y}}_i^{m,k}\le\mathbf{Y}_i^{m,k}\le\overline{\mathbf{Y}}_i^{m,k} \\
& \underline{\mathbf{y}}_i^{m,k}\le\mathbf{y}_i^{m,k}\le\overline{\mathbf{y}}_i^{m,k} \\
& \underline{\mathbf{z}}_i^{m,k}\le\mathbf{z}_i^{m,k}\le\overline{\mathbf{z}}_i^{m,k},
\end{aligned}\tag{3}$$

where $w_{X,i}^{m,k}$, $\mathbf{w}_x^{m,k}$, $\mathbf{w}_{Y,i}^{m,k}$, and $\mathbf{w}_y^{m,k}$ are the weighting coefficients. The discrepancy functions associated with the target points from one level above are minimized while the functions in terms of the responses from one level below are minimized at the same time.

The discrepancy functions can be formulated by either the L_2-norm differences [4] between the variables and target points or the squared L_2-norm functions [6]. However, the multiobjective formulations are very difficult for ATC to find the proper solutions in the MDO problems. Some researchers [16, 17] recognized the importance of the weighting coefficients in the discrepancy functions for finding the optimal solutions and diminishing the inconsistency simultaneously. Kim et al. [21] noticed that the proper selection of weighting coefficients is crucial for obtaining the reasonable optimal solutions in ATC. There are two approaches to resolving the problems associated with the weighting coefficients: one is to find the proper choice of weights and the other is not to use the weighted terms in the multiobjective formulations. Many researches have been conducted to appropriately determine the weighting coefficients, and the design problem will converge to different Pareto solution with different weighting coefficients. BTC tries to formulate the target cascading problem into a single objective design framework, and the optimization iteration information in each design level is shared to make the design points converge to the global optimum.

3. Bounded Target Cascading

In this Section, a new bounded target cascading (BTC) method is proposed to solve the MDO problem in (1) without using the weighted multiobjective formulations. Instead of assigning the point targets to the subsystems in ATC, the bounded targets are introduced in the new method. The target and response bounds are defined to avoid the weighted discrepancy formulations. Furthermore, the common variables are coordinated based on sensitivity analysis.

3.1. Target Bounds in BTC for Subsystem. In the bounded target cascading method, target bounds for the common variables in the ith subsystem are defined as

$$\left|Y_{i,j}^{m,k}-Y_{i,j}^{m-1,k}\right|\le d_{i,j}^{m,k}\tag{4}$$

which are centered at the target points $Y_{i,j}^{m-1,k}$ and ranged by $d_{i,j}^{m,k}$. Instead of using the weighted term of $w_{Y_{i,j}}^{m,k}\|Y_{i,j}^{m,k}-Y_{i,j}^{m-1,k}\|^2$ in the multiobjective formulation, the target bounds in (4) are given to bound the common variables. In order to converge at the correct position of the common variables, the ranges of the target bound $d_{i,j}^{m,k}$ are defined by the difference between the optimum in the previous iteration and the current target points. Therefore, the target bound is defined as

$$\left|Y_{i,j}^{m,k}-Y_{i,j}^{m-1,k}\right|\le d_{i,j}^{m,k};\qquad d_{i,j}^{m,k}=\left|Y_{i,j}^{m,k-1}-Y_{i,j}^{m-1,k}\right|,\tag{5}$$

where $Y_{i,j}^{m,k-1}$ denotes the previous optimal solution and $Y_{i,j}^{m-1,k}$ stands for the target in the kth iteration.

In the coordination processes, if the discrepancy between design target and design point for any subsystem is less than one percent of the design response, the target bounds for common variables will be redefined using the bounds of the subsystem, which is calculated by

$$\left[\underline{Y}_{i,j}^{m,k},\overline{Y}_{i,j}^{m,k}\right]=\bigcap_{i=1}^{n_m}\left[Y_{i,j}^{m-1,k}-d_{i,j}^{m,k},Y_{i,j}^{m-1,k}+d_{i,j}^{m,k}\right].\tag{6}$$

The design optimization problem in subsystem is to minimize the deviations of subsystem performances and common variables from assigned targets. The design problem in subsystem converges with the two deviations are small enough.

3.2. Target Bounds in BTC for System. In system level, the weighted discrepancy function of the design responses is replaced by the new response bound which is defined as

$$\begin{aligned}
\mathbf{x}_i^{m,k}&\le\mathbf{x}_i^{m+1,k-1}\quad\text{for }\boldsymbol{\delta}_i^{m,k}<0,\\
\mathbf{x}_i^{m,k}&\ge\mathbf{x}_i^{m+1,k-1}\quad\text{for }\boldsymbol{\delta}_i^{m,k}>0,
\end{aligned}\tag{7}$$

where $\boldsymbol{\delta}_i^{m,k}=\mathbf{x}_i^{m+1,k-1}-\mathbf{x}_i^{m,k-1}$ is the difference between the design response and optimal solution in the previous iteration. When the response is found to be smaller than the optimal solution in the previous iteration, in order to fulfill the feasibility and optimality conditions in the subsystem, the new design point should be bounded above by the response. Contrarily, the design point should be bounded below by the response when the response is larger than the optimal solution in the previous iteration.

If the common variables of the responses exist and then constrained by their sensitivity relations with the design variables. Therefore, approximated linear equations are established by the first-order Taylor expansion, which is given by

$$\mathbf{x}_i^{m,k} \cong \mathbf{x}_i^{m+1,k-1} + \left(\mathbf{y}_i^{m,k} - \mathbf{y}_i^{m+1,k-1}\right) \cdot \frac{\Delta \mathbf{x}_i^{m,k-1}}{\Delta \mathbf{y}_i^{m,k-1}}. \tag{8}$$

If there are c_j common variables in subsystem i, the updating information for common variables with the ith subsystem is provided in system by

$$x_{\text{sub},i} \cong x_{\text{sub},i}^{\text{sub},k-1} + \sum_{j=1}^{c_i} \left(y_{\text{sub},ij}^{\text{sys}} - y_{\text{sub},ij}^{\text{sub},k-1} \right) \cdot \frac{\partial x_{\text{sub},i}}{\partial y_{\text{sub},ij}}. \tag{9}$$

In system level, for coordinating common variables between subsystem i and subsystem j, the target for common variable will be calculated based on the updating information and the sensitivity of system objective function to the subsystem performance, which is given by

$$y^{\text{sys}} = \frac{\left(\partial f_{\text{sys}} / \partial x_{\text{sub},i}\right) y_{\text{sub},i}^{\text{sub},k} + \left(\partial f_{\text{sys}} / \partial x_{\text{sub},j}\right) y_{\text{sub},j}^{\text{sub},k}}{\partial f_{\text{sys}} / \partial x_{\text{sub},i} + \partial f_{\text{sys}} / \partial x_{\text{sub},j}}. \tag{10}$$

Therefore, in BTC method, instead of considering the weighted discrepancy term of $\mathbf{w}_i^{m,k} \| \mathbf{y}_i^{m,k} - \mathbf{y}_i^{m+1,k-1} \|^2$, the common variable in system level is subject to the sensitivity relation in (8)–(10).

3.3. Solution Process. The optimization process using the proposed bounded target cascading method follows the divide-and-conquer strategy. The system level minimizes the cost function subject to the system-level constraints, the new response bounds, and the sensitivity relations of common variables, which is given by

$$\begin{aligned}
\text{min:} \quad & f\left(\mathbf{x}^{m,k}, \mathbf{z}\right) \\
\text{w.r.t.} \quad & \mathbf{x}^{m,k}, \mathbf{y}^{m,k}, \mathbf{z} \\
\text{s.t.} \quad & \mathbf{g}^{m}\left(\mathbf{x}^{m,k}, \mathbf{z}\right) \le \mathbf{0} \\
& \mathbf{h}^{m}\left(\mathbf{x}^{m,k}, \mathbf{z}\right) = \mathbf{0} \\
& \mathbf{x}^{m,k} \cong \mathbf{x}^{m+1,k-1} + \left(\mathbf{y}_i^{m,k} - \mathbf{y}_i^{m+1,k-1}\right) \cdot \frac{\Delta \mathbf{x}^{m,k-1}}{\Delta \mathbf{y}_i^{m,k-1}} \\
& \mathbf{y}^{m,k} = \frac{\sum \left(\partial f\left(\mathbf{x}^{m,k}, \mathbf{z}\right) / \partial x_i^{m,k}\right) \mathbf{y}_i^{m,k}}{\sum \left(\partial f\left(\mathbf{x}^{m,k}, \mathbf{z}\right) / \partial x_i^{m,k}\right)} \\
& \mathbf{x}^{m,k} \le \mathbf{x}^{m+1,k-1} \quad \text{for } \boldsymbol{\delta}^{m,k} < 0 \\
& \mathbf{x}^{m,k} \ge \mathbf{x}^{m+1,k-1} \quad \text{for } \boldsymbol{\delta}^{m,k} > 0 \\
& \underline{\mathbf{x}}^{m,k} \le \mathbf{x}^{m,k} \le \overline{\mathbf{x}}^{m,k} \\
& \underline{\mathbf{y}}^{m,k} \le \mathbf{y}^{m,k} \le \overline{\mathbf{y}}^{m,k} \\
& \underline{\mathbf{z}} \le \mathbf{z} \le \overline{\mathbf{z}}.
\end{aligned} \tag{11}$$

The target bounds are then formulated based on the optimal solution and assigned to the subsystems. In general, the subsystem finds the local optimal solution subject to subsystem constraints and new target bounds:

$$\begin{aligned}
\text{min:} \quad & \left\| X_i^{m,k} - X_i^{m-1,k} \right\|^2 \\
\text{w.r.t.} \quad & X_i^{m,k}, \mathbf{x}_i^{m,k}, \mathbf{Y}_i^{m,k}, \mathbf{y}_i^{m,k}, \mathbf{z}_i \\
\text{s.t.} \quad & \mathbf{g}_i^{m}\left(X_i^{m,k}, \mathbf{x}_i^{m,k}, \mathbf{Y}_i^{m,k}, \mathbf{z}_i\right) \le \mathbf{0} \\
& \mathbf{h}_i^{m}\left(X_i^{m,k}, \mathbf{x}_i^{m,k}, \mathbf{Y}_i^{m,k}, \mathbf{z}_i\right) = \mathbf{0} \\
& \mathbf{x}^{m,k} \cong \mathbf{x}^{m+1,k-1} + \left(\mathbf{y}_i^{m,k} - \mathbf{y}_i^{m+1,k-1}\right) \cdot \frac{\Delta \mathbf{x}^{m,k-1}}{\Delta \mathbf{y}_i^{m,k-1}} \\
& \left| Y_{i,j}^{m,k} - Y_{i,j}^{m-1,k} \right| \le d_{i,j}^{m,k} \\
& \mathbf{x}_i^{m,k} \le \mathbf{x}_i^{m+1,k-1} \quad \text{for } \boldsymbol{\delta}_i^{m+1,k} < 0 \\
& \mathbf{x}_i^{m,k} \ge \mathbf{x}_i^{m+1,k-1} \quad \text{for } \boldsymbol{\delta}_i^{m+1,k} > 0 \\
& \underline{X}_i^{m,k} \le X_i^{m,k} \le \overline{X}_i^{m,k} \\
& \underline{\mathbf{x}}_i^{m,k} \le \mathbf{x}_i^{m,k} \le \overline{\mathbf{x}}_i^{m,k} \\
& \underline{\mathbf{Y}}_i^{m,k} \le \mathbf{Y}_i^{m,k} \le \overline{\mathbf{Y}}_i^{m,k} \\
& \underline{\mathbf{y}}_i^{m,k} \le \mathbf{y}_i^{m,k} \le \overline{\mathbf{y}}_i^{m,k} \\
& \underline{\mathbf{z}}_i \le \mathbf{z}_i \le \overline{\mathbf{z}}_i.
\end{aligned} \tag{12}$$

The local optimal solutions are then updated back to the system level as the information of the response bounds. For the subsystems at the bottom of the hierarchy structure, (12) is simplified as

$$\begin{aligned}
\text{min:} \quad & \left\| X_i^{m,k} - X_i^{m-1,k} \right\|^2 \\
\text{w.r.t.} \quad & X_i^{m,k}, \mathbf{Y}_i^{m,k}, \mathbf{z}_i \\
\text{s.t.} \quad & \mathbf{g}_i^{m}\left(X_i^{m,k}, \mathbf{Y}_i^{m,k}, \mathbf{z}_i\right) \le \mathbf{0} \\
& \mathbf{h}_i^{m}\left(X_i^{m,k}, \mathbf{Y}_i^{m,k}, \mathbf{z}_i\right) = \mathbf{0} \\
& \left| Y_{ij}^{m,k} - Y_{ij}^{m-1,k} \right| \le d_{ij}^{m,k}
\end{aligned}$$

$$\underline{X}_i^{m,k} \le X_i^{m,k} \le \overline{X}_i^{m,k}$$
$$\underline{\mathbf{Y}}_i^{m,k} \le \mathbf{Y}_i^{m,k} \le \overline{\mathbf{Y}}_i^{m,k}$$
$$\underline{\mathbf{z}}_i \le \mathbf{z}_i \le \overline{\mathbf{z}}_i. \tag{13}$$

The iteration process of BTC method continues until the convergence criterion is satisfied. In this paper, the convergence criterion is defined by the absolute difference between the current and the previous design points to be smaller than the acceptable limit. In the next Section, some numerical examples solved by the proposed BTC method and the ATC method with various settings of the weighting coefficients are presented and discussed.

3.4. Convergence of BTC. The convergence properties of ATC have been discussed by [16], which proved that ATC process converges to the optimal point that satisfies the necessary optimality conditions of the original design problem. BTC solves decomposed MDO problems using the same coordinate principle with ATC in the targets cascading process, that is, hierarchical overlapping coordination, which is achieved by the exchange of information between different decompositions. The properties of BTC are presented as the following.

3.4.1. The Upper or Lower Bound for Design Point in System Level. In the system level for general ATC, the consistency constraints $c_i^{m,k} = \|\mathbf{x}_i^{m+1,k} - \mathbf{x}_i^{m,k}\|$ are penalized in the objective function using various penalty functions, which minimizes the deviation of design points with response from subsystems. However, these methods may lead to the optimization problem converge to a local optimum or converge earlier. Lower bounds or upper bounds based on the response from subsystems are defined to consider the performance of each subsystem and provide more flexibility for system problem.

3.4.2. The Linear Approximation for Coordinating Common Variable in System Level. The sensitivity of each subsystem to common variables is calculated to coordinate common variable between subsystems in the system level. The gradient calculation is the key issue to guarantee the accuracy of this approximation. In this paper, pseudo finite difference $\Delta\mathbf{x}_i^{m,k-1}/\Delta\mathbf{y}_i^{m,k-1}$ and reduced gradient method is introduced to calculate the gradient.

Suppose the equality relation between the design variable, $x_{\text{sub},i}$, and a common variable, $y_{\text{sub},ij}$, exist:

$$h\left(x_{\text{sub},i}, y_{\text{sub},ij}\right) = 0. \tag{14}$$

The partial derivative of the equality constraint with respect to common variables is

$$\left.\frac{\partial h(x_{\text{sub},i}, y_{\text{sub},ij})}{\partial y_{\text{sub},ij}}\right|_{y_{\text{sub},ij}^{\text{sub},k}} + \frac{\partial h(x_{\text{sub},i}, y_{\text{sub},ij})}{\partial x_{\text{sub},i}} \left.\frac{\partial x_{\text{sub},i}}{\partial y_{\text{sub},ij}}\right|_{x_{\text{sub},i}^{\text{sub},k}} = 0, \tag{15}$$

where $x_{\text{sub},i}^{\text{sub},k}$ and $y_{\text{sub},ij}^{\text{sub},k}$ are the design points at kth iteration, and the sensitivity relation between the design variable and common variable is derived as

$$\left.\frac{\partial x_{\text{sub},i}}{\partial y_{\text{sub},ij}}\right|^{k} = -\left.\frac{\partial h(x_{\text{sub},i}, y_{\text{sub},ij})/\partial y_{\text{sub},ij}}{\partial h(x_{\text{sub},i}, y_{\text{sub},ij})/\partial x_{\text{sub},i}}\right|^{k}. \tag{16}$$

If the equality relation is not available, surrogate models will be formulated by simulation, and for high dimensionality engineering problem, the efficiency of simulation is the most considerable problem.

3.4.3. Interval Convergence for Common Variables in Subsystems. During the optimization iteration in subsystems, the deviation between targets and response should be decreased:

$$\begin{aligned}\left|Y_{i,j}^{m,k} - Y_{i,j}^{m-1,k}\right| &\le r\left|Y_{i,j}^{m,k-1} - Y_{i,j}^{m-1,k}\right| \\ &\le \cdots \le r\left|Y_{i,j}^{m,0} - Y_{i,j}^{m-1,1}\right|,\end{aligned} \tag{17}$$

where $0 \le r \le 1$, in this paper, $r = 1$. The convergence property of subsystem will change with different r.

4. Numerical Examples

In this Section, two mathematical problems are decomposed into hierarchical structures and solved by the proposed bounded target cascading method and the analytical target cascading method with various settings of the weighting coefficients.

4.1. Convex Quadratic Programming Problem. The undecomposed convex minimization problem is stated as in

$$\begin{aligned}\min:\quad & x_1^2 + x_2^2 \\ \text{s.t.}\quad & g_1 = x_3^{-2} + x_4^2 - x_5^2 \le 0 \\ & g_2 = x_5^2 + x_6^{-2} - x_7^2 \le 0 \\ & h_1 = x_1^2 - x_3^2 - x_4^{-2} - x_5^2 = 0 \\ & h_2 = x_2^2 - x_5^2 - x_6^2 - x_7^2 = 0 \\ & x_1, \ldots, x_7 \ge 0.\end{aligned} \tag{18}$$

Using the analytical target cascading method, the convex problem is decomposed into the structure with two levels, constraints g_1 and h_1 in subsystem 1 and constraints g_2 and h_2 in subsystem 2. For general ATC, The system level is formulated as (19), while the first and the second subsystems are given by (20) and (21), respectively:

$$\begin{aligned}\min_{x_1,x_2,x_3}: \quad & x_1^2 + x_2^2 + w_1\left(x_1 - x_1^{k-1,\text{sub}_1}\right)^2 + w_2\left(x_2 - x_2^{k-1,\text{sub}_2}\right)^2 \\ & + w_3\left(x_5 - x_5^{k-1,\text{sub}_1}\right)^2 + w_4\left(x_5 - x_5^{k-1,\text{sub}_2}\right)^2 \\ \text{s.t.} \quad & 0 \le x_1, x_2, x_5,\end{aligned} \tag{19}$$

$$\begin{aligned}\min_{x_1,x_3}: \quad & w_1\left(x_1 - x_1^{k,\text{sys}}\right)^2 + w_3\left(x_5 - x_5^{k,\text{sys}}\right)^2 \\ \text{s.t.} \quad & h_1 = x_1^2 - x_3^2 - x_4^{-2} - x_5^2 = 0 \\ & g_1 = x_3^{-2} + x_4^2 - x_5^2 \le 0 \\ & 0 \le x_1, x_3, x_4, x_5,\end{aligned} \tag{20}$$

$$\begin{aligned}\min_{x_2,x_3}: \quad & w_2\left(x_2 - x_2^{k,\text{sys}}\right)^2 + w_4\left(x_5 - x_5^{k,\text{sys}}\right)^2 \\ \text{s.t.} \quad & h_2 = x_2^2 - x_5^2 - x_6^2 - x_7^2 = 0 \\ & g_2 = x_5^2 + x_6^{-2} - x_7^2 \le 0 \\ & 0 \le x_2, x_5, x_6, x_7,\end{aligned} \tag{21}$$

where x_1^{k-1,sub_1} and x_5^{k-1,sub_1} are the responses from the first subsystem in the $(k-1)$th iteration; x_2^{k-1,sub_2} and x_5^{k-1,sub_2} are the ones from the second subsystem; $x_1^{k,\text{sys}}$, $x_2^{k,\text{sys}}$, and $x_5^{k,\text{sys}}$ are the optimal solutions of the system level in the kth iteration as well as the targets for the both subsystems. The initial points $\mathbf{x}^{k=0,\text{sys}}$ and $\mathbf{x}^{k=0,\text{sub}_i}$ follow $\mathbf{x}^{k=0} = [1\ \ 1\ \ 1]^T$ as well as the initial parameters $\mathbf{x}^{k=1,\text{sub}_i}$. In the formulation of ATC, the proper selection of the weighting coefficients w_1, w_2, w_3, and w_4 is crucial for obtaining the correct optimal solution.

In the proposed bounded target cascading method, the discrepancy terms of common variables are displaced by the sensitivity relations in the subsystems while the ones of design variables are displaced by the response bounds. Therefore, the system level is

$$\begin{aligned}\min: \quad & f_{\text{sys}} = x_1^2 + x_2^2 \\ \text{s.t.} \quad & x_i = x_i^{k-1,\text{sub}_i} + \left(y_i - x_5^{k-1,\text{sub}_i}\right)\frac{dx_1\left(x_i^{k-1,\text{sub}_i}\right)}{dx_5} \\ & y^{\text{sys}} = \frac{\left(\partial f_{\text{sys}}/\partial x_{\text{sub},1}\right)y_1 + \left(\partial f_{\text{sys}}/\partial x_{\text{sub},2}\right)y_2}{\partial f_{\text{sys}}/\partial x_{\text{sub},1} + \partial f_{\text{sys}}/\partial x_{\text{sub},2}} \\ & x_i \le x_i^{k-1,\text{sub}_i} \quad \text{for } \delta_i = x_i^{k-1,\text{sub}_1} - x_i^{k-1} < 0 \\ & x_i \ge x_i^{k-1,\text{sub}_i} \quad \text{for } \delta_i > 0 \\ & 0 \le x_1, x_2, x_5, y_i, \quad i = 1, 2.\end{aligned} \tag{22}$$

In the first iteration, the sensitivity relations are omitted because no common variations have been made in x_i^{k,sub_i}. When the response x_i^{k-1,sub_i} is smaller than the previous design $x_i^{k-1,\text{sys}}$, that is, $\delta_i < 0$, the new design should be smaller than or equal to the response. On the other hand, $x_i \ge x_i^{k-1,\text{sub}_i}$ is considered when $\delta_i > 0$. In the special case of $x_i^{k-1,\text{sys}} = x_i^{k-1,\text{sub}_i}$, the direction of the response bound is determined based on the most recent nonzero δ_i. Furthermore, the ith subsystem is reformulated as

$$\begin{aligned}\min: \quad & \left(x_1 - x_1^{k,\text{sys}}\right)^2 \\ \text{s.t.} \quad & \left|x_5 - y^{k,\text{sys}}\right| \le d_{\text{sub},1}; \qquad d_{\text{sub},1} = \left|x_5^{k-1,\text{sub}} - y^{k,\text{sys}}\right| \\ & h_1 = x_1^2 - x_3^2 - x_4^{-2} - x_5^2 = 0 \\ & g_1 = x_3^{-2} + x_4^2 - x_5^2 \le 0 \\ & 0 \le x_1, x_2, x_3, \\ \min: \quad & \left(x_2 - x_2^{k,\text{sys}}\right)^2 \\ \text{s.t.} \quad & \left|x_5 - y^{k,\text{sys}}\right| \le d_{\text{sub},2}; \qquad d_{\text{sub},2} = \left|x_5^{k-1,\text{sub}} - y^{k,\text{sys}}\right| \\ & h_2 = x_2^2 - x_5^2 - x_6^2 - x_7^2 = 0 \\ & g_2 = x_5^2 + x_6^{-2} - x_7^2 \le 0 \\ & 0 \le x_1, x_2, x_3\end{aligned} \tag{23}$$

while the common variable is constrained by a target bound centered at the $x_5^{k,\text{sys}}$ and ranged by the difference between the current target points and the previous optimal solution. When the current target point agrees with the previous optimal solution, the difference term $d_{\text{sub},i}$ equals zero, and the common variable will be located at the target point $x_3^{k,\text{sys}}$.

In the proposed BTC method, the weighted discrepancy terms in the multiobjective formulations have been avoided and displaced by the bounds associated with the targets and responses. The optimization process follows the divide-and-conquer strategy while target bounds are assigned to the subsystems, and the response bounds are updated back to the system level. The iteration stops when the termination criterion, the absolute difference between $\mathbf{x}^k$ and $\mathbf{x}^{k-1} \le 10^{-3}$, is satisfied. The detailed iteration process of the system level is shown in Figure 1. The system-level optimal solutions of $[x_1^*, x_2^*, x_5^*] = [2.1529, 2.0725, 1.0707]$ are determined inside the updated bounds of responses illustrated by the solid and dash lines, respectively. Figure 2 demonstrates the iteration processes in the subsystems. The optimal solutions of $x_5^* = 1.0707$ are found inside the updated target bounds, shown by the solid lines.

Moreover, the problem has been solved by the ATC using various settings of weighting coefficients. Two constant weights $w = 1$ or 150 and two increasing weights, $w^{k=0} = 1$; $w^k = 2w^{k-1}$ or $5w^{k-1}$ for $k \ge 1$ are considered. Table 1 lists the results with the starting point of $[3\ \ 2.2\ \ 1]^T$. BTC uses only 12 iterations (Iter.) and 7569 function evaluations (FE) to find the reasonable solutions while ATC requires more

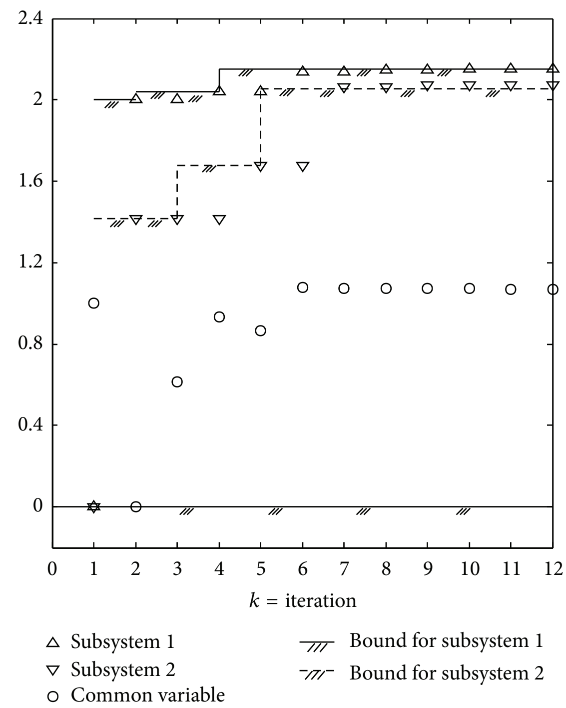

FIGURE 1: System-level iteration process of solving example 1 by BTC.

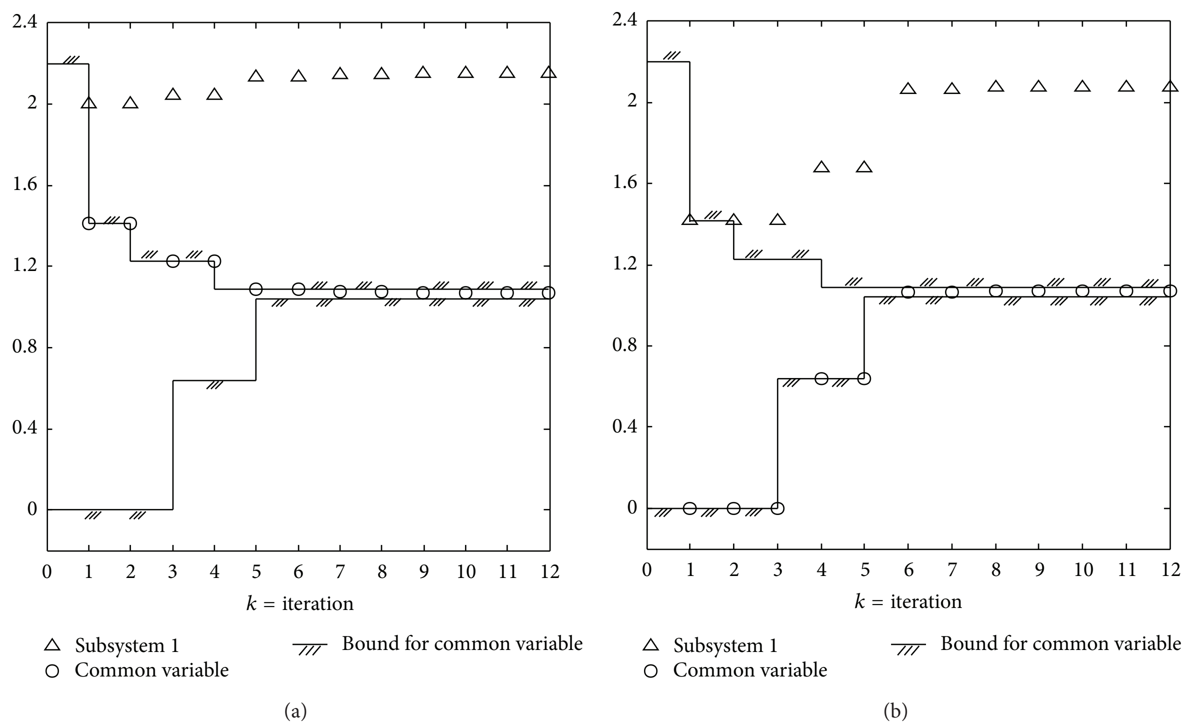

FIGURE 2: Iteration processes of solving example 1 by BTC in the (a) first and (b) second subsystems.

Table 1: Results for example 1.

Methods	$\mathbf{x}^*$	Iter.	FE	Error
AIO	[2.1491, 2.0759, 1.0746]	—	—	—
BTC	[2.1529, 2.0725, 1.0707]	12	7659	0.85%
ATC[a]	Cannot converge			
ATC[b]	Cannot converge			
ATC[c]	[2.183, 2.0415, 1.0419]	15	2265	7.8%
ATC[d]	[2.1177, 2.1053, 1.1055]	309	34863	7.2%

[a] $w = 1$.
[b] $w = 150$.
[c] $w^{k=0} = 1; w^k = 2^{k-1}$ for $k \geq 1$.
[d] $w^{k=0} = 1; w^k = 5w^{k-1}$ for $k \geq 1$.

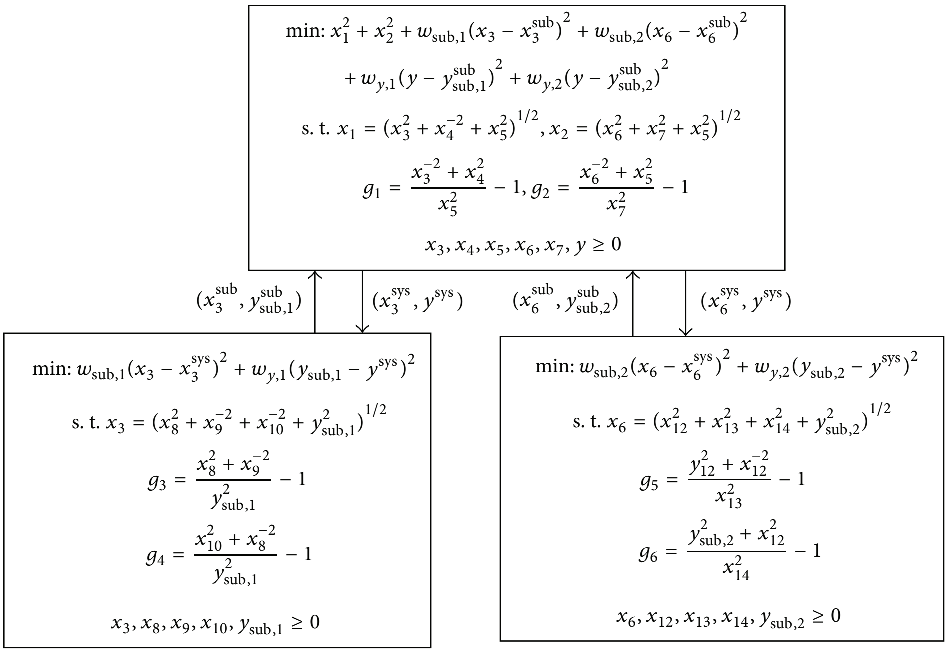

Figure 3: Design problem in general ATC method.

to converge. The accuracy of the solutions is evaluated by the absolute difference with the results by all-in-one (AIO) methods, denoted as Error. The results show the BTC has 0.85% Error but the ATC produces Errors with more than 7%.

4.2. Nonconvex Geometric Programming Problem. The second example is a nonconvex geometric problem, which was used in [5, 18, 19, 21, 22], and the original programming problem is shown in (24).

$$
\begin{aligned}
\min: \quad & x_1^2 + x_2^2 \\
\text{s.t.} \quad & g_1 = \left(x_3^{-2} + x_4^2\right) \times x_5^{-2} - 1 \leq 0 \\
& g_2 = \left(x_6^{-2} + x_5^2\right) \times x_7^{-2} - 1 \leq 0 \\
& g_3 = \left(x_8^2 + x_9^{-2}\right) \times x_{11}^{-2} - 1 \leq 0 \\
& g_4 = \left(x_{10}^2 + x_8^{-2}\right) \times x_{11}^{-2} - 1 \leq 0 \\
& g_5 = \left(x_{11}^2 + x_{12}^{-2}\right) \times x_{13}^{-2} - 1 \leq 0 \\
& g_6 = \left(x_{11}^2 + x_{12}^2\right) \times x_{14}^{-2} - 1 \leq 0 \\
& h_1 = x_1 - \left(x_3^2 + x_4^{-2} + x_5^2\right)^{1/2} = 0 \\
& h_2 = x_2 - \left(x_6^2 + x_7^2 + x_5^2\right)^{1/2} = 0 \\
& h_3 = x_3 - \left(x_8^2 + x_9^{-2} + x_{10}^{-2} + x_{11}^2\right)^{1/2} = 0 \\
& h_4 = x_6 - \left(x_{12}^2 + x_{13}^2 + x_{14}^2 + x_{11}^2\right)^{1/2} = 0 \\
& x_1, x_2, \ldots, x_{14} \geq 0.
\end{aligned}
\tag{24}
$$

The geometric problem is decomposed into the structure two levels, with constraints g_1, g_2, h_1, and h_2 in the system level, constraints g_3, g_4, and h_3 in subsystem 1, and constraints g_5, g_6, and h_4 in subsystem 2. The formulations for system level and subsystem level of the decomposed design problem in general ATC method and the proposed BTC method are provided in Figures 3 and 4.

The BTC method converges with 15 iterations and 7632 function evaluations. The optimal solution and comparisons with other methods are shown in Table 2. Figure 5 presents

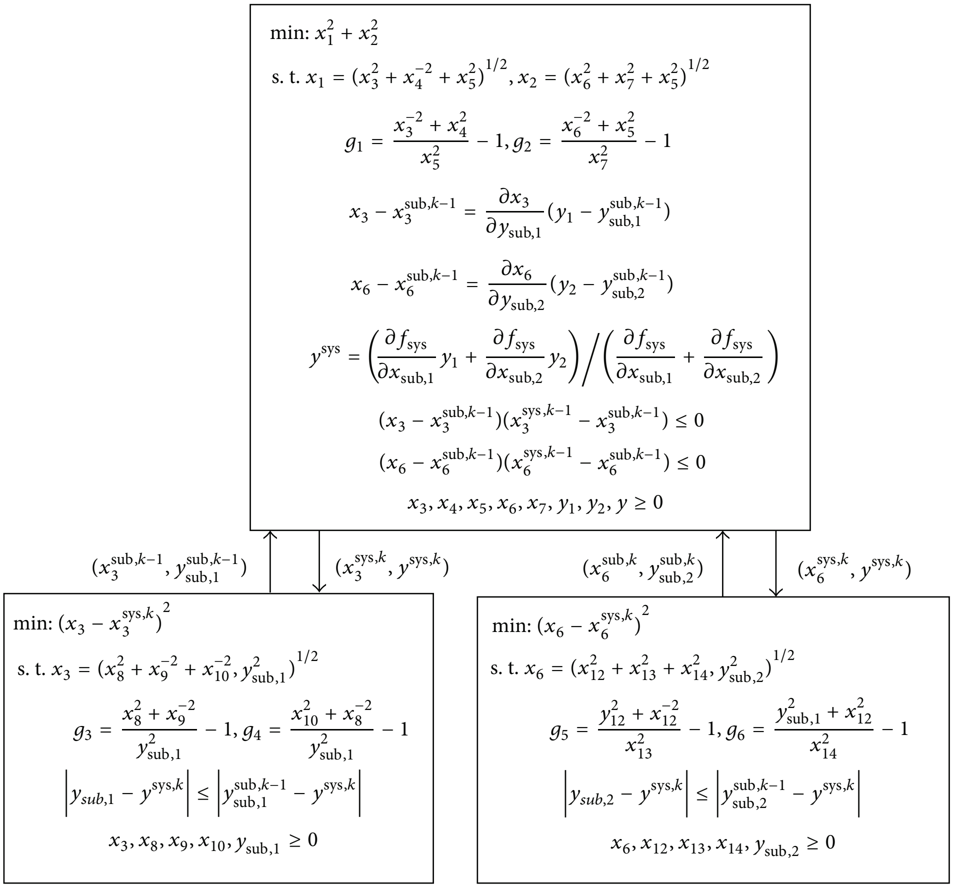

Figure 4: Design problem in BTC method.

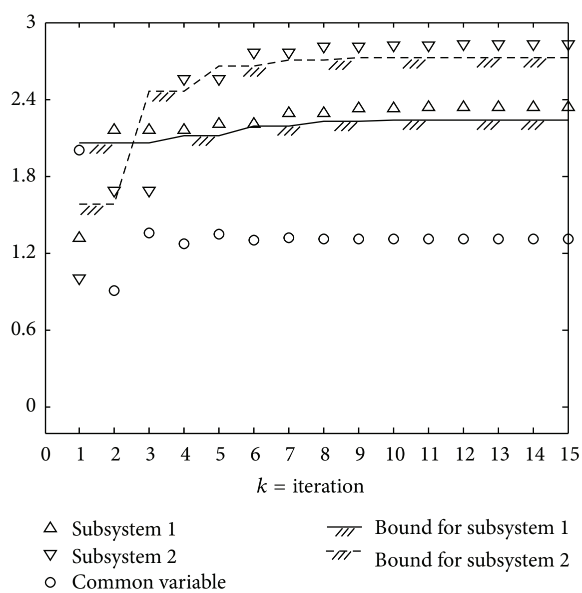

Figure 5: System-level iteration process of solving example 2 by BTC.

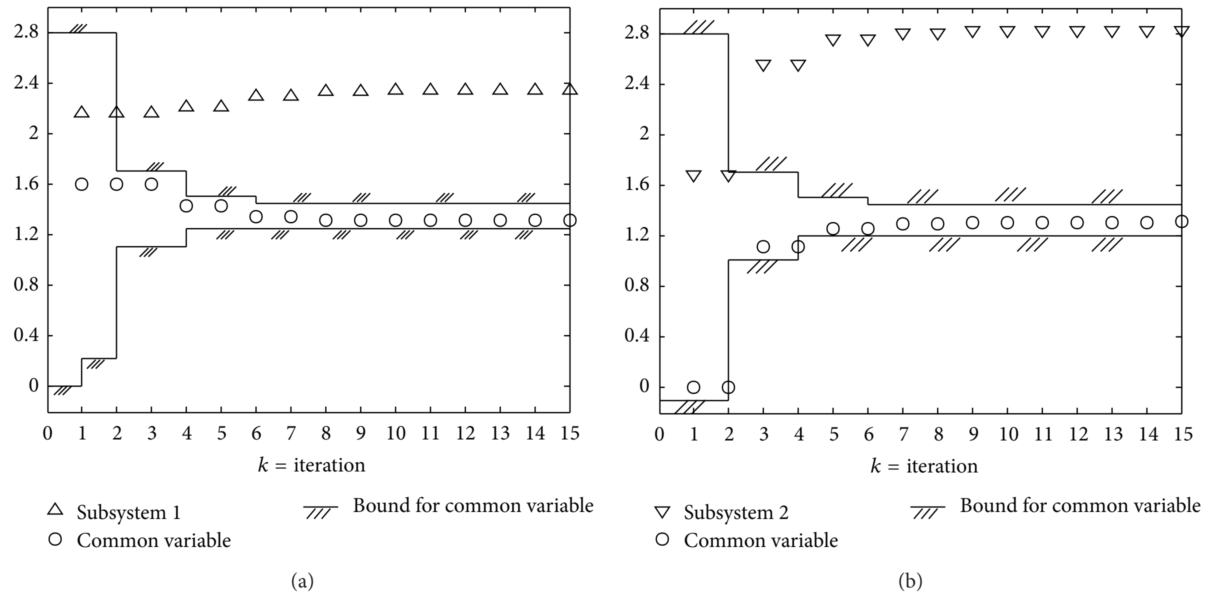

Figure 6: Iteration processes of solving example 2 by BTC in the (a) first and (b) second subsystems.

Table 2: Results for example 2.

Solution	AIO	BTC	LATC [21]	
			$m = 100$	$m = 10$
x_1	2.84	2.82	2.84	2.84
x_2	3.09	3.10	3.07	3.09
x_3	2.36	2.34	2.36	2.36
x_4	0.76	0.76	0.76	0.76
x_5	0.87	0.87	0.87	0.87
x_6	2.81	2.82	2.80	2.81
x_7	0.94	0.94	0.94	0.94
x_8	0.97	0.97	0.97	0.97
x_9	0.87	0.88	0.86	0.87
x_{10}	0.80	0.80	0.79	0.80
x_{11}	1.30	1.31	1.29	1.30
x_{12}	0.84	0.84	0.84	0.84
x_{13}	1.76	1.77	1.75	1.76
x_{14}	1.55	1.56	2.84	1.55
Iter.	—	15	101	556
FE	—	7632	8124	136162
Function value	17.61	17.57	17.49	17.61
Error	—	0.56%	0.49%	0.15%

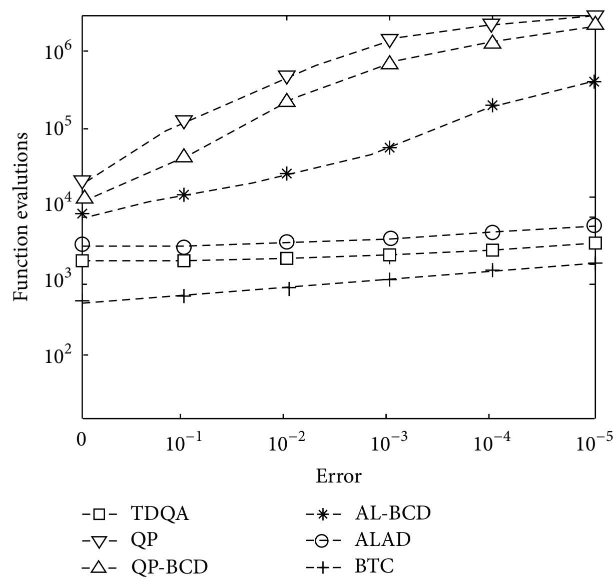

Figure 7: Computational cost versus solution accuracy.

the iteration process of solving the problem using the proposed BTC in the system level. On the other hand, Figure 6 demonstrates the iteration processes in the subsystems using the BTC. Similarly, the optimal common variables are found inside the target bounds, illustrated by the solid lines.

The comparisons of computational cost between the BTC method and methods proposed in [4, 18, 20–22] is shown in Figure 7, including the quadratic penalty method (QP), the quadratic penalty method with block coordinate descent method (QP-BCD), the augmented Lagrangian method (AL), the augmented Lagrangian method with BCD (AL-BCD), the augmented Lagrangian with alternating direction method of multipliers (ALAD), and truncated diagonal quadratic approximation method (TDQA).

5. Conclusions

MDO problems have been decomposed into hierarchical structures with one system level and multiple subsystems. The main task in the decomposed multidisciplinary design optimization algorithm is to minimize the cost function and diminish the discrepancy between subsystems simultaneously. The well-known method, analytical target cascading, formulates the multiobjective formulations in terms of the weighted discrepancy functions in order to maintain the consistency between subsystems. However, the choice of the weighting coefficients is very problem dependent and improper selections of the weighting coefficients will lead to incorrect solutions.

In this paper, a new bounded target cascading method is proposed to solve the MDO to avoid the problems associated with the weighting coefficients. Instead of the point targets assigned for design variables in the analytical target cascading method, bounded targets are introduced in the new method. The target bounds are obtained from the optimal solutions in each level. Furthermore, the response bounds are established based on the optimal solutions from the subsystems and utilized to replace the weighted discrepancy functions of responses in the analytical target cascading method. If the common variables exist, they are coordinated based on their sensitivities with respect to design variables. The numerical examples validate that the single-objective bounded target cascading can efficiently and accurately find the optimal solutions.

How to efficiently formulate a hybrid framework of hierarchical ATC and nonhierarchical ATC for large scale MDO problems is the further work.

Nomenclature

f: Cost function
$\mathbf{x}_i$: Variables in the ith subsystem coupling with the below level
$\mathbf{y}_i$: Variables from one level below coordinated in the ith subsystem; the component $y_{i,pq}$ is the common variable between subsystems p and q
X_i: Variable in the ith subsystem coupling with the above level
$\mathbf{Y}_i$: Common variables in the ith subsystem; the component Y_{ij} is shared with the jth subsystem
$\mathbf{z}_i$: Local variables in the ith subsystem
$\mathbf{g}_i$: Inequality constraints in the ith subsystem
$\mathbf{h}_i$: Equality constraints in the ith subsystem
n_m: Number of subsystems in the mth level
$w_X, \mathbf{w}_x, \mathbf{w}_Y, \mathbf{w}_y$: Weighting coefficients in analytical target cascading.
$\mathbf{d}$: Target bounds for common variables
$\boldsymbol{\delta}$: Response bounds for coupling variables.

Superscripts

m: Index of level
k: Design point in the kth iteration
sys: System
sub: Subsystem.

Subscripts

i, j: Index of subsystem in the mth level; $i, j = 1, \ldots, n_m$
p, q: Index of subsystem in the $(m+1)$th level; $p, q = 1, \ldots, n_{m+1}$.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgments

This research was partially supported by the National Natural Science Foundation of China under the Grant no. U1330130 and Fundamental Research Funds for the Central Universities under the Grant no. ZYGX2012J098.

References

[1] J. R. R. A. Martins and A. B. Lambe, "Multidisciplinary design optimization: a survey of architectures," *AIAA Journal*, vol. 51, no. 9, pp. 2049–2075, 2013.

[2] S. Tosserams, L. F. P. Etman, and J. E. Rooda, "A classification of methods for distributed system optimization based on formulation structure," *Structural and Multidisciplinary Optimization*, vol. 39, no. 5, pp. 503–517, 2009.

[3] N. M. Alexandrov and R. M. Lewis, "Analytical and computational aspects of collaborative optimization for multidisciplinary design," *AIAA Journal*, vol. 40, no. 2, pp. 301–309, 2002.

[4] H. M. Kim, M. Kokkolaras, L. S. Louca et al., "Target cascading in vehicle redesign: a class VI truck study," *International Journal of Vehicle Design*, vol. 29, no. 3, pp. 199–225, 2002.

[5] H. M. Kim, N. F. Michelena, P. Y. Papalambros, and T. Jiang, "Target cascading in optimal system design," *Journal of Mechanical Design*, vol. 125, no. 3, pp. 474–480, 2003.

[6] J. Allison, *Complex system optimization: a review of analytical target cascading, collaborative optimization, and other formulations [Ph.D. thesis]*, University of Michigan, Ann Arbor, Mich, USA, 2004.

[7] M. Kokkolaras, R. Fellini, H. M. Kim, N. F. Michelena, and P. Y. Papalambros, "Extension of the target cascading formulation to the design of product families," *Structural and Multidisciplinary Optimization*, vol. 24, no. 4, pp. 293–301, 2002.

[8] H. M. Kim, D. G. Rideout, P. Y. Papalambros, and J. L. Stein, "Analytical target cascading in automotive vehicle design," *Journal of Mechanical Design*, vol. 125, no. 3, pp. 481–489, 2003.

[9] M. Kokkolaras, L. S. Louca, G. J. Delagrammatikas et al., "Simulation-based optimal design of heavy trucks by model-based decomposition: an extensive analytical target cascading case study," *International Journal of Heavy Vehicle Systems*, vol. 11, no. 3-4, pp. 403–433, 2004.

[10] Z. J. Li, M. Kokkolaras, P. Papalambros, and S. J. Hu, "Product and process tolerance allocation in multistation compliant assembly using analytical target cascading," *Journal of Mechanical Design*, vol. 130, no. 9, Article ID 091701, 9 pages, 2008.

[11] J. Sobieszczanski-Sobieski, "A linear decomposition method for large optimization problems—blueprint for development," Tech. Rep. NASA-TM-83248, 1982.

[12] R. J. Balling and J. Sobieszczanski-Sobieski, "An algorithm for solving the system-level problem in multilevel optimization," *Structural Optimization*, vol. 9, no. 3-4, pp. 168–177, 1995.

[13] R. J. Balling and J. Sobieszczanski-Sobieski, "Optimization of coupled systems: a critical overview of approaches," *AIAA Journal*, vol. 34, no. 1, pp. 6–17, 1996.

[14] A.-V. Demiguel and W. Murray, "An analysis of collaborative optimization methods," in *Proceedings of the 8th AIAA/USAF/NASA/ISSMO Symposium on Multidisciplinary Analysis and Optimization*, Paper no. AIAA-2000-4720, Long Beach, Calif, USA, 2000.

[15] X. Y. Gu, J. E. Renaud, and C. L. Penninger, "Implicit uncertainty propagation for robust collaborative optimization," *Journal of Mechanical Design*, vol. 128, no. 4, pp. 1001–1013, 2006.

[16] N. Michelena, H. Park, and P. Y. Papalambros, "Convergence properties of analytical target cascading," *AIAA Journal*, vol. 41, no. 5, pp. 897–905, 2003.

[17] J. Allison, M. Kokkolaras, M. Zawislak, and P. Y. Papalambros, "On the use of analytical target cascading and collaborative optimization for complex system design," in *Proceedings of the 6th World Congress on Structural and Multidisciplinary Optimization*, Rio de Janeiro, Brazil, 2005.

[18] J. J. Michalek and P. Y. Papalambros, "An efficient weighting update method to achieve acceptable consistency deviation in analytical target cascading," *Journal of Mechanical Design*, vol. 127, no. 2, pp. 206–214, 2005.

[19] J. J. Michalek and P. Y. Papalambros, "Weights, norms, and notation in analytical target cascading," *Journal of Mechanical Design*, vol. 127, no. 3, pp. 499–501, 2005.

[20] S. Tosserams, L. F. P. Etman, P. Y. Papalambros, and J. E. Rooda, "An augmented Lagrangian relaxation for analytical target cascading using the alternating direction method of multipliers," *Structural and Multidisciplinary Optimization*, vol. 31, no. 3, pp. 176–189, 2006.

[21] H. M. Kim, W. Chen, and M. M. Wiecek, "Lagrangian coordination for enhancing the convergence of analytical target cascading," *AIAA Journal*, vol. 44, no. 10, pp. 2197–2207, 2006.

[22] Y. J. Li, Z. S. Lu, and J. J. Michalek, "Diagonal quadratic approximation for parallelization of analytical, target cascading," *Journal of Mechanical Design*, vol. 130, no. 5, Article ID 051402, 11 pages, 2008.