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An efficient life prediction methodology for low cycle fatigue–creep based on ductility exhaustion theory

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Abstract

Low cycle fatigue–creep is the main reason for the failures of many engineering components under high temperature and cyclic loading. Based on the exhaustion of the static toughness and dissipation of the plastic strain energy during fatigue failure, a new low cycle fatigue–creep life prediction model that is consistent with the fatigue–creep damage mechanism and sensitive to the fatigue damage process is presented in an attempt to develop viscosity-based approaches for general use in isothermal and thermo-mechanical loading. In this model, the theory of ductility exhaustion is used to describe the process of fatigue–creep interaction. It was assumed that the ductility exhaustion related only to the plastic strain and creep strain caused by tensile stress under stress-controlled conditions. In addition, the mechanisms of loading waveform, creep and mean stress effects were taken into account in a low cycle fatigue–creep regime. The predicted lives by the proposed model agree well with the reported experimental data from literature under different temperature loading conditions.

Keywords

High temperature low cycle fatigue, creep, ductility, life prediction, viscosity

Introduction

As the operating temperature of high temperature structures has increased to satisfy the demands for their high efficiency, such structures operating in petrochemical, power, aeronautics and astronautics are subjected to cyclically varying loads under low cycle fatigue (LCF, typically below about 10^5 cycles) at high temperatures. For these structures, not only is there repeated loading, but also

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depending on temperature, strain rate and hold time, time-dependent damage processes such as creep, dynamic strain aging and environmental attack become more important when compared to those under room temperature conditions (Evans et al., 2005; Pineau and Antolovich, 2009). These abnormal mechanisms at high temperature make the materials' behavior complicated by inducing unexpected hardening or softening effects. Moreover, this leads to increased difficulty in the life prediction of these structures. Therefore, the mechanisms of LCF, creep and their interaction behavior at high temperature should be considered when damage assessment and/or life prediction are carried out for these high temperature structures.

Life assessment of high temperature structures under LCF loading has become a major challenge for both the scientific and industrial community (Pineau and Antolovich, 2009; Zhang, 2010). Deformation and failure mechanisms of low cycle fatigue–creep (LCF–C) at high temperature are very complex and depend on numerous test and material parameters. Until now, a considerable amount of effort has been devoted to develop a suitable method for LCF–C life prediction, such as linear damage summation (Zhang, 2010), frequency modified Manson–Coffin equation (Coffin, 1976), frequency separation (FS) technique (Coffin, 1974), strain range partition (Manson et al., 1971), frequency modified damage function (FMDF) model (Ostergren, 1967), damage rate approach (Majumdar and Maiya, 1976), and ductility exhaustion (DE) approach (Goswami, 1997; Hales, 1983; Priest and Ellison, 1981). These methods are commonly used for life prediction, but there are many difficulties in obtaining various parameters and constants in these equations. Moreover, from those studies using a variety of techniques, the results are based either directly on data generated from fatigue–creep tests or indirectly on the fatigue–creep test data, which show that these methods can only predict life within a factor of 2–4 at best (Viswanathan, 1995). In order to predict LCF–C lives within a factor of 2, particularly at low total-strain ranges, all of these methods need additional sophistication, although this can detract from their original simplicity. Hence, life prediction under such conditions is still a complex situation and current well-established models are lacking.

In recent years, the study of fatigue and creep interaction has received increasing attention in either isothermal or thermal-mechanical fatigue conditions (Boismier and Sehitoglu, 1990). The following problems need to be carefully solved for LCF–C life prediction: quantification of the fatigue–creep interaction; description of temperature, loading waveform and mean stress effects; and an appropriate damage accumulation rule. In order to improve the LCF–C life prediction accuracy and make the design effectively, different approaches have been proposed to handle these problems. Robustness of a life prediction method is a crucial key point and it has triggered renewed interest in strain energy based approaches (Chiou and Yip, 2006; Fan et al., 2007; Gocmez et al., 2010; Koh, 2002; Lee, 2008; Maurel et al., 2009; Payten et al., 2010; Zhu and Huang, 2010; Zhu et al., 2011a, 2011b). Energy-based approaches have been developed for fatigue and/or LCF–C life prediction using the hysteresis loops area (Chiou and Yip, 2006; Fan et al., 2007; Koh, 2002; Lee et al., 2008; Zhu et al., 2011a). Recently, based on DE theory and the effective stress concept, Fan et al. (2007) investigated the fatigue–creep interaction behavior of 1.25Cr0.5Mo steel and presented a mean strain rate (MSR) model under stress control mode. The predicted lives by the MSR model showed a good agreement with the observed ones when DE was the dominant mechanism. Currently, most of life prediction methodologies use the Manson–Coffin law as a starting point and modify it in some way to account for time-dependent damage mechanisms such as creep and environmental effects. For example, Zhu and Huang (2010) developed a generalized strain energy damage function (GSEDF) model to reduce the difference between the approximate strain energy and the real strain energy absorbed during the fatigue failure process. This model provides a more accurate life prediction for fatigue–creep interaction than the FS and

FMDF methods. By assuming that creep damage is directly proportional to absorbed internal energy density, Payten et al. (2010) put forward a strain energy density exhaustion approach, which is derived from considerations of mechanistic cavity growth and is based on rupture elongation to failure data using true strain.

All these models have their own capabilities and have shown their validity on limited number of alloys and/or loading conditions. However, models reviewed above each have one or more of the following limitations: (a) cannot accurately describe the effects of loading waveform and mean strain or mean stress on the fatigue life; (b) not favorable under limited test data and some certain experimental conditions, such as stress control mode; (c) involve too many 'to be determined parameters or constants' and/or need additional tests to obtain these parameters, design engineers need efficient and simple models for structural analysis, thus further improvements on models for LCF–C life prediction are expected.

A viscosity-based life prediction model for strain control mode was derived in the authors' previous work (Zhu et al., 2011b) and applied to a number of LCF tests on a GH4133 Superalloy. Since the existing ductility model was not suitable for life prediction under stress control mode, this article extends the approach and presents a practical model to predict LCF–C life under stress control mode, where the fatigue–creep toughness acts as the damage parameter defining the dynamic viscosity. This model needs to comply with industrial requirements for high efficiency, ease of use and fewer constants which can be directly obtained from fatigue–creep tests rather than a series of pure fatigue and/or creep tests. Several sets of experimental data under stress-controlled loading are used to validate the proposed model. Following this, the prediction results of the proposed method were compared with the MSR (Fan et al., 2007) and GSEDF (Zhu and Huang, 2010) models and was found to be more efficient and accurate. The capabilities of this model are then further discussed. Finally, some concluding remarks are made.

Ductility exhaustion during cyclic loading

Fatigue is a damage accumulation process in which material property deteriorates continuously undergoing fluctuating stresses and strains. Previous studies have demonstrated that the intrinsic ductility degrades during cyclic loading, which corresponds to the energy absorbed by the material. Further, there is an intrinsic relationship between the fatigue life and material ductility, as well as fatigue damage accumulation and the exhaustion of material ductility. Among the fatigue–creep life prediction methods using inelastic strains as a principal parameter, the ductility exhaustion approach developed by Priest and Ellison (1981) and Hales (1983), has been employed in the life assessment of fracture-critical components (Ainsworth, 2003; Takahashi et al., 2008; Wood et al., 1988). Ductility exhaustion theory assumes that, during the fatigue and creep fracture process, damage evolution usually goes with continuous ductility exhaustion of the material, leading to failure once the accumulated strain reaches a critical ductility. Based on this idea, a ductility model was developed in Goswami (1997, 2004) by assuming that deformation and damage under LCF at high temperatures can be characterized in forms of viscous flow. This concept has been applied in modeling steady-state creep behavior. The mechanisms of steady-state creep behavior between the deformation and the viscous flow were extended in the dwell fatigue situation in which fatigue and creep processes interact. Under cyclic loading at high temperatures, LCF failure is inelastic strain dominated and deformation, which depends upon time at a rate measured by strain rate is produced. During the fatigue analysis of mechanical components, dynamic viscosity was often used as a damage parameter (Fan et al., 2006; Zhu et al., 2011b). Under dwell fatigue loading, the deformation depends upon the strain range and time, the rate of damage relates to the strain rate of the cycle. Therefore, dynamic viscosity υ_d should account for the strain range effects, and is defined using the fundamental viscosity concept (Goswami, 2004)

$$\upsilon_d = \Delta\sigma \cdot (\Delta\varepsilon_t/\dot{\varepsilon}) \tag{1}$$

where $\Delta\sigma$, $\Delta\varepsilon_t$ and $\dot{\varepsilon}$ are the stress range, the total strain range and the strain rate, respectively.

The viscous flow accommodation process in a material initiates under cyclic and/or static loading. When the ability of a material to accommodate any further viscous flow ceases, failure occurs when the dynamic viscosity reaches a critical value. Material toughness represents the ability of a material to accommodate permanent deformation to a certain extent. When the dynamic viscosity becomes equal to the material toughness is reached, it is defined as a failure criterion, which is expressed as

$$\sum f(\upsilon_d) = T_m \tag{2}$$

where T_m is the fatigue toughness, which represents the total energy dissipated by the material/component cumulatively up to failure. The material toughness is defined as a product of ductility and cyclic strength (Goswami, 2004). Under cyclic loading, the continuous reduction of the material toughness indicates the progressive exhaustion of the ability to absorb energy inherent in the material. Moreover, it is directly associated with the raising of the material's internal energy resulting from the irreversible energy dissipation process of fatigue failure (Ye and Wang, 2001). In general, the internal energy stored in the material by the formation of point defects, the generation and rearrangement of dislocations and the formation of internal surfaces such as voids and cracks often leads to material damage (Harvey et al., 1998). It implies that the material toughness is a material property that relates to the physical mechanism of fatigue damage, which is more sensitive to the fatigue damage process than others (Ye and Wang, 2001).

Material ductility depends not only upon the experimental parameters (including stress range, strain rate and temperature, etc.), but also upon impurity content, grain size, relaxed stresses and other factors such as cyclic hardening/softening effects (Fan et al., 2007). Using the Edmund and White equation, the fatigue ductility is given as (Priest and Ellison, 1981)

$$Ductility = \Delta\varepsilon_p N_f \tag{3}$$

Then, material toughness can be determined by

$$T_m = \Delta\varepsilon_p N_f \Delta\sigma_{sat} \tag{4}$$

where σ_{sat} is the saturated tension stress at half-life.

Based on the experimental results of Hart and Solomon (1973), there is a scaling relationship when constructing curves for strain range to strain rate ratios and cycles to failure on log–log scales. This behavior produced a linear equation with a slope of m and a ductility model for LCF life prediction at high temperature was obtained by (Goswami, 1997, 2004)

$$\Delta\sigma(\Delta\varepsilon_t/\dot{\varepsilon})^m = A\Delta\varepsilon_p\Delta\sigma_{sat}N_f \tag{5}$$

where A is a material parameter which balances the units of this equation. The value of A depends on test parameters (Goswami, 1997).

The cyclic stress–strain response is an important material property in the design of enhanced fatigue resistance, which is depicted by hysteresis loops (Zhu et al., 2011). Based on the Ramberg–Osgood relation (Ramberg and Osgood, 1943), the cyclic stress–strain curve is expressed as

$$\frac{\Delta\sigma}{2} = K'\left(\frac{\Delta\varepsilon_p}{2}\right)^{n'} \tag{6}$$

where K' and n' are the cyclic strength coefficient and the cyclic strain hardening exponent, respectively, which can be obtained from a log–log linear regression analysis of cyclic strain amplitudes and corresponding cyclic stress amplitudes under fully reversed fatigue tests.

Equation (6) describes the relationship between the flow stress and the plastic strain amplitude under cyclic loading. Combining with equation (6) and rearranging equation (5) results in

$$N_f = \left[K'\left(\Delta\varepsilon_p\right)^{n'-1}(\Delta\varepsilon_t/\dot{\varepsilon})^m\right]/(A\cdot\Delta\sigma_{sat}) \tag{7}$$

It should be noted that equation (7) relates to parameters such as stress, strain range, strain rate and cyclic stress–strain relations. However, LCF life prediction using this model is only applicable for continuous fatigue, slow-fast waveforms and strain-controlled conditions (Goswami, 1997), so the applicability of this model is restricted within narrow limits. Moreover, many mechanical components are subjected to a certain degree of mean stress or mean strain. Most engineering materials tend to exhibit a reduction in fatigue life when undergoing fatigue tests with increasing mean stress. Consequently, mean stress plays an important role in determining the fatigue behavior of high temperature structures. Thus, the mean stress effects must be evaluated when developing fatigue life curves for these components. To account for the effects of mean stress accurately and expand the ductility model's application to other conditions such as different loading waveforms and stress control mode, a new model based on DE theory is developed in the following section.

Proposed model for LCF–C life prediction

According to the mechanism of fatigue–creep interaction at high temperature, a good life prediction method must consider not only the effects of stress/strain level, loading history, impurity content, but also creep factors such as hold time, strain rate and temperature. Until now, many earlier approaches are mainly suitable for life predictions under strain control mode and few models can be applied to stress-controlled tests. So far as stress control mode is concerned, the LCF–C life is mainly influenced by stress range, mean stress, maximum stress, temperature, tensile/compressive hold time and stress rate. The reduction of the material ductility indicates the progressive exhaustion of the ability to absorb energy inherent in the material due to fatigue damage evolution, which is associated with the irreversible energy dissipation during fatigue failure. Fatigue–creep toughness acts as the damage parameter to define the dynamic viscosity based on ductility exhaustion theory. In this section, a new viscosity-based model under stress control mode is proposed for LCF–C life prediction.

Under LCF–C at high temperature, it has become apparent that fatigue life depends not only on the testing temperature but also on the loading waveform due to the creep damage. Besides, creep and transient effects such as cyclic hardening/softening have significant effects on fatigue damage accumulation. Compared with compressive dwell effects, creep damage is much more sensitive to the tensile stress and tensile dwell. In general, the compressive dwell effects on the LCF–C life are

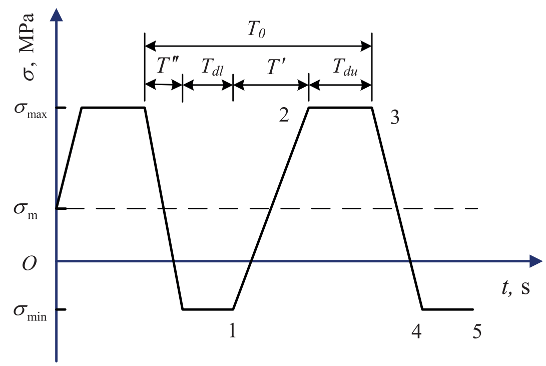

Figure 1. Trapezoidal loading waveform used in fatigue–creep interaction loading conditions.

complex (Zhang, 2010). One of the key factors leading to the failure of high temperature structures is the fatigue–creep interaction, which results from thermally induced stress and strain. The effects of time-dependent test parameters on fatigue life are complex, leading to complex interactions between fatigue and creep with environmental factors. These interactions cannot be characterized accurately in a life prediction model. Hence, a suitable life prediction model to account for the loading waveform, temperature and mean stress effects is needed, particularly when the results of LCF–C tests are applied to simulate thermal fatigue conditions. The trapezoid load diagram was used to analyze the LCF–C behavior of materials under high temperatures, which is depicted in Figure 1.

In Figure 1, T_{du},T_{dl},T' and T'' are the tensile hold time, compressive hold time, tension-going time and compression-going time, respectively, in one loading cycle when $\sigma_{\max} > 0$ and $\sigma_{\min} < 0$. T_{dl} is the tensile hold-time when $\sigma_{\min} > 0$. T_0 and $T$are the total time period and the period time not including the hold time, where $T = T' + T''$.

It should be pointed out that the material toughness is a mechanical property parameter sensitive to the fatigue–creep damage process (Ye and Wang, 2001). A certain quantity of energy, actually the ductility is gradually exhausted during fatigue failure. The energy dissipated in the material during one loading cycle or the cycles up to the failure, is usually calculated from the history of the changes in cyclic strain and stress, together with the number of cycles. The more accumulated damage leads to the more exhausted ductility, and failure occurs once a critical toughness threshold is reached. Using this relationship, the exhausted ductility has been viewed as an indicator of the accumulated damage under LCF–C (Fan et al., 2007; Goswami, 1997; Maurel et al., 2009; Payten et al., 2010; Zhu and Huang, 2010; Zhu et al., 2011a, 2011b).

As aforementioned, damage accumulation during LCF, creep and their interaction is actually a ductility exhaustion process in response to cyclic and static creep under stress control mode. According to the stress cycle shown in Figure 1, the elastic or plastic strain produced at the beginning of cyclic loading is the instantaneous elastic strain or instantaneous plastic strain, so there is no accumulated plastic strain. In the stage of loading (or tension-going period) $1 \rightarrow 2$, the produced strain consists of elastic strain $\Delta\varepsilon_e$ and plastic strain$\Delta\varepsilon_p$. The former is recoverable, thus only the plastic strain leads to a serious reduction in ductility (ductility exhaustion). In the period of tension hold $2 \rightarrow 3$, the produced strain is mainly the creep strain $\Delta\varepsilon_c$ which changes with time and causes the ductility exhaustion. During the stage of unloading (or compression-going period) $3 \rightarrow 4$, the deformation occurs in the form of recovered elastic strain and the possible irrecoverable plastic deformation, the latter will cause a reduction in ductility under tensile stresses. In the period of compression hold $4 \rightarrow 5$, the produced strain is mainly the compression creep strain. For the trapezoid loading waveform, due to plastic flow

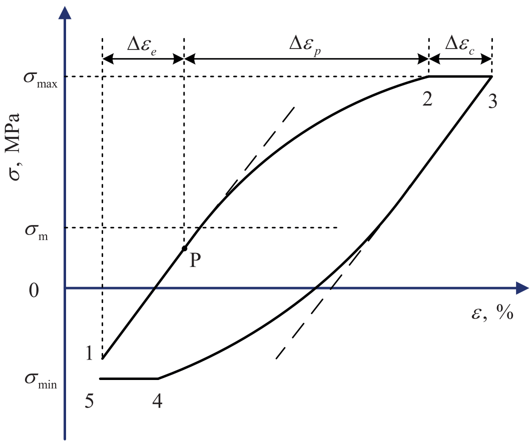

Figure 2. Hysteresis loop under stress-controlled trapezoidal loading waveform.

and creep deformation, the ductility is gradually exhausted along with the latter stage of loading, the whole tension hold phase and the former stage of unloading for most metallic materials. One of the main issues concerns with when the plastic strain will occur in the tension-going period, see P point as shown in Figure 2. From the viewpoint of plastic mechanics, there is not a one-to-one correspondence between the stress and strain under cyclic loading, but rather it is determined by the loading path. With different stress ratios, the stress needed for producing plastic strain will be different. Under cyclic loading, the interactive behavior between stress and strain during fatigue failure can be characterized by hysteresis loops. Recently, research has shown that a perspective based on viscosity can be used to quantify this interaction for LCF–C life prediction (Cheng and Plumtree, 1998; Fan et al., 2006, 2007; Goswami, 1997, 2004). The corresponding hysteresis loop under stress control mode with trapezoidal waveform is shown in Figure 2.

The determination of actual start value of the stress for producing plastic strain under high temperature (the stress magnitude σ_P at P point in Figure 2) has been of interest to industry and academia. Based on the exhaustion of material toughness and the dissipation of plastic strain energy during fatigue failure, ductility exhaustion is assumed to be related only to the plastic strain and the creep strain. To avoid the problem of determination of P point, an attempt has been made to deduce a new viscosity-based life prediction method under stress control mode, in which the fatigue–creep toughness is used as the control parameter.

Similar to the energy criterion proposed in Stowell (1966), a viscosity-based parameter E_p per cycle under fatigue–creep interaction was developed in the authors' previous work (Zhu and Huang, 2010)

$$E_p = T_{du}\sigma_{\max} + (T_{dl} + T)\sigma_{\min}H(\sigma_{\min}) + \frac{T}{2}f(\sigma_{\max},\sigma_{\min}) \tag{8}$$

and

$$f(\sigma_{\max},\sigma_{\min}) = \begin{cases} \Delta\sigma, & \sigma_{\min} > 0 \\ \frac{\sigma_{\max}^2}{\Delta\sigma}, & \sigma_{\min} \leq 0 \end{cases} \tag{9}$$

where $H(\sigma_{\min})$ is the unit step function of $\sigma_{\min}$, $f(\sigma_{\max},\sigma_{\min})$ is the stress conversion function, which depends on the maximum and minimum stress and material properties, $\Delta\sigma$ is the stress range. Based on the fact that all materials are sensitive to the tensile stress and tensile dwell, an increased dwell time often results in a decreased life. However, the effects of compressive stress and compressive dwell on the fatigue–creep life are complex and vary from material to material (Goswami and Hänninen, 2001a, 2001b; Zhang, 2010). According to the assumption that only tensile stress can induce fatigue–creep damage made in Chrzanowski (1976), $H(\sigma_{\min})$ is expressed as

$$H(\sigma_{\min}) = \begin{cases} 1, & \sigma_{\min} \geq 0 \\ 0, & \sigma_{\min} < 0 \end{cases} \tag{10}$$

As aforementioned, plastic strain occurs only when tensile stress equals σ_P for the trapezoidal loading waveform under different stress ratios. The dynamic viscosity υ_d per cycle is actually the stress area between the loading waveform and the σ_P-stress line. Comparing the viscosity-based parameter E_p in equation (8) with the dynamic viscosity υ_d in equation (1), note that the latter is included in the former which includes the extra elastic strain caused by tensile stress. According to the analysis of ductility exhaustion per cycle, both elastic strain energy density, ΔW_e, and plastic strain energy density, ΔW_p, are involved in fatigue damage accumulation for crack initiation and propagation. Based mainly on the idea of strain energy density exhaustion (Payten et al., 2010) and the above analysis, a new definition of dynamic viscosity υ_d has been presented in Zhu et al. (2012a) by using the viscosity-based parameter E_p and the tensile elastic energy input ΔW_{FL} per cycle

$$\upsilon_d = E_p - T_0 \Delta W_{FL} \tag{11}$$

where the tensile elastic energy causes no damage during fatigue failure and can be expressed as (Golos, 1995)

$$\Delta W_{FL} = \sigma_{\lim}^2 / 2E \tag{12}$$

where $\sigma_{\lim}$ represents the fatigue limit of material.

For consistency and simplicity, the relationship between the stress cycle period and fatigue life under stress control mode are assumed to follow a power law and the dynamic viscosity during fatigue failure is given as

$$\sum f(\upsilon_d) = C\left(E_p - T_0 \Delta W_{FL}\right)^{\alpha} \tag{13}$$

As fatigue toughness describes both strength and plasticity of a material, the dynamic viscosity υ_d is associated with the stress and strain by the material toughness based on the ductility exhaustion theory. Considering that only tensile inelastic strain energy can induce crack initiation and propagation, a generalized energy based fatigue–creep damage parameter has been developed to account for the effects of creep and mean stress on the fatigue life in (Zhu et al., 2012b)

$$\left(\Delta W_p \sigma_{\max}^{1+n'}\right) \cdot N_f^{\beta(1+n')} = C_1 \tag{14}$$

where ΔW_p is the plastic strain energy density measured from stabilized hysteresis loops.

As can be seen by equation (14), LCF life has a certain dependency on the fatigue–creep damage parameter $\Delta W_p \sigma_{\max}^{1+n'}$. Comparing equation (14) with equation (4), note that these two equations which are derived from different theoretical backgrounds have the similar form. Different from the empirical definition of fatigue toughness T_m in Zhu et al. (2012a), it can be given in a physics-based form

$$T_m = \left(\Delta W_p \sigma_{\max}^{1+n'}\right) \cdot N_f^{\beta(1+n')} \tag{15}$$

Based on the failure criterion of DE theory in equation (2), a new LCF–C life prediction model can be derived by equating the dynamic viscosity (equation (13)) and the fatigue toughness equation (equation (15)) as follows

$$C\left(E_p - T_0 \Delta W_{FL}\right)^{\alpha} = \left(\Delta W_p \sigma_{\max}^{1+n'}\right) \cdot N_f^{\beta(1+n')} \tag{16}$$

where C is a material parameter used to balance the units of this equation.

The number of cycles to failure under LCF–C can be calculated by rearranging equation (16)

$$N_f = C_2\left(\Delta W_p \sigma_{\max}^{1+n'}\right)^{\frac{-1}{\beta(1+n')}}\left(E_p - T_0 \Delta W_{FL}\right)^{\frac{\alpha}{\beta(1+n')}} \tag{17}$$

Before using equation (17) to predict LCF–C life, the cyclic stress strain parameter n' was easily derived by appropriate data fitting for each material using equation (6). Owing to only three material parameters (C_2, α andβ) in equation (17), which can also be fitted from test data (such as block loading test data), it is convenient to use this model for LCF–C life prediction at high temperature.

Studies have shown that material ratcheting behavior depends upon factors such as mean stress, stress amplitude, frequency, loading history and micro-structural characteristics (Xia et al., 1996). Similar to the damage parameters developed in Smith et al. (1970) and Xia et al. (1996), equation (17) incorporates most of these factors and describes the ratcheting effect on fatigue life by inelastic strain range (including plastic strain range and creep strain range) and the mean stress effect by maximum stress. From the above analysis, it is clear that both mean stress and ratcheting effects are taken into account within the proposed LCF–C life prediction model.

For actual components under stress-controlled LCF–C with hold time working conditions, the fatigue lives predicted by equation (17) represent the degradation of the materials' strength and plasticity. Compared with the published models in Goswami (1997, 2004), Fan et al. (2006) and Ye and Wang (2001), the proposed model considers not only the cyclic hardening effects on LCF–C life, but also the effects of mean stress under stress control mode. In addition, it provides a practical way to calculate the dynamic viscosity per cycle under LCF–C interaction conditions. In the next section, the predication results from the proposed model are compared with experimental data available in the literature.

Experimental validation

In this section, the prediction results using the proposed method are compared with experimental data available in the literature. The objective is to examine the applicability of the model to LCF–C life prediction under stress control mode. Data from a number of stress-controlled fatigue–creep interaction tests for 1.25Cr0.5Mo steel (Chen et al., 2007; Fan et al., 2007) were used to verify the feasibility and life prediction capability of the proposed model by comparing with the MSR (Fan et al., 2007) and GSEDF (Zhu and Huang, 2010) methods. The tests were conducted using stress-controlled trapezoid waveform with a hold period of 5s duration at $\sigma_{\max}$ and $\sigma_{\min}$ respectively, as

$T_{du} = T_{dl} = 5$s and total cycle time of 20 s as seen in Figure 1. The main factors influencing fatigue life and creep life are σ_a and σ_m, respectively. So fatigue–creep interaction behavior depends on σ_a and σ_m. Under different stress ratios and σ_m, the fatigue–creep tests were performed under two temperatures (540°C and 520°C), stress amplitudes ranging from 25 to 190 MPa, and four predetermined maximum stresses (200, 210, 220 and 230 MPa). More detailed mechanical properties of the materials and test conditions can be found in Chen et al. (2007) and Fan et al. (2007).

According to the results of simple tensile fatigue tests of 1.25Cr0.5Mo steel, it exhibits a slender cyclic hardening characteristic at 540°C. Using the least square method, the calculated value of K' and n' in equation (6) are 446.3 MPa and 0.09472, respectively. Life predictions for LCF–C were conducted using equation (17). Combining the experimental results and the loading parameters, under different maximum stress and stress ratio, the fitted life prediction model for 1.25Cr0.5Mo steel at 540°C is given by

$$N_f = 2.47628 \times 10^{36}\left(\Delta W_p \sigma_{\max}^{1.09472}\right)^{-0.849069}\left(E_p - 20\Delta W_{FL}\right)^{-2.37225} \tag{18}$$

Similarly at 520°C, the fatigue life is approximately expressed as

$$N_f = 1.46487 \times 10^{39}\left(\Delta W_p \sigma_{\max}^{1.3487}\right)^{-0.996514}\left(E_p - 20\Delta W_{FL}\right)^{-2.23421} \tag{19}$$

The comparison between experimental data and fatigue life predicted using equation (17) are made and depicted in Figure 3. The dashed line in the graph represents the ±1.5 factor indictors and the solid line for the ±2 factor indictors. From Figure 3, the result shows that prediction by the new model matches well with all the experimental data within ±2 scatter band and 32 out of 34 cyclic lives are predicted within a factor of ±1.5. Obviously, a good agreement is found between the experimental results and the theoretical predictions under different temperatures.

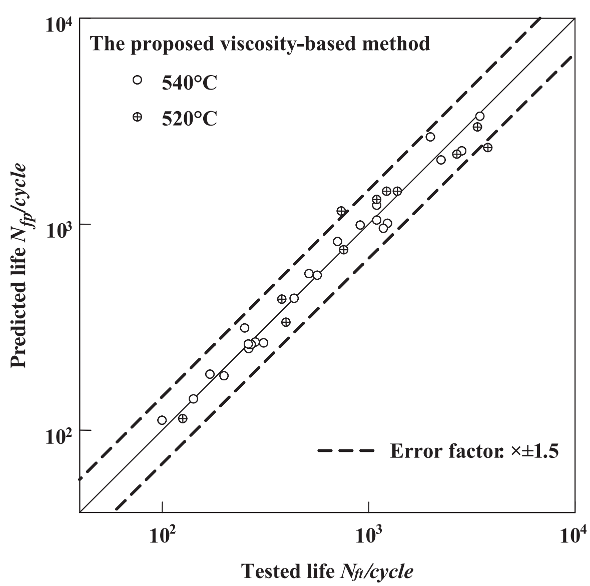

Figure 3. Comparison between lives predicted by the proposed viscosity-based method and lives tested.

To reflect the capability of the new method as explained in the preceding section and evaluate its applicability to account for the effects of mean stress and creep, two other methods, the MSR (Fan et al., 2007) and the GSEDF (Zhu et al., 2012b), were employed for comparison purposes, respectively.

The LCF–C lives predicted by these three methods are in accordance with experimental results, the correlations between the experimental and the predicted fatigue lives are given in Figure 4, in which comparisons between test and prediction by the GSEDF and the MSR methods are made.

The results show that nearly all the predicted cyclic lives by the GSEDF and the MSR methods fall into a range within a scatter band of ±2, while 30 out of 34, 26 out of 33 cyclic lives predicted by the GSEDF and the MSR model are within a factor of ±1.5, respectively. Comparing the scatter band and the standard deviation of these methods from Figures 3 and 4, it is found that the proposed model had the best predictability. The proposed life prediction model in equation (17) can predict the LCF–C behavior of 1.25Cr0.5Mo steel well at a certain temperature, but whether it can be consolidated into one equation for a certain temperature interval or not will be evaluated as follows.

In many cases, the failure of high-temperature structures comes from thermally induced stress and strain. Hence, developing a suitable life prediction model to account for temperature effects is very important, especially when the results of LCF tests can give a reference for the thermal fatigue analysis. If temperature effects on fatigue life can be properly characterized, life prediction at specific elevated temperatures will be available with reference fatigue data normally obtained at room temperature. In general, thermal fatigue tests are difficult to perform and also take a long time due to the slow temperature variation rate. At this point, high temperature isothermal fatigue data

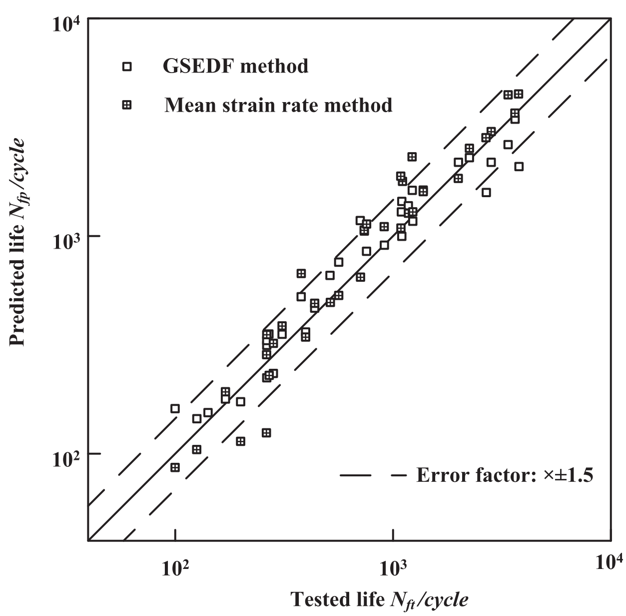

Figure 4. Comparison between lives predicted by GSEDF, MSR methods and lives tested. GSEDF: generalized strain energy damage function; MSR: mean strain rate.

were used for thermal fatigue analysis. Based on equation (17), the LCF–C life for 1.25Cr0.5Mo steel under different temperatures is predicted by

$$N_f = 1.05099 \times 10^{34}\left(\Delta W_p \sigma_{\max}^{1.09472}\right)^{-0.845813}\left(E_p - 20\Delta W_{FL}\right)^{-2.11692} \tag{20}$$

The comparisons are shown in Figure 5 by plotting the LCF–C life obtained by equation (20) and experimental data together at different temperatures.

It was observed that the prediction results agree with the experimental data very well for different temperature loading conditions. The fatigue life correction factor range is ±2 or better, with which 35 out of 36 and 31 out of 36 cyclic lives are predicted within a factor of ±2 and ±1.5, respectively. It is worth noting that equation (17) can be used to predict lives under complex loading conditions, in which the temperature effect on fatigue life is described. Based on DE theory, this model transforms the complex correlation between N_f and loading parameters (including $\sigma_{\max}$, σ_{m}, σ_a, $\Delta\varepsilon_p$, strain rate) into a rational relation, by which life prediction for other conditions can easily be made. Thus, the proposed model describes the time dependencies of deformation characteristics.

As both the proposed method and the GSEDF model considered the mean stress and loading waveform effects, the differences between the experimental and calculated LCF–C life by these two methods are relatively small. By introducing the dynamic viscosity υ_d, moreover, the proposed method considers the temperature effect rather than the GSEDF model. In practical engineering, a certain degree of mean stress exists in these components. Thus the proposed model can provide a reference for life evaluation of these components in the actual loading under thermal fatigue conditions. According to the development of the new model and the comparisons of the prediction results by these methods as shown in Figures 3 to 5, it should be noted that the proposed model has

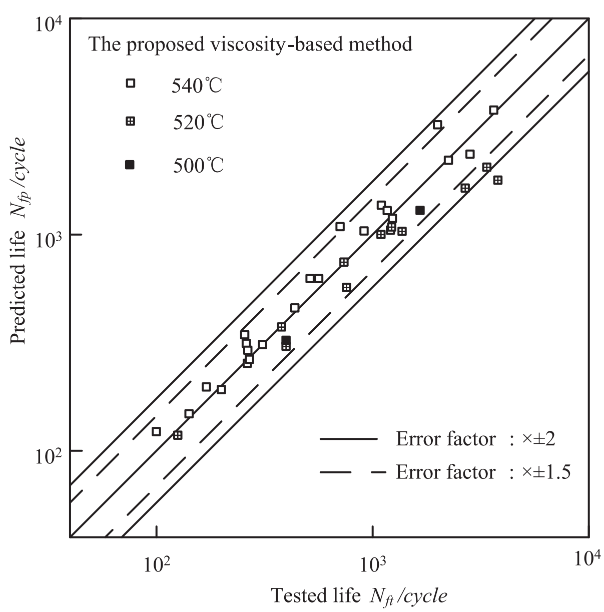

Figure 5. Comparison between lives predicted by the proposed method and lives tested under different temperatures.

a better life prediction capability than others, which support a ductility exhaustion approach for designing against creep/fatigue.

According to the proposed model, it is valid for most metallic materials. Moreover, it has the following advantages when compared to other approaches: few 'to be determined parameters or constants', considers mean stress, temperature and loading waveform effects and has a higher life prediction precision. Compared with the viscosity-based model developed in Zhu et al. (2012a), the fatigue toughness given in the proposed model is based on physical process rather than empirical definition. Thus, in summary, although it has been shown that a ductility exhaustion approach can predict the LCF–C life within a factor of two (in those cases where ductility exhaustion is the dominant mechanism under stress-controlled conditions), uncertainties in the detailed evaluations include: definition of cycles to failure, inclusion of a rapid strain rate component of creep which may not be damaging, definition and scatter of ductility. Further validation and modification are required to consider these uncertainties and other types of waveforms: shape, strain rate and hold time, etc. Application of the proposed model to life prediction for different materials and multiaxial loading also needs further study.

Conclusions

Based on the comparison of the predicted and experimental fatigue lives of tested material, the following conclusions can be drawn:

(1) A model describing the materials' life as a combination of LCF and creep was proposed based on ductility exhaustion theory and a viscosity-based method suitable for calculating the fatigue life under stress control mode was elaborated. The proposed model correlates and describes the fatigue–creep damage with the loading parameters in a certain temperature interval.
(2) Compared with the GSEDF and the MSR methods, the proposed model is more suitable for LCF–C life prediction of high temperature structures. By this method, all the test data were predicted within a factor of ± 2 and nearly 94.1% of the test data were predicted within a factor of ± 1.5. It has higher prediction accuracy and offers a useful reference for thermal fatigue analysis by calibration using isothermal fatigue tests only, which shows a great potential of cost savings for future test programs on the basis of similar materials and loading conditions.
(3) Compared with the existing strain energy based LCF–C life prediction methods, the benefits of the proposed model are achieved in the accuracy of fatigue life prediction by considering the creep and mean stress effects under stress-controlled loading. It provides an efficient way to describe the dynamic deterioration behavior of high temperature materials under fatigue–creep interaction. For thermal fatigue loading, further research work is required for experimental validation.

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Appendix

Notation

A	model parameter
C, C_1, C_2	material-specific constants
E	Young's modulus
E_p	viscosity-based parameter

K'	cyclic strength coefficient
n'	cyclic strain hardening exponent
N_f	number of cycles to failure
N_{ft}	number of tested life cycles
N_{fp}	number of predicted life cycles
ΔW_e	elastic strain energy density
ΔW_{FL}	tensile elastic strain energy input per cycle which causes no damage
ΔW_p	plastic strain energy density
T_m	fatigue toughness
α,β	model parameters
$\dot{\varepsilon}$	strain rate
υ_d	dynamic viscosity
$\Delta\varepsilon_t$	total strain range
$\Delta\varepsilon_e$,$\Delta\varepsilon_p$	elastic strain range and plastic strain range
$\Delta\varepsilon_{in}$	inelastic strain range
σ_a	stress amplitude
$\sigma_{\max}$,$\sigma_{\min}$	maximum and minimum stress
σ_m	mean stress
$\Delta\sigma_{sat}$	saturated stress range
$\Delta\sigma$	stress range