A Bayesian Approach for System Reliability Analysis With Multilevel Pass-Fail, Lifetime and Degradation Data Sets

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***Abstract*—Reliability analysis of complex systems is a critical issue in reliability engineering. Motivated by practical needs, this paper investigates a Bayesian approach for system reliability assessment and prediction with multilevel heterogeneous data sets. Two major imperatives have been handled in the proposed approach, which provides a comprehensive Bayesian framework for the integration of multilevel heterogeneous data sets. In particular, the pass-fail data, lifetime data, and degradation data at different system levels are combined coherently for system reliability analysis. This approach goes beyond the alternatives that deal with solely multilevel pass-fail or lifetime data, and presents a more practical tool for real engineering applications. In addition, the indices for reliability assessment and prediction are constructed coherently within the proposed Bayesian framework. It gives rise to a natural manner of incorporating this approach into a decision-making procedure for system operation and management. The effectiveness of the proposed approach is illustrated with reliability analysis of a navigation satellite.**

***Index Terms*—Bayesian reliability, multilevel heterogeneous data sets (MHDS), reliability assessment, reliability prediction.**

ACRONYMS

MCMC	Markov chain Monte Carlo
MHDS	multilevel heterogeneous data sets
RBD	reliability block diagram
PDF	probability density function
CDF	cumulative distribution function
i.i.d.	s-independent and identically distributed
r.v.	random variable

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NOTATION

S_0	node in the highest system level, e.g., system
$S_j, j = 1, \ldots, m$	nodes in the intermediate system level, e.g., subsystems
$C_i, i = 1, \ldots, n,$	nodes in the lowest system level, e.g., components
A_j	set of direct subordinates of node S_j
$\Psi(\cdot)$	structure function defined by the RBD
$R(t\|\boldsymbol{\theta})$	reliability function
$f(t\|\boldsymbol{\theta})$	probability density function
$L(D\|\boldsymbol{\theta})$	likelihood function
$N^{P/F}$	node with pass-fail data
N^{Life}	node with lifetime data
N^{Deg}	node with degradation data
$A^{P/F}$	set of nodes with pass-fail data
A^{Life}	set of nodes with lifetime data
A^{Deg}	set of nodes with degradation data
$D^{P/F} = (\boldsymbol{n}^{P/F}, \boldsymbol{y}^{P/F}, \boldsymbol{t}^{P/F})$	pass-fail data set
$D^{Life}(\boldsymbol{t}^F, \boldsymbol{t}^C)$	lifetime data set
$D^{Deg}(\boldsymbol{d}^{Deg}, \boldsymbol{t}^{Deg})$	degradation data set
$Y^{P/F}$	r.v. for pass-fail data
G^{Deg}	r.v. for degradation measurement
Y^{Deg}	degradation curve
$\pi(\boldsymbol{\Theta})$	prior distribution
$p(\boldsymbol{\Theta}\|D^{P/F}, D^{Life}, D^{Deg})$	posterior distribution
$T_{1-\alpha}$	predicted lifetime with failure quantile α

I. Introduction

MODERN products and systems are becoming increasingly complex. Reliability assessments and predictions about such systems have attracted great attention in reliability engineering [1], [2]. One of the critical challenges induced by complex systems is to implement system reliability analysis with limited full-system tests. Nevertheless, there may be abundant heterogeneous data at different system levels that need to be incorporated. Taking a satellite as an example, before it is launched into space, engineers must figure out whether the satellite is reliable enough to be launched. However, due to limited time and resources, engineers cannot conduct full-system tests. The only available reliability information for system reliability analysis is gathered from various stakeholders, each responsible for various subsystems and components. Such data sets are collected from different reliability tests, and distributed in different system levels. The pass-fail, lifetime, and degradation data sets are generally included. To perform a reliable system analysis of the satellite, one needs to utilize these multilevel heterogeneous data sets (MHDS) in a comprehensive way. Moreover, the indices for reliability assessment and prediction should be derived coherently based on these data sets. In other words, it is important to incorporate the MHDS into the system reliability analysis process.

Such examples are pervasive throughout different industries, e.g., the reliability analysis of high-speed trains [3], [4], nuclear power plants [5], [6], remote wind generators [7], and all kinds of munitions [8], [9]. All of these systems are facing the challenge of integrating MHDS for reliability analysis. This challenge has also been highlighted in a panel discussion in Technometrics [10], and the work by Anderson-Cook [11]. There is also a strong demand in system operational research and engineering design to deal with reliability analysis of complex systems with MHDS. This demand has been identified as one of the most critical problems in the field of engineering design, which has not been resolved sufficiently [12], [13]. Accordingly, the objective of this paper is to develop a method for system reliability analysis with multilevel pass-fail, lifetime, and degradation data sets. Two important analytical concerns have been addressed in the proposed approach: 1) a coherent framework for integrating MHDS using the Bayesian approach, and 2) a natural manner of deriving the indices for reliability assessment and prediction within the proposed framework.

To highlight our contributions, it is useful to review the relevant research in Bayesian reliability analysis by integrating MHDS. Such problems were initially studied by Mastran [14], Mastran & Singpurwalla [15], Barlow [16], Martz *et al.* [17], and Martz & Waller [18]. They put forward methods for incorporating multilevel binomial data or lifetime data using a Bayesian method that was mainly carried out in an approximation. However, the coherency and efficiency of their methods become growing concerns with increases in system complexity and data types. Benefiting from the implementation of the Markov chain Monte Carlo (MCMC) methods for Bayesian analysis, fully Bayesian methods for system reliability analysis have been extensively studied. Methods for system reliability analysis with lifetime or degradation data have been put forward by Huang *et al.* [19], Wang *et al.* [20], and Ye *et al.* [21]. Additionally, Huang & An [22] presented a discrete stress-strength interference model for system reliability analysis with dependent strength. Wang *et al.* [23] presented a Bayesian updating mechanism to deal with reliability assessment with evolving, insufficient, and subjective data sets. These methods were mainly constructed based on the lifetime or degradation data at the system level, which were effective for a system with lots of system-level data. However, when dealing with a system with limited full-system tests, the effectiveness and the precision of these methods were challenged.

Considering the research on system reliability analysis with multilevel data sets, Johnson *et al.* [24] proposed a fully hierarchical Bayesian method for reliability assessment of multi-component systems with binomial data. This work was extended by Hamada *et al.* [25] to a system described in a fault tree with binomial data as well. Wilson *et al.* [26] briefly summarized system reliability analysis with multilevel data. Later, Anderson-Cook *et al.* [27], Shane Reese *et al.* [28], and Jackson & Mosleh [29] carried on this research by separately focusing on situations with multilevel pass-fail data and lifetime data. The majority of these methods dealt with a specific type of reliability data, such as binomial or lifetime data. However, when facing situations with MHDS, these methods cannot be used because their model frameworks were limited for specific data types.

The review of related literature reveals that little attention has been paid to system reliability analysis with MHDS. Limited exceptions are the methods proposed by Wilson *et al.* [26], Anderson-Cook *et al.* [30], and Guo & Wilson [31]. In particular, the methods proposed by Wilson *et al.* [26], and Anderson-Cook *et al.* [30] were mainly used to demonstrate the possibility of incorporating MHDS. Both of them were developed under particular system structures with specific assumptions about the data types. On the other hand, the method developed by Guo & Wilson [31] provided a generic framework for reliability estimation with MHDS. However, the indices for reliability analysis within their method have not been studied, which makes the applicability of their method quite limited, especially for the practical imperatives highlighted above. As one can see, reliability analysis by incorporating MHDS as considered in this paper has not been extensively studied in the literature.

The aim of this paper is to develop a fully Bayesian approach for reliability analysis by integrating multilevel pass-fail, lifetime, and degradation data. To overcome the limitation of data types which can be analyzed using the methods proposed by Johnson *et al.* [24], Wang *et al.* [20], and Shane Reese *et al.* [28], the proposed approach provides a more generic method for reliability analysis of complex systems with MHDS. A coherent information integration framework has been developed to carry out reliability analysis by integrating multilevel pass-fail, lifetime, and degradation data. On the other hand, rather than developing another framework limited to reliability estimation, such as those proposed by Wilson *et al.* [26], Anderson-Cook *et al.* [30], and Guo & Wilson [31], we carry out a coherent study of reliability assessment and prediction with MHDS. By developing the reliability indicators within the Bayesian framework, the proposed approach can be naturally incorporated into a decision-making procedure for system operation and management.

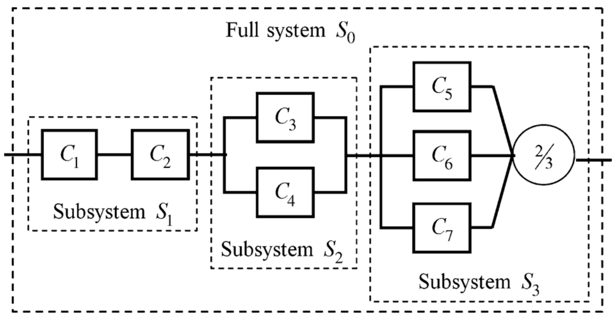

Fig. 1. Reliability block diagram of a simplified satellite system.

The remainder of this paper is organized as follows. We begin by introducing a general system, and the related MHDS in Section II. In Section III, the Bayesian approach for reliability analysis with MHDS is presented in a graphic form with the kernel strategy. Then, the model within the proposed Bayesian approach for information integration is developed step by step in Section IV. The indices for reliability assessment and prediction derived based on the information integration model are presented in Section V. We illustrate the proposed approach with an application to a navigation satellite in Section VI. Finally, we conclude the paper with a discussion.

II. A General System and MHDS

A. A General System Structure

Generally, a system may consist of multiple components which are connected in a series, parallel, or series-parallel structure. Without loss of generality, a series k-out-of-n: G system is studied for illustration. By making k equal to n or 1, the k-out-of-n: G structure respectively becomes a series or a parallel structure. Various system structures can be included as special cases of the series k-out-of-n: G structure. In this paper, we develop the proposed approach by demonstrating on a simplified system structure, which is derived from a satellite in a satellite navigation system. The reliability block diagram (RBD) of the satellite is depicted in Fig. 1. It consists of three subsystems in the form of series k-out-of-n: G structures with S_1 being a 2-out-2 structure, S_2 being a 1-out-2 structure, and S_3 being a 2-out-3 structure. Note that a fault tree that contains AND and OR gates can be easily transformed to a RBD [32].

In this paper, we use the following structure. The components, subsystems, and system in the RBD are referred to as nodes, such as C_i and S_j in Fig. 1. The direct subordinates of S_j are the nodes in the next lower level, which constitute node S_j. The set of direct subordinates of S_j is denoted as A_j. In Fig. 1, for instance, system S_0 has a direct subordinates set $A_0 = (S_1, S_2, S_3)$.

B. The Multilevel Heterogeneous Data Sets

Reliability data for a complex system are generally gathered from various sources in different forms. The MHDS investigated in this paper include the pass-fail, lifetime, and degradation data sets collected from separate reliability tests for the components, subsystems, and system. Specifically, the first group of data sets is the pass-fail, lifetime, and degradation data sets collected for individual components. The second group is the pass-fail, and lifetime data sets collected for the subsystems, and the overall system. The third source of information is prior information regarding the reliability of particular nodes before these data sets are collected. The paper is dedicated to modeling and incorporating the multilevel pass-fail, lifetime, and degradation data sets. The third source of information is combined illustratively in the case study. Three kinds of prior information are incorporated, including the similarity of reliability for groups of components, the informative priors for particular components, and diffuse prior information (non-informative priors) for the remaining components. For more information about the prior information derivation, please refer to Seth [33], Bedford *et al.* [34], and Gutierrez-Pulido *et al.* [35].

III. The Proposed Bayesian Approach

The proposed approach is originated for integrating the information contained in MHDS. The approach is developed by addressing two major concerns: 1) parametrically modeling the multilevel system structure to preserve the probabilistic constructs defined by the RBD, and 2) coherently combining the heterogeneous data sets through the derivation of their joint likelihood function and the formulation of the parameters' joint posterior distribution. Accordingly, the proposed approach consists of three indispensable frameworks: the framework for modeling a multilevel system structure, the framework for combining heterogeneous data sets, and the framework for deriving indices for reliability analysis. A descriptive flowchart of the proposed Bayesian approach is given in Fig. 2.

The first framework is a substitution strategy for modeling multilevel system structures. It is carried out by re-expressing the reliability function of high level node S_j in terms of the corresponding functions of its direct subordinates, which are contained in set A_j. The structure function Ψ_{Sj} derived from the RBD is used to construct the inherent functional relationship.

The second framework is a combining strategy for integrating the heterogeneous data sets. It is implemented by multiplying together the likelihood contribution of each data type to obtain a joint likelihood function. These likelihood contributions are separately developed according to the specific data types and parametric models of particular nodes.

The third framework is a Bayesian inference strategy for information integration and indices derivation. It is carried out by the construction of a Bayesian model for MHDS, and the formulization of the model based reliability indices for system reliability analysis. The Bayesian model is constructed by deriving the posterior distribution of model parameters using the joint likelihood function and specified prior distributions. The model based reliability indices are developed in a fully Bayesian way. These indices are generally presented s-conditionally on particular model parameters. As the joint posterior distribution of model parameters are obtained, the inferences of these indices are generated by averaging over the posterior distribution of related model parameters.

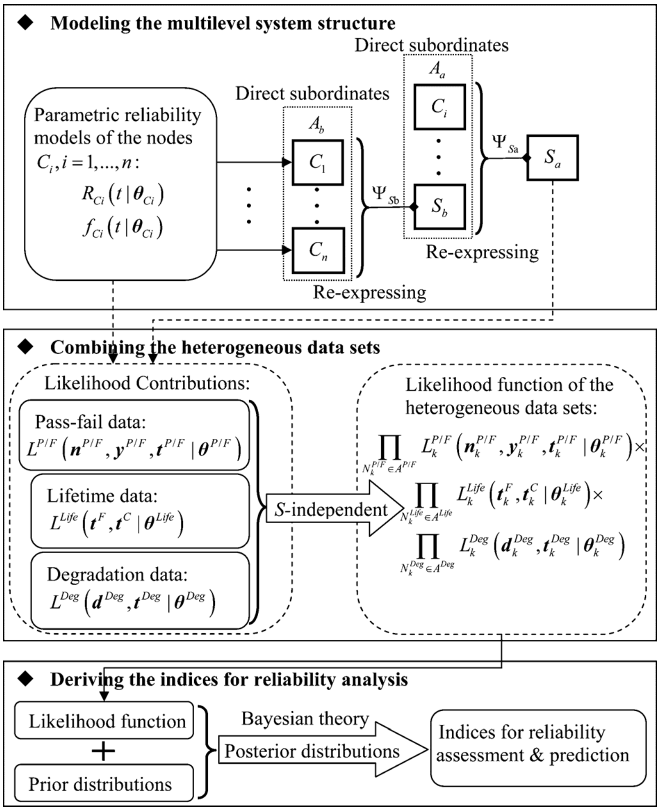

Fig. 2. Descriptive flowchart of the proposed approach.

IV. Bayesian Approach for MHDS

In this section, the Bayesian model for MHDS is constructed following the basic framework of the proposed approach: (i) modeling a multilevel system structure, (ii) deriving a joint likelihood function for heterogeneous data sets, and (iii) constructing a Bayesian model for MHDS.

A. Modeling a Multilevel System Structure

The multilevel system structure is modeled parametrically to provide a coherent information transition platform. As depicted in Fig. 2, the multilevel system structure is modeled based on parametric models of components $C_i, i = 1, \dots, n$. A substitution strategy is developed for modeling the high level nodes $S_j, j = 0, 1, \dots, m$. This substitution is implemented by re-expressing the reliability function of S_j with reliability functions of its direct subordinates. For instance, in Fig. 2, the reliability function of S_a is re-expressed by the reliability functions of its direct subordinates in set A_a based on the structure function Ψ_{Sa}. Respectively, let $R_{Ci}(t|\boldsymbol{\theta}_{Ci})$, and $f_{Ci}(t|\boldsymbol{\theta}_{Ci})$ denote the reliability function, and probability density function (PDF) of component C_i with parameters $\boldsymbol{\theta}_{Ci}$. The reliability function, and PDF of the higher level node S_j are expressed as

$$\begin{aligned} R_{Sj}(t|\boldsymbol{\theta}_{Sj}) &= \Psi_{Sj}(R_{Sk}, R_{Ck} : \forall S_k \in A_j, \forall C_k \in A_j) \\ f_{Sj}(t|\boldsymbol{\theta}_{Sj}) &= -\sum_{\forall S_k \in A_j} \frac{\partial R_{Sj}}{\partial R_{Sk}} \frac{dR_{Sk}(t|\boldsymbol{\theta}_{Sk})}{dt} \\ &\quad - \sum_{\forall C_k \in A_j} \frac{\partial R_{Sj}}{\partial R_{Ck}} f_{Ck}(t|\boldsymbol{\theta}_{Ck}) \end{aligned} \tag{1}$$

where $\boldsymbol{\theta}_{Sj}$ denotes model parameters involved in the reliability function of node S_j. $\Psi_{Sj}(\cdot)$ is the structure function of node S_j defined by the RBD, which describes the reliability relationship between the node S_j and its direct subordinates. S_k, and C_k are respectively the subordinate subsystems, and components of node S_j within the direct subordinates set A_j.

B. Combining MHDS

The heterogeneous data sets are incorporated through their contributions to the joint likelihood of the Bayesian model, as depicted in Fig. 2. In this subsection, the reliability models and the likelihood contributions of the heterogeneous data sets are modeled and analyzed.

1) Pass-Fail Data, and Their Likelihood Contributions: Let $N^{P/F}$ denote the node C or S with pass-fail data. Suppose L lots of $N^{P/F}$ are subject to test with random sampling. At every m_i different observation time points t_{ij}, n_{ij} samples are tested separately, where $i = 1, \dots, L$, and $j = 1, \dots, m_i$. We observe that the number of units that pass each test is y_{ij}. Let $Y_{ij}^{P/F}$ be the r.v. following a s-binomial distribution. The probability that a unit $N^{P/F}$ passes the test at time t_{ij} is determined by its reliability at that time, as $R(t_{ij}|\boldsymbol{\theta}^{P/F})$. The model for the pass-fail data is described as

$$\begin{aligned} &Y^{\frac{P}{F}}\left(t_{ij}|\boldsymbol{\theta}^{\frac{P}{F}}\right) \sim Binomial\left(n_{ij}, R\left(t_{ij}|\boldsymbol{\theta}^{\frac{P}{F}}\right)\right), \\ &i = 1, \dots, L, j = 1, \dots, m_i \end{aligned} \tag{2}$$

For node $N^{P/F}$ with reliability function $R(t_{ij}|\boldsymbol{\theta}^{P/F})$, the likelihood of the pass-fail data set $D^{P/F} = (\boldsymbol{n}^{P/F}, \boldsymbol{y}^{P/F}, \boldsymbol{t}^{P/F})$ can be described as

$$\begin{aligned} L^{\frac{P}{F}}\left(\boldsymbol{n}^{\frac{P}{F}}, \boldsymbol{y}^{\frac{P}{F}}, \boldsymbol{t}^{\frac{P}{F}}|\boldsymbol{\theta}^{\frac{P}{F}}\right) = \prod_{i=1}^{L}\prod_{j=1}^{m_i}\Bigg\{ &\binom{n_{ij}}{y_{ij}}\left[R\left(t_{ij}|\boldsymbol{\theta}^{\frac{P}{F}}\right)\right]^{y_{ij}} \\ &\times\left[1 - R\left(t_{ij}|\boldsymbol{\theta}^{\frac{P}{F}}\right)\right]^{n_{ij}-y_{ij}}\Bigg\} \end{aligned} \tag{3}$$

where the n_{ij}, y_{ij}, and t_{ij} with $i = 1, \dots, L$, and $j = 1, \dots, m_i$ in this pass-fail data set for node $N^{P/F}$ are separately included in the matrices $\boldsymbol{n}^{P/F}$, $\boldsymbol{y}^{P/F}$, and $\boldsymbol{t}^{P/F}$. The parameters involved in the reliability function of node $N^{P/F}$ as described in (1) are included in the parameter vector $\boldsymbol{\theta}^{P/F}$.

2) Lifetime Data and Their Likelihood Contributions: Let N^{Life} denote the nodes C or S with lifetime data. Suppose n units are tested. A group of right-censored lifetime data is collected, which includes n^F exact failure time points t_i^F with $i = 1, \dots, n^F$, and n^C right-censored time points t_i^C with $i = 1, \dots, n^C$, where $n = n^F + n^C$. The likelihood contribution of the exact failure time t_i^F is $f(t_i^F|\boldsymbol{\theta}^{Life})$. The likelihood contribution of the right-censored time t_i^C is in the form of $R(t_i^C|\boldsymbol{\theta}^{Life})$. Then, the likelihood function for the lifetime data $D^{Life} = (\boldsymbol{t}^F, \boldsymbol{t}^C)$ can be obtained as

$$L^{Life}(\boldsymbol{t}^F, \boldsymbol{t}^C|\boldsymbol{\theta}^{Life}) = \prod_{i=1}^{n^F} f\left(t_i^F|\boldsymbol{\theta}^{Life}\right)\prod_{i=1}^{n^C} R\left(t_i^C|\boldsymbol{\theta}^{Life}\right) \tag{4}$$

where the observed lifetime t_i^F with $i = 1, \dots, n^F$ and t_i^C with $i = 1, \dots, n^C$ are separately include in the vectors $\boldsymbol{t}^F$ and $\boldsymbol{t}^C$.

The parameters for the reliability function of node N^{Life} are included in the parameter vector $\boldsymbol{\theta}^{Life}$.

The likelihood contribution of lifetime data described above can be extended to the situation with left-censored or interval-censored lifetime data. This extension can be implemented by replacing the likelihood contribution of $R(t_i^{RC}|\boldsymbol{\theta}^{Life})$ by $1-R(t_i^{LC}|\boldsymbol{\theta}^{Life})$ for left-censored time t_i^{LC}, and by $R(t_i^{LC}|\boldsymbol{\theta}^{Life}) - R(t_i^{RC}|\boldsymbol{\theta}^{Life})$ for interval-censored time points (t_i^{LC}, t_i^{RC}).

3) Degradation Data and Their Likelihood Contributions: Let N^{Deg} denote the component C with degradation data. Suppose n components have been tested. The degradation measurements g_{ij} have been observed at each observation time points t_{ij}, where $i = 1, \ldots, n$, and $j = 1, \ldots, m_i$. These degradation measurements g_{ij} include the measurement errors ξ_i for each tested component. Let G^{Deg} be the r.v. for the degradation measurement. Given the measurement error $\xi_i \sim Normal(0, \sigma^2)$, the degradation process can be modeled by the distribution

$$G^{Deg}\left(t_{ij}|\boldsymbol{\beta}^{Deg}, \sigma^2\right) \sim Normal\left(Y^{Deg}(t_{ij}|\boldsymbol{\beta}^{Deg}), \sigma^2\right) \tag{5}$$

where $Y^{Deg}(t_{ij}|\boldsymbol{\beta}^{Deg})$ denotes the degradation curve of the component N^{Deg}, which is a function of testing time t_{ij} with parameters $\boldsymbol{\beta}^{Deg}$. It is the real degradation of this component aside from the measurement error. This degradation curve is generally defined by a specific failure mechanism of the component N^{Deg}. A general review of this degradation model can be found in the work by Bae *et al.* [36].

The component N^{Deg} fails when the degradation curve $Y^{Deg}(t_{ij}|\boldsymbol{\beta}^{Deg})$ first crosses a predefined threshold value Y^D. As a convention, with the assumption of a monotonically increasing degradation path, the reliability of the unit N^{Deg} is the probability that the value of $Y^{Deg}(t_{ij}|\boldsymbol{\beta}^{Deg})$ is higher than the lower critical threshold Y^D [36].

$$R(t_{ij}|\boldsymbol{\beta}^{Deg}) = \mathrm{P}\left[Y^{Deg}(t_{ij}|\boldsymbol{\beta}^{Deg}) \geq Y^D\right] \tag{6}$$

Based on the degradation model in (5), the likelihood function for degradation data $D^{Deg} = (\boldsymbol{d}^{Deg}, \boldsymbol{t}^{Deg})$ can be expressed as

$$L^{Deg}(\boldsymbol{g}^{Deg}, \boldsymbol{t}^{Deg}|\boldsymbol{\theta}^{Deg}) = \prod_{i=1}^{n}\prod_{j=1}^{m_i} \phi\left(\left(g_{ij} - Y^{Deg}(t_{ij}|\boldsymbol{\beta}^{Deg})\right)\sigma^{-1}\right) \tag{7}$$

where $\phi(\cdot)$ is the PDF of the standard s-normal distribution. The degradation measurements g_{ij} and t_{ij} with $i = 1, \ldots, n$ and $j = 1, \ldots, m_i$ are included in the matrices $\boldsymbol{g}^{Deg}$ and $\boldsymbol{t}^{Deg}$. Parameters for the reliability function of N^{Deg} are included in the parameter vector $\boldsymbol{\beta}^{Deg}$. All the model parameters involved in the degradation data are included in the parameter vector $\boldsymbol{\theta}^{Deg}$.

The likelihood contribution derived above can be extended to other degradation models with slight changes. The degradation models that are reviewed by Bae *et al.* [36] or applied in the literature by Ye *et al.* [37] can be easily incorporated by substituting (5) and (7) with the corresponding probability functions and the implied likelihood functions.

C. Bayesian Model for MHDS

The likelihood function for the MHDS is obtained by multiplying together the respective likelihood contributions of these data sets. Let $A^{P/F}$, A^{Life}, and A^{Deg} separately denote the sets of C and S that related to the pass-fail, lifetime, and degradation data. Then the likelihood function for the MHDS is given by

$$\begin{aligned} L\left(D^{\frac{P}{F}}, D^{Life}, D^{Deg}|\boldsymbol{\Theta}\right) &= \prod_{N_k^{\frac{P}{F}} \in A^{\frac{P}{F}}} L_k^{\frac{P}{F}}\left(\boldsymbol{n}_k^{\frac{P}{F}}, \boldsymbol{y}_k^{\frac{P}{F}}, \boldsymbol{t}_k^{\frac{P}{F}}|\boldsymbol{\theta}_k^{\frac{P}{F}}\right) \\ &\times \prod_{N_k^{Life} \in A^{Life}} L_k^{Life}\left(\boldsymbol{t}_k^F, \boldsymbol{t}_k^C|\boldsymbol{\theta}_k^{Life}\right) \\ &\times \prod_{N_k^{Deg} \in A^{Deg}} L_k^{Deg}\left(\boldsymbol{g}_k^{Deg}, \boldsymbol{t}_k^{Deg}|\boldsymbol{\theta}_k^{Deg}\right) \end{aligned} \tag{8}$$

where $\boldsymbol{\Theta}$ includes all the parameters involved in the whole model.

According to the Bayesian theory, given the prior distribution of model parameters $\boldsymbol{\Theta}$, the Bayesian model for the MHDS is obtained as

$$\begin{aligned} p\left(\boldsymbol{\Theta}|D^{\frac{P}{F}}, D^{Life}, D^{Deg}\right) &\propto \pi(\boldsymbol{\Theta}) \times L\left(D^{\frac{P}{F}}, D^{Life}, D^{Deg}|\boldsymbol{\Theta}\right) \\ &= \pi(\boldsymbol{\Theta}) \times \prod_{N_k^{\frac{P}{F}} \in A^{\frac{P}{F}}} L_k^{\frac{P}{F}}\left(\boldsymbol{n}_k^{\frac{P}{F}}, \boldsymbol{y}_k^{\frac{P}{F}}, \boldsymbol{t}_k^{\frac{P}{F}}|\boldsymbol{\theta}_k^{\frac{P}{F}}\right) \\ &\times \prod_{N_k^{Life} \in A^{Life}} L_k^{Life}\left(\boldsymbol{t}_k^F, \boldsymbol{t}_k^C|\boldsymbol{\theta}_k^{Life}\right) \\ &\times \prod_{N_k^{Deg} \in A^{Deg}} L_k^{Deg}\left(\boldsymbol{g}_k^{Deg}, \boldsymbol{t}_k^{Deg}|\boldsymbol{\theta}_k^{Deg}\right) \end{aligned} \tag{9}$$

where $\pi(\boldsymbol{\Theta})$ are the joint prior distributions for system model parameters. As described in Section II part B, three kinds of prior distributions have been considered in this paper, which reflect different considerations for the derivation and choice of priors. $p(\boldsymbol{\Theta}|D^{P/F}, D^{Life}, D^{Deg})$ is the joint posterior distribution of model parameters $\boldsymbol{\Theta}$, which represents the result of integrating MHDS. The construction of indices for reliability assessment and prediction is based on this posterior distribution.

Obviously, the joint posterior distribution in (9) cannot be obtained analytically. The MCMC method is utilized to generate samples of model parameters from this joint posterior distribution. Meanwhile, the sample-based posterior analysis can be easily carried out based on these generated posterior samples. There are two series of MCMC algorithms, namely the Metropolish algorithms (e.g., Gelfand & Smith [38]), and the Gibbs samplers (e.g., Smith & Roberts [39]). In addition, a well-developed software package, the WinBUGS [40], is used to implement the MCMC for the Bayesian model derived above. For details regarding MCMC and BUGS, readers are referred to the works by Ntzoufras [41], and Kruschke [42].

D. Model Validation and Justification

As described in Fig. 2, the construction of indices for reliability analysis and prediction are based on the joint posterior distribution of model parameters $\boldsymbol{\Theta}$. To present a reliable construction of these reliability indices, it is necessary to test the adequacy of the proposed Bayesian approach for the MHDS. In this subsection, we describe a model diagnostic approach based on the Bayesian χ^2 goodness-of-fit test introduced by Johnson [43].

Let $T_s = (t_1, t_2, t_3, \ldots, t_n)$ denote an i.i.d. data set with the CDF $F(t|\boldsymbol{\Theta})$. Let $0 = a_0 < a_1 < \ldots < a_K = 1$ denote K equally spaced quantiles from a s-uniform distribution, and define $p_j = a_j - a_{j-1}$ and $K \approx n^{0.4}$, where n is the sample size of the observation T_s. The procedure of this test is described as follows.

1) Generate a random sample $\tilde{\boldsymbol{\Theta}}$ from the joint posterior distribution $p(\boldsymbol{\Theta}|D^{P/F}, D^{Life}, D^{Deg})$.
2) Calculate the number of observations $t_i, i = 1, \ldots, n$ that fall into the interval $[a_{j-1}, a_j]$ with $a_{j-1} < F(t|\hat{\Theta}) \leq a_j$ for all the K intervals as $m_j(\tilde{\boldsymbol{\Theta}}), j = 1, \ldots, K$.
3) Calculate the Bayesian χ^2 test statistic for the random sample $\tilde{\boldsymbol{\Theta}}$ defined by

$$R^B(\tilde{\boldsymbol{\Theta}}) = \sum_{j=1}^{K} \frac{\left(m_j(\tilde{\boldsymbol{\Theta}}) - np_j\right)^2}{np_j}. \quad (10)$$

4) Repeat steps 1 to 3 for N times, and calculate the probability that $B_p = \Pr(R^B(\tilde{\boldsymbol{\Theta}}) < \chi^2_{K-1,0.95})$ with $\chi^2_{K-1,0.95}$, denoting the 0.95 quantile of a chi-square distribution on $K-1$ degrees of freedom.

The fitness of the model can be calibrated by this probability B_p. According to the procedure, the probability B_p is calculated in a simulation way. It is based on the posterior distribution of the proposed Bayesian model and the χ^2 goodness-of-fit test. Hence, it is generally reported that $B_p\%$ of the samples from the joint posterior distribution of parameters $\boldsymbol{\Theta}$ fit the data well. The higher the probability is, the better the model fits the data sets.

V. Indices For Reliability Analysis With MHDS

In this section, the indices for system reliability assessment and prediction are constructed. Following the strategy depicted in Fig. 2, these indices are derived based on the joint posterior distribution.

A. Indices for Reliability Assessment

When the MHDS are available, it is interesting to assess the reliability of the system at the present time. Moreover, it is necessary to estimate the residual life of the system with available data sets. Such assessments are usually adopted to set the operation and management of the system. In this subsection, the failure rate and residual life distribution are studied in a fully Bayesian way.

Based on the system reliability function in (1), and the joint posterior distribution of the parameters $\boldsymbol{\Theta}$ in (9), the failure rate of the system at the present time t_p can be obtained as

$$\lambda(t_p|D) = \int_{\Theta} \frac{f(t_p|\boldsymbol{\Theta})}{R(t_p|\boldsymbol{\Theta})} p(\boldsymbol{\Theta}|D) d\boldsymbol{\Theta} \quad (11)$$

where $D = \{D^{P/F}, D^{Life}, D^{Deg}\}$ denotes the available MHDS. $f(t_p|\boldsymbol{\Theta})$, $R(t_p|\boldsymbol{\Theta})$, and $\lambda(t_p|D)$ are separately the PDF, reliability function, and failure rate of the system.

Given that the system has survived up to the present time t_p, the probability that the system will survive another interval of mission time x can be calculated by

$$\begin{aligned} R(x + t_p|t_p, D) &= P(T > x + t_p | T > t_p, D) \\ &= \int_{\Theta} \frac{R(x + t_p|\boldsymbol{\Theta})}{R(t_p|\boldsymbol{\Theta})} p(\boldsymbol{\Theta}|D) d\boldsymbol{\Theta} \end{aligned} \quad (12)$$

The PDF of the residual life X can be obtained based on (12) as

$$g_X(x + t_p|t_p, D) = \int_{\Theta} \frac{f(x + t_p|\boldsymbol{\Theta})}{R(t_p|\boldsymbol{\Theta})} p(\boldsymbol{\Theta}|D) d\boldsymbol{\Theta}. \quad (13)$$

Similar to the joint posterior distribution in (9), these reliability indices based on the joint posterior distribution cannot be specified analytically. The simulation based integration is used to facilitate their calculations. By substituting the generated posterior samples $\tilde{\boldsymbol{\Theta}}$ into the corresponding PDF and reliability functions above, i.i.d. samples for these reliability indices are obtained. Summary statistics can be easily obtained based on these random i.i.d. samples. For instance, the integrations above are approximated by the mean of relative i.i.d. samples. Moreover, the variances and confidence intervals for these indices can be obtained within this Bayesian framework as well.

B. Indices for Reliability Prediction

When a further system is launched, it is necessary to predict system reliability at future time points. Such predictions are usually adopted to set strategies for system operation and warranty. In this subsection, the reliability as a function of mission time, and the lower percentile of time-to-failure distribution of the system, are constructed. The former is used to predict system reliability in a whole lifetime scale. The latter can be adopted to describe dependability of the system in a specific mission time interval.

Based on the Bayesian model for the MHDS, the reliability of the system as a function of mission time can be obtained as

$$R(t|D) = \int_{\Theta} R(t|\boldsymbol{\Theta}) p(\boldsymbol{\Theta}|D) d\boldsymbol{\Theta}. \quad (14)$$

Additionally, based on (14), the $1 - \alpha$ lower-bound on the system time-to-failure (time-to-first-failure) can be obtained as

$$T_{1-\alpha} = \inf \left\{ t' \geq 0 : \int_{\Theta} R(t'|\boldsymbol{\Theta}) p(\boldsymbol{\Theta}|D) d\boldsymbol{\Theta} \geq 1 - \alpha \right\} \quad (15)$$

where $T_{1-\alpha}$ is the predicted lifetime of the system defined by the $1 - \alpha$ lower-bound on system time-to-first-failure. α is the failure quantile, which is determined based on the consequence and the risk of the early failure of the system.

Similar to (11)–(13), the indices for reliability prediction derived above have no analytical forms. The calculations are based

TABLE I
MULTILEVEL HETEROGENEOUS RELIABILITY DATA FOR THE SATELLITE

Nodes	Pass-Fail data as numbers of pass vs. sample (observation time)
C_1	23/25 (2), 21/25 (4), 20/25 (6), 18/25 (8), 17/25 (10), 15/25 (12), 14/25 (14);
C_2	18/20 (2), 16/20 (4), 15/20 (6), 13/20 (8), 12/20 (10), 11/20 (12), 10/20 (14); 36/40 (2), 33/40 (4), 30/40 (6), 27/40 (8), 24/40 (10), 22/40 (12), 20/40 (14);
C_5	9/10 (2), 9/10 (4), 8/10 (6), 8/10 (8), 7/10 (10), 7/10 (12), 6/10 (14); 14/15 (2), 13/15 (4), 12/15 (6), 12/15 (8), 11/15 (10), 10/15 (12), 10/15 (14);
S_1	27/30 (2), 22/30 (4), 16/30 (6), 15/30 (8), 14/30 (10), 10/30 (12), 8/30 (14);

Nodes	Lifetime data as failure time and '+' for censored time
(C_3, C_4)	(8.80, 10.28), (17.55, 30.82), (36.15, 36.15), (7.54, 7.54), (14.93, 25.91), (18.06, 12.61), (17.73, 13.00), (25.29, 25.29), (22.36, 22.36), (27.95, 46.61), (27.07, 24.84), (24.84, 10.88), (43.68, 34.16), (32.76, 32.83), (20.17, 20.95), (20.81, 16.03), (31.18, 5.60), (22.79, 12.92), (19.36, 31.49), (22.01, 22.77), (7.13, 24.57), (22.01, 16.41), (28.54, 15.47), (7.14, 7.14), (9.05, 9.05), (27.35, 27.35), (30.88, 5.51), (7.50, 20.16), (6.12, 35.29), (16.58, 10.27)
C_6	37.40, 46.60, 39.66, 50+, 30.56, 30.34, 50+, 19.05, 50+, 59.73, 16.48, 23.82, 50+, 44.40, 36.28, 50+, 50+, 29.92, 36.79, 9.39, 24.87, 9.74, 29.94, 50+, 50+
S_0	5.40, 8.59, 6.34, 6.08, 9.26, 5.11, 10.55, 6.37, 10.56, 4.73
S_2	21.31, 15.74, 5.00, 7.50, 12.32, 14.48, 17.33, 29.43, 39.70, 6.06, 30.95, 9.90, 1.17, 6.90, 41.69, 54.84, 27.81, 31.77, 31.33, 17.88, 25.02, 11.67, 24.63, 16.84, 33.16, 34.96, 7.05, 53.60, 18.59, 28.58
S_3	45.28, 47.08, 68.89, 22.68, 24.90, 38.12, 16.60, 21.45, 30.15, 56.08, 34.79, 26.90, 20.23, 50.60, 18.51, 15.78, 41.50, 78.61, 11.75, 28.42

Nodes	Degradation data									
Time	2	4	6	8	10	12	14	16	18	20
C_7	98.56	97.15	95.71	94.27	92.85	91.42	90.00	88.56	87.14	85.71
	97.21	94.40	91.63	88.82	86.03	83.22	80.43	77.64	74.85	72.03
	99.17	98.32	97.48	96.65	95.79	94.96	94.12	93.28	92.43	91.60
	99.00	97.97	97.00	96.00	95.00	94.01	93.02	92.04	91.02	90.02
	97.44	94.92	92.39	89.82	87.27	84.74	82.18	79.64	77.10	74.54
	96.77	93.56	90.32	87.09	83.86	80.65	77.42	74.19	70.96	67.73

on the posterior samples of model parameters using simulation based integration.

VI. ILLUSTRATIVE EXAMPLE

Reliability assessment and prediction for satellites in a satellite navigation system is carried out in this section to demonstrate the proposed Bayesian approach. The satellite constellation for a navigation system consists of a number of satellites, which work in concert for the service of global positioning. These satellites are distributed in multiple orbits, and categorized in groups. In each group, a predefined number of satellites are under operation, and the rest are for spare. The performance of the constellation is determined by the available number of operational satellites in the constellation. Accordingly, the logistics for operation and management of this constellation requires precise assessment and prediction of the satellite reliability using available data and information.

A. System Structure and Reliability Data for the Satellite

In general, system level reliability data and information for a satellite system are sparse, and at times even nonexistent. However, the test data for its critical subsystems and components are sufficient. Moreover, as navigation satellites are evolved over each generation, the basic framework of the satellite is inherited from former satellites (e.g., the satellite bus). Prior information from these former satellites about subsystems and components that the new satellites inherited are available and abundant. These data can be incorporated for reliability assessment of newly developed satellites. Besides, during the operation of the constellation, lifetime data are obtained from failed satellites in particular orbits. The satellites for the navigation constellation in the same orbit are similar. They are all composed of the same components, and subjected to the same space environment. As a result, these lifetime data are used as field data of similar satellites in the same orbit.

The RBD of the satellite is given in Fig. 1. The MHDS which are collected during the development, production, and operation of the satellite are given in Table I. To avoid proprietary issues, the system structure is simplified, the units of values are omitted, and the data are modified in a certain way. Largely, however, the nature of the RBD, the MHDS, and the application of the proposed approach are the same as the original from the perspective of demonstration of the proposed method for reliability analysis with MHDS.

B. Bayesian Model for the MHDS

As described in Fig. 2, the first step is to define parametric models for the components of the satellite system. These models are chosen according to the characteristics of related data sets and the testimony of experts. The s-exponential distribution is adopted to model the reliability of components C_1, C_2, and C_5 as $T_i \sim Exponential(\lambda_i)$, $i = 1,2,5$. The Marshall-Olkin bivariate Weibull distribution [44] is employed to model the lifetime of the components C_4 and C_5 as $(T_3, T_4) \sim MOBW(\alpha_{34}, \lambda_{34a}, \lambda_{34b}, \lambda_{34c})$. Its CDF is given as $F(t_3, t_4) = 1-\exp\{-[\lambda_{34a} t_3^{\alpha_{34}} +$

$\lambda_{34b}t_4^{\alpha_{34}} + \lambda_{34c}\max(t_3^{\alpha_{34}}, t_4^{\alpha_{34}})]\}$. The Weibull distribution is used to model the reliability of component C_6 as $T_6 \sim Weibull(\alpha_6, \lambda_6)$ with $R_{C6}(t|\alpha_6, \lambda_6) = 1 - \exp(-\lambda_6 t^{\alpha_6})$. The degradation data for component C_7 is modeled with a linear degradation curve as $Y^{Deg}(t_{ij}|\boldsymbol{\beta}^{Deg}) = \alpha_7 - t_{ij}\beta_i^{-1}$, where the unit-to-unit variance is modeled with $\beta_i \sim Weibull(\nu_7, \lambda_7)$ [26], [31], and $\boldsymbol{\beta}^{Deg} = (\alpha_7, \nu_7, \lambda_7)$. Then, the reliability function for this component is obtained as $T_7 \sim Weibull(\nu_7, \lambda_7(\alpha_7 - Y^D)^{\nu_7})$. The selection of these reliability models for the components is based on their respective goodness-of-fit test of these models, and the testimony of experts. The generality of reliability models included in the example has also been considered in the selection of these models.

Meanwhile, the prior information is quantified into prior distributions for the model parameters given above. As described in Section II-B, three kinds of prior distributions are included to account for three types of prior information considered in this paper. The first type of information is the consideration of the resemblance of reliability between similar components, which is based on the testimony of experts and information from previous satellites. The components C_1 and C_2 are confirmed sharing some resemblance in their reliability. Their prior distributions are specified in a hierarchical way, where the gamma distribution is used as the prior distributions for parameters λ_1 and λ_2. In addition, the hyperparameters of the gamma priors share the same hyper-prior distributions, which are adopted to model their similarity in terms of reliability [29]. The second type of information is the past data and accumulated knowledge of reliability for components C_3, C_4, C_5, and C_6. Prior distributions are obtained by fitting the past data to some representative distributions for prior derivation, where the gamma, lognormal, and s-normal distributions are generally used as representative priors. Methods for subjective information quantification are adopted to quantify the accumulated knowledge of reliability, for instance the probability encoding method. Bayesian information fusion toolkits are used to combine these priors generated from past data with the ones obtained from subjective knowledge [45]. The third type of prior distribution is the diffuse prior distribution, which is used to describe the situation that no specific prior information is available. Generally adopted is a representative prior with large variance, or a uniform distribution with a relatively large interval, that is also referred to as a non-informative prior. In this case study, because component C_7 is a newly developed component, prior information is lacking for this component. Diffuse prior distributions are adopted for the model parameters of this component. More specifically, the prior distributions used for model parameters of the satellite system are summarized in Table II.

The multilevel system structure of the satellite is modeled following the substitution strategy depicted in Fig. 2 and (1). The reliability function of the satellite system S_0, and subsystems S_1, S_2, S_3, are obtained as follows.

$$\begin{aligned}R_{S0}(t|\boldsymbol{\theta}_{S0}) &= R_{S1}(t)R_{S2}(t)R_{S3}(t) = R_1(t)R_2(t) \times R_{34}(t)\\ &\quad \times [R_5(t)R_6(t) + R_5(t)R_7(t) + R_6(t)R_7(t)\\ &\quad -2R_5(t)R_6(t)R_7(t)];\\ R_{S1}(t|\boldsymbol{\theta}_{S1}) &= \exp[-(\lambda_1+\lambda_2)t];\end{aligned}$$

TABLE II
PRIOR DISTRIBUTIONS FOR THE PARAMETERS OF THE SATELLITE SYSTEM

Parameter	Prior distributions
λ_1, λ_2	$\lambda_i \sim Gamma(a_\lambda, b_\lambda), i=1,2, \begin{cases} a_\lambda \sim Lognormal(\mu_{a\lambda}, \sigma_{a\lambda}) \\ b_\lambda \sim Lognormal(\mu_{b\lambda}, \sigma_{b\lambda}) \end{cases}$
α_{34}, λ_{34a}, λ_{34b}, λ_{34c}	$\alpha_{34} \sim Gamma(a_{34}, b_{34}), \lambda_{34a} \sim Gamma(a_{34a}, b_{34a})$, $\lambda_{34b} \sim Gamma(a_{34b}, b_{34b}), \lambda_{34c} \sim Gamma(a_{34c}, b_{34c})$
λ_5	$\lambda_5 \sim Normal(\mu_{\lambda5}, \sigma_{\lambda5}), \lambda_5 > 0$
α_6, λ_6	$\alpha_6 \sim Gamma(a_6, b_6)$, $\lambda_6 \sim Lognormal(\mu_{\lambda6}, \sigma_{\lambda6})$
α_7, λ_7, ν_7, σ_7	$\alpha_7 \sim Gamma(a_{a7}, b_{a7}), \lambda_7 \sim Gamma(a_{\lambda7}, b_{\lambda7})$, $\nu_7 \sim Uniform(a_{\nu7}, b_{\nu7}), \sigma_7 \sim Uniform(a_{\sigma7}, b_{\sigma7})$

$$\begin{aligned}R_{S2}(t|\boldsymbol{\theta}_{S2}) &= \exp[-(\lambda_{34a}+\lambda_{34c})t^{\alpha_{34}}]\\ &\quad + \exp[-(\lambda_{34b}+\lambda_{34c})t^{\alpha_{34}}]\\ &\quad - \exp[-(\lambda_{34a}+\lambda_{34b}+\lambda_{34c})t^{\alpha_{34}}];\\ R_{S3}(t|\boldsymbol{\theta}_{S3}) &= \exp(-\lambda_5 t - \lambda_6 t^{\alpha_6})\\ &\quad + \exp\left(-\lambda_5 t - \lambda_7\left(\frac{t}{(\alpha_7 - Y^D)}\right)^{\nu_7}\right)\\ &\quad + \exp\left(-\lambda_6 t^{\alpha_6} - \lambda_7\left(\frac{t}{(\alpha_7 - Y^D)}\right)^{\nu_7}\right)\\ &\quad - 2\exp\Bigg[-\lambda_5 t - \lambda_6 t^{\alpha_6}\\ &\qquad - \lambda_7\left(\frac{t}{(\alpha_7 - Y^D)}\right)^{\nu_7}\Bigg]\end{aligned} \tag{16}$$

where $\boldsymbol{\theta}_{S0} = (\boldsymbol{\theta}_{S1}, \boldsymbol{\theta}_{S2}, \boldsymbol{\theta}_{S3})$ with $\boldsymbol{\theta}_{S1} = (\lambda_1, \lambda_2)$, $\boldsymbol{\theta}_{S2} = (\alpha_{34}, \lambda_{34a}, \lambda_{34b}, \lambda_{34c})$, and $\boldsymbol{\theta}_{S3} = (\lambda_5, \alpha_6, \lambda_6, \alpha_7, \nu_7, \lambda_7)$.

Using the combining strategy depicted in Fig. 2 and (8), the joint likelihood function of the MHDS is obtained. With the prior distribution given in Table II, the joint posterior distribution for model parameters of the satellite is given as

$$\begin{aligned}&p\left(\boldsymbol{\Theta}|D^{\frac{P}{F}}, D^{Life}, D^{Deg}\right) \propto \pi(\boldsymbol{\Theta})\\ &\times \prod_{N_k^{\frac{P}{F}} \in A^{\frac{P}{F}}} L^{\frac{P}{F}}\left(\boldsymbol{n}_k^{\frac{P}{F}}, \boldsymbol{y}_k^{\frac{P}{F}}, \boldsymbol{t}_k^{\frac{P}{F}}|\boldsymbol{\theta}_k^{\frac{P}{F}}\right)\\ &\times \prod_{N_k^{Life} \in A^{Life}} L_k^{Life}\left(\boldsymbol{t}_k^F, \boldsymbol{t}_k^C|\boldsymbol{\theta}_k^{Life}\right)\\ &\times \prod_{N_k^{Deg} \in A^{Deg}} L_k^{Deg}\left(\boldsymbol{g}_k^{Deg}, \boldsymbol{t}_k^{Deg}|\boldsymbol{\theta}_k^{Deg}\right)\end{aligned} \tag{17}$$

where $\boldsymbol{\Theta} = (\lambda_1, \lambda_2, \alpha_{34}, \lambda_{34a}, \lambda_{34b}, \lambda_{34c}, \lambda_5, \alpha_6, \lambda_6, \alpha_7, \lambda_7, \nu_7, \sigma_7)$, $A^{P/F} = \{C_1, C_2, C_5, S_1\}$, $A^{Life} = \{C_3, C_4, C_6, S_0, S_2, S_3\}$, and $A^{Deg} = \{C_7\}$. The contributions of the priors and the likelihood function to this joint posterior distribution are separately given as

$$\begin{aligned}&\pi(\boldsymbol{\Theta})\\ &= \prod_{i=1}^{2} \frac{b_\lambda^{a_\lambda}\lambda_i^{a_\lambda - 1}}{\Gamma(a_\lambda)}\exp(-b_\lambda\lambda_i)\cdot\frac{1}{a_\lambda}\phi\left(\frac{\ln a_\lambda - \mu_{a\lambda}}{\sigma_{a\lambda}}\right)\\ &\quad \cdot\frac{1}{b_\lambda}\phi\left(\frac{\ln b_\lambda - \mu_{b\lambda}}{\sigma_{b\lambda}}\right)\end{aligned}$$

TABLE III
SUMMARY STATISTICS OF THE POSTERIOR SAMPLES FOR THE PARAMETERS

Param	Posterior		Posterior percentiles		Param	Posterior		Posterior percentiles	
	Mean	SD	2.5%	97.5%		Mean	SD	2.5%	97.5%
λ_1	0.0395	0.0050	0.0299	0.0497	α_6	2.6870	0.4507	1.8590	3.6560
λ_2	0.0495	0.0040	0.0419	0.0576	λ_6	1.47E-4	3.30E-4	9.95E-7	0.0010
α_{34}	0.7167	0.0676	0.5881	0.8535	α_7	100	0.0028	99.99	100
λ_{34a}	0.0724	0.0214	0.0383	0.1215	λ_7	0.3860	0.1751	0.1292	0.8002
λ_{34b}	0.1089	0.0306	0.0591	0.1784	ν_7	1.7780	0.4999	0.9564	2.9080
λ_{34c}	0.1125	0.0315	0.0614	0.1840	σ_7	0.0101	0.0010	0.0080	0.0124
λ_5	0.0314	0.0049	0.0226	0.0418					

$$
\begin{aligned}
&\times \alpha_{34}^{a_{34}-1}\exp(-b_{34}\alpha_{34})\cdot\lambda_{34a}^{a_{34a}-1}\exp(-b_{34a}\lambda_{34a})\\
&\cdot\lambda_{34b}^{a_{34b}-1}\exp(-b_{34b}\lambda_{34b})\cdot\lambda_{34c}^{a_{34c}-1}\exp(-b_{34c}\lambda_{34c})\\
&\times\phi\left(\frac{\lambda_5-\mu_{\lambda 5}}{\sigma_{\lambda 5}}\right)\times\alpha_6^{a_6-1}\exp(-b_6\alpha_6)\\
&\cdot\frac{1}{\lambda_6}\phi\left(\frac{\ln\lambda_6-\mu_{\lambda 6}}{\sigma_{\lambda 6}}\right)\\
&\times\alpha_7^{a_{\alpha 7}-1}\exp(-b_{\alpha 7}\alpha_7)\cdot\lambda_7^{a_{\lambda 7}-1}\exp(-b_{\lambda 7}\lambda_7)
\end{aligned}
$$

$$
\begin{aligned}
&L\left(D^{\frac{P}{F}},D^{Life},D^{Deg}|\boldsymbol{\Theta}\right)\\
&=L_{C1}^{\frac{P}{F}}\left(\boldsymbol{n}_{C1}^{\frac{P}{F}},\boldsymbol{y}_{C1}^{\frac{P}{F}},\boldsymbol{t}_{C1}^{\frac{P}{F}}|\lambda_1\right)L_{C2}^{\frac{P}{F}}\left(\boldsymbol{n}_{C1}^{\frac{P}{F}},\boldsymbol{y}_{C1}^{\frac{P}{F}},\boldsymbol{t}_{C1}^{\frac{P}{F}}|\lambda_2\right)\\
&\times L_{C3,C4}^{Life}\left(\boldsymbol{t}_{C3,C4}^{F},\boldsymbol{t}_{C3,C4}^{C}|\alpha_{34},\lambda_{34a},\lambda_{34b},\lambda_{34c}\right)L_{C5}^{\frac{P}{F}}\\
&\times\left(\boldsymbol{n}_{C5}^{\frac{P}{F}},\boldsymbol{y}_{C5}^{\frac{P}{F}},\boldsymbol{t}_{C5}^{\frac{P}{F}}|\lambda_5\right)\\
&\times L_{C6}^{Life}\left(\boldsymbol{t}_{C6}^{F},\boldsymbol{t}_{C6}^{C}|\alpha_6,\lambda_6\right)L_{C7}^{Deg}\\
&\times\left(\boldsymbol{g}_{C7}^{Deg},\boldsymbol{t}_{C7}^{Deg}|\alpha_7,\nu_7,\lambda_7,\sigma_7\right)\\
&\times L_{S1}^{\frac{P}{F}}\left(\boldsymbol{n}_{C1}^{\frac{P}{F}},\boldsymbol{y}_{C1}^{\frac{P}{F}},\boldsymbol{t}_{C1}^{\frac{P}{F}}|\boldsymbol{\theta}_{S1}\right)L_{S2}^{Life}\left(\boldsymbol{t}_{S2}^{F},\boldsymbol{t}_{S2}^{C}|\boldsymbol{\theta}_{S2}\right)\\
&\times L_{S3}^{Life}\left(\boldsymbol{t}_{S3}^{F},\boldsymbol{t}_{S3}^{C}|\boldsymbol{\theta}_{S3}\right)\times L_{S0}^{Life}\left(\boldsymbol{t}_{S0}^{F},\boldsymbol{t}_{S0}^{C}|\boldsymbol{\theta}_{S0}\right)
\end{aligned}
$$

where the likelihood contributions of relative pass-fail, lifetime, and degradation data are obtained by substituting relative reliability functions of components, subsystems, and the system as given in Section V part B into the corresponding (3), (4), and (7).

C. Calculation: Sampling From the Posterior Distribution of the Bayesian Model

As described in Section IV, the assessment and prediction of satellite reliability are carried out by generating samples from the joint posterior distribution in (17). The WinBUGS software is used to implement the sampling procedure. 20,000 samples are generated from this joint posterior distribution with 1,000 samples for burn-in. The posterior quantities and the posterior confidence intervals of the model parameters are summarized in Table III.

D. Model Validation, and Justification

As reliability assessment and prediction of the satellite are based on the joint posterior distribution of the parameters $\boldsymbol{\Theta}$, a model diagnostic is implemented to test the fitness of the proposed Bayesian model. We carry out this testing using the Bayesian χ^2 goodness-of-fit test introduced in Section IV part D. By applying the 20,000 posterior samples to the Bayesian χ^2 goodness-of-fit test procedure, the testing result for the lifetime data of S_0 is 0.9789. It suggests that 97.89% posterior samples fit the available system lifetime data well. As the model diagnostics are based on the lifetime data in the system level of the satellite, it is sufficient enough to indicate the fitting of the proposed model for the satellite system. The samples from the joint posterior distribution can be used to derive indices for satellite reliability assessment and prediction.

TABLE IV
SUMMARY STATISTICS OF INDICES FOR SATELLITE RELIABILITY ASSESSMENT

Reliability indices ($t_p=3$)	Mean	SD	Median	2.5%	97.5%
$R_{S0}(t_p)$	0.6237	0.0290	0.6249	0.6438	0.6762
$\lambda_{S0}(t_p)$	0.2229	0.0186	0.2212	0.1914	0.2642
$R(t_p+2\mid t_p)$	0.7458	0.0138	0.7459	0.7184	0.7721

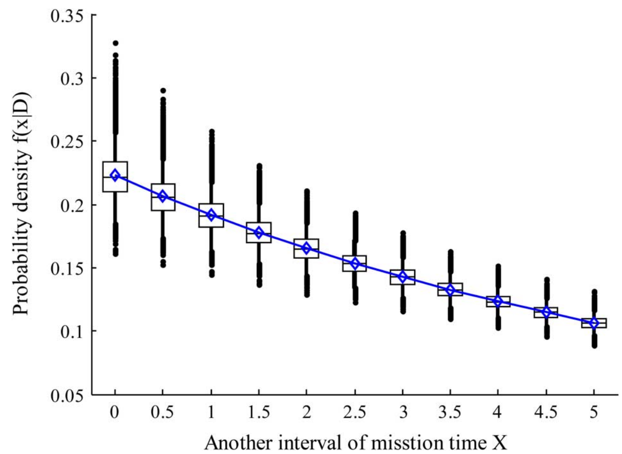

Fig. 3. The PDF of the residual life X given $t_p = 3$ with boxplots of the samples at each time points.

E. Reliability Assessment and Prediction of the Satellite

Considering the operation and management of the satellite constellation, given the system has survived up to the present time $t_p = 3$ with the accumulation of the MHDS in Table I, our primary interest is on the reliability of the satellite at this point in time. According to the indices derived in Section V part A, the reliability and the failure rate of the satellite system at the present time $t_p = 3$, and the probability that the system will survive another mission time $x = 2$ are obtained and presented in Table IV. The results are obtained based on the 20,000 posterior samples. The simulation based integration method described in Section V is implemented. In addition, the PDF of the residual life X given that the satellite has survived up to the present time $t_p = 3$ is presented in Fig. 3.

Suppose a new satellite for the satellite constellation is going to be launched, and we are interested in the reliability of this new satellite. According to the constructed indices for reliability prediction in (14), the predicted reliability distribution of the new satellite is obtained and presented in Fig. 4. It is generated based on the 20,000 posterior samples using simulation based integration. Similarly, according to (15), the lower-bounds on

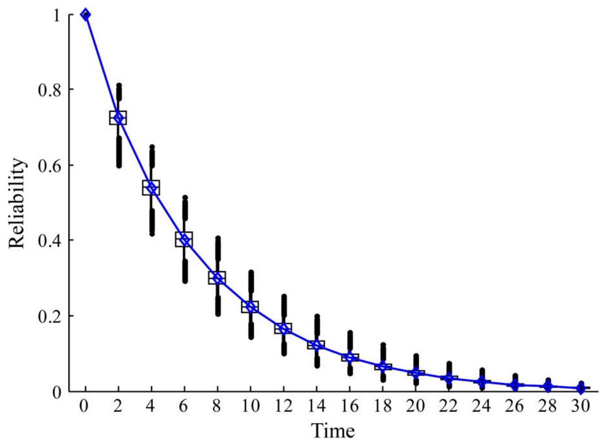

Fig. 4. The predicted reliability distribution of the new satellite with boxplots of the samples at each time points.

TABLE V
THE PREDICTED LOWER-BOUND PERCENTILE LIFETIME FOR THE SATELLITE

Lower-bound values of α	Mean	SD	Median	2.5%	97.5%
0.05	0.2681	0.0597	0.2650	0.1550	0.3850
0.25	1.7800	0.2248	1.7850	1.3350	2.2150
0.5	4.5296	0.4053	4.5350	3.7450	5.3150
0.75	9.2507	0.6321	9.2350	8.0450	10.4950
0.95	19.6233	1.0657	09.5950	17.6250	21.7600

failure percentiles for the satellite with the failure quantile $\alpha = 0.05$, 0.25, 0.5, 0.75, and 0.95 are obtained and presented in Table V.

VII. DISCUSSION, AND CONCLUSION

In this paper, a fully Bayesian approach for integrating multilevel heterogeneous data sets for reliability analysis is developed. Two generic engineering concerns have been addressed: 1) a coherent framework for integrating multilevel heterogeneous data sets, which is emphasized on the combination of different types of reliability data within different system levels, where the pass-fail data, the lifetime data, and the degradation data are included; and 2) an effective way of obtaining the indices for reliability assessment and prediction based on the proposed Bayesian framework. Various reliability indices have been constructed within the proposed framework for the sake of system operation and management.

Moreover, the reliability analysis with multilevel heterogeneous data sets is implemented through a fully Bayesian perspective. The proposed Bayesian approach overcomes the sole objective of reliability assessment, and actually provides useful reliability indices for system operation and management. These indices are embedded in the proposed Bayesian framework, and calculated based on posterior samples using a simulation based integration method. In addition, a Bayesian χ^2 goodness-of-fit test model diagnostic method is introduced to facilitate the testing of model fitting, which is critical for reliability analysis with multilevel heterogeneous data sets.

However, this paper cannot incorporate all possible ways of considering the Bayesian reliability method for multilevel heterogeneous data sets. In particular, the derivation of the prior distribution and the modeling of degradation data for nodes in the higher levels of systems are not properly addressed in this paper. Moreover, the improvement of the MCMC method for situations with complex data sets is of interest for our future works.

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