Standard Article

Interaction balance optimization in multidisciplinary design optimization problems

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Abstract

As a bi-level optimization method, collaborative optimization can solve multidisciplinary design optimization problems in practical engineering effectively. However, if there are high-dimensional couplings in a multidisciplinary design optimization problem, a large number of compatibility constraints will be required in collaborative optimization. In this situation, collaborative optimization will not be suitable to be utilized because of low computational efficiency or divergence issue. To solve this problem, an efficient interaction balance optimization method is proposed in this article. In interaction balance optimization method, the simple coordination strategy of interaction balance principle and the distributed optimization strategy of collaborative optimization can be integrated effectively. Lagrange multipliers are used instead of compatibility constraints to maintain the consistency between any two coupled disciplines. Two examples are given to show the effectiveness of the proposed method.

Keywords

collaborative optimization, multidisciplinary design optimization, compatibility constraint, interaction balanced optimization, Lagrange multiplier

Introduction

With the development of engineering systems, concurrent engineering (CE) has received increasing attention. Recently, many CE technologies have been developed to solve integrated, simultaneous design problems of engineering systems (Chhabra and Emami, 2014; Deshmukh et al., 2013; Huang et al., 2011; Hwang et al., 2014; Meng et al., 2014; Ostrosi et al., 2014; Zhang et al., 2011, 2013). As a specific implementation of CE technologies, multidisciplinary design optimization (MDO) is a powerful method to design complex and coupled engineering systems (Huang et al., 2011; Hwang et al., 2014; Meng et al., 2014; Sobieszczanski-Sobieski and Haftka, 1997; Zhang et al., 2011, 2013). In MDO, a cost-type objective function is minimized while multidisciplinary constraints are satisfied. Furthermore, a coordination strategy is used to make interaction variables (including shared design variables and linking variables) satisfying the consistency requirements between any two coupled disciplines (Jeremy and Panos, 2005). Based on the coordination strategies, MDO methods can be categorized into two types: single-level methods and multilevel methods (Balling and Wilkinson, 1997; Chen et al., 2002; Hulme and Bloebaum, 2000; Martins and Lambe, 2013; Tosserams et al., 2009). Single-level methods generally have a single optimizer and use a non-hierarchical structure directly. Multidisciplinary feasible (MDF; Cramer, 1992; Lambe and Martins, 2012), individual discipline feasible (IDF; Cramer et al., 1994; Dener and Hicken, 2014; Kodiyalam and Sobieszczanski-Sobieski, 2001; Yu et al., 2014), and all-at-once (AAO; Haftka, 1994; Roshanian et al., 2014) are single-level methods. Multilevel methods modify the relationship of a non-

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hierarchical structure into a hierarchical structure. Generally, they have system and subsystem levels. Each level has an optimizer. Therefore, each discipline has a separate analyzer and can determine design solutions. Concurrent subspace optimization (CSSO; Liang et al., 2012; Renaud and Gabriele, 1994; Wujek et al., 1996; Zhang and Tang, 2015), bi-level integrated system synthesis (BLISS; Barthelemy and Sobieszczanski-Sobieski, 1983; Sobieszczanski-Sobieski et al., 2003; Yi et al., 2008; Zhao and Cui, 2011; Zhou et al., 2014), and collaborative optimization (CO; Alexandrov and Lewis, 2002; Braun, 1996; Braun and Kroo, 1995; Li et al., 2014; Tappeta and Renaud, 1997; Zhao et al., 2012) are multilevel methods.

Although many methods have been proposed, the developments of MDO are still necessary. On one hand, the development of engineering systems may make the existing MDO methods difficult to deal with all new challenges efficiently. On the other hand, because of the limited time and cost, designers have to select an appropriate method among various MDO methods to solve their problems. More new MDO methods should be developed to provide more options for designers. For example, the CO method is suitable to solve a MDO problem for the distributed engineering system with low couplings. However, if the distributed engineering system is large scale, there will be a large number of interaction variables in the corresponding MDO model. In this situation, CO method will be low efficient or even cannot converge.

This work is aimed to address the low-efficient issue of CO. We will focus on the situation where the architecture of engineering system is distributed and CO is feasible to use. The question is how can we enhance the efficiency of CO to solve MDO problems for large-scale engineering systems? To answer the question, we integrate the coordination strategy of interaction balance principle (IBP) and the collaborative mechanism of CO, and propose a new MDO method, which is named as interaction balance optimization (IBO).

This article is organized as follows. In section "The formulation of MDO," the general MDO formulation is given. In section "The CO method," the CO method is briefly reviewed. In section "IBO," the basic idea of IBO is discussed in detail. Then the solution procedure of IBO is proposed. In section "Examples," a numerical example and an aircraft conceptual design problem are used to show the effectiveness of the proposed method. Some conclusions are presented in section "Conclusion."

The formulation of MDO

In this study, we consider the case that the system objective is a linear sum of all discipline objectives. Then a MDO problem can be formulated as

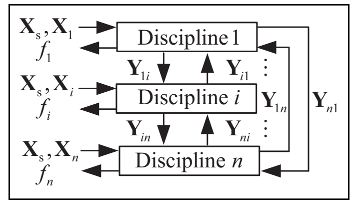

Figure 1. Interactions between coupled disciplines.

$$
\begin{aligned}
&\min_{\mathbf{X}_s, \mathbf{X}_i, \mathbf{Y}_{ji}, \mathbf{Y}_{ij}} f = \sum_{i=1}^{n} f_i(\mathbf{X}_s, \mathbf{X}_i, \mathbf{Y}_{ji}, \mathbf{Y}_{ij}) \\
&\text{s.t.} \quad \boldsymbol{g}_i(\mathbf{X}_s, \mathbf{X}_i, \mathbf{Y}_{ji}, \mathbf{Y}_{ij}) > 0 \\
&\qquad \boldsymbol{h}_i(\mathbf{X}_s, \mathbf{X}_i, \mathbf{Y}_{ji}, \mathbf{Y}_{ij}) = 0 \\
&\mathbf{X}_i^L \le \mathbf{X}_i \le \mathbf{X}_i^U, \ \mathbf{X}_s^L \le \mathbf{X}_s \le \mathbf{X}_s^U \\
&\mathbf{Y}_{ij}^L \le \mathbf{Y}_{ij} \le \mathbf{Y}_{ij}^U, \ \mathbf{Y}_{ji}^L \le \mathbf{Y}_{ji} \le \mathbf{Y}_{ji}^U, \\
&i, j = 1, 2, \ldots, n \ \ (i \ne j)
\end{aligned}
\tag{1}
$$

where $f(\bullet)$ is the system objective; $f_i(\bullet)$ is the ith discipline objective; $\boldsymbol{g}_i(\bullet)$ is the vector of inequality constraints in the ith discipline, $\boldsymbol{g}_i(\bullet) > \boldsymbol{0}$ denotes the feasible region; $h_i(\bullet)$ is the vector of equality constraints in the ith discipline; $\mathbf{X}_s$ is the vector of shared design variables; $\mathbf{X}_i$ is the vector of local design variables in the ith discipline; $\mathbf{Y}$ is the vector of linking variables, $\mathbf{Y}_{ij}$ denotes the inputs of the jth discipline and the outputs of the ith discipline, $\mathbf{Y}_{ji}$ denotes the inputs of the ith discipline and the outputs of the jth discipline, the relationship of $\mathbf{Y}_{ij}$ and $\mathbf{Y}_{ji}$ can be denoted as $\mathbf{Y}_{ij} = \mathbf{Y}_{ij}(\mathbf{X}_s, \mathbf{X}_i, \mathbf{Y}_{ji})$; superscripts L and U denote the lower and upper bounds, respectively; n is the total number of disciplines. The interactions between the coupled disciplines in a MDO problem is shown in Figure 1.

The CO method

In MDO, the performance of an engineering system is driven not only by the performance of individual disciplines but also by their interactions. Considering these interactions in an optimization problem generally requires a special coordination strategy. In CO, a complex system is decomposed into multiple disciplines. Then, the original MDO problem in equation (1) is modified into a system optimization problem as

$$
\begin{aligned}
&\min_{\mathbf{X}'_s, \mathbf{X}'_i, \mathbf{Y}'_{ji}, \mathbf{Y}'_{ij}} f = \sum_{i=1}^{n} f_i\left(\mathbf{X}'_s, \mathbf{X}'_i, \mathbf{Y}'_{ji}, \mathbf{Y}'_{ij}\right) \\
&\text{s.t.} \quad J_i = (\mathbf{X}'_i - \mathbf{X}_i)^2 + (\mathbf{X}'_s - \mathbf{X}_s)^2 \\
&\qquad + \left(\mathbf{Y}'_{ji} - \mathbf{Y}_{ji}\right)^2 + \left(\mathbf{Y}'_{ij} - \mathbf{Y}_{ij}\right)^2 \le \varepsilon, \\
&\qquad i, j = 1, 2, \ldots, n \ \ (i \ne j)
\end{aligned}
\tag{2}
$$

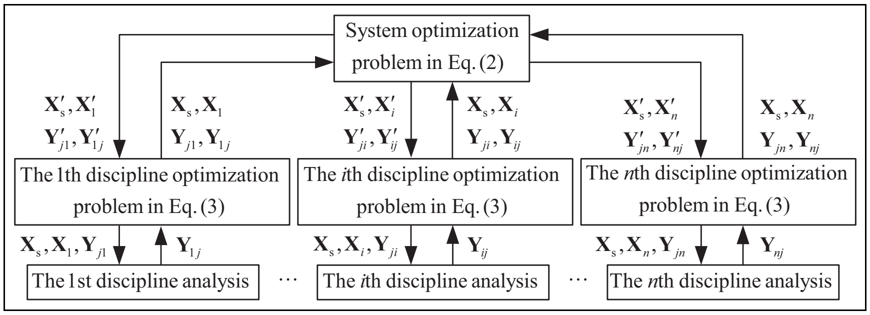

Figure 2. Optimization strategy of CO.

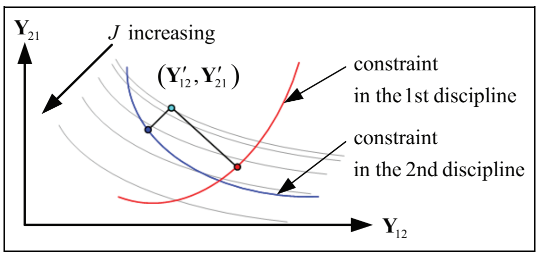

Figure 3. Coordination strategy of CO (Kroo, 2004).

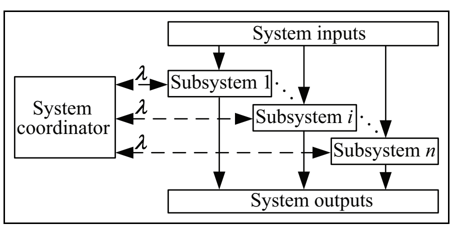

Figure 4. Coordination strategy of IBP.

and some corresponding discipline optimization problems as

$$
\begin{aligned}
\min_{\mathbf{X}_s, \mathbf{X}_i, \mathbf{Y}_{ji}} \quad & J_i = \left(\mathbf{X}_i' - \mathbf{X}_i\right)^2 + \left(\mathbf{X}_s' - \mathbf{X}_s\right)^2 \\
& \quad + \left(\mathbf{Y}_{ji}' - \mathbf{Y}_{ji}\right)^2 + \left(\mathbf{Y}_{ij}' - \mathbf{Y}_{ij}\right)^2 \\
\text{s.t.} \quad & g_i\left(\mathbf{X}_s, \mathbf{X}_i, \mathbf{Y}_{ji}, \mathbf{Y}_{ij}\right) > 0 \\
& h_i\left(\mathbf{X}_s, \mathbf{X}_i, \mathbf{Y}_{ji}, \mathbf{Y}_{ij}\right) = 0 \\
& \mathbf{X}_i^L \leq \mathbf{X}_i \leq \mathbf{X}_i^U, \ \mathbf{X}_s^L \leq \mathbf{X}_s \leq \mathbf{X}_s^U \\
& \mathbf{Y}_{ij}^L \leq \mathbf{Y}_{ij} \leq \mathbf{Y}_{ij}^U, \ \mathbf{Y}_{ji}^L \leq \mathbf{Y}_{ji} \leq \mathbf{Y}_{ji}^U, \\
& i, j = 1, 2, \ldots, n \ \ (i \neq j)
\end{aligned} \tag{3}
$$

where J_i is the compatibility constraint in the ith discipline and ε is a very small positive number.

The system optimization problem in equation (2) manages the overall optimization process at system level. It minimizes the system objective f with the results $(\mathbf{X}_s, \mathbf{X}_i, \mathbf{Y}_{ji}, \mathbf{Y}_{ij})$ from subsystem level and determines the target values $(\mathbf{X}_s', \mathbf{X}_i', \mathbf{Y}_{ji}', \mathbf{Y}_{ij}')$ for discipline optimization problems. Each discipline optimization problem finds the values of $(\mathbf{X}_s, \mathbf{X}_i, \mathbf{Y}_{ji}, \mathbf{Y}_{ij})$ which can satisfy their own constraints while trying to match the target values from system level as closely as possible. The optimization strategy and the coordination strategy of CO are shown in Figures 2 and 3, respectively.

The optimization strategy of CO fits the architecture of distributed engineering systems (Perez et al., 2004). Thus, it can be applied in practical easily. However, if there are a large number of interaction variables in a MDO problem, the CO method will be low efficient or even cannot converge (Kim, 2001). It is because that the high dimensionality of interaction variables makes it difficult or even impossible to satisfy compatibility constraints.

IBO

To solve the above problem in CO, IBP, a coordination method for large-scale systems, is developed in this study. Similar to CO, IBP decomposes a large-scale system into multiple subsystems and has system and subsystem levels (Amgai and Abdelwahed, 2014; Findeisen et al., 1980; Mehrotra and Abdelwahed, 2014; Sadati and Marvast, 2006). At subsystem level, each subsystem problem is solved concurrently while the Lagrange multipliers $\boldsymbol{\lambda}$ are considered as coordination parameters. The system coordinator manipulates $\boldsymbol{\lambda}$ at system level to manage the overall coordination process. The coordination strategy of IBP is shown in Figure 4.

We combine the coordination strategy of IBP and the distributed optimization strategy of CO and propose the IBO method to solve MDO problems. In IBO, we treat sharing design variable $\mathbf{X}_s$ as linking variables which are denoted as $\mathbf{X}_{s,ij}$ and $\mathbf{X}_{s,ji}$, respectively. The coupled relationship between $\mathbf{X}_{s,ij}$ and $\mathbf{X}_{s,ji}$ can be considered as $\mathbf{X}_{s,ij} = \mathbf{X}_{s,ji}$. Moreover, we use $\mathbf{I}$ to denote the interaction variables in a MDO problem, $\mathbf{I}_{ij} = \{\mathbf{X}_{s,ij}, \mathbf{Y}_{ij}\}$.

Then, based on the linear relationship $f = \sum_{i=1}^{n} f_i$, the MDO problem in equation (1) can be decomposed into multiple coupled discipline optimization problems as in equation (4)

$$\begin{aligned}
\min_{\mathbf{X}_i, \mathbf{I}_{ji}} \quad & f_i = f_i\left(\mathbf{X}_i, \mathbf{I}_{ji}, \mathbf{I}_{ij}\left(\mathbf{X}_i, \mathbf{I}_{ji}\right)\right) \\
\text{s.t.} \quad & g_i\left(\mathbf{X}_i, \mathbf{I}_{ji}, \mathbf{I}_{ij}\left(\mathbf{X}_i, \mathbf{I}_{ji}\right)\right) > 0 \\
& h_i\left(\mathbf{X}_i, \mathbf{I}_{ji}, \mathbf{I}_{ij}\left(\mathbf{X}_i, \mathbf{I}_{ji}\right)\right) = 0 \\
& \mathbf{X}_i^L \le \mathbf{X}_i \le \mathbf{X}_i^U, \; \mathbf{I}_{ji}^L \le \mathbf{I}_{ji} \le \mathbf{I}_{ji}^U \\
& \mathbf{I}_{ij}^L \le \mathbf{I}_{ij}\left(\mathbf{X}_i, \mathbf{I}_{ji}\right) \le \mathbf{I}_{ij}^U, \quad i,j = 1, 2, \ldots, n \;\; (i \neq j)
\end{aligned} \tag{4}$$

In equation (4), the value of $\mathbf{I}_{ij}$ can be obtained by the ith discipline analysis $\mathbf{I}_{ij} = \mathbf{I}_{ij}(\mathbf{X}_i, \mathbf{I}_{ji})$. To control the discipline optimization process at subsystem level, we introduce $\boldsymbol{\lambda}$ to discipline optimization problems. Moreover, the discipline Lagrange function L_i which is corresponding to the original discipline objective $f_i(\bullet)$ is constructed as

$$\begin{aligned}
L_i\left(\mathbf{X}_i, \mathbf{I}_{ji}, \mathbf{I}_{ij}\left(\mathbf{X}_i, \mathbf{I}_{ji}\right)\right) &= f_i\left(\mathbf{X}_i, \mathbf{I}_{ji}, \mathbf{I}_{ij}\left(\mathbf{X}_i, \mathbf{I}_{ji}\right)\right) \\
&\quad + \boldsymbol{\lambda}_i^T \times \mathbf{I}_{ji} - \sum_{j=1, j \neq i}^{n} \boldsymbol{\lambda}_j^T \times \mathbf{I}_{ij}\left(\mathbf{X}_i, \mathbf{I}_{ji}\right)
\end{aligned} \tag{5}$$

Then, the discipline optimization problems in equation (4) are changed to

$$\begin{aligned}
\min_{\mathbf{X}_i, \mathbf{I}_{ji}} \quad & L_i = L_i\left(\mathbf{X}_i, \mathbf{I}_{ji}, \mathbf{I}_{ij}\left(\mathbf{X}_i, \mathbf{I}_{ji}\right)\right) \\
\text{s.t.} \quad & g_i\left(\mathbf{X}_i, \mathbf{I}_{ji}, \mathbf{I}_{ij}\left(\mathbf{X}_i, \mathbf{I}_{ji}\right)\right) > 0 \\
& h_i\left(\mathbf{X}_i, \mathbf{I}_{ji}, \mathbf{I}_{ij}\left(\mathbf{X}_i, \mathbf{I}_{ji}\right)\right) = 0 \\
& \mathbf{X}_i^L \le \mathbf{X}_i \le \mathbf{X}_i^U, \; \mathbf{I}_{ji}^L \le \mathbf{I}_{ji} \le \mathbf{I}_{ji}^U \\
& \mathbf{I}_{ij}^L \le \mathbf{I}_{ij}\left(\mathbf{X}_i, \mathbf{I}_{ji}\right) \le \mathbf{I}_{ij}^U, \quad i,j = 1, 2, \ldots, n \;\; (i \neq j)
\end{aligned} \tag{6}$$

It should be noted that the isolated discipline optimization problems in equation (6) can be solved concurrently at subsystem level and $\boldsymbol{\lambda}$ are considered as design parameters. At system level, we construct the system Lagrange function L as a linear sum of the discipline Lagrange functions

$$\begin{aligned}
L\left(\mathbf{X}_i, \mathbf{I}_{ji}, \mathbf{I}_{ij}\right) &= \sum_{\substack{i,j=1 \\ (i \neq j)}}^{n} L_i\left(\mathbf{X}_i, \mathbf{I}_{ji}, \mathbf{I}_{ij}\right) = \sum_{i=1}^{n} \{ f_i\left(\mathbf{X}_i, \mathbf{I}_{ji}, \mathbf{I}_{ij}\right) \\
&\quad + \boldsymbol{\lambda}_j^T \times \left(\mathbf{I}_{ij} - \mathbf{I}_{ij}\left(\mathbf{X}_i, \mathbf{I}_{ji}\right)\right) \}
\end{aligned} \tag{7}$$

In equation (6), each discipline optimization problem is independent. Thus

$$\sum_{i=1}^{n} \min L_i = \min \sum_{i=1}^{n} L_i = \min L \tag{8}$$

Moreover, we denote the Lagrange dual function of the system Lagrange function in equation (7) as

$$\phi(\boldsymbol{\lambda}) = L\left(\mathbf{X}_i(\boldsymbol{\lambda}), \mathbf{I}_{ji}(\boldsymbol{\lambda}), \mathbf{I}_{ij}(\boldsymbol{\lambda})\right) \tag{9}$$

Then, the equivalence relationship in equation (10) can be obtained based on the Lagrange duality theorem if there is a saddle point in the system Lagrange function

$$\min_{\mathbf{X}_i, \mathbf{I}_{ji}, \mathbf{I}_{ij}} L\left(\mathbf{X}_i, \mathbf{I}_{ji}, \mathbf{I}_{ij}\right) = \max_{\boldsymbol{\lambda}} \phi(\boldsymbol{\lambda}) \tag{10}$$

Substituting equation (8) into equation (10), we obtain $\sum_{i=1}^{n} \min_{\mathbf{X}_i, \mathbf{I}_{ji}} L_i(\mathbf{X}_i, \mathbf{I}_{ji}) = \max_{\boldsymbol{\lambda}} \phi(\boldsymbol{\lambda})$. We assume that equation (9) is derivable. Then $\phi'(\boldsymbol{\lambda})$ will be equal to zero if the maximum of $\phi(\boldsymbol{\lambda})$ is obtained, $\phi'(\boldsymbol{\lambda}) = \mathbf{I}_{ij}(\boldsymbol{\lambda}) - \mathbf{I}_{ij}(\mathbf{X}_i(\boldsymbol{\lambda}), \mathbf{I}_{ji}(\boldsymbol{\lambda})) = 0$. Moreover, the system Lagrange function will be equal to the system objective function

$$\begin{aligned}
L\left(\mathbf{X}_i, \mathbf{I}_{ji}, \mathbf{I}_{ij}\right) &= \sum_{i=1}^{n} \left\{ f_i\left(\mathbf{X}_i, \mathbf{I}_{ji}, \mathbf{I}_{ij}\right) + \boldsymbol{\lambda}_j^T \times \left(\mathbf{I}_{ij} - \mathbf{I}_{ij}\left(\mathbf{X}_i, \mathbf{I}_{ji}\right)\right) \right\} \\
&= \sum_{i=1}^{n} \left\{ f_i\left(\mathbf{X}_i, \mathbf{I}_{ji}, \mathbf{I}_{ij}\right) \right\} = f\left(\mathbf{X}_i, \mathbf{I}_{ji}, \mathbf{I}_{ij}\right)
\end{aligned} \tag{11}$$

Thus, we can update λ according to the solutions in equation (6) at system level and manage the discipline optimization problems at subsystem level. The consistency requirements between coupled disciplines will be satisfied if the optimum of system objective is obtained in IBO. The distributed optimization strategy of IBO is illustrated in Figure 5.

The detailed solution procedure of IBO is given as follows:

Step 1. Set the initial values for design variables $\mathbf{X}_i^{(k-1)}$, $\mathbf{I}_{ji}^{(k-1)}$, and $\mathbf{I}_{ij}^{(k-1)}$; the initial values for Lagrange multipliers $\lambda_i^{(k-1)}$; and the cycle number $k = 1$. To keep the original gradient information in the revised optimization problems, $\lambda_i^{(k-1)}$ can be equal to zero at the beginning of optimization process.

Step 2. Solve the system optimization problem $\max_{\lambda} \phi(\lambda)$ in equation (10) at system level. The design variables $\mathbf{X}_i^{(k-1)}$, $\mathbf{I}_{ji}^{(k-1)}$, and $\mathbf{I}_{ij}^{(k-1)}$ are taken as design parameters during the system optimization process. After the optimization convergences, the solutions $\lambda_i^{(k)}$ are sent to the discipline optimization problems at subsystem level.

Step 3. Solve the discipline optimization problems in equation (6) at subsystem level. Using the distributed optimization strategy of IBO, discipline optimizations can be performed concurrently. $\lambda_i^{(k)}$ are used as design parameters during the discipline optimization process. After the optimization convergences, the solutions $\mathbf{X}^{(k)}$ and $\mathbf{I}^{(k)}$ can be obtained.

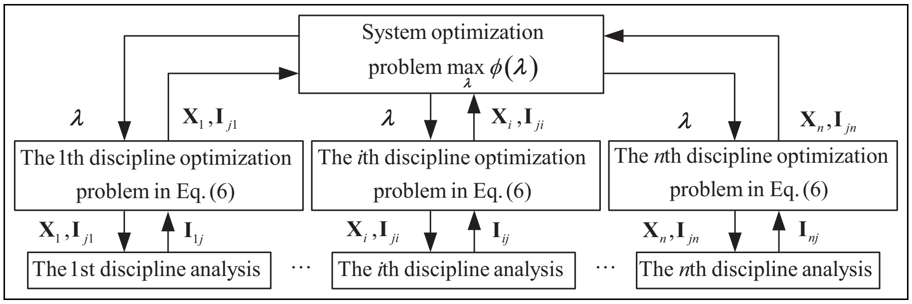

Figure 5. Optimization strategy of IBO.

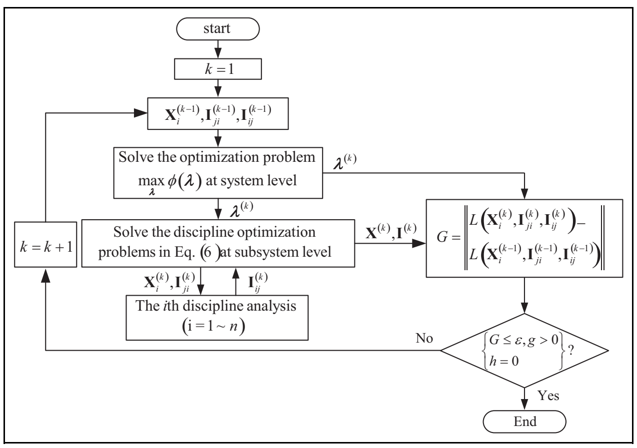

Figure 6. Flowchart of IBO.

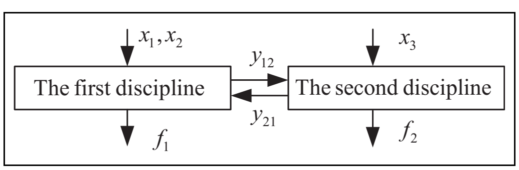

Figure 7. Coupled relationship of two disciplines.

Step 4. Check the convergence. Calculate $G = \left\| L(\mathbf{X}_i^{(k)}, \mathbf{I}_{ji}^{(k)}, \mathbf{I}_{ij}^{(k)}) - L(\mathbf{X}_i^{(k-1)}, \mathbf{I}_{ji}^{(k-1)}, \mathbf{I}_{ij}^{(k-1)}) \right\|$, $i,j = 1, 2, \ldots, n$ and $i \neq j$. If $G \le \varepsilon$, $g > 0$, $h = 0$, and the value of system objective is stable, go to Step 5; otherwise, set $k = k + 1$ and go to Step 2.
Step 5. Stop the overall optimization process. Output the solutions $\mathbf{X}^{(k)}$ and $\mathbf{I}^{(k)}$.

The flowchart of IBO is illustrated in Figure 6.

Examples

In this section, the effectiveness of the proposed method is illustrated by a numerical example and an aircraft conceptual design problem. We also use other multilevel MDO methods, CO, CSSO, and BLISS, as comparisons to solve these MDO problems. All optimization processes are conducted under the platform of iSIGHT™.

A numerical example

A numerical example is provided to show the proposed method in detail. The formulation of this example is given in equation (12)

$$\begin{aligned} \min \quad & f = (y_{21} - 2)^2 + x_1^2 + x_2^2 + (y_{12} - 1)^2 + x_3^2 \\ \text{s.t.} \quad & y_{12} = x_1 - x_2 + 2y_{21}, y_{21} = x_3 - y_{12} \\ & -1 \le x_1 \le 1, \ -1 \le x_2 \le 1, 0 \le x_3 \le 3 \\ & 0 \le y_{12} \le 1, 0 \le y_{21} \le 2 \end{aligned} \tag{12}$$

where f is the system objective; x_1, x_2, x_3, y_{12}, and y_{21} are the design variables.

We modify the original optimization problem into a MDO problem including two coupled disciplines. The discipline optimization problems are given in equations (13) and (14), and the coupled relationship is shown in Figure 7.

The first discipline optimization problem is

$$\begin{aligned} \min_{x_1, x_2, y_{21}} \quad & f_1 = (y_{21} - 2)^2 + x_1^2 + x_2^2 \\ \text{s.t.} \quad & -1 \le x_1 \le 1, \quad -1 \le x_2 \le 1 \\ & 0 \le y_{12} \le 1, \quad 0 \le y_{21} \le 2 \end{aligned} \tag{13}$$

where $y_{12} = x_1 - x_2 + 2y_{21}$.

The second discipline optimization problem is

$$\begin{aligned} \min_{x_3, y_{12}} \quad & f_2 = (y_{12} - 1)^2 + x_3^2 \\ \text{s.t.} \quad & 0 \le x_3 \le 3, \ 0 \le y_{12} \le 1, \ 0 \le y_{21} \le 2 \end{aligned} \tag{14}$$

where $y_{21} = x_3 - y_{12}$.

Using the distributed optimization strategy of IBO, there are a system optimization problem and two modified discipline optimization problems as follows.

Table 1. Solutions of the numerical example.

	The first point				The second point			
	IBO	CO	CSSO	BLISS	IBO	CO	CSSO	BLISS
x_1	−0.2999	−0.3001	−0.2999	−0.2993	−0.3000	−0.3000	−0.3000	−0.2993
x_2	0.2995	0.2999	0.2995	0.2990	0.2995	0.2998	0.2995	0.2990
x_3	0.8989	0.8998	0.8989	0.9005	0.8988	0.8998	0.8988	0.9005
y_{12}	0.3996	0.3999	0.3995	0.3995	0.3995	0.3999	0.3994	0.4009
y_{21}	0.4995	0.4999	0.4994	0.4989	0.4995	0.4999	0.4994	0.4996
f	3.6005	3.6000	3.6000	3.5977	3.6005	3.6000	3.6000	3.6000
Time (s)	66	91	151	127	74	106	138	130

IBO: interaction balance optimization; CO: collaborative optimization; CSSO: concurrent subspace optimization; BLISS: bi-level integrated system synthesis.

Table 2. List of variables in the aircraft conceptual design problem.

	Description (unit)		Description (unit)
x_1	Aspect ratio of the wing	y_1	Total aircraft wetted area (ft^2)
x_2	Wing area (ft^2)	y_2	Maximum lift to drag ratio
x_3	Fuselage length (ft)	y_3	Stall speed (ft/s)
x_4	Fuselage diameter (ft)	y_4	Aircraft range (miles)
x_5	Density of air at cruise altitude (slug/ft^3)	y_5	Gross take-off weight (lbs)
x_6	Cruise speed (ft/s)	y_6	Empty weight (lbs)
x_7	Fuel weight (lbs)	–	–

The system optimization problem at system level is

$$\max_{\lambda_1,\lambda_2} L(\lambda_1,\lambda_2) = (y_{21}-2)^2 + x_1^2 + x_2^2 + (y_{12}-1)^2 + x_3^2 + \lambda_1 \times (y_{21} - (x_3 - y_{12})) + \lambda_2 \times (y_{12} - (x_1 - x_2 + 2\times y_{21})) \tag{15}$$

The first modified discipline optimization problem at subsystem level is

$$\begin{aligned} \min_{x_1,x_2,y_{21}} \quad & L_1(x_1,x_2,y_{21}) = (y_{21}-2)^2 + x_1^2 + x_2^2 + \lambda_1 \times y_{21} - \lambda_2 \times (x_1 - x_2 + 2\times y_{21}) \\ \text{s.t.} \quad & -1 \le x_1 \le 1, \quad -1 \le x_2 \le 1, \quad 0 \le x_1 - x_2 + 2\times y_{21} \le 1, \quad 0 \le y_{21} \le 2 \end{aligned} \tag{16}$$

The second modified discipline optimization problem at subsystem level is

$$\begin{aligned} \min_{x_3,y_{12}} \quad & L_2(x_3,y_{12}) = (y_{12}-1)^2 + x_3^2 + \lambda_2 \times y_{12} - \lambda_1 \times (x_3 - y_{12}) \\ \text{s.t.} \quad & 0 \le x_3 \le 3, \quad 0 \le y_{12} \le 1, \quad 0 \le x_3 - y_{12} \le 2 \end{aligned} \tag{17}$$

We select two initial points to show the effectiveness of the proposed method. The first point $(x_1, x_2, x_3, y_{12}, y_{21}) = (0, 0, 0, 0, 0)$ is in the feasible region and the second point $(x_1, x_2, x_3, y_{12}, y_{21}) = (-1, 1, -1, 1, -1)$ is in the non-feasible region. The initial Lagrange multipliers are $\lambda = (\lambda_1, \lambda_2) = (0, 0)$. The solutions obtained by different methods are shown in Table 1. We can see that all methods can obtain reasonable solutions. Compared with CSSO and BLISS, CO and IBO enjoy the higher computational efficiency. This is because that both CSSO and BLISS use the system analysis to maintain the consistency between two coupled

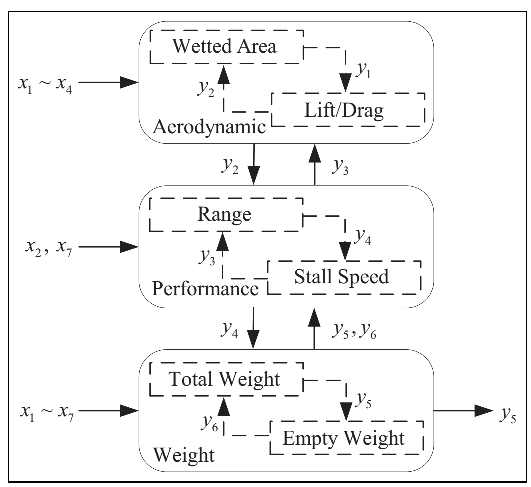

Figure 8. Aircraft conceptual design problem.

Table 3. Solutions of the aircraft conceptual design problem.

	IBO	CO	CSSO	BLISS		IBO	CO	CSSO	BLISS
x_1	5	5	5	5	y_1	711.48	710.68	709.14	711.11
x_2	176.62	177.14	175.93	176.28	y_2	10.997	11.021	10.975	10.819
x_3	20	20	20	20	y_3	70.002	70.014	69.994	70.000
x_4	4	4	4	4	y_4	561.76	560.91	560.47	562.01
x_5	0.0017	0.0015	0.0018	0.0014	y_5	1744.2	1738.6	1746.1	1744.8
x_6	200	200	200	200	y_6	1207.2	1201.6	1210.4	1207.8
x_7	142.88	141.79	143.02	140.57	Time (s)	556	784	920	874

IBO: interaction balance optimization; CO: collaborative optimization; CSSO: concurrent subspace optimization; BLISS: bi-level integrated system synthesis.

disciplines. The system analysis is effective, however less efficiency. On the other hand, instead of system analysis, compatibility constraints and Lagrange multipliers are used in CO and IBO, respectively, which can enhance the efficiency.

Aircraft conceptual design problem

The second example is an aircraft conceptual design problem which is developed by the MDO research group at the University of Notre Dame (Agarwal et al., 2004). The optimization problem involves aerodynamic discipline, weight discipline, and performance discipline. The tightly coupled relationships among these disciplines are shown in Figure 8.

There are five shared design variables $x_1 \sim x_4, x_7$, six linking variables $y_1 \sim y_6$, and two discipline design variables x_5, x_6. The descriptions of the variables are given in Table 2.

The optimal objective in this example is to determine the least gross take-off weight within the bounded design space subject to two performance constraints. The constraints are the range and stall speed of the aircraft. The optimization model of this example can be formulated as follows (Agarwal et al., 2004)

$$\begin{aligned} \min_{x_1 \tilde{x}_7} \quad & f = \text{Weight} = y_5 \\ \text{s.t.} \quad & g_1 = 1 - \frac{y_3}{V_{\text{req}}^{\text{stall}}} \geq 0 \\ & g_2 = 1 - \frac{\text{Range}_{\text{req}}}{y_4} \geq 0 \end{aligned} \tag{18}$$

where $V_{\text{req}}^{\text{stall}} = 70\,\text{ft/s}$ and $\text{Range}_{\text{req}} = 560\,\text{miles}$. The solutions of this example from IBO, CO, CSSO, and BLISS are shown in Table 3. We can see that the similar results can be obtained using different MDO methods. IBO converges using 556 s, while CO, CSSO, and BLISS require 784, 920, and 874 s, respectively. The investigation shows that IBO can enjoy higher efficiency. CSSO and BLISS need longer computational time because of the system analysis. The extra computational burden is brought into MDO problems by system analysis, leading to low efficiency. Furthermore, the high dimensionality of interaction variables increases the non-linearity of compatibility constraints, which makes it difficult for CO to converge. Thus, the efficiency of CO is lower than IBO in this example.

Conclusion

In this article, we propose an effective IBO method to solve MDO problems for large-scale distributed engineering systems. The main difference of IBO from CSSO and BLISS is that IBO eliminates system analysis. Thus, IBO can enjoy higher efficiency. The main difference of IBO from CO is that IBO uses Lagrange multipliers instead of compatibility constraints to maintain the consistency between coupled disciplines. Thus, IBO is more effective than CO to solve a MDO problem with high-dimensional couplings. Furthermore, the simple coordination strategy of IBP and the distributed optimization strategy of CO are integrated effectively in IBO. Thus, IBO fits the architecture of distributed engineering systems. Using the distributed optimization strategy of IBO and taking the advantage of advanced computational analysis tools, designers can simultaneously improve the design and reduce the time and cost of the design cycle. However, the assumption that the system objective is a linear sum of all discipline objectives may limit the application of IBO. In practical engineering, the relationship between system objective and discipline objectives are non-linear generally. To solve this problem, we will try to use powerful multi-objective optimization methods to enhance IBO in future works.

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References

Agarwal H, Renaud JE, Preston EL, et al. (2004) Uncertainty quantification using evidence theory in multidisciplinary design optimization. *Reliability Engineering and System Safety* 85(1): 281–294.

Alexandrov NM and Lewis RM (2002) Analytical and computational aspects of collaborative optimization for multidisciplinary design. *AIAA Journal* 40(2): 301–309.

Amgai R and Abdelwahed S (2014) Power management of shipboard power systems using Interaction Balance Principle. In: *Proceedings of the North American power symposium (NAPS)*, Pullman, WA, 7–9 September, pp. 1–6. New York: IEEE.

Balling R and Wilkinson C (1997) Execution of multidisciplinary design optimization approaches on common test problems. *AIAA Journal* 35(1): 178–186.

Barthelemy JF and Sobieszczanski-Sobieski J (1983) Optimum sensitivity derivatives of objective functions in nonlinear programming. *AIAA Journal* 21(6): 910–915.

Braun RD (1996) *Collaborative optimization: an architecture for large-scale distributed design*. PhD Thesis, Stanford University, Stanford, CA.

Braun RD and Kroo IM (1995) Development and application of the collaborative optimization architecture in a multidiscipline design environment. Available at: http://ntrs.nasa.gov/archive/nasa/casi.ntrs.nasa.gov/20040111288.pdf

Chen S, Zhang F and Khalid M (2002) Evaluation of three decomposition MDO algorithms. In: *Proceedings of the 23rd international congress of aerospace sciences*, Toronto, ON, Canada, 8–13 September, pp. 113.1–113.10. Bonn, Germany: International Council of the Aeronautical Sciences (ICAS).

Chhabra R and Emami MR (2014) A holistic approach to concurrent engineering and its application to robotics. *Concurrent Engineering: Research and Applications* 22(1): 48–61.

Cramer EJ (1992) On alternative problem formulations for MDO. In: *Proceedings of the 4th symposium on multidisciplinary analysis and optimization*, Cleveland, OH, 21–23 September 1992.

Cramer EJ, Dennis JJE, Frank PD, et al. (1994) Problem formulation for multidisciplinary optimization. *SIAM Journal on Optimization* 4(4): 754–776.

Dener A and Hicken JE (2014) Revisiting individual discipline feasible using matrix-free Inexact-Newton-Krylov. In: *Proceedings of the 10th AIAA multidisciplinary design optimization conference*, National Harbor, MD, 13–17 January, pp. 1–13. Reston, VA: AIAA.

Deshmukh M, Schaus V, Fischer PM, et al. (2013) Decision support tool for concurrent engineering in space mission design. In: Stjepandić J, Rock G and Bil C (eds) *Concurrent Engineering Approaches for Sustainable Product Development in a Multi-Disciplinary Environment*. London: Springer, pp. 497–508.

Findeisen W, Bailey FN, Brdys M, et al. (1980) *Control and Coordination in Hierarchical Systems*. London: John Wiley & Sons.

Haftka RT (1994) Simultaneous analysis and design. *AIAA Journal* 23(7): 1099–1103.

Huang HZ, Zhang X, Yuan W, et al. (2011) Collaborative reliability analysis under the environment of multidisciplinary design optimization. *Concurrent Engineering: Research and Applications* 19(3): 245–254.

Hulme KF and Bloebaum CL (2000) A simulation-based comparison of multidisciplinary design optimization solution strategies using CASCADE. *Structural and Multidisciplinary Optimization* 19(1): 17–35.

Hwang JT, Lee DY, Cutler JW, et al. (2014) Large-scale multidisciplinary optimization of a small satellite's design and operation. *Journal of Spacecraft and Rockets* 51(5): 1648–1663.

Jeremy JM and Panos YP (2005) An efficient weighting update method to achieve acceptable consistency deviation in analytical target cascading. *Journal of Mechanical Design* 127(2): 206–214.

Kim HM (2001) *Target cascading in optimal system design*. PhD Thesis, University of Michigan, Ann Arbor, MI.

Kodiyalam S and Sobieszczanski-Sobieski J (2001) Multidisciplinary design optimization-some formal methods, framework requirements, and application to vehicle design. *International Journal of Vehicle Design* 25(1): 3–22.

Kroo I (2004) Distributed multidisciplinary design and collaborative optimization. Available at: http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.140.138&rep=rep1&type=pdf

Lambe AB and Martins JRRA (2012) Extensions to the design structure matrix for the description of multidisciplinary design, analysis, and optimization processes. *Structural and Multidisciplinary Optimization* 46(2): 273–284.

Li H, Ma M and Zhang W (2014) Improving collaborative optimization for MDO problems with multi-objective subsystems. *Structural and Multidisciplinary Optimization* 49(4): 609–620.

Liang H, Zhu M, Guo X, et al. (2012) Conceptual design optimization of high altitude airship in concurrent subspace optimization. In: *Proceedings of the 50th AIAA aerospace sciences meeting including the new horizons forum and aerospace exposition*, Nashville, TN, 9–12 January.

Martins JRRA and Lambe AB (2013) Multidisciplinary design optimization: a survey of architectures. *AIAA Journal* 51(9): 2049–2075.

Mehrotra R and Abdelwahed S (2014) Towards autonomic performance management of large scale data centers using interaction balance principle. *Cluster Computing* 17(3): 979–999.

Meng D, Zhang X, Huang HZ, et al. (2014) Interaction prediction optimization in multidisciplinary design optimization problems. *The Scientific World Journal* 2014: 698453 (7 pp.).

Ostrosi E, Stjepandić J, Fukuda S, et al. (2014) Modularity: new trends for product platform strategy support in concurrent engineering. In: *Proceedings of the 21st ISPE international conference on concurrent engineering*, Beijing, China, 9–11 September, pp. 414–423. Amsterdam, The Netherlands: IOS Press.

Perez RE, Liu HHT and Behdinan K (2004) Evaluation of multidisciplinary optimization approaches for aircraft conceptual design. In: *Proceedings of the 10th AIAA/ISSMO multidisciplinary analysis and optimization conference*, Albany, NY, 30 August–1 September, pp. 1–11. AIAA.

Renaud JE and Gabriele GA (1994) Approximation in nonhierarchic system optimization. *AIAA Journal* 32(1): 198–205.

Roshanian J, Ebrahimi M, Taheri E, et al. (2014) Multidisciplinary design optimization of space transportation control system using genetic algorithm. *Journal of Aerospace Engineering* 228(4): 518–529.

Sadati N and Marvast ED (2006) Stochastic control of two-level nonlinear large-scale systems; Part II—interaction balance principle. In: *Proceedings of international conference on systems, man and cybernetics (IEEE)*, Taipei, Taiwan, 8–11 October, pp. 3856–3861. New York: IEEE.

Sobieszczanski-Sobieski J, Altus DT, Phillips M, et al. (2003) Bi-level system synthesis (BLISS) for concurrent and distributed processing. *AIAA Journal* 41(10): 1996–2003.

Sobieszczanski-Sobieski J and Haftka RT (1997) Multidisciplinary aerospace design optimization: survey of recent developments. *Structural and Multidisciplinary Optimization* 14(1): 1–23.

Tappeta RV and Renaud JE (1997) Multiobjective collaborative optimization. *Journal of Mechanical Design* 119(3): 403–411.

Tosserams S, Etman LFP and Rooda JE (2009) A classification of methods for distributed system optimization based on formulation structure. *Structural and Multidisciplinary Optimization* 39(5): 503–517.

Wujek BA, Renaud JE and Batill SM (1996) Concurrent subspace optimization using design variable shared in a distributed computing environment. *Concurrent Engineering: Research and Applications* 4(4): 361–377.

Yi SI, Shin JK and Park GJ (2008) Comparison of MDO methods with mathematical examples. *Structure and Multidiscipline Optimization* 35(5): 391–402.

Yu BY, Honda T and Zubair S (2014) Multi-disciplinary design optimization for large-scale reverse osmosis systems. In: *Proceedings of ASME 2014: international design engineering technical conferences and computers and information in engineering conference*, Buffalo, NY, 17–20 August, pp. 1–9. New York: ASME.

Zhang D and Tang S (2015) Concurrent subspace design optimization and analysis of hypersonic vehicles based on response surface models. *Aerospace Science and Technology* 42: 39–49.

Zhang X, Huang HZ, Wang Z, et al. (2011) A new method for achieving flexibility in hierarchical multilevel system design. *Concurrent Engineering: Research and Applications* 19(2): 187–196.

Zhang X, Huang HZ, Wang Z, et al. (2013) A Pareto set coordination method for analytical target cascading. *Concurrent Engineering: Research and Applications* 21(4): 286–295.

Zhao M and Cui W (2011) On the development of bi-level integrated system collaborative optimization. *Structural and Multidisciplinary Optimization* 43(1): 73–84.

Zhao WZ, Xu XH and Wang CY (2012) Multidiscipline collaborative optimization of differential steering system of electric vehicle with motorized wheels. *Science China Technological Sciences* 55(12): 3462–3468.

Zhou H, Jiang P, Shao X, et al. (2014) An improved bi-level integrated system collaborative optimization method for multidisciplinary design optimization. In: *Proceedings of the 6th international conference on modeling, identification & control (ICMIC)*, Melbourne, VIC, Australia, 3–5 December, pp. 365–370. IEEE.

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