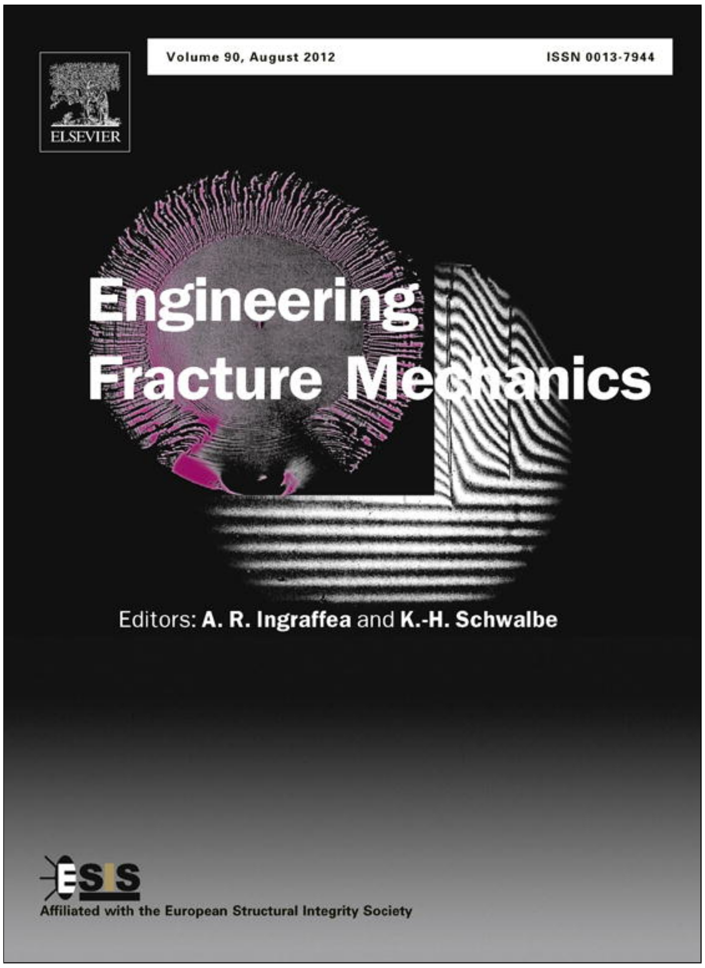
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A generalized energy-based fatigue–creep damage parameter for life prediction of turbine disk alloys

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ABSTRACT

In this paper, a new model for life prediction of turbine disk alloys (GH4133) is proposed. Based on plastic strain energy density, a generalized energy-based fatigue–creep damage parameter is developed to account for creep and mean strain/stress effects in the low cycle fatigue regime. Moreover, the mechanism of cyclic hardening is taken into account within this model. It provides a better prediction of GH4133's fatigue behavior when compared to the Smith–Watson–Topper and plastic strain energy density methods. Under mean strain conditions, the proposed model provides a more accurate life prediction of GH4133 than that under zero-mean strain conditions.

1. Introduction

Disk failure in an aircraft engine often leads to a catastrophic failure making the disk one of the most critical flight safety components with respect to both fracture and durability. Low cycle fatigue (LCF) is the principal failure mode of these aircraft engine disks. With increased performance and thrust-to-weight ratios of aircraft engines, these disks undergo higher stresses and temperatures resulting in a need for higher reliability. The critical nature of the aircraft engine disks under increased stresses and temperatures creates new challenges to the precise prediction of LCF life and experiment assessment methodology [1].

LCF behavior at high temperature is of considerable interest in the selection, design, and safety assessment of many engineering components used in aviation and gas turbines. These components often fail prematurely under high temperature and cyclic loading. This phenomenon is often called high temperature low cycle fatigue (HTLCF), when the cycles to failure are lower than 10,000 cycles [1]. To accurately evaluate the lives for these key components, HTLCF prediction methods have received a great deal of interest. HTLCF is an interactive mechanism of different processes including time-independent plastic strain, time-dependent creep and dynamic strain aging (DSA), environment corrosion, and the complex interactions between them. These damage mechanisms and their interactions make it difficult to predict the HTLCF life, thus, no unified model has been developed [1]. Extensive research have been done to reveal the relationship between the cyclic frequency [2,3], hold period [4], strain range [5], strain rate [6], heat treatment [7], testing temperature [8] and environment [9,10] on the cyclic stress response, deformation mode and fatigue life of superalloys for HTLCF. It has become apparent that HTLCF life depends not only on the testing temperature but also the loading waveform resulting from the creep effect. Therefore,

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Nomenclature

C_1, C_2, C_3	material constants representing the material energy absorption capacity
E	elastic modulus
m	model parameter
K'	cyclic strength coefficient
n'	cyclic strain hardening exponent
N_f	number of cycles to failure
R_ε	strain ratio
ΔW_p	plastic strain energy density (PSED)
ΔW_s	strain energy
W_f	true toughness
W_{f+}	pseudo-true toughness
α, β	model parameters
ε_a	strain amplitude
$\varepsilon_{\max}$, $\varepsilon_{\min}$	maximum and minimum strain
ε_m	mean strain
$\Delta\varepsilon_t$	strain range
$\Delta\varepsilon_e$, $\Delta\varepsilon_p$	elastic strain range and plastic strain range
$\Delta\varepsilon_{in}$	inelastic strain range
ε_f	fracture strain
σ_a	stress amplitude
$\sigma_{\max}$, $\sigma_{\min}$	maximum and minimum stress
σ_m	mean stress
$\Delta\sigma$	stress range
σ_f	fracture stress
σ_{UTS}	ultimate tensile strength
υ	frequency factor
$\sigma_{\max}\varepsilon_a$	SWT parameter

the studies of the complicated behaviors inherent to HTLCF are recognized to be essential with respect to the effects of different time-dependent damage mechanisms and the remaining life to failure.

Increasing attention has been paid to the study of fatigue and creep interaction in both isothermal and thermal–mechanical fatigue conditions [2–18]. Using the traditional evaluation methods of fatigue life such as the Coffin–Manson law, the effects of loading frequency and hold time on fatigue cannot be described under HTLCF. Therefore, some other methods for HTLCF life prediction have been developed. Examples of these methods include the linear damage summation (LDS) [1], frequency modified Coffin–Manson equation (FMCM) [11], frequency separation technique (FS) [12], strain range partition (SRP) [13], frequency modified damage function model (FMDF) [14], ductility exhaustion approach (DE) [15], and damage rate approach (DRA) [16]. In the practical application of these models, there are many difficulties in obtaining the various parameters and/or constants required for each equation. As a result, highly precise methods of fatigue life estimation are needed to effectively design a component. Recently, Shang et al. [6] developed a time-dependent fatigue damage model based on LDS, in which the computing time of time-dependent damage is defined with tension–compression strain rate and cycle period. However, it is not applicable for life prediction under loading conditions with hold-time. Payten et al. [17] postulated that on a macroscopic level, creep damage is measured by the absorbed internal energy density, and then proposed the strain energy density exhaustion (SEDE) approach from considerations of mechanistic cavity growth. In order to reflect the effects of different time-dependent damage mechanisms on HTLCF life, Zhu and Huang [18,19] used the dynamic viscosity to correlate the fatigue–creep damage and the life, and then proposed a viscosity-based model (VBM) for life prediction, in which the mechanisms of the loading waveform, temperature and mean stress effects were taken into account within the HTLCF regime.

Unfortunately, the models reviewed above have the following limitations: (a) they cannot accurately describe the effects of different loading waveforms (LDS [1], FMCM [11] and FMDF models [14]); (b) many of the existing models are not favorable under finite test data and certain experimental conditions, (SRP [13], and DE approach [15]), these methods obviously restrict their applications for life prediction under stress control; SEDE [17] and VBM [18,19] are limited to the usage for evaluating the HTLCF life for complex loading conditions; (c) most of them ignore the influences of mean strain or stress effects on the fatigue life, (LDS [1], FS [12], DRA [16] and model presented in [6]). In summary, further improvements on HTLCF life prediction models are needed.

In the present investigation, a review of the earlier energy-based fatigue life prediction models is made. Next, a new energy-based fatigue life prediction model is presented by combining the strain energy damage function model [14,20] and the frequency separation technique. In this model a creep-compensating parameter is introduced to account for the creep and mean strain/stress effects on the LCF lives of hot section components based on plastic strain energy density (PSED). The approach used in the proposed model reflects the effects of time-dependent damage mechanisms on HTLCF life and it is different from those used in models published previously. By introducing the loading waveform, strain rate and frequency, the proposed model can describe the effects of different time-dependent damage mechanisms on HTLCF life more accurately than previous models. To verify this method, the LCF life is assessed using experimental data of turbine disk material GH4133 in existing literature. The lives predicted by the proposed model are compared with experimental results resulting in a good agreement.

2. Energy-based fatigue life prediction models

A certain quantity of energy is gradually dissipated by cyclic fatigue and creep during HTLCF. Under the energy-based life approach once the critical energy is reached, fracture will occur, the more damage that is accumulated, the more energy that is dissipated. Following a proposed one to one relation between the accumulated damage and dissipated energy, a measurement of the damage done to a material is made. Numerous energy criteria and corresponding assessments of fatigue life are based on this idea, which has been used to describe ductile and LCF behavior [15].

Under cyclic loading conditions, the relationship between the stress and the strain during deformation can be represented by hysteresis loops. Thus an energy perspective can be used to quantify this relationship. The accumulated interaction effects in a material result in its eventual failure. Using stress–strain hysteresis loops at high temperatures, various researchers have developed fatigue life curves by adopting an energy parameter to estimate the life of a material under different loading conditions. These methods are often called energy-based fatigue life prediction models.

A considerable amount of effort has been done to define suitable damage parameters which correlate life to failure in the previously mentioned models. Up to now, the damage parameters have been represented by stress, strain, inelastic strain energy and strain energy density. Under LCF conditions, the alternating plastic strain range, $\Delta\varepsilon_p$, is used to develop fatigue-life curves by relating $\Delta\varepsilon_p$ to N_f using the Coffin–Manson Law [1]

$$\Delta\varepsilon_p N_f^{\alpha} = C_1 \tag{1}$$

Generally the plastic strain range ($\Delta\varepsilon_p$) is used as a control parameter to construct the Coffin–Manson curve, by which LCF life can be predicted. However previous studies have showed that these models were not applicable for the following conditions: First, many tests were carried out under total strain control rather than plastic strain control mode. Within this group of total strain-controlled tests, a group of experimental data with the same $\Delta\varepsilon_e$ or $\Delta\varepsilon_p$ could not be collected. This results from the fact that $\Delta\varepsilon_e$ and $\Delta\varepsilon_p$ changed under the same $\Delta\varepsilon_t$ for different specimens. Second, high temperature thermal fatigue results from thermal expansion strain fluctuations. Therefore, total strain-controlled tests can be used to simulate thermal fatigue [21]. Material ductility depends not only on test and material parameters, such as strain rate, hold time and temperature, but also on impurity content, grain size and other factors, such as cyclic hardening/softening. During total strain controlled fatigue deformation, cyclic hardening/softening occurs. Plastic strain range variations with the number of cycles were observed. For stainless steels undergoing LCF deformation, a noticeable change in plastic strain range with the number of cycles was observed [22]. According to the evolution of four fatigue parameters (plastic strain range, peak stress, Tomkins parameter and PSED) with the number of cycles in 429EM stainless steel [21], the plastic strain amplitude decreased continuously when cyclic hardening occurs. Also, increases in the Tomkins parameter and peak stress were observed in the early cycles. However, the PSED (ΔW_p) is nearly independent throughout the fatigue process. The studies in [22] indicated that the variation of PSED through the whole fatigue process was relatively small in some steels. Compared with the plastic strain range, it is better to use PSED as damage parameter for life prediction under cyclic hardening/softening conditions.

PSED, is defined as the inner area of the cyclic stress–strain hysteresis loop, is commonly applied as a damage parameter when predicting fatigue life. Up to now, many studies have been attempted to establish fatigue criteria using PSED measured from stabilized hysteresis loops [21,23–26]. In order to evaluate the fatigue behavior of high pressure tube steel in the high cycle fatigue regime, Koh [24] investigated the fatigue damage using the total cyclic strain energy density parameter under strain-controlled conditions, which took mean stress effects into account. Based on the stable PSED under tension conditions, Chiou and Yip [26] developed a modified energy parameter $(\Delta W_p)_T$ to account for the mean stress effects in the LCF regime. Since its magnitude is equal to one half of the PSED, there are no intrinsic differences between $(\Delta W_p)_T$ and ΔW_p when using these two parameters to evaluate fatigue life. Recently, based on the universal slope method, Lee et al. [21] used PSED normalized by material toughness in life prediction to account for temperature effects on fatigue life. In this model, the fatigue lives were predicted within a factor of 2.5, which shows an overestimating tendency in 316L and 429EM stainless steels with temperature increases. Similar to the universal slope method, creep effects, DSA and interactions among these factors were not properly accounted for in this model.

Under cyclic loading, the plastic strain energy per cycle is considered a measure of the amount of fatigue damage per cycle. The movement of dislocations is described by the cyclic plastic strain and the resistance against their motion is described by the cyclic stress [23]. In LCF, considerable amount of plastic strain by the material and the hysteresis energy absorbed during cyclic loading has been postulated as a basis for failure analysis. Morrow in his analysis [23] of cyclic plastic strain energy showed the relation between PSED and the fatigue life to be

$$\Delta W_p \cdot N_f^{\beta} = C_2 \tag{2}$$

According to the earlier introduced damage parameter [21,26], it is worth noting that degradation mechanisms such as creep are not sufficient enough to be considered. As aforementioned, to accurately account for the creep effect on the LCF lives of hot section components, including different loading waveforms, mean stress effects, and expand the application to stress-controlled tests, a new life prediction model is developed in the next section.

3. A generalized energy-based fatigue life prediction model

3.1. Fatigue life evaluation using strain energy parameter

Many high temperature structures are subjected to loading that requires resistance to creep. The key factor determining the failure of these components under a certain environment is the fatigue–creep interaction, which comes from thermally induced stresses and strain. Time-dependent and environmental effects (such as hold time, load waveform and temperature) are very difficult to incorporate into a life prediction model due to the complexity of fatigue–creep interaction. Therefore, defining a damage parameter, which accounts for creep and mean strain/stress effects, is very important when LCF test results are used to simulate fatigue–creep interaction conditions. If these effects on fatigue life can be described, life prediction under complex interactions of fatigue and creep processes within LCF regime will be reasonably obtained under high temperature. Only a few attempts have been performed to account for the creep and mean strain/stress effects in life predictions based on the energy principle. However, the applicability of these models was found to be limited because the time-dependent factors for creep are not sufficiently considered in these models. In this study, a creep-compensating parameter was introduced to estimate these effects on the LCF life.

The ability of a material to accommodate permanent deformation was defined in terms of the material's toughness. The energy dissipated by a material in one loading cycle can be characterized by the PSED. Fig. 1 provides a graphical interpretation of the damage parameter ΔW_p. Note that its magnitude is determined by integrating the enclosed area within the curve *ABCDEFA*. This parameter reflects the interaction between the stress and strain under cyclic loading conditions which causes premature failure or damage to the material. Based on the energy concept, the corresponding damage of material in one loading cycle is

$$\frac{1}{N} = \frac{\Delta W_p}{W_f} \tag{3}$$

where N is the predicted life and W_f is the total energy dissipated by the material cumulatively up to failure and is also called the fatigue toughness. The toughness of a material is a product of ductility and strength [15]:

$$\textit{Material toughness} = \textit{ductility} \times \textit{strength} \tag{4}$$

Since strength in a cyclic fatigue test is in terms of saturated stress range at a particular strain range, the strength in Eq. (4) can be evaluated by the saturated stress range. Using the Edmund and White equation [1], ductility can be determined as follows:

$$\textit{Ductility} = \Delta\varepsilon_p N_f \tag{5}$$

Then, material toughness can be obtained by substituting Eq. (5) and saturated stress range into Eq. (4). Comparing Eq. (2) and Eq. (3), C_2 is equal to W_f, and Eq. (2) can be rewritten as:

$$\left(\frac{\Delta W_p}{W_f}\right)^{1/\beta} N_f = 1 \tag{6}$$

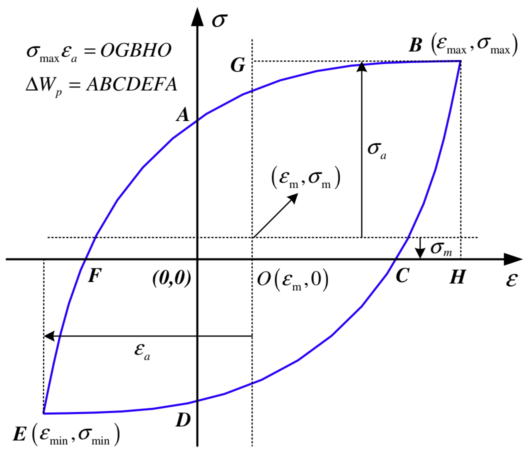

Fig. 1. Schematic interpretation of fatigue damage parameter ΔW_p.

The total energy absorbed until final fracture can be measured from the true stress–strain curve and this value is described by true toughness (W_f) of a material. For the materials exhibiting necking behavior, however, the complicated stress state after necking leads to the fact that strain beyond necking is realistically difficult to measure. Thus, a linear approximation is obtained by connecting the necking point to the final fracture point as shown in Fig. 2. This linear extrapolation of the true stress–strain curve (Fig. 2) beyond necking was found to be the upper bound for the true stress–strain relation [27]. The energy associated with this enlarged area is defined as "pseudo-true toughness (W_{f+})". In Fig. 2, σ_f and ε_f are the fracture stress and fracture strain at the fracture point respectively and pseudo-true toughness can be separated into two energy terms:

$$W_{f+} = W_t + W_n \tag{7}$$

where W_t is the tensile toughness which can be measured from the area below the engineering stress–strain curve and W_n is defined in Fig. 2. By using Eq. (7) and rearranging Eq. (6), the following relation was determined

$$\left(\frac{\Delta W_p}{W_{f+}}\right)^{1/\beta} N_f \approx 1 \tag{8}$$

Due to the difference between W_{f+} and W_f, a loading factor was used to account for this difference in Eq. (8), which depends on the loading condition. Combining the loading conditions factor, Eq. (6) can be modified into

$$\left(\frac{\Delta W_p}{C_f W_{f+}}\right)^{1/\beta} N_f = 1 \tag{9}$$

where C_f is the loading coefficient. Comparing Eq. (2) and Eq. (9), C_2 is equal to $C_f W_{f+}$. When the accumulated PSED under fatigue deformation is equal to the pseudo-true fatigue toughness, the material will fracture and C_f is equal to 1. The actual value of C_f generally becomes smaller than 1 because of the difference between the engineering stress–strain and true stress–strain curves. If the energy from the post necking process can be correctly evaluated, it can be used as the value for true toughness, in place of the pseudo-true toughness. As a result the value of C_f will become close to 1. Using Eq. (9) the HTLCF life can be well predicted, but the values of C_f and β should be known beforehand. It is very difficult to estimate fatigue–creep toughness W_{f+} under different loading conditions as mentioned above. Hence, a methodology for determining the value of material constants for the calculation of fatigue life efficiently using Eq. (9) is needed, if possible, without further fatigue test. In order to reduce the difference between the pseudo-true toughness and true toughness absorbed during the process of damage and get higher precision, a new energy-based fatigue damage parameter model is presented on the basis of a combination of theoretical computation and empirical analysis in the following section.

3.2. A generalized energy-based fatigue damage parameter considering the creep effect

According to the interaction between the different failure mechanisms present at high temperatures, the damage parameter should consider not only the effects of mean strain/stress, but also those factors present during creep such as frequency, strain rate and temperature [19]. The principle of Ostergren's model is that only the inelastic strain energy can induce crack opening and propagation. In Ostergren's model the strain energy controls the LCF damage evolution and is expressed approximately by the damage function [20]. In this section a new model is proposed, based on the PSED model used to describe the relationship between the stress–strain and failure life as well as Ostergren's model and the frequency separation technique.

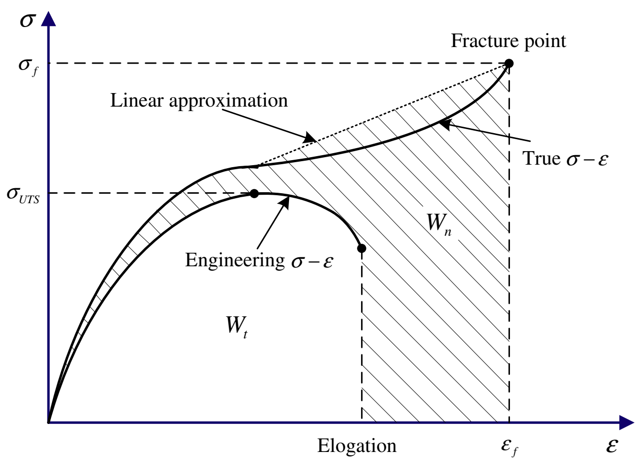

Fig. 2. Schematic illustration of the engineering stress–strain curve and true stress–strain curve.

Morrow [23] found that the total plastic strain energy required for fatigue failure is not constant, moreover it increases with a decrease in stress amplitude. Additionally, the total energy depends on the stress or plastic strain amplitude found via the cyclic stress–strain behavior. The cyclic stress–strain response is an important material property in designing for enhanced fatigue resistance [28]. It describes the relationship between flow stress and plastic strain amplitude under cyclic loading. Based on the Ramberg–Osgood relation [1], the cyclic stress–strain curve can be described by

$$\frac{\Delta\sigma}{2} = K'\left(\frac{\Delta\varepsilon_p}{2}\right)^{n'} \tag{10}$$

where $\Delta\sigma$ is the stress range with $\Delta\sigma = \sigma_{\max} - \sigma_{\min}$.

Based on Eq. (10), K' and n' can be obtained from the log–log linear regression analysis of the cyclic strain amplitudes and corresponding cyclic stress amplitudes for fully reversed fatigue tests.

For a material that satisfies the Masing's hypothesis, the PSED absorbed during a cycle is the area of the hysteresis loop [23]:

$$\Delta W_p = \frac{1-n'}{1+n'} \cdot \Delta\sigma \cdot \Delta\varepsilon_p \tag{11}$$

Materials for which the hysteresis loop can be described by magnifying the cyclic stress–strain curve by a factor of 2 are said to exhibit Masing behavior [29]. Recent research in [30,31] has revealed that material used in this study exhibits Masing behavior. Substituting Eq. (10) into Eq. (11) gives

$$\Delta W_p = 4\frac{1-n'}{1+n'}K'\left(\frac{\Delta\varepsilon_p}{2}\right)^{1+n'} \tag{12}$$

The PSED can be measured from the stabilized (half-life) hysteresis loops or be calculated using Eq. (12) and the LCF properties. In general, the measured plastic strain energy is in accordance with the calculated strain energy for turbine disk alloys exhibiting Masing behavior. Experimental test parameters were used in a physics-based model to predict LCF life at high temperatures. Due to time-dependent damage mechanisms under high temperature such as creep and environment corrosion, the experimental results from Sun et al. [6] showed that the shape and size of the hysteresis loop are influenced by the cyclic frequency, loading waveforms and strain rates. So the PSED can not be calculated directly by means of Eq. (12). Therefore, an attempt has been made to deduce a new damage parameter model for life prediction, in which fatigue–creep toughness is used as the control parameter.

Ostergren proposed a strain energy damage function model [14,20] similar to the material toughness equation shown in Eq. (4). The strain energy damage function ΔW_s was expressed approximately by multiplication of the inelastic strain range $\Delta\varepsilon_{in}$ and maximum of tension stress $\sigma_{\max}$

$$\Delta W_s = \Delta\varepsilon_{in}\sigma_{\max} \tag{13}$$

where $\Delta\varepsilon_{in}$ can be replaced by the plastic strain range $\Delta\varepsilon_p$ under pure fatigue mode. The relationship between strain energy and fatigue life can be expressed by the following power exponent function

$$\Delta W_s N_f^m = C_3 \tag{14}$$

The introduction of hold time in a LCF test at high temperature can be considered a frequency effect or an effect of a time dependent process and is a widely used method of studying the fatigue–creep interaction of high temperature alloys. The effect of environment at high temperatures is reflected by a frequency factor υ, so the frequency modified strain energy damage function for HTLCF life prediction is [20]

$$N_f = \left(\frac{C_3}{\Delta\varepsilon_{in}\sigma_{\max}}\right)^{1/m}\upsilon^{1-k} \tag{15}$$

where k is a material constant related to the environmental conditions, such as temperature. Eq. (15) reduces to the strain energy damage function model for low temperature fatigue, where $k = 1$ at low frequencies. The selection of υ depends on the sensitivity of the material to different waveforms and some suggestions were given by Ostergren [20].

Similar to Coffin [12], a frequency separation technique was used to reflect the effects of frequency and loading waveforms on the life prediction for HTLCF. A frequency separation-strain energy damage function model was deduced as follows:

$$N_f = \left(\frac{C_3}{\Delta\varepsilon_{in}\sigma_{\max}}\right)^{1/m}\left(\frac{\upsilon_t}{2}\right)^{1-k}\left(\frac{\upsilon_c}{\upsilon_t}\right)^{n} \tag{16}$$

where C_3, m and k are material constants which can be obtained from the experimental data under a balanced waveform, n is a material constant which can be obtained from the unbalanced waveform test data and υ_t is the theoretical frequency of tension at half-period. When the strain rate of tension is equal to that of compression, this is called a balanced waveform, otherwise it is an unbalanced waveform. The theoretical frequency of tension at half-period is the frequency of the balanced waveform with the same tension strain rate. According to Eq. (16), the predicted life depends on the degree of unbalance υ_c/υ_t and tension half-period time $1/\upsilon_t$, this equation can be simplified into:

$$N_f = \left(\frac{C_3}{\Delta\varepsilon_{in}\sigma_{\max}}\right)^{1/m}\left(\frac{\upsilon_c}{\upsilon_t}\right)^n(\upsilon_t)^{1-k} \tag{17}$$

Under LCF, $\Delta\varepsilon_{in}$ is approximately obtained by $\Delta\varepsilon_p$. Substituting Eq. (12) into Eq. (17) gives

$$N_f = \left(\frac{C_3^{1+n'}}{2^{n'-1}\sigma_{\max}^{1+n'}\frac{1+n'}{(1-n')K'}\Delta W_p}\right)^{\frac{1}{m(1+n')}}\left(\frac{\upsilon_c}{\upsilon_t}\right)^n(\upsilon_t)^{1-k} \tag{18}$$

Eq. (18) can now be simply expressed with the same form as Eq. (8)

$$\left(\frac{\Delta W_p}{T'}\right)^{\frac{1}{m(1+n')}}N_f = 1, \tag{19}$$

$$T' = \frac{C_3^{1+n'}(1-n')K'}{2^{n'-1}(1+n')}\cdot\frac{\upsilon_c^{mn(1+n')}}{\upsilon_t^{m(n+k-1)(1+n')}\sigma_{\max}^{1+n'}} \tag{20}$$

The material constant T' is the total energy accumulated up to failure as described in Eq. (3) and therefore $T' = W_{f+}$. The main factor influencing fatigue life is σ_a and the main factor influencing creep life is σ_m.

Hence, T', a creep-compensating parameter based on frequency separation factors can be defined. Under these assumptions, Eq. (9) can be expressed as

$$\left(\frac{\Delta W_p}{C_f W_{f+}}\right)^{\frac{1}{m(1+n')}}N_f = 1 \tag{21}$$

It is worth noting that Eqs. (8) and (19) have similar forms even though they are derived from different theoretical backgrounds. Comparing these two equations, the fatigue exponent β in Eq. (8) can be written as $m(1 + n')$. The loading coefficient C_f, which depends on the loading conditions, is given empirically as a correction factor

$$C_f(\sigma_{\max}, \sigma_m) = \left(\frac{\sigma_{\max} - \sigma_1(\sigma_m)}{\sigma_{UTS} - \sigma_{\max}}\right)^a \tag{22}$$

where a is a temperature-dependent constant. The fatigue limit function, σ_1, is expressed as

$$\sigma_1(\sigma_m) = \sigma_{10} + (1 - b\sigma_{10}/\sigma_{UTS})\bar{\sigma} \tag{23}$$

where σ_{10} is the fatigue limit under zero mean stress and b is a material constant. Substituting Eq. (22) into Eq. (21) gives

$$\left(\frac{\Delta W_p}{\left(\frac{\sigma_{\max}-\sigma_1(\bar{\sigma})}{\sigma_{UTS}-\sigma_{\max}}\right)^a W_{f+}}\right)^{\frac{1}{m(1+n')}}N_f = 1 \tag{24}$$

$$W_{f+} = \frac{C_3^{1+n'}(1-n')K'}{2^{n'-1}(1+n')}\cdot\frac{\upsilon_c^{mn(1+n')}}{\upsilon_t^{m(n+k-1)(1+n')}\sigma_{\max}^{1+n'}} \tag{25}$$

The equation shown above relates parameters such as stress, strain range, strain rate, frequency effect and cyclic stress–strain relations. It is very easy to use Eq. (24) to predict the life for HTLCF, because most of the constants can be determined by data fitting.

Under balanced waveform loading, $\upsilon_t = \upsilon_c$, Eq. (24) can be rewritten as

$$(C_4\upsilon_t^{m(1+n')(k-1)}\cdot\Delta W_p\sigma_{\max}^{1+n'})^{\frac{1}{m(1+n')}}N_f = 1, \tag{26}$$

$$C_4 = \frac{2^{n'-1}(1+n')}{C_3^{1+n'}C_f(1-n')K'} \tag{27}$$

Based on Eq. (26), it should be noted that LCF life has a certain dependency with $\Delta W_p\sigma_{\max}^{1+n'}$, which includes factors that influence both fatigue life and creep life. $\Delta W_p\sigma_{\max}^{1+n'}$ can be called a generalized energy-based fatigue–creep damage parameter. The combination of Eqs. (23)–(27) gives the degree of unbalance υ_c/υ_t which characterizes the effect of frequency on HTLCF life. To a certain extent, the degree of unbalance reflects mechanisms of creep, frequency and mean stress effect on LCF life at high temperature.

4. Validations of the energy-based fatigue life prediction model

To verify the feasibility and prediction capability of the energy-based fatigue–creep damage parameter for life prediction, the proposed model is applied to experimental results of turbine disk material GH4133 [32]. GH4133 is a nickel-based superalloy, which is used for jet engine turbine disks. The heat treatment conditions for this alloy include austenitization (8 h at 1080 °C, air-cooled) and tempering (16 h at 750 °C, air-cooled). HTLCF data were obtained from the Beijing Institute of Aeronautical Materials (China). Detailed mechanical properties of tested materials, test conditions, and strain-life data are reported in [32,33]. This paper will focus only on the assessment of the test data using the new proposed model.

In this section, the applicability of the proposed method will be assessed using the HTLCF data. Under different strain ratios and σ_m, all of the tests were performed in axial total strain control mode with a triangular fully reversed waveform, using an axial extensometer placed on the specimen. Numerous tests were carried out under various conditions: mechanical strain range of 0.5–1.4% for isothermal LCF at temperatures (400, 500 °C) with a strain ratio, R_ε, equal to -1 and a strain range of 0.5–1.2% for isothermal LCF at temperature 400 °C with $R_\varepsilon = 0$, respectively. Test parameters and results are shown in Figs. 3–6.

According to the experimental conditions and data, the strain rate of tension is equal to the stain rate of compression and remains constant. So $(\upsilon_t)^{m(1+n')(k-1)}$ can be merged into the material constant C_4, Eq. (26) can be simplified as

$$(C_4 \cdot \Delta W_p \sigma_{\max}^{1+n'})^{\frac{1}{m(1+n')}} N_f = 1 \tag{28}$$

According to the results of fully reversed fatigue tests, the Superalloy GH4133 exhibits a slender cyclic hardening characteristic at 500 °C. The calculated value of K' and n' in Eq. (10) are 1716.5 MPa and 0.1107, respectively. Life predictions of LCF were conducted using Eq. (28). Combining the experimental data and parameters listed in [33], under different total strain conditions with $R_\varepsilon = -1$, the fitted life prediction model for GH4133 at 500 °C is as follows:

$$(\Delta W_p \sigma_{\max}^{1.1107})^{0.5825} N_f = 1.22885 \times 10^{14} \tag{29}$$

A comparison between experimental results and the proposed predictions is shown in Fig. 3. Two evaluating parameters for life assessments were used: scatter band and standard deviation. The dashed line in the graph represents a ±1.5 factor indictor and the solid line represents a ±2 factor indictor. From Fig. 3, 20 out of 27 cyclic lives are predicted within a factor of ±1.5 and the predicted results are in good agreement with these observed ones.

Similarly at 400 °C and $R_\varepsilon = -1$, the fitted life prediction model for GH4133 is

$$(\Delta W_p \sigma_{\max}^{1.1267})^{0.5583} N_f = 5.72095 \times 10^{13} \tag{30}$$

To reflect the capability of this new model and evaluate $\Delta W_p \sigma_{\max}^{1+n'}$ as a fatigue damage parameter considering creep effects, two other popular parameters, the SWT and PSED were employed for comparison respectively. Fatigue damage has been evaluated by using the product of maximum stress and strain amplitude, a form of strain energy proposed by Smith et al. [34]. And

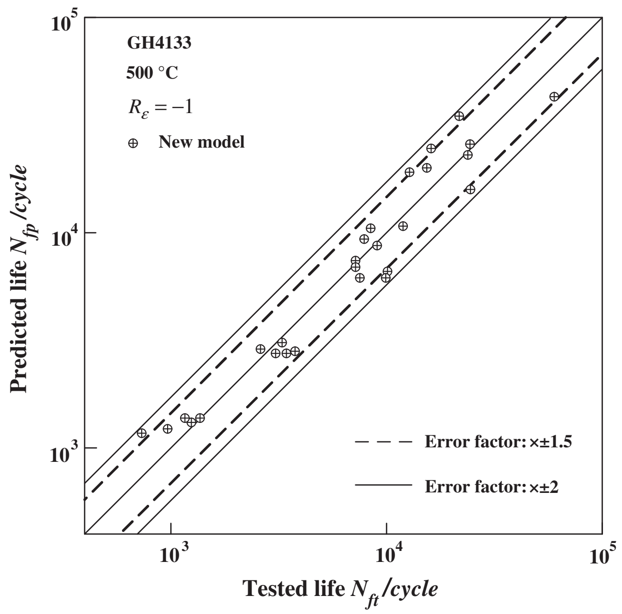

Fig. 3. Predicted life vs. tested life for GH4133 at 500 °C.

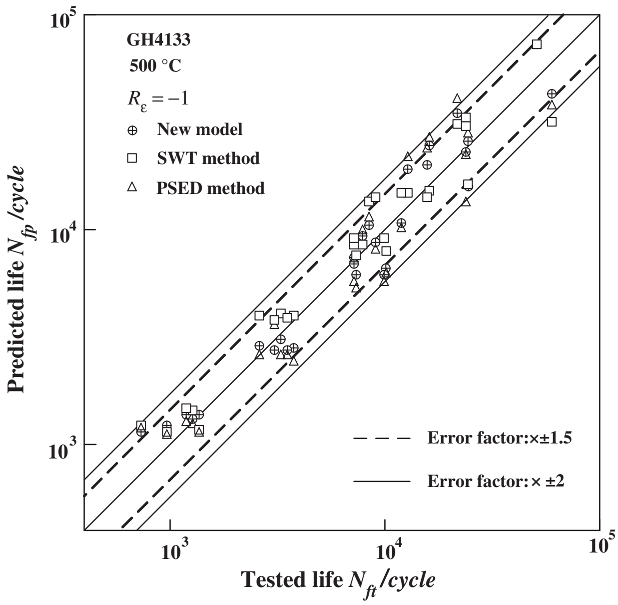

Fig. 4. Comparison between lives predicted by new model, SWT, PSED method and lives tested for GH4133 at 500 °C.

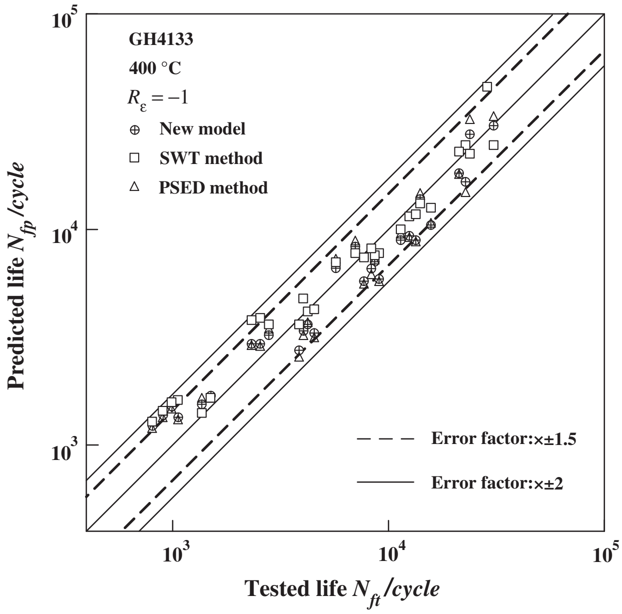

Fig. 5. Comparison between lives predicted by new model, SWT, PSED method and lives tested for GH4133 at 400 °C.

this product of $\sigma_{max}\varepsilon_a$ is conventionally referred to as the SWT parameter. Its magnitude is determined by integrating the enclosed area within the curve OGBHO as shown in Fig. 1, which can also be applied for life prediction under strain controlled tests for HTLCF and can be simplified as

$$(\sigma_{\max}\varepsilon_a)N_f^{\gamma} = C_5 \tag{31}$$

where C_5 and γ are material constants.

When using the SWT and PSED methods (Eq. (2)), all of the test data is fitted into one curve. It is first necessary to construct a fatigue life curve based on the SWT and PSED parameters for fully reversed cyclic loading. Eq. (31) can be used to predict the effect of mean strain on the fatigue life of GH4133 for different values of the product $\sigma_{max}\varepsilon_a$. The predicted fatigue life for each of the product $\sigma_{max}\varepsilon_a$ can be plotted against the experimental data, as shown in Figs. 4 and 5, in which comparisons between test data and data predicted by the new model.

The results show that all of the predicted cyclic lives given by the new method are within a factor of ±2 to the test ones. Not all of the test data predicted by the SWT and PSED methods were within a factor of ±2 compared to the test results. From the test results, it was determined that the proposed model and PSED model cannot be applied when the plastic strain range

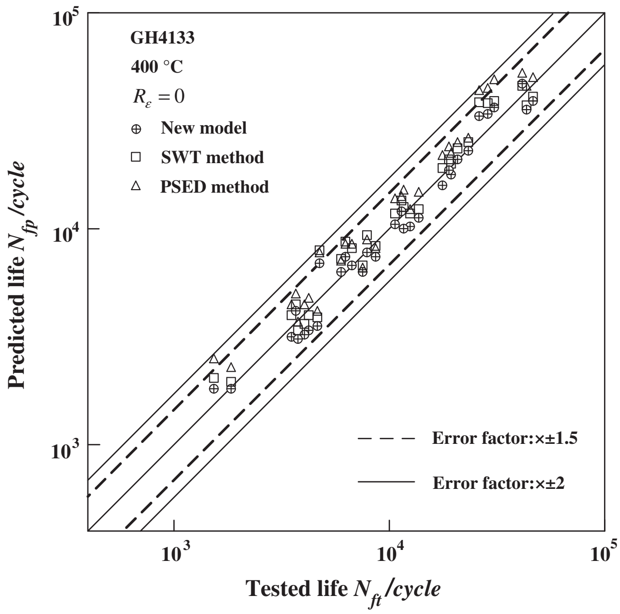

Fig. 6. Comparison between lives predicted by new model, SWT, PSED method and lives tested for GH4133 under different mean strain level at 400 °C.

is so small (even negligible). Substituting the calculated plastic strain range into Eq. (11), the strain energy is also small (even negligible). The points with small (even negligible) plastic strain ranges will be considered invalid data points when fitting the strain energy-fatigue life curve. Using the filtered test data, both the proposed model and the SWT method made a better life prediction than the PSED method, where 42/55, 44/57 and 37/55 of the predicted cyclic lives were within a factor of ±1.5 respectively. Comparing the scatter band and standard deviation of those methods indicates that the proposed model has a better predictability than the others.

In order to investigate the effect of mean strain on GH4133's fatigue response and verify the prediction effect of the new damage parameter developed in this paper, a series of strain-controlled LCF tests was performed at mean strain level of ε_m = 0.6%, 0.5%, 0.4%, 0.35%, 0.3% and 0.25%, respectively, with strain amplitudes ranging from 0.25% to 0.6%. The fatigue tests were conducted under strain control mode and a triangular waveform was used for all tests. The heat treatment conditions of the GH4133 were the same as mentioned above. The experimental parameters and results of fatigue tests conducted with a non-zero mean strain are summarized in Fig. 6.

Taking the experimental results from [33], the fatigue life curve for GH4133 at 400 °C under R_ε = 0 based on the proposed damage parameter is given by

$$(\Delta W_p \sigma_{\max}^{1.2184})^{1.4011} N_f = 1.39628 \times 10^{30} \tag{32}$$

Comparisons between the experimental results and theoretical predictions by the new model, SWT and PSED method under different mean strain levels are shown in Fig. 6.

Figs. 3–6 illustrate the relationship between the fatigue life and mean strain effect where different combinations of mean strain and strain amplitude have different effects on the fatigue life. From the test results, it can be concluded that an applied mean strain causes a change in the fatigue life. Generally speaking, due to mean stress relaxations, the imposed mean strain has no effect on the fatigue life at constant strain amplitude. Moreover, a stable non-zero tensile mean stress often reduces the fatigue life but a compression one increases it due to mean strain action. Hence, the mean strain effect on the fatigue life of GH4133 is mainly attributed to the presence of a stable non-zero mean stress. In other words, mean stress is the main factor influencing life rather than the mean strain. The effects of mean strains on the fatigue life are not considerable unless they accompany the mean stresses.

Therefore, it is important to understand the correlation between the mean stress effect and the fatigue life. This correlation can be interpreted using the proposed damage parameter, $\Delta W_p \sigma_{\max}^{1+n'}$. Using this parameter for life prediction under an applied mean strain as shown in Fig. 6, the results show that all of the predicted cyclic lives by the new method are within a factor of ±1.5 to the test ones, which is better than the SWT and PSED methods. It should also be noted that about 74.2% of cyclic lives predicted by the proposed model are within a factor of ±1.25. Obviously, the proposed damage parameter and SWT model have better life prediction capability over the PSED method under an applied mean strain. From the discussions above, clearly, the mean stress effect on the fatigue life has been incorporated into the damage parameters, $\Delta W_p \sigma_{\max}^{1+n'}$ and SWT parameter under mean strain conditions.

The differences between the experimental and calculated LCF life based on the proposed method and SWT model are relatively small, because both of them consider the effect of mean stress. Moreover, the new method considers the creep effect where the SWT model does not. According to the theoretical derivation of $\Delta W_p \sigma_{\max}^{1+n'}$ and comparisons of prediction results by these parameters, it is better to use $\Delta W_p \sigma_{\max}^{1+n'}$ as a damage parameter rather than PSED for drastic hardening or softening conditions. According to the experimental results, the GH4133 exhibits a more obvious cyclic hardening characteristic at 400 °C than 500 °C without mean strain effect, for which the value of cyclic strain hardening exponent n' increases from 0.1107 at 500 °C to 0.1267 at 400 °C. Fatigue tests with an applied mean strain exhibited a more obvious cyclic hardening characteristic than those without an applied mean strain, where the value of n' at 400 °C without mean strain effect increases from 0.1267 to 0.2184 at 400 °C with an applied mean strain. To a great extent, the mechanism of cyclic hardening effect has also been introduced into the proposed damage parameter when using it for HTLCF life prediction.

Recently, Sadananda et al. [35] investigated the role of stress range $\Delta\sigma$ and maximum stress $\sigma_{\max}$ in the fatigue crack nucleation and propagation. Sadananda also made an effort to investigate the relationship between these two parameters to both stress-life and fracture-mechanics descriptions. The two-parameter approach in terms of $\sigma_{\max}$ and $\Delta\sigma$ forms a physical basis for the *S–N* analysis. Most life prediction methodologies, including the simple LDS [1], ignore this two-parameter nature of fatigue, hence it is empirical. It is believed to have a better prediction of fatigue life in terms of a single parameter that incorporates both $\sigma_{\max}$ and $\Delta\sigma$ in each cycle [35], which is consistent with the proposed damage parameter ($\Delta W_p \sigma_{\max}^{1+n'}$) in this paper for life prediction. Similar to the SWT parameter [34], it considers the effects of mean stress on the predicted life through the maximum stress, where $\sigma_{\max} = \sigma_a + \sigma_m$. Therefore, by including factors that influence both fatigue life and creep life, the proposed damage parameter leads to a tighter scatter band compared with SWT and PSED methods. Based on this physical basis for the *S–N* analysis, $\Delta W_p \sigma_{\max}^{1+n'}$ can describe the effects of different time-dependent damage mechanisms on HTLCF life more accurately than the others.

Figs. 3–6 highlight that accurate life prediction results for GH4133 with or without an applied mean strain can be obtained using both the proposed damage parameter and the SWT parameter. Fig. 3–6 also show that the new method presented in this paper has higher HTLCF life prediction precision than SWT and PSED methods. Note that the proposed damage parameter has a higher predictive accuracy for GH4133 tests with an applied mean strain than those tested without mean strain conditions. According to the practical applications of hot section components, a certain degree of mean stress/strain exists in these components. Thus it can be better used for life evaluation of these components in the actual loading with mean stress/strain conditions rather than fully reversed cyclic loading. The applicability of this model in other cases such as different high-temperature structural materials, different loading conditions and uncertainty analysis needs to be further evaluated.

5. Conclusions

By combining the cyclic PSED, Ostergren's model and the frequency separation technique for HTLCF, a generalized energy-based fatigue–creep damage parameter was developed to account for the effects of creep and mean stress/strain on the fatigue life between these two regimes. To verify the feasibility and validity of this proposed model, the LCF test data of GH4133 used for turbine disks were compared with the predicted results. For comparison purposes, the SWT and PSED parameters have also been applied to predict the fatigue lives. Based on the experimental results, some conclusions can be drawn:

(1) A fatigue–creep damage parameter, $\Delta W_p \sigma_{\max}^{1+n'}$, has been proposed for the life prediction of HTLCF undergoing deformation with or without mean stress/strain. It was introduced by modifying the PSED by a creep-compensating parameter and the fatigue exponent was obtained from the Ostergren's model. To reflect the effects of frequency and loading waveforms on HTLCF life, the frequency separation technique was introduced to deal with the new energy-based damage model.
(2) The PSED parameter cannot properly account for the effects of mean stress and creep on LCF life. According to the proposed damage parameter, the mechanism of cyclic hardening was taken into account with the effects of creep and mean strain on the fatigue life. Through the energy-based damage parameter ($\Delta W_p \sigma_{\max}^{1+n'}$), this model can transform the complex correlation between N_f and $\sigma_{\max}$, σ_m, σ_a, $\Delta\varepsilon_p$, strain rate into a rational relation.
(3) Both the SWT and proposed damage parameter yield more satisfactory life prediction results for GH4133 than the PSED one. Compared with the SWT and PSED methods, all of the test data were predicted within a factor of ±2 and nearly 84.9% predicted by the new damage parameter were within a factor of ±1.5. The new damage parameter has a higher precision of life prediction over the SWT and PSED methods, respectively. Under mean strain conditions, it also provides more accurate predictions of GH4133 than those under non-mean strain conditions, where all of the test data was within a factor of ±1.5 to predicted values.
(4) It should be pointed out that the proposed model is applicable for both strain-controlled and stress-controlled tests. Moreover, its application can be extended to thermal and/or thermal–mechanical fatigue conditions, which can provide a new method for both design against fatigue of new structures and estimation of the remaining fatigue life of existing structures.

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References

[1] Zhang JS. High temperature deformation and fracture of materials. Beijing: Science; 2007. p. 386–416 [chapter 22].

[2] Shyam A, Milligan WW. Effects of deformation behavior on fatigue fracture surface morphology in a nickel-base superalloy. Acta Mater 2004;52(6):1503–13.

[3] Kiyak Y, Fedelich B, May T, Pfennig A. Simulation of crack growth under low cycle fatigue at high temperature in a single crystal superalloy. Engng Fract Mech 2008;75(8):2418–43.

[4] Lu YL, Chen LY, Wang GY, et al. Hold time effects on low cycle fatigue behavior of HAYNES 230® superalloy at high temperatures. Mater Sci Engng A 2005;409(1–2):282–91.

[5] Findley KO, Saxena A. Low cycle fatigue in Rene 88DT at 650 °C: Crack nucleation mechanisms and modeling. Metall Mater Trans A 2006;37(5):1469–75.

[6] Sun GQ, Shang DG, Li CS. Time-dependent fatigue damage model under uniaxial and multiaxial loading at elevated temperature. Int J Fatigue 2008;30(10–11):1821–6.

[7] Praveen KVU, Singh V. Effect of heat treatment on Coffin–Manson relationship in LCF of superalloy IN718. Mater Sci Engng A 2008;485(1–2):352–8.

[8] Rho BS, Nam SW, Xie X. The effect of test temperature on the intergranular cracking of Nb-A286 alloy in low cycle fatigue. J Mater Sci 2002;37(1):203–9.

[9] Wan JS, Yue ZF. A low-cycle fatigue life model of nickel-based single crystal superalloys under multiaxial stress state. Mater Sci Engng A 2005;392(1–2):145–9.

[10] Bache MR, Jones JP, Drew GL, et al. Environment and time dependent effects on the fatigue response of an advanced nickel based supperalloy. Int J Fatigue 2009;31(11–12):1719–23.

[11] Coffin LF. International symposium on creep–fatigue interactions. In: Curran KM, editor. MPC.3. New York: ASME; 1976. p. 349–63.

[12] Coffin LF. Fatigue at high temperature-prediction and interpretation. In: James Clayton Memorial Lecture at the University of Sheffield, 1st April 1974. London: Institute of Mechanical Engineers; 1974, p. 9, 74, 188.

[13] Manson SS, Halford GR, Hirschberg MH. Creep–fatigue analysis by strain-range partitioning. In: First symposia on design for elevated temperature environment. ASME; 1971. p. 12–28.

[14] Ostergren WJ. A damage foundation hold time and frequency effects in elevated temperature low cycle fatigue. J Test Evaluat 1967;4:327–39.

[15] Goswami T. Low cycle fatigue life prediction – a new model. Int J Fatigue 1997;19(2):109–15.

[16] Majumdar S, Maiya PS. A damage equation for creep–fatigue interaction. In: the winter annual meeting of ASME. New York: ASME; 1976. p. 323–36.

[17] Payten WM, Dean DW, Snowden KU. A strain energy density method for the prediction of creep–fatigue damage in high temperature components. Mater Sci Engng A 2010;527(7–8):1920–5.

[18] Zhu SP, Huang HZ. A generalized frequency separation-strain energy damage function model for low cycle fatigue–creep life prediction. Fatigue Fract Engng Mater Struct 2010;33(4):227–37.

[19] Zhu SP, Huang HZ, Li Y, He L. A novel viscosity-based model for low cycle fatigue–creep life prediction of high-temperature structures. Int J Damage Mech, in press. http://dx.doi.org/10.1177/1056789511432789.

[20] Viswanathan R. Damage mechanisms and life assessment of high temperature components. ASM Int 1995:132–48 [chapter 4].

[21] Lee KO, Hong SG, Lee SB. A new energy-based damage parameter in life prediction of high-temperature structural materials. Mater Sci Engng A 2008;496(1–2):471–7.

[22] Hong SG, Lee KO, Lim JY, et al. Experimental investigation on the proper fatigue parameter of cyclically non-stabilized materials. Key Engng Mater 2005;297–300:2477–82.

[23] Morrow J. Cyclic plastic strain energy and fatigue of metals. In: Internal friction, damping, and cyclic plasticity. ASTM STP 378; 1965. p. 45–87.

[24] Koh SK. Fatigue damage evaluation of a high pressure tube steel using cyclic strain energy density. Int J Press Vess Pip 2002;79(12):791–8.

[25] Varvani-Farahani A, Kodric T, Ghahramani A. A method of fatigue life prediction in notched and un-notched components. J Mater Process Technol 2005;169(1):94–102.

[26] Chiou YC, Yip MC. An energy-based damage parameter for the life prediction of AISI 304 stainless steel subjected to mean strain. J Chin Inst Engng 2006;29(3):507–17.

[27] Ling Y. Uniaxial true stress–strain after necking. AMP J Technol 1996;5:37–48.

[28] Zhu SP, Huang HZ, Wang ZL. Fatigue life estimation considering damaging and strengthening of low amplitude loads under different load sequences using fuzzy sets approach. Int J Damage Mech 2011;20(6):876–99.

[29] Plumtree A, Abdel-Raouf H. Cyclic stress–strain response and substructure. Int J Fatigue 2001;23(9):799–805.

[30] Lu HP, Lu S, Tang JX. Research on the unsymmetrical cyclic fatigue life and reliable life analysis of a turbine disk. J Mech Strength 2007;29(1):92–6.

[31] Sun JH. The research on mechanism of GH4133B superalloy after fatigue at elevated temperature. Master's dissertation, Department of Materials and Metallurgical Engineering, University of Science and Technology Liaoning; 2008.

[32] Study on the Material Properties of turbine Disk and Case of an Aeroengine Series. Beijing Institute of Aeronautical Materials; 1996.

[33] Wang WG. Research on prediction model for disc LCF life and experiment assessment methodology. PhD dissertation, College of Energy & Power Engineering, Nanjing University of Aeronautics and Astronautics; 2006.

[34] Smith KN, Watson P, Topper TH. A stress–strain function for the fatigue of metals. J Mater 1970;5(4):767–78.

[35] Sadananda K, Sarkar S, Kujawski D, et al. A two-parameter analysis of *S–N* fatigue life using $\Delta\sigma$ and σ_{max}. Int J Fatigue 2009;31(11–12):1648–59.