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A generalized frequency separation–strain energy damage function model for low cycle fatigue–creep life prediction

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ABSTRACT Fatigue–creep interaction is a key factor for the failures of many engineering components and structures under high temperature and cyclic loading. These fatigue–creep life prediction issues are significant in selection, design and safety assessments of those components. Based on the frequency-modified Manson–Coffin equation and Ostergren's model, a new model for high temperature low cycle fatigue (HTLCF), a generalized frequency separation–strain energy damage function model is developed. The approach used in this model to reflect the effects of time-dependent damaging mechanisms on HTLCF life is different from those used in all the earlier models. A new strain energy damage function is used to reduce the difference between the approximate strain energy and real strain energy absorbed during the damage process. This proposed model can describe the effects of different time-dependent damaging mechanisms on HTLCF life more accurately than others. Comparing traditional frequency separation technique (FS) and strain energy frequency-modified approach (SEFS), the proposed model is widely applicable and more precise in predicting the life of fatigue–creep interaction. Experimental data from existing literature are used to demonstrate the feasibility and applicability of the proposed model. A good agreement is found between the predicted results and experimental data.

Keywords frequency separation; high temperature low cycle fatigue–creep; hold time; life prediction; strain energy.

INTRODUCTION

Many engineering components operating in aviation, power-generating, petrochemical and other industries are subject to cyclically varying loads such as low cycle fatigue stress at high temperature and corrosive environment. These components often fail within a limited number of load cycles, e.g. below about 10^4 cycles.[1] This phenomenon is often called high temperature low cycle fatigue (HTLCF). HTLCF is an interactive mechanism of different processes such as time-independent plastic strain, time-dependent creep and environment corrosion, oxidation, and the complex interaction between them. These damage mechanisms make it difficult to predict life for HTLCF and so far there is not a unified model that can make accurate life prediction for fatigue–creep interaction.[1] Even for simple high-temperature fatigue, the life and evolution of damage is influenced by the factors of loading waveforms, frequency, hold-time and environment which can be ignored at room temperature. Fournier[2–4] investigated the effects of tensile/compressive holding periods on the low cycle fatigue lifetime under high temperature conditions, and the effects of tensile/compressive holding periods to fatigue lifetimes were compared by tests considering with the mean stress effect and environmentally-assisted. The fracture behaviour of oxide layers was studied to understand the difference between tensile/compressive holding periods. A new model was then put forward based on the physical mechanisms, which contained the cycles for the initiation and propagation of transgranular fatigue cracks under the interaction of creep, fatigue and oxidation. Therefore, HTLCF is the process of fatigue–creep interaction in a broad sense.[1] Fatigue–creep life prediction issues are very important for selection, design, and safety assessments of these components.[5]

Life prediction for fatigue–creep interaction of components under high temperature has been of interest

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to industry and academia. Dozens of models have been put forward,[4–28] but HTLCF life prediction using these models is still difficult because they involve many constants and parameters. These values are related to damage and difficult to determine, so the applicability of these reported models is restricted. In this paper, based on the frequency-modified Manson–Coffin equation and Ostergren's model,[12] through analysing the impacts of hold-time, strain rate, loading waveforms and the failure process of fatigue–creep interaction, a new model of HTLCF-C life prediction is proposed after providing a new definition of strain energy damage function. It is modified by the frequency separation technique and takes the effects of time durations into consideration. The approach used in proposed model to reflect the effects of time-dependent damaging mechanisms on HTLCF-C life is different from those used in reported models. By introducing the tension and compression hold-time, strain rate and frequency, this model can describe the effects of different time-dependent damaging mechanisms on the HTLCF-C life more accurately than others. To verify this method, the fatigue–creep life is assessed using experimental data in existing literatures. The predicted lives are compared with tested ones and a good agreement is achieved between them.

A REVIEW OF EXISTING MODELS

Abundant life prediction methods have been developed for fatigue–creep interaction. The most well-known ones based on one-dimensional condition are as follows: linear damage rule (LDR),[6,7] frequency-modified Manson–Coffin equation,[8] frequency separation technique (FS),[9] strain range partition (SRP),[10] strain energy partition (SEP),[11] frequency-modified damage function model (FMDF),[12] time-modified energy model (TME),[13] ductility exhaustion approaches (DE),[14–16] generic equations (GE),[17,18] damage rate approach (DRA),[19] and damage parameter approach (DPA).[20] Recently, Jeong[21] developed a new model called stress relaxation range (SRR), in which the SRR is used as main control parameter. By this method Manson–Coffin curves under different conditions can be normalized to a master curve. This method is valid for fatigue–creep interaction life prediction under strain-controlled condition with long tensile hold-time. Nam[22–24] proposed a model for the fatigue–creep life prediction based on a new damage function in terms of nucleation and growth of grain boundary cavities. This method has a good accuracy for life prediction, but it is limited to the usage for evaluating the strain-controlled fatigue–creep interaction. Under stress-controlled mode, Fan *et al*.[25] put forward a mean strain rate model based on the ductility exhaustion theory and effective stress concept. In this method, the mean strain rate was used as main factor associated with the fracture life. They performed the assessment of 1.25Cr0.5Mo steel with this model. The lives predicted reached a good agreement with tested ones when ductility exhaustion was the dominant mechanism. Liang[26] presented a new model for entire range of low cycle fatigue which covers the extremely low cycle fatigue regime (less than 100 cycles). Liang proposed a new expression by combining the exponential damage function and power law Manson–Coffin relationship to describe the nonlinearity and provided a smooth transition between them. The fatigue–creep interaction in the two and three-dimensional conditions can be generally expanded and described on the basis of one-dimensional theory through introducing corresponding parameters.

The existing models reviewed above cannot accurately describe the effects of different loading waveforms, for example, frequency-modified Manson–Coffin equation,[8] FS method[9] and FMDF model.[12] Many of the existing models are not favourable under finite test data and some certain experimental conditions, for example, the DE[14–16] and SRR method.[21] These methods obviously restrict their applications for life prediction under stress control. It is also difficult to apply the SRP[10] and SEP[11] methods to stress-controlled conditions due to unstable and not-close hysteresis loops induced by cyclic creep deformations. To reflect the effects of different loading waveforms accurately, get higher utilization of test data, and expand the application to stress-controlled tests, we have developed a new model based on the discussions of following two models. Further improvements on high temperature low cycle fatigue–creep (HTLCF-C) life prediction models are expected.

Frequency-modified Manson–Coffin equation

Manson–Coffin law was proposed more than 50 years ago by two authors independently.[27,28] So far this method has been widely used in life prediction for low cycle fatigue. Manson–Coffin law uses strain amplitudes to describe fatigue life under low temperature (the temperature with no creep damage, i.e. under 300 degree Celsius for carbon steel and 400 degree Celsius for alloy steel). The total strain range can be separated into the elastic and plastic strain ranges and written as.[29]

$$\Delta\varepsilon_t/2 = \Delta\varepsilon_e/2 + \Delta\varepsilon_p/2, \tag{1}$$

where $\Delta\varepsilon_t$, $\Delta\varepsilon_e$ and $\Delta\varepsilon_p$ are the total strain range, elastic and plastic strain range. Most tests showed that the relationship between total strain range and life for low cycle fatigue can be expressed by the Manson–Coffin equation.[29] The first part of Eq. (1) can be further expressed in terms of Basquin equation, and the second term can be replaced by the Coffin–Manson relation,[29] then,

$$\Delta\varepsilon_t/2 = \Delta\varepsilon_e/2 + \Delta\varepsilon_p/2 = \frac{\sigma_f'}{E}(2N_f)^b + \varepsilon_f'(2N_f)^c, \quad (2)$$

where σ_f' and ε_f' are the fatigue strength coefficient and fatigue ductility, c and b are the fatigue ductility exponent and fatigue strength exponent, E is the Young's modulus, N_f is the number of cycles to failure.

Equation (2) is generally useful for low cycle fatigue under room temperature. Under high temperature (i.e. above 300 degree Celsius for carbon steel and 400 degree Celsius for alloy steel), Coffin[8] proposed the frequency-modified Manson–Coffin equation to account for the environmental and other time-dependent effects. This approach considers the effects of frequency for the induced creep damage at HTLCF, which is only applicable for the continuous fatigue with equal tensile and compressive frequency. The expression of the frequency-modified Manson–Coffin equation is as below

$$\Delta\varepsilon_t = \Delta\varepsilon_e + \Delta\varepsilon_p = C_1(N_f \upsilon^{k_1-1})^{-\beta_1} + C_2(N_f \upsilon^{k_2-1})^{-\beta_2}, \quad (3)$$

where C_1, C_2, β_1 and β_2 are material parameters, υ is the frequency factor, and k_1, k_2 are material constants related to the environment conditions, such as temperature. Equation (3) has been used for life prediction of various steels at high temperature, and the lives were predicted within a factor of 2–4.[30]

Therefore, the relationship between frequency-modified plastic strain range and fatigue–creep life from Eq. (3) can be expressed by the following equation

$$(N_f \upsilon^{k_2-1})^{\beta_2}\Delta\varepsilon_p = C_2. \quad (4)$$

Equation (4) reduces to the Manson–Coffin law in low temperature fatigue when the effect of frequency vanished at $k_2 = 1$.

The selection of υ depends on the properties of materials. For the creep with no sensitivity to loading waveforms, the relationship between υ and the total time period T_0 (see Fig. 1) can be described by[30]

$$\frac{1}{\upsilon} = T_0 + T_h, \quad (5)$$

where T_0 and T_h represent the total time period and total hold-time with $T_h = T_{du} + T_{dl}$.

The time-dependent damage induced by creep and environment corrosion is often affected by the loading waveform. This damage was caused by the tension waveform when the tensile hold-time T_{du} is longer than the compressive hold-time T_{dl}. To account for the waveform effects,

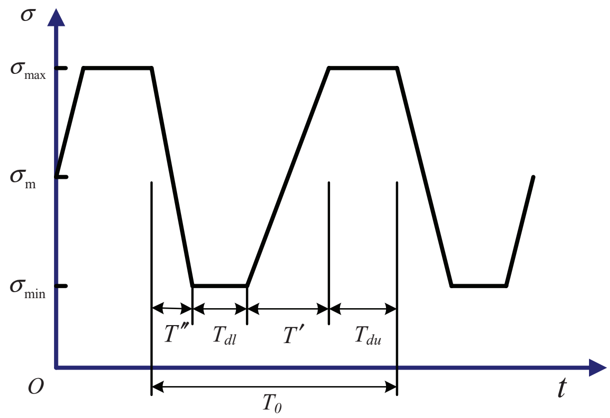

Fig. 1 The loading waveform.

let the υ

$$\begin{cases} \dfrac{1}{\upsilon} = T_0 + T_{du} - T_{dl} & T_{du} > T_{dl} \\ \dfrac{1}{\upsilon} = T_0 & T_{du} \le T_{dl} \end{cases}, \quad (6)$$

where T_{du} and T_{dl} represent the tensile hold-time and compressive hold-time in one loading cycle when $\sigma_{max} > 0$ and $\sigma_{min} < 0$, as shown in Fig. 1. T_{dl} is tensile hold-time when $\sigma_{min} > 0$.

However, it has been well recognized that some predictive errors are found when adopting the method outlined above. The Manson–Coffin equation, widely used in engineering, gives prediction results with great error due to some shortcomings as follows:[31] (1) For some metal such as aluminium alloys, the elastic and plastic curves needed by Manson–Coffin equation cannot be constructed well by linear fitting in log-log scale coordinate system, but the curves tend to bend down instead of linear fitting, which is inconsistent with the reality.[1,32] (2) According to the strain–life curve obtained by Manson–Coffin equation, $\Delta\varepsilon_e$ and $\Delta\varepsilon_p$ can be obtained from the stable hysteresis loops or at half-life, but the hysteresis loops often cannot achieve stability at half-life or even up to failure. (3) Within the group strain-controlled texts, we cannot get a group of experimental data under the same $\Delta\varepsilon_e$ or $\Delta\varepsilon_p$. Since that $\Delta\varepsilon_e$ and $\Delta\varepsilon_p$ changed under the same $\Delta\varepsilon_t$ for different specimen, and this makes the obtaining of probability–strain–life curve difficult.

The Manson–Coffin equation has been widely used for life prediction at room temperature, but it is not as accurate at high temperature. The calculation error of elastic strain range caused by the determinate error of Young's modulus is evident in tests of HTLCF. This often leads to an instance that the value of plastic strain range is so small and sometimes even negative.[31,33] The frequency-modified Manson–Coffin equation cannot accurately describe the effects of hold-time, loading waveforms, and different tension/compression strain rates, which may cause serious errors where the effect of creep is dominating.[26,34] Based on these observations, we propose a new model to describe the effects of fatigue–creep interaction.

Strain energy damage function model

Considering that fatigue crack grows only in the tension stage, Ostergren proposed the strain energy damage function model.[12,30] This model assumes that only tensile inelastic strain energy can induce the crack opening and propagation. The strain energy damage function ΔW_t can be expressed approximately by multiplication of the inelastic strain range $\Delta\varepsilon_{in}$ and maximum of tension stress $\sigma_{\max}$, i.e.

$$\Delta W_t = \Delta\varepsilon_{in}\sigma_{\max}, \tag{7}$$

where ΔW_t is the strain energy, $\sigma_{\max}$ is the maximum of tension stress and $\Delta\varepsilon_{in}$ is the inelastic strain range of stable hysteresis loop or half-life, which can be replaced by the plastic strain range $\Delta\varepsilon_p$ under pure fatigue mode. The relationship between strain energy and fatigue life can be expressed by the power exponent function, i.e.

$$\Delta W_t N_f^{\alpha} = C, \tag{8}$$

where N_f is the low cycle fatigue life, and α and C are material constants.

Based on effect of environment at high temperature reflected by the frequency factor υ, the frequency-modified strain energy damage function is proposed for HTLCF life prediction,[30] i.e.

$$N_f = C_3(\Delta\varepsilon_{in}\sigma_{\max})^{\beta_3}\upsilon^m, \tag{9}$$

where υ is the frequency factor, and C_3, β_3 and m are material constants. The selection of υ depends on the sensibility of materials to different waveforms, some suggestions were given by Ostergren.[30,35,36]

It is, however, recognized that some predictive errors are found when adopting the model outlined above. In application of strain energy damage function model, the reasons for prediction error are as follows:[31] (1) The plastic strain range is so small even negative calculated from test results of high temperature alloys at HTLCF. Substituting the calculated plastic strain range into Eq. (7), the strain energy is also small and even negative. The points with those small plastic strain ranges will be considered invalid data points when fitting the strain energy–fatigue life curve, resulting in a great waste of test data. (2) The fatigue–creep life is controlled by the real strain energy. Mean stress is the main factor influencing life rather than the mean strain. When considering the effect of mean stress, the difference between the defined strain energy in Eq. (7) and real strain energy absorbed during the damage process is so evident for high temperature alloys at HTLCF by the existence of mean stress.

This model uses inelastic strain energy as the damage function, and the effect of mean stress on fatigue life has been taken into consideration. However, the model still cannot accurately describe the effect of loading waveforms at high temperature, because the selection of frequency factor υ is somewhat subjective.[13,37]

In summary, the fatigue–creep life prediction depends not only upon the strain range, strain rate, frequency, hold-time and loading waveforms, but also the environment and material properties. The function of fatigue–creep interaction should not be limited to just the plastic strain range, it is also closely linked to the hold-time, properties of deformation (including plastic deformations and creep deformations) and environmental factors. Through comparing with the advantages and disadvantages of frequency-modified Manson–Coffin equation and strain energy damage function model, a new model of HTLCF-C life prediction is proposed based on the investigation of large amounts of experimental data. In this paper, we put an effort into getting a simple expression, better life prediction capability and applicability of the new model, which has considered different hold-time and strain rate through modification by frequency separation.

THE PROPOSED MODEL FOR LIFE PREDICTION OF HTLCF-C

In Manson–Coffin equation, the plastic strain range $\Delta\varepsilon_p$ is one kind of damage function, which is used to describe the fatigue life. According to the interaction mechanisms of failure at high temperature, the damage function should consider not only the effect of plastic strain range, but also frequency, hold-time and temperature, etc. The main idea of Ostergren's model is that only the inelastic strain energy can induce the crack opening and propagation, the strain energy controls the low cycle fatigue damage evolution and was expressed by the damage function approximately.[30] Based on the way of frequency-modified Manson–Coffin equation used to describe the effects of hold-time, loading waveforms and main idea of Ostergren's model, a new model is proposed.

The effect of compressive hold-time to the life at HTLCF can usually be ignored, as creep damage is sensitive to the tensile hold-time instead of compressive hold-time.[1] The key factor determining the failure of hot section components under certain environment is the fatigue–creep interaction. To account for the interaction of fatigue and time-dependent damage caused by creep,

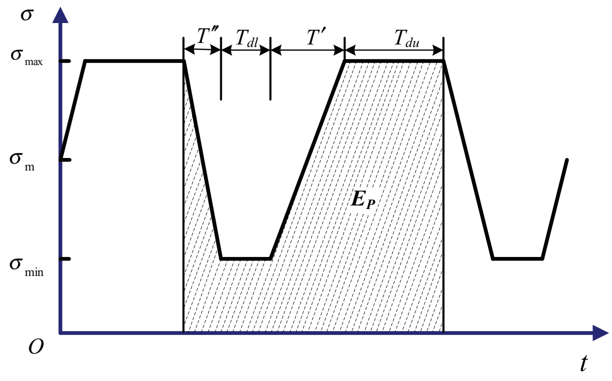

Fig. 2 The waveform used in fatigue–creep interaction loading conditions.[1]

we use the characteristic of 'between place cut in' load diagram to analyse the conditions of most alloys under high temperature, pressure and cyclic loading. The load diagram of fatigue–creep interaction is plotted in Fig. 2.

In the stress cycle showed in Fig. 2, σ_m is the mean stress, $\sigma_{\max}$ and $\sigma_{\min}$ are the maximum and minimum stress respectively, where $\sigma_m = (\sigma_{\max} + \sigma_{\min})/2$, T_{du}, T_{dl}, T' and T'' represent the tensile hold-time, compressive hold-time, tension-going time and compression-going time in one loading cycle when $\sigma_{\max} > 0$ and $\sigma_{\min} < 0$, T is the period time not including the hold time, where $T = T' + T''$. T_{dl} becomes the tensile hold-time in the situation of $\sigma_{\min} > 0$.

A certain quantity of energy is dissipated along with the damage process of material. The more damage that is cumulated, the more energy is dissipated. According to the one to one correspondence relation between them, we can use the dissipated energy of material to measure the damage of material. This idea has been used to describe ductile and low cycle fatigue.[13,16,18,37] Based on the above description, we assume that the energy parameter accumulated under fatigue–creep interaction can be described by the stress area under loading waveforms, and above the zero-stress line (this means only tension stress can cause damage). The energy parameter E_p per cycle with the shadow in Fig. 2 can be calculated by the function below

$$E_p = T_{du}\sigma_{\max} + (T_{dl} + T)\sigma_{\min} H(\sigma_{\min}) + \frac{T}{2} f(\sigma_{\max}, \sigma_{\min}) \tag{10}$$

and

$$f(\sigma_{\max}, \sigma_{\min}) = \frac{\sigma_{\max}^2}{\sigma_{\max} - \sigma_{\min} H(-\sigma_{\min})} - \sigma_{\min} H(\sigma_{\min}), \tag{11}$$

where $H(\sigma_{\min})$ is the unit step function of $\sigma_{\min}$. According to the assumption that only tension stress can induce fatigue–creep damage while compression stress cannot, we can define $H(\sigma_{\min})$ as follows:

$$H(\sigma_{\min}) = \begin{cases} 1, & \sigma_{\min} > 0 \\ 0, & \sigma_{\min} \le 0 \end{cases}, \tag{12}$$

where $f(\sigma_{\max}, \sigma_{\min})$ is termed 'stress conversion function' determined by the maximum and minimum stress and material properties. In principle, the real form of $f(\sigma_{\max}, \sigma_{\min})$ should be obtained from experiments, which is very difficult to put into practice.

However, based on the definition of E_p in Eq. (10) and Fig. 2, the physical meaning of $\frac{T}{2} f(\sigma_{\max}, \sigma_{\min})$ (corresponds to the third part on the right side of E_p in Eq. (10)) is the energy parameter accumulated under tension-going time and compression-going time in one loading cycle. In addition, considering the following facts from the tests: (1) the boundary conditions obtained from the waveform itself in Fig. 2 based on its physical meaning, i.e. $\frac{T}{2} f(\sigma_{\max}, \sigma_{\min}) \propto \Delta\sigma$ for $\sigma_{\min} > 0$ and $\frac{T}{2} f(\sigma_{\max}, \sigma_{\min}) \propto \frac{\sigma_{\max}^2}{\Delta\sigma}$ for $\sigma_{\min} \le 0$; (2) to a great extent, the energy parameter E_p increases with increasing in maximum stress; (3) the assumption that fatigue–creep damage can only be induced by tension stresses rather than compression stresses, we suggest a simple form of $f(\sigma_{\max}, \sigma_{\min})$ in Eq. (11) which can be derived from the above.

Introducing Eq. (12) into Eq. (11), we can get

$$f(\sigma_{\max}, \sigma_{\min}) = \begin{cases} \Delta\sigma, & \sigma_{\min} > 0 \\ \dfrac{\sigma_{\max}^2}{\Delta\sigma}, & \sigma_{\min} \le 0 \end{cases}, \tag{13}$$

where $\Delta\sigma$ is the stress range with $\Delta\sigma = \sigma_{\max} - \sigma_{\min}$.

Based on the main idea of strain energy damage function model and analysis of large amounts of test data, in order to reduce the difference between the approximate and real strain energy absorbed during the process of damage and get a higher precision, a new definition of strain energy damage function is presented in terms of following equation

$$\Delta W_t = \Delta\varepsilon_{in}(E_p)^{\phi}, \tag{14}$$

where ΔW_t is the strain energy damage function, and ϕ is the stress damage exponent related to environment conditions.

Similar to Ostergren's model, the strain energy and fatigue life follow the relationship of power law index, substituting Eq. (14) into Eq. (8), i.e.

$$\Delta\varepsilon_{in}(E_p)^{\phi} N_f^{\alpha} = C. \tag{15}$$

At high temperature, the shape and size of the hysteresis loop are influenced by the cyclic frequency and loading waveform. For the reason of time-dependent damaging mechanisms such as creep and environment corrosion, it is suggested that a frequency factor υ should be introduced into the formula to modify Eq. (15), which results in

$$N_f = C_3(\Delta\varepsilon_{in}(E_p)^{\phi})^{\beta_3}\upsilon^m. \tag{16}$$

To reflect the effects of frequency and loading waveform to the life prediction of HTLCF, we introduce the frequency separation technique to deal with Eq. (16) as like Coffin.[9] When the strain rate of tension is equal to that of compression, which is known as either balanced waveform or unbalanced waveform, the relationship between the stress range and plastic strain range can be written as

$$\Delta\sigma = A(\Delta\varepsilon_p)^r\upsilon^q, \tag{17}$$

where r and q are material constants. For the unbalanced waveforms, we can get two different stress ranges from Eq. (17). We can assume that real stress range is the average of two different stress ranges, namely stress averaging. The frequency of unbalance waveform can be described by the tension-stage frequency υ_t and the compression-stage frequency υ_c, namely frequency averaging. The results of stress averaging and frequency averaging are different except at $\upsilon_t = \upsilon_c$. Combining with the stress range by stress averaging and Eq. (16), we can get the equivalent plastic strain range $\Delta\varepsilon'_p$

$$\Delta\varepsilon'_p = \Delta\varepsilon_p\left[\frac{(\upsilon_c/\upsilon_t)^q+1}{2}\right]^{1/r}. \tag{18}$$

Then, substituting Eq. (18) and $\upsilon_t/2$ into Eq. (16), i.e.

$$N_f = C_3\left(\Delta\varepsilon_{in}(E_p)^{\phi}\left[\frac{(\upsilon_c/\upsilon_t)^q+1}{2}\right]^{1/r}\right)^{\beta_3}\left(\frac{\upsilon_t}{2}\right)^m. \tag{19}$$

According to Eq. (19), the life predicted depends on the degree of unbalance υ_c/υ_t and tension half-period time $1/\upsilon_t$, this equation can be simplified as follows:

$$N_f = C_3(\Delta\varepsilon_{in}(E_p)^{\phi})^{\beta_3}\left[\frac{\upsilon_c}{\upsilon_t}\right]^{\varphi}(\upsilon_t)^m. \tag{20}$$

Substituting Eq. (10) into Eq. (20), we can easily get

$$N_f = C_3\left(\Delta\varepsilon_{in}\left(T_{du}\sigma_{\max} + (T_{dl}+T)\sigma_{\min}H(\sigma_{\min}) + \frac{T}{2}f(\sigma_{\max},\sigma_{\min})\right)^{\phi}\right)^{\beta_3}\left[\frac{\upsilon_c}{\upsilon_t}\right]^{\varphi}(\upsilon_t)^m, \tag{21}$$

where C_3, β_3 and m are material constants which can be obtained from the experimental data under balance waveform, φ is a material constant which can be obtained from the unbalance waveform test data, and υ_t is the hypothetic frequency of tension half-period. The hypothetic frequency of tension half-period is the frequency of balance waveform with the same tension strain rate. Combining with Fig. 2, we can have $\upsilon_t = \frac{1}{2T'}$, in a similar way, the hypothetic frequency of compression half-period $\upsilon_c = \frac{1}{2T''}$. Equation (21) is the formula of the new model, which has taken it into consideration that effects of different hold-time, frequency, strain rate, mean stress and waveform to life prediction for HTLCF-C.

Considering that without hold-time in the loading waveform, viz. $T_{du} = T_{dl} = 0$, it is noted that the model reduces to the expression similar to Ostergren's model when $\phi = \frac{1}{2}$. So, it will be called a generalized frequency separation–strain energy damage function model. The expression of low cycle fatigue life without hold-time can be written as

$$N_f = \begin{cases} C_3\left(\Delta\varepsilon_{in}\sigma_{\max}\sqrt{T/2\Delta\sigma}\right)^{\beta_3}\left[\dfrac{\upsilon_c}{\upsilon_t}\right]^{\varphi}(\upsilon_t)^m, & \sigma_{\min}\le 0 \\ C_3\left(\Delta\varepsilon_{in}\sqrt{T\sigma_{\mathrm{m}}}\right)^{\beta_3}\left[\dfrac{\upsilon_c}{\upsilon_t}\right]^{\varphi}(\upsilon_t)^m, & \sigma_{\min}>0 \end{cases}. \tag{22}$$

The relationship between υ and υ_t, υ_c can be expressed by the equation

$$\upsilon = \frac{1}{T'+T''} = \frac{2(\upsilon_c/\upsilon_t)}{\upsilon_c/\upsilon_t+1}. \tag{23}$$

By the combination of Eqs (16), (21) and (23), the effect of frequency to fatigue–creep life can be characterized by the degree of unbalance υ_c/υ_t. To a certain extent, it reflects the mechanism of frequency affect to low cycle fatigue life at high temperature.

EVALUATION OF THE MODEL

Up to now, many earlier approaches are mainly suitable for life predictions under strain control, and few models can be applied to stress-controlled tests.[25] The application of the SEP and SRP methods to stress-controlled cyclic conditions is also difficult, because the unstable and not-close hysteresis loops were induced by cyclic creep deformations. To verify the feasibility and prediction effect of the new model developed in this paper, fatigue–creep tests were carried out under stress-controlled at different temperature.

We obtained HTLCF data from the National Technology Research Centre on PVP Safety Engineering, Hefei, China. Details of mechanical properties of the materials, test conditions, and strain–life data are reported in references.[25,38,39] Readers are directed to original references for these data.[40–42] We will discuss the assessments of data with the new model below.

In this section, applicability of the proposed method will be assessed with HTLCF data. The main factor influencing fatigue life is σ_a, and the main factor influencing creep life is σ_{m}. Therefore, fatigue–creep interaction behaviour will be changed with σ_a and σ_{m}. Under the

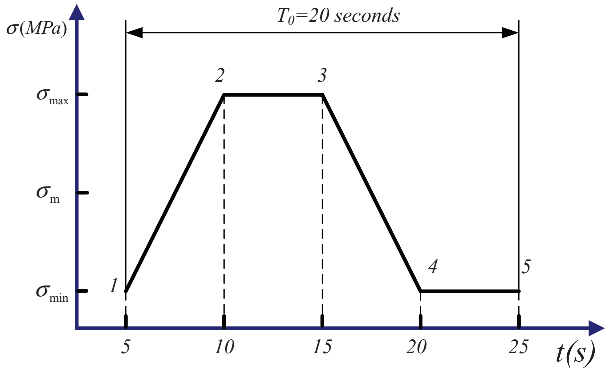

Fig. 3 Trapezium waveform in fatigue–creep interaction loading conditions.

different stress ratio and σ_m, details of the test are as follows. The data comprised two temperatures: 540 degree Celsius and 520 degree Celsius, stress amplitudes: from 25 MPa to 190 MPa, four certain maximum stresses: 200 MPa, 210 MPa, 220 MPa and 230 MPa, one certain frequency and mode of waveform: the trapezoid waveform in 0.05Hz. The loading waveform with a hold-time of 5s duration at σ_{max} and σ_{min} respectively is shown in Fig. 3. Test parameters and results are given in Tables 1 and 2.

The strains of point 1 ∼ 5 in Fig. 3 were obtained by the extensometer, and the inelastic strain range per cycle can be expressed as

$$\Delta\varepsilon_{in} = \varepsilon_5 - \varepsilon_1, \tag{24}$$

where ε_1 is the initial strain in a cycle, and ε_5 is the final strain in a cycle.

According to the experimental conditions and data, the hold-time and strain rate of tension is equal to that of compression and keeping fixed. So $(\upsilon_t)^m$ can be merged into the material constant C_3, Eq. (21) can be simplified as follows

$$N_f = C_3\left(\Delta\varepsilon_{in}\left(T_{du}\sigma_{max} + (T_{dl} + T)\sigma_{min}H(\sigma_{min}) + \frac{T}{2}f(\sigma_{max},\sigma_{min})\right)^{\phi}\right)^{\beta_3}. \tag{25}$$

The hold-time of tension and compression shown in Fig. 3 was 5s, viz. $T_{du} = T_{dl} = 5s$. Fatigue–creep life predictions were conducted by Eq. (25). Combining the experimental data from Table 1 and the loading waveform in Fig. 3, for different maximum stress and stress ratio, the fitted life prediction model for *1.25Cr0.5Mo* steel at 540 degree

Table 1 Experimental parameters and life predictions of new model, FS, SEFS methods at 540 °C for 1.25Cr0.5Mo steel

Maximum stress of tests	Stress range $\Delta\sigma$ ($\sigma_{min} \sim \sigma_{max}$/Mpa)	Inelastic strain range of half-life $\Delta\varepsilon_{in}$ (%)	Cyclic life tested N_{ft} (cycle)	N_{fp} predicted by the new method (cycle)	N_{fp} predicted by FS method (cycle)	N_{fp} predicted by SEFS method (cycle)
Max Stress = 200 MPa	50 (150 ∼ 200)	0.002700095	1952	2173	3321	3503
	150 (50 ∼ 200)	0.001855671	3688	3458	4764	4741
	300 (−100 ∼ 200)	0.012053573	908	908	787	1047
	250 (−50 ∼ 200)	0.004487762	2914	2086	2036	2325
	350 (−150 ∼ 200)	0.085710149	169	177	119	215
	350 (−150 ∼ 200)	0.092444568	198	167	110	204
	350 (−150 ∼ 200)	0.06484573	264	225	156	269
	350 (−150 ∼ 200)	0.042463609	255	321	234	379
Max Stress = 210 MPa	210 (0 ∼ 210)	0.010071726	1177	1002	935	792
	210 (0 ∼ 210)	0.005951911	1143	1563	1552	1174
	210 (0 ∼ 210)	0.003694547	2280	2340	2456	1679
	260 (−50 ∼ 210)	0.024450503	439	485	398	407
	260 (−50 ∼ 210)	0.016635129	515	672	577	544
	160 (50 ∼ 210)	0.005328134	1169	1392	1726	1276
	110 (100 ∼ 210)	0.005969052	1227	1175	1547	1172
	360 (−150 ∼ 210)	0.097583211	142	155	105	145
	60 (150 ∼ 210)	0.007175124	735	1183	1296	1021
	310 (−100 ∼ 210)	0.048775574	274	234	205	243
Max Stress = 220 MPa	220 (0 ∼ 220)	0.013327132	566	775	714	488
	170 (50 ∼ 220)	0.025915479	268	359	377	294
	320 (−100 ∼ 220)	0.091740718	101	157	112	112
	120 (100 ∼ 220)	0.023230701	308	366	418	319
	70 (150 ∼ 220)	0.024522025	261	328	397	307

Table 2 Experimental parameters and life predictions of new model, FS, SEFS method at 520 °C for 1.25Cr0.5Mo steel

Maximum stress of tests	Stress range $\Delta\sigma$ ($\sigma_{\min} \sim \sigma_{\max}$/Mpa)	Inelastic strain range of half-life $\Delta\varepsilon_{in}$ (%)	Cyclic life tested N_{ft} (cycle)	N_{fp} predicted by the new method (cycle)	N_{fp} predicted by FS method (cycle)	N_{fp} predicted by SEFS method (cycle)
Max Stress = 220 MPa	370 (−150 ∼ 220)	0.103501	129	143	99	132
	320 (−100 ∼ 220)	0.036789	400	352	269	381
	270 (−50 ∼ 220)	0.008326	1106	1278	1123	1750
	120 (100 ∼ 220)	0.003962	3753	1985	2296	3748
	170 (50 ∼ 220)	0.003161	3397	2542	2853	4725
	70 (150 ∼ 220)	0.005256	2745	1486	1749	2805
Max Stress = 230 MPa	280 (−50 ∼ 230)	0.022058	376	536	440	233
	230 (0 ∼ 230)	0.006258	1218	1590	1479	1201
	180 (50 ∼ 230)	0.005293	1377	1600	1737	1493
	130 (100 ∼ 230)	0.010084	755	867	934	645
	80 (150 ∼ 230)	0.007469	735	1083	1247	954
	380 (−150 ∼ 230)	–	–	–	–	–

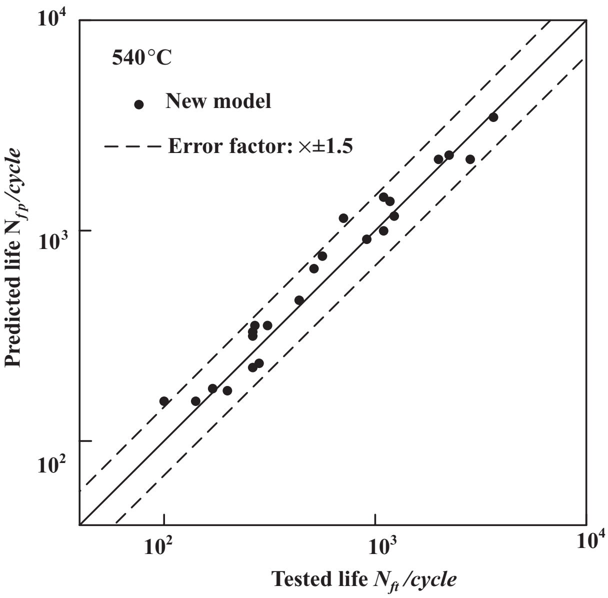

Fig. 4 Predicted life vs. tested life at 540 °C.

Celsius is as follows:

$$N_f = 2867.72282\left(\Delta\varepsilon_{in}\left(5\sigma_{\max} + 15\sigma_{\min}H(\sigma_{\min}) + \frac{5}{2}f(\sigma_{\max}, \sigma_{\min})\right)^{0.49325}\right)^{-0.84671}. \tag{26}$$

A comparison between experimental results and the proposed predictions is shown in Fig. 4. The broken line in the graph means ±1.5 factor indictor, and 21 out of 23 cyclic lives are predicted within a factor of ±1.5 and the predicted results are in good agreement with these observed ones.

To reflect the capability of this new model, the test data are also assessed by the FS and SEFS methods, respectively. The FS and SEFS methods can also be applied to stress-controlled tests at HTLCF. Assuming the hold time of tension is equal to that of compression, these methods can be simplified as follows:

$$N_f = C_4\Delta\varepsilon_{in}^{-\alpha}, \tag{27}$$

$$N_f = C_5(\sigma_{\max}\Delta\varepsilon_{in})^{-\beta}, \tag{28}$$

where C_4, C_5, α and β are material constants, $\Delta\varepsilon_{in}$ is the inelastic strain range of stable stage or half-life.

When using the FS method (Eq. (27)), all of the test data are fitted into one curve. When using the SEFS method (Eq. (28)), the test data with different temperature and $\sigma_{\max}$ are fitted, respectively. The inelastic strain range $\Delta\varepsilon_{in}$ measured and cyclic lives predicted by three methods are listed in Table 1. Comparisons between test and prediction by the FS, SEFS and new model are shown in Fig. 5. Two evaluating parameters of life assessments are used: scatter band and standard deviation. The former describes the scatter of the predicted extreme point and the degree of deviation between test and prediction, and the latter shows the degree of data points that deviate from the average. The results show that all the predicted cyclic lives are in a factor of ±2 to the test ones, and nearly 91.3% of the test data are predicted by the proposed model within a factor of ±1.5, which is better than the 72.7% and 80.6% of that predicted by the FS and SEFS methods,[25] respectively. From Table 1, it should be noted that about 70% of cyclic lives predicted by the proposed model are in a factor of ±1.25. Comparing with the scatter band and standard deviation of those methods, results indicate that the proposed model has a better predictability than the others.

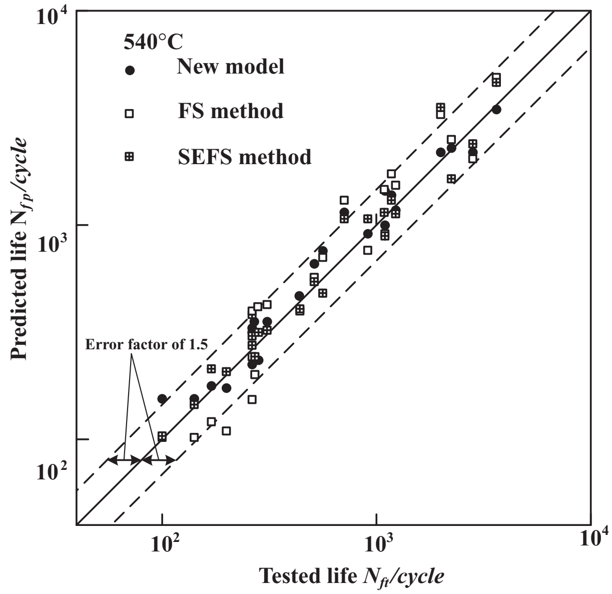

Fig. 5 Comparison between lives predicted by new model, FS, SEFS methods at 540 °C.

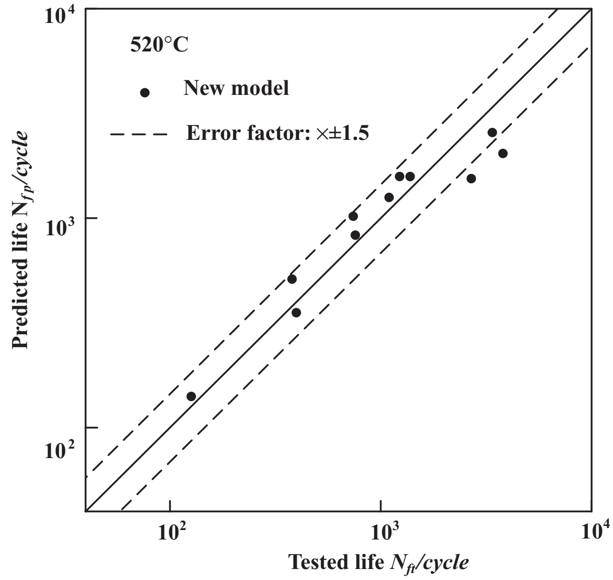

Fig. 6 Predicted life vs. tested life at 520 °C.

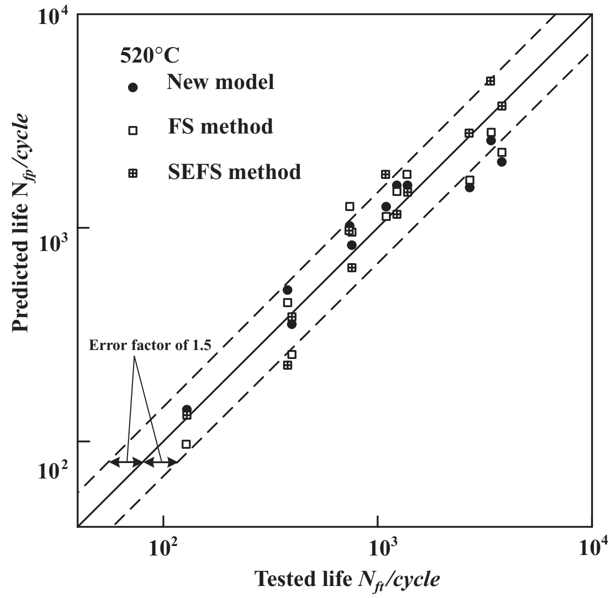

Fig. 7 Comparison between lives predicted by new model, FS, SEFS methods at 520 °C.

Similarly, the fitted life prediction model for *1.25Cr0.5Mo* steel at 520 degree Celsius can be written as follows:

$$N_f = 156.98889\left(\Delta\varepsilon_{in}\left(5\sigma_{\max} + 15\sigma_{\min}H(\sigma_{\min}) + \frac{5}{2}f(\sigma_{\max},\sigma_{\min})\right)^{0.33086}\right)^{-0.8765}. \tag{29}$$

From Eqs (26) and (29), material constants ϕ and β_3 are affected by temperature. From Table 2, all the predicted results fall into a range within a scatter band of ± 2 and most of test data are predicted within a factor of ± 1.5 which is shown in Fig. 6, a good agreement is achieved between them. Lives predicted by the new model, FS and SEFS methods at 520 degree Celsius are compared in Fig. 7.

It can be concluded from Figs 5–7 that the new model presented in this paper has higher precision of life prediction under HTLCF-C except individual abnormal points. Besides, to examine the application of this model in other cases such as with stress relaxation under the strain-controlled tests, different materials, strain rate and hold-time will be further evaluated.

CONCLUSIONS

Based on the investigation of frequency modification Manson–Coffin equation and Ostergren's model for low cycle fatigue, a new model for HTLCF-C life prediction, a generalized frequency separation–strain energy damage function model, is presented on the basis of strain energy damage function. To check the feasibility and validity of this proposed model, the fatigue–creep interaction tests have been performed with data from literatures. Some conclusions can be drawn from the present investigation.

1 Comparing with the mean strain rate model,[25] the proposed model involves not only the effects of maximum stress, stress range and mean stress, but also the effects

of hold-time, tension/compression strain rate and waveform on fatigue–creep behaviour.

2 From Eqs (21) and (22), the lifetime of fatigue–creep interaction can be assessed for the failure of hot section components. For the low cycle fatigue without hold-time, fatigue–creep interaction only occurs on condition that the frequency is sufficiently low, namely the period time T is sufficiently long that the time-dependent creep cannot be restrained to pure fatigue, the life is mainly affected by the maximum stress and stress range at minimum stress $\sigma_{\min} \leq 0$ and mean stress at minimum stress $\sigma_{\min} > 0$.

3 The proposed model is applicable for both the strain-controlled tests and stress-controlled tests. Moreover, the expression of this model needs fewer life prediction parameters for application. It can also be applied under finite test data with higher utilization of test data.

4 By employing and optimizing this model, the fatigue–creep life is assessed with experimental data from existing literatures. With the same predictability of the mean strain rate model,[25] nearly 91% of the test data are predicted within a factor of ± 1.5 by the new model, which has higher precision of life prediction than FS and SEFS methods.

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