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Fatigue Life Estimation of an Aircaft Engine Under Different Load Spectrums

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Abstract. Aircraft engine components are subjected to variable amplitude load conditions usually they tend to experience fatigue damage. Many fatigue damage accumulation theories have been put forward to predict the fatigue lives of structure components. Under different load spectra, the aircraft engine has different working life. In this paper, based on the linear damage rule, the fatigue life of an aircraft engine was estimated by considering the load spectrum difference factor. According to one aircraft engine used for different load spectrums, the relationship between the load spectrum and the life is discussed. Moreover, the fatigue life of this engine used in A-type bomber is estimated based on the different load spectrum of the B-type bomber. A good agreement is found between the estimated results and real life data.

Keywords. Fatigue life, linear damage rule, aircraft engine, load spectrum.

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Nomenclature

s_L stress level L

n_L corresponding numbers of load cycles at stress level L

N_L cycles to failure under the stress level L

D fatigue damage

S_i corresponding load

n_i load cycles

N_i number of cycles

R^* stress ratio

X_p peak load

K_t stress concentration factor

σ_a stress amplitude

σ_{a0} stress amplitude when the mean value is 0

σ_m mean stress

σ_{m0} mean stress when the stress amplitude is 0

L_f specified reliability coefficient

V_2 high-pressure rotor speed

S_a^* symmetrical cyclic stress

S_a stress amplitude

S_m mean stress

S_b ultimate tensile strength

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1 Introduction

Aircraft engine components are subjected to variable amplitude load conditions usually they tend to experience fatigue damage [1]. A reliable lifetime prediction is particularly important in the design, safety assessments and optimization of engineering materials and structures. Many fatigue damage accumulation theories have been put forward to predict the fatigue lives of structure components, such as linear damage rule, Grover–Manson theory and Corten–Dolan theory [2]. The linear damage rule, also called the Palmgren–Miner rule (just the Miner rule for short), is commonly used in analyzing cumulative fatigue damage. The basic assumptions of fatigue damage cumulative theory [3] is:

(1) The damage occurs and accumulates only when the stress is higher than the fatigue limit. The fatigue damage relates not only to the number of cycles under the applied stress, but also to the number of cycles to failure.

(2) The failure of a component is assumed to occur when the cumulative damage reaches 100%.

The aircraft engine components, such as turbine disks and blades are used at harsh conditions such as high temperature or high pressure. The high reliability and safety of these components are largely due to a combination of improved materials, improved life prediction capabilities and highly conservative design. Thus, a better understanding of the loading conditions to which today's high-temperature structures are subjected as well as developing better analytical tools are necessary [4, 5]. Recently, energy based approaches have been used to predict damage for fatigue and fatigue-creep cycling [6].

About the reliable lifetime evaluation for engine components, researchers have presented several models [7–13]. For example, Shen [7] put forward a probability-based procedure to predict the reliability of gas turbine engine blades subjected to high cycle fatigue, which provides lifetime design guide lines for blades and an optimal maintenance strategy in management. On the basis of the linear damage accumulation rule and the simulated probability density distributions of the stress and the strain, Yue et al. [8] developed a reliability model for the low cycle fatigue life analysis of turbine disk structure, which considers the randomness in the life characteristic of the material. Salam et al. [9] investigated the failure cause of one engine in fighter plane during flight, which resulted from the 2nd stage fractured turbine blade. By Measuring the residual stresses, Silveira et al. [10] analyzed the effect induced by those scratches produced during the handling and mounting of an aircraft turbine disk on their fatigue life. Recently, Zhu and Huang [11–13] developed two models for life prediction considering fatigue-creep interaction for high-temperature structures One model uses the generalized strain energy damage function to predict the life of continuous cyclic loading at elevated temperature [11]. The second one is a ductility exhaustion in which damage accrues by means of viscous flow and ductility consumption is only related to plastic strain and creep strain for turbine disk alloys [12]. Moreover, a probabilistic methodology for low cycle fatigue life prediction using Bayes' theorem was developed to quantify model uncertainty resulting from creation of different deterministic model parameters [13]. These models have wider applications and are more precise to predict the life of fatigue-creep interaction. Right now, based on the load spectra of an aircraft engine used in B-Type bomber, we need to estimate its service life when equipped in A-type bomber and provide a guideline for its use.

Under different load spectra, the aircraft engine has different working life. In this paper, an aircraft engine's fatigue life will be estimated using the cumulative damage theory based on the high-pressure rotor speed. In order to reduce the flight accident risk and improve working life without compromise of reliability, an accurate algorithm for fatigue life estimation of aircraft engine is needed based on the load spectrum difference factor, which is the main purpose of this paper.

2 Linear Damage Accumulation Rule

For a load spectrum that includes stress level $s_1, s_2, \ldots, s_L$, the corresponding numbers of load cycles at these stress level are $n_1, n_2, \ldots, n_L$. $N_1, N_2, \ldots, N_L$ represent the cycles to failure under each stress level. According to the linear damage accumulation rule, the fatigue damage caused by each stress level can be characterized by the ratios $\frac{n_1}{N_1}, \frac{n_2}{N_2}, \ldots, \frac{n_L}{N_L}$ The failure of component is assumed to occur when the cumulative damage D reaches 100% According to the above assumptions, fatigue life prediction under variable amplitude block loading can be predicted by the Miner rule

$$D = \sum_{i=1}^{L} \frac{n_i}{N_i}. \qquad (1)$$

Once the loading spectrum and the working S-N curve (as shown in Fig.1) have been obtained, Eq. (1) can be used to estimate the safe-life for a component To estimate the safe-life for an aircraft engine, it follows a numbers of steps outlined as follows [14–16]:

(1) Select a flight condition from the mission spectrum and obtain the corresponding load, S_i.

(2) From the working S-N curve, determine the number of cycles, N_i, available at load level S_i before component safe-life is expected.

(3) Determine the number of cycles occurred at the load S_i in the load spectrum for a given time period (say one hour).

(4) Calculate the fatigue damage due to load S_i as n_i/N_i per hour.

(5) Repeat steps (1) to (4) for the other load level in the load spectrum.

(6) If there are L levels of loads in the spectrum, then the total fatigue damage D induced in the component can be estimated by Eq. (1).

(7) The safe-life of the component is then the reciprocal, with Safe-life $=1/D$ hours.

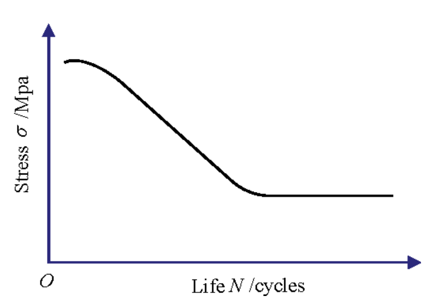

Figure 1. Schematic interpretation of S-N Curve.

However, most metallic materials exhibit more complex behaviors than the ones modeled by a linear damage rule. Therefore, the linear damage rule sometimes gives inaccurate prediction results due to the following shortcomings. First, it neglects the damage induced by stresses below the fatigue limit. Second, it is not sensitive to the load sequence by the linear accumulation theorem of fatigue damage.

In the authors' previous study a new linear damage accumulation rule was proposed to consider the damaging of low amplitude loads with different sequences using fuzzy sets theory [14]. Recent studies indicate that during the crack propagation phase, the high-frequency low amplitude loads were not affected by the load sequence effect. Eq. (1) can be used to estimate the working life of mechanical components under different load spectrums based on the load spectrum and the S-N curve.

3 Determining the Parameters in the Miner Rule

To use the Miner rule, the number of occurrences, n_i of load S_i must be found. This section details the three methods [15, 16] to count the load cycles n_i.

3.1 Block Counting Method

According to the block counting method, the loads generated in the flight condition are taken as a single block and the peak load (X_p) is assumed to occur over the entire time. When using the S-N data, X_p is usually taken as $0.5X_p$ or depending on the manufacturer's design policy. Therefore, there is only one load associated with the flight condition and the number of cycles is n_x

This is the most conservative of cycle counting methods. If the calculated lives do not yield satisfactory results (i.e. they do not meet minimum requirements), then manufacturers might make use of a less conservative cycle counting scheme

3.2 Cycle Counting Method I – The Sub-block Approach

For the sub-block approach, the loads in the flight condition are considered in more detail than that in the block counting method. The flight condition is divided into several blocks and then the block counting method is applied to each of the resulting "sub-blocks". The actual method of deciding how to create the sub-blocks is not very rigorous and each manufacturer has its own preferred schemes. If the lives produced by this method are still not acceptable, then the number of sub blocks may be increased, with a consequent decrease in the conservatism of the calculated lives.

3.3 Cycle Counting Method II – The Exact Cycle Counting approach

This is the least conservative of these three methods and is the limit case of the subblock approach in that each individual cycle is counted separately so $n = 1$.

Though many fatigue damage accumulation theories have been published, the Miner rule is most widely used in the fatigue design and fatigue life estimation in engineering due to its simplicity. Moreover, the use of Miner rule has been generally successful because of the vast experience in its application built up since the 1940s along with the use of appropriate empirical correction factors. Therefore, by choosing working S-N curve, the Miner rule will be used to estimate the remaining life of an aircraft engine in next section.

4 Life Estimation of an Aircraft Engine Using the High-pressure Rotor Speed

To estimate the fatigue life of an engine used in different types of aircrafts, this paper follows steps of nominal stress method [17].

4.1 Steps of the Nominal Stress Method

4.1.1 Determine the Stress Spectrum of Fatigue Critical Part

Stress spectrum is determined by the fatigue load spectrum and the stress analysis of key parts, such as 1st stage turbine disk.

4.1.2 Obtain the S-N Curve Under a Stress Ratio R^* of the Key Structural Part (or The S_a -N Curve Under a Mean Stress)

The S-N curve of key part based on the same material S-N curve of the key part which has the same stress concentration factor K_t will be obtained.

4.1.3 Choose the Constant Life Curve

Constant life curve (as shown in Fig. 2) is usually described by the following formula (linear formula, the modified Goodman diagram):

$$\sigma_a = \sigma_{a0}\left(1 - \frac{\sigma_m}{\sigma_{m0}}\right) \tag{2}$$

where σ_a is the stress amplitude; σ_{a0} is the stress amplitude when the mean value is 0, which is equal to the stress peak when the ratio R $= -1$, can also be labeled as σ_{-1}; σ_m is mean stress; σ_{m0} is the mean stress when the stress amplitude is 0 (the ultimate tensile strength of material).

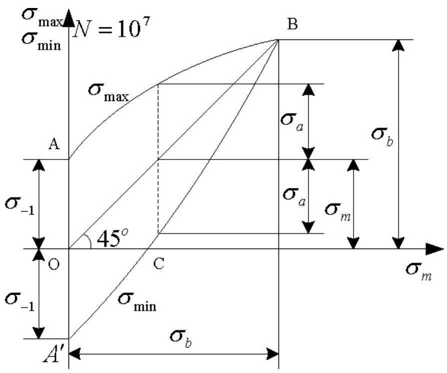

Figure 2. Constant life curve.

Other points on the constant life curve, except static load point ($\sigma_m = 0$), have the same level of fatigue damage. Symmetrical loop point ($\sigma_m = 0$) corresponds to the cyclic fatigue limit σ_a, which relates to the value of life cycle.

4.1.4 Calculate the Fatigue Life Under Different Levels of Stress Cycle

Stress cycle for each level can be characterized as $(\sigma_{ai}, \sigma_{mi})$ or $(\sigma_{\max i}, R_i)$, in engineering the latter form was often used, and $\sigma_{\max i}$ is abbreviated to σ_i. Based on the constant life curve, the peak stress σ_i^* at constant stress is R^*can be calculated by:

$$\sigma_i^* = \frac{(1 - R_i)\sigma_{m0}\sigma_i}{\sigma_{m0}(1 - R_i^*) + S_i(R^* - R_i)}. \tag{3}$$

Substituting σ_i^* into R^*leads to S in S-N curve, the fatigue life at the level of stress cycles (σ_i, R_i)can be obtained by:

$$N_i = \left(\frac{S_\infty A}{\sigma_i^* - S_\infty}\right)^{\frac{1}{\alpha}}. \tag{4}$$

4.1.5 Use the Linear Damage Rule to Estimate the Mean Life of Structures

Based on the Miner's rule, if the load spectrum is a spectrum block (base period) on behalf of N_0 flight hours, the median fatigue life (flight hours) of the structure is:

$$N_{50} = \frac{N_0}{\sum_{i=1}^{k} \frac{n_i}{N_i}}, \tag{5}$$

where k is the load series, n_i is a block of spectrum in the i-th group of load (stress) cycles, compared with it N_i is the mean life of this load (stress) under the action of constant amplitude.

4.1.6 Calculate the Fatigue Life with Reliability

Fatigue life with reliability can be calculated by the following two methods use the specified reliability Coefficient L_f corresponding to the fatigue life, the life with reliability of the structure is:

$$N_p = \frac{N_{50}}{L_f} = \frac{N_0}{L_f \cdot \sum_{i=1}^{k} \frac{n_i}{N_i}}. \tag{6}$$

In the above calculation, S-N curve can be replaced by the P-S-N curve with reliability then a fatigue life of the component with reliability can be obtained.

4.2 Fatigue Life Estimation of an Aircraft Engine

One type of aircraft engine were equipped in two types of bomber A and B. Respectively the load spectrums were collected from real-time data acquisition in the aircrafts The load matrix of high-pressure rotor speed (V_2) are shown in Table 1 and Table 2 In these tables 0% corresponds to the lowest rotor speed, 100% corresponds to the maximum rotor speed of the engine.

Valley (%)	Peak (%)							
	97–100	94.5–97	92.5–94.5	91–92.5	87.5–91	85.5–87.5	81–85.5	60–81
0–60	290	17	2	1	6	0	1	16
60–81	111	200	282	225	828	452	240	174
81–85.5	1	16	35	36	87	31		
85.5–87.5	2	36	44	90	28			
87.5–91	2	38	49					
91–92.5	1							
92.5–94.5	3							

Table 1. Cyclic load matrix V_2 for A-type bomber (Cycles, 1000 Hours).

Valley (%)	Peak (%)							
	97–100	94.5–97	92.5–94.5	91–92.5	87.5–91	85.5–87.5	81–85.5	60–81
0–60	134	64	10	0	11	2	1	8
60–81	54	144	163	101	531	440	916	334
81–85.5	3	10	17	11	32	12		
85.5–87.5	0	6	13	17	1			
87.5–91	1	11	19					
91–92.5	0	0						
92.5–94.5	0							

Table 2. Cyclic load matrix V_2 for B-type bomber (Cycles, 1000 Hours).

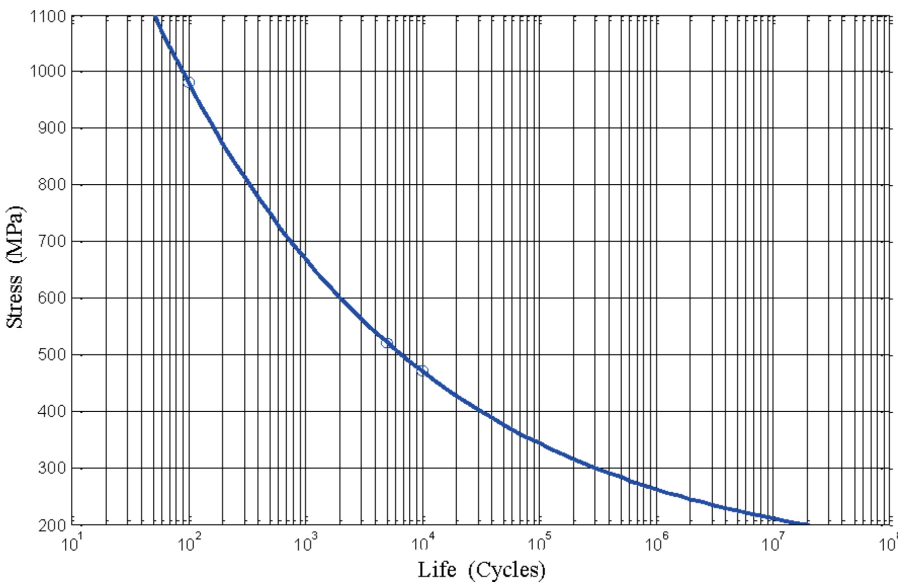

Figure 3. Working S-N curve of an aircraft engine.

According to these two types of aircraft engine high pressure rotor speed (V_2) cyclic matrices, V_2 were used as a nominal stress.

$$S = k \cdot V_2^2 \,, \tag{7}$$

where k is a conversion factor.

Based on the constant life curves, the load of rotor speed is converted to a symmetrical cyclic stress [17] as follows:

$$S_a^* = \frac{S_a}{(1 - \frac{S_m}{S_b})} \,, \tag{8}$$

where S_a^* is the symmetrical cyclic stress, S_a is the stress amplitude, S_m is the mean stress, S_b is the ultimate tensile strength of material. For simplicity, it is assumed that S_b is the corresponding nominal stress of 300% load. Cyclic stress is taken as the minimum valley to maximum peak load, mean stress is one half of the sum of maximum peak and minimum valley load. For the B-type bomber shown in Table 2, the corresponding symmetrical cyclic stress was calculated as listed in Table 3.

The stress-life (S-N) curve of an aircraft engine is shown in Fig. 3.

The S-N curve can be given by a power function

$$S = S_\infty \left(1 + \frac{A}{N^\alpha}\right) . \tag{9}$$

According to Eq. (9), the fatigue life can be obtained from the equation below:

$$N_i = \left(\frac{AS_\infty}{S_i - S_\infty}\right)^{\frac{1}{\alpha}} . \tag{10}$$

From Fig. 3, the engine life equation is obtained as

$$N_i = \left(\frac{4.828 \times 10^5}{S_i - 10^6}\right)^2 , \tag{11}$$

where $S_\infty = 10^6$, A=0.4828 and $\alpha = 0.5$.

No.	Stress cycle (%)	Stress amplitude S_a (%)	Mean stress S_m (%)	Symmetrical cyclic stress amplitude S_a^* (%)	n_i	N_i	Equivalent damage D_i
1	0–81	40.5	40.5	46.82081	8	4002	0.001999
2	0–85.5	42.75	42.75	49.85423	1	3530	0.000283
3	0–87.5	43.75	43.75	51.21951	2	3344	0.000598
4	0–91	45.5	45.5	53.63458	11	3050	0.003607
5	0–92.5	46.25	46.25	54.6798	0	2934	0
6	0–94.5	47.25	47.25	56.08309	10	2789	0.003585
7	0–97	48.5	48.5	57.85288	64	2621	0.024416
8	0–100	50	50	60	134	2437	0.054985
9	60–81	10.5	70.5	13.72549	334	46569	0.007172
10	60–85.5	12.75	72.75	16.83168	916	30967	0.02958
11	60–87.5	13.75	73.75	18.23204	440	26393	0.016671
12	60–91	15.5	75.5	20.71269	531	20450	0.025966
13	60–92.5	16.25	76.25	21.78771	101	18481	0.005465
14	60–94.5	17.25	77.25	23.23232	163	16255	0.010028
15	60–97	18.5	78.5	25.05643	144	13974	0.010305
16	60–100	20	80	27.27273	54	11795	0.004578
17	81–87.5	3.25	84.25	4.519119	12	429588	2.79E–05
18	81–91	5	86	7.009346	32	178568	0.000179
19	81–92.5	5.75	86.75	8.089097	11	134079	8.2E–05
20	81–94.5	6.75	87.75	9.540636	17	96384	0.000176
21	81–97	8	89	11.37441	10	67811	0.000147
22	81–100	9.5	90.5	13.60382	3	47406	6.33E–05
23	85.5–91	2.75	88.25	3.896104	1	577960	1.73E–06
24	85.5–92.5	3.5	99	5.223881	17	321494	5.29E–05
25	85.5–94.5	4.5	90	6.428571	13	212290	6.12E–05
26	85.5–97	5.75	91.25	8.263473	6	128479	4.67E–05
27	85.5–100	7.25	92.75	10.49457	0	79658	0
28	87.5–94.5	3.5	91	5.023923	19	347595	5.47E–05
29	87.5–97	4.75	92.25	6.859206	11	186471	5.9E–05
30	87.5–100	6.25	93.75	9.090909	1	106156	9.42E–06
31	91–97	3	94	4.368932	0	459630	0
32	91–100	4.5	95.5	6.601467	0	201316	0
33	92.5–100	3.75	96.25	5.521472	0	287773	0

Table 3. Fatigue damage estimates using high-pressure rotor speed V_2 for B-type bomber.

No.	Stress cycle (%)	Stress amplitude S_a (%)	Mean stress S_m (%)	Symmetrical cyclic stress amplitude S_a^* (%)	n_i	N_i	Equivalent damage D_i
1	0–81	40.5	40.5	46.82081	16	4002	0.003998
2	0–85.5	42.75	42.75	49.85423	1	3530	0.000283
3	0–87.5	43.75	43.75	51.21951	0	3344	0
4	0–91	45.5	45.5	53.63458	6	3050	0.001967
5	0–92.5	46.25	46.25	54.6798	1	2934	0.000341
6	0–94.5	47.25	47.25	56.08309	2	2789	0.000717
7	0–97	48.5	48.5	57.85288	17	2621	0.006486
8	0–100	50	50	60	290	2437	0.118794
9	60–81	10.5	70.5	13.72549	174	46569	0.003736
10	60–85.5	12.75	72.75	16.83168	240	30967	0.00775
11	60–87.5	13.75	73.75	18.23204	452	26393	0.017126
12	60–91	15.5	75.5	20.71269	828	20450	0.050611
13	60–92.5	16.25	76.25	21.78771	225	18481	0.012175
14	60–94.5	17.25	77.25	23.23232	282	16255	0.017349
15	60–97	18.5	78.5	25.05643	200	13974	0.014312
16	60–100	20	80	27.27273	111	11795	0.009411
17	81–87.5	3.25	84.25	4.519119	31	429588	7.22E−05
18	81–91	5	86	7.009346	87	178568	0.000487
19	81–92.5	5.75	86.75	8.089097	36	134079	0.000268
20	81–94.5	6.75	87.75	9.540636	35	96384	0.000363
21	81–97	8	89	11.37441	16	67811	0.000236
22	81–100	9.5	90.5	13.60382	1	47406	2.11E−05
23	85.5–91	2.75	88.25	3.896104	8	577960	1.38E−05
24	85.5–92.5	3.5	99	5.223881	90	321494	0.00028
25	85.5–94.5	4.5	90	6.428571	44	212290	0.000207
26	85.5–97	5.75	91.25	8.263473	36	128479	0.00028
27	85.5–100	7.25	92.75	10.49457	2	79658	2.51E−05
28	87.5–94.5	3.5	91	5.023923	49	347595	0.000141
29	87.5–97	4.75	92.25	6.859206	38	186471	0.000204
30	87.5–100	6.25	93.75	9.090909	2	106156	1.88E−05
31	91–97	3	94	4.368932	0	459630	0
32	91–100	4.5	95.5	6.601467	1	201316	4.97E−06
33	92.5–100	3.75	96.25	5.521472	3	287773	1.04E−05

Table 4. Fatigue damage estimates using high-pressure rotor speed V_2 for A-type bomber.

Substituting the symmetric cycle stress amplitude S^*_{ai} in Table 3 into Eq. (11), the cyclic life at each level stress can be obtained as shown in Table 3.

Based on the Miner rule, the cumulative damage is

$$D_{\text{total}} = \sum_{i=1}^{k} \frac{n_i}{[\frac{4.828\times 10^5}{(S^*_{ai}-10^6)}]^2}\,. \tag{12}$$

The life cycle N_i and the equivalent damage D_i were listed in Table 3, and $D_{\text{total}} = 0.2$.

The fatigue life of the same engine used in A-type bomber can be estimated using Eq. (12), the life cycle N_i for A-type bomber and equivalent damage were listed in Table 4. The cumulative damage $D_{\text{total}} = \sum_{i=1}^{40} D_i = 0.268$.

According to the damage estimated in bomber A and B, due to different load spectrums between two bombers, the damage ratio of 1st stage turbine disk in these two bombers is:

$$K_d = \frac{D_k}{D_y} = \frac{0.268}{0.2} = \frac{1.34}{1}\,. \tag{13}$$

It is shown that the life of the engine equipped in bomber A is shorter than that in bomber B.

According to Table 4, the equivalent cumulative damage is $D_{\text{total}} = \sum_{i=1}^{33} D_i = 0.267694$. For this engine used in A-type bomber, when the cumulative damage reaches one under the current load spectrum, the estimated fatigue life of this engine is

$$T = \frac{1000}{0.267694} \times 1.0 = 3735.6 \quad (\text{Hours})\,. \tag{14}$$

Using Eq. (14), the estimated life is obtained without considering the scatter of test data. For these two types of bombers, the fatigue life of engine in A-type bomber can be estimated based on the load spectrum of B-type bomber. As there is a similar scatter factor exists for this engine equipped in the B-type bomber, the estimated life of engine in A-type bomber has certain analogies compared with the condition of engine equipped in the B-type bomber. When the scatter factor of engine used in A-type bomber differs greatly from that in B-type bomber, the scatter of test data of two bombers must be considered respectively.

5 Conclusions

Estimation of the aircraft engine life is a very complex issue which involves many factors. Based on the working S-N curve and linear cumulative damage theory, the relationship between the load spectrum and the life of an aircraft engine are investigated by using a key part of the engine-1st stage turbine disk as a research object. Under different load spectrums, the working life of the same engine is different for using in A and B-type bombers. The results show that the working life of 1st stage turbine disk equipped in A-type bomber is shorter than that equipped in B-type bomber, which is about 25% ($(5000 - 3735.6)/5000 = 25.2\%$).

Acknowledgments

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