Condition-Based Maintenance With Scheduling Threshold and Maintenance Threshold

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***Abstract*—In order to arrange maintenance resources according to system condition, the lead time needs to be considered within the context of condition-based maintenance (CBM). Therefore, a scheduling threshold is introduced to replace the time to schedule, which is used as a decision variable in combination with a maintenance threshold and a failure threshold. The long-run expected cost rate for maintenance considers the maintenance cost, the cost of the waiting time of suppliers and customers. In this way, suppliers can schedule maintenance services in advance when the system condition reaches the scheduling threshold, and perform maintenance when the system condition exceeds the maintenance threshold. Furthermore, the optimal maintenance plan is updated dynamically in the framework of Prognostics and Health Management (PHM). Finally, a numerical example is provided to demonstrate the effectiveness and the dynamic nature of the proposed method.**

***Index Terms*—Condition-based maintenance (CBM), lead threshold, lead time, maintenance threshold, scheduling threshold.**

NOTATION

X_F	Failure threshold.
X_M	Maintenance threshold.
X_S	Scheduling threshold.
X_L	Lead threshold $= X_M - X_S$.
Δt	Unit time interval.
j	Time index for time period from $(j-1)\,\Delta t$ to $j\Delta t$.
T_F	Time to failure, where $T_F = \inf(t\,\lvert X\,(t) \geq X_F\,)$.
T_M	Time to maintenance, where $T_M = \inf(t\,\lvert X\,(t) \geq X_M\,)$.
T_S	Time to schedule, where $T_S = \inf(t\,\lvert X\,(t) \geq X_S\,)$.
T_L	Lead time $T_L = L\Delta t$.
T_{WS}	Waiting time of suppliers, suppliers are waiting.
T_{WC}	Waiting time of customers, customers are waiting.
C_k	Cost of maintenance in system types k, $k = 1, 2, 3$.
C_{WS}	Cost of waiting time of suppliers per unit time.
C_{WC}	Cost of waiting time of customers per unit time.
X_i^j	Cumulative degradation from time $i\Delta t$ to $j\Delta t$.
Y_i	Degradation damage in time $j\Delta t$.
P_l	Probability of system types l, $l = 1, 2, \ldots, 5$.

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I. INTRODUCTION

FROM the suppliers' perspective, the lead time, as the duration of maintenance resources preparation, is required within the context of condition-based maintenance (CBM). In order to improve the maintenance resources management, especially the spare parts management, instead of using the time to schedule maintenance resources, a scheduling threshold is introduced to provide the optimal maintenance plan with the minimum cost based on system conditions to arrange maintenance resources. For example, to arrange the spare parts for one particular part of an aircraft, we need to decide when to order the spare part and when to replace it in the aircraft. Then, there is a lead time between the time to order and the time to replace. In order to obtain the optimal plan for the particular part, we dynamically need a maintenance threshold to replace it and a scheduling threshold to order it both based on the estimated condition of that product. Furthermore, if the spare part comes early, there is the cost of warehousing and human resources; if the spare part comes late, there is the cost of the expensive delayed flight or even the cancellation of the flight. When the maintenance resources are not available when the system fails, there is the extra cost of waiting time of customers such as the penalty cost of downtime. When the maintenance resources are available before performing the maintenance, there is the cost of waiting time of suppliers such as inventory cost of planned parts. Meanwhile, maintenance costs are different based on the condition when all the maintenance resources are available. The objective is to reduce the overall expected cost rate [1].

In the literature, maintenance policies [2] are classified into the corrective maintenance, the preventive maintenance, the CBM [3]–[6], and the predictive maintenance [7]–[10]. There are different system models for various CBM practices, such as the nonhomogeneous Poisson process model [11], the two-stage failure process model [12], the fatigue crack degradation model [13], the proportional hazards model [14], the belief rule-based prognostic model [15], and the maintenance models for warranty [16]. A survey of the gamma process in maintenance is presented in [17], and the application of the gamma process in continuously monitored degrading systems is studied in [18]. In particular, for a gamma process degradation in the finite time horizon, the probability distribution of the cost rate is provided by the means of the discrete Fourier transformation of a characteristic function [19]. Because the gamma process is positive and strictly increasing, it is widely used for positive and strictly monotone deterioration processes [20].

Nowadays, there are a lot of applications of CBM, such as commercial-heavy vehicles with grouping maintenance operations [21], the gas turbine with the health index [22], traction batteries with the impedance measurement [23], the gas-insulated switchgear with the accuracy of sensor information [24], marine engine cylinder liners with a state-dependent wear model [25], and the wind turbine [26]–[30]. However, in these applications, the lead time [31] is not considered.

Here, to compensate for the effects of the lead time, a three-threshold maintenance scheme is presented. The scheme contains three critical condition thresholds.

1) The failure threshold is a deterministic constant [32]. A system exceeding the failure threshold is failed.
2) The maintenance threshold is a condition threshold for maintenance execution [33]. If the system's degradation reaches this threshold, the maintenance is carried out immediately when all resources are ready.
3) The scheduling threshold is the condition threshold for resource arrangement. Maintenance resources start to be prepared when the degradation exceeds this threshold.

In time-based maintenance (TBM), the lead time is the duration between the time to schedule and the time to perform the maintenance [34], not including the period of the maintenance actions and decision-makings. In CBM, the lead threshold is defined as the difference between the scheduling threshold and the maintenance threshold. Then the optimal lead threshold is determined by two factors: the lead time from service providers and the system degradation from the field operations. In this way, we convert the lead time to the lead threshold by the scheduling threshold and the maintenance threshold.

Using the three-threshold maintenance scheme, instead of considering system stages, we propose the concept of system types. It is important to distinguish the system stages [35] and the proposed system types in two aspects: 1) one degradation path can have different degradation stages at different time points, while one degradation path can only belong to one system type with the information of the whole life time, and 2) we use system stages to describe the system condition at a single time point, whereas, system types are used to describe the system for a time line including the system stages at each time point. Five system types are considered in this article at each time step. In types 1–3, at each time step, the system condition reaches the scheduling threshold where the maintenance cost is calculated. Then in these three types, the system types are also the maintenance cost types, classified by the future condition when resources are prepared: 1) in type 1, the system condition is lower than the maintenance threshold; 2) in type 2, the system condition is greater than the maintenance threshold, and lower than the failure threshold; and 3) in type 3, the system condition is greater than the failure threshold. In types 4–5, the system types are classified by the past and current system conditions: 1) in type 4, the current system condition is still lower than the scheduling threshold and 2) in type 5, the past system condition was higher than the scheduling threshold. Above all, the five system types contain all possible kinds of degradation paths at each time step.

According to the system types, there are three types of maintenance cost corresponding to the system types 1–3, respectively. Moreover, there are extra cost of waiting time of suppliers and customers. In type 1, there is waiting time of suppliers, from the time when the resources are prepared to the time when the degradation reaches the maintenance threshold. In type 3, there is waiting time of customers, from the failure time to the time when the necessary maintenance resources are available. Both costs of waiting time of suppliers and customers are integrated in the overall cost.

The main contributions of this paper are as follows.

- A scheduling threshold as an essential decision variable is introduced to replace the time to schedule for suppliers. It combines with a maintenance threshold and a failure threshold.
- System types, classified by these thresholds, are proposed based on the past and present information, and the prediction for the future.
- The expected waiting time of suppliers and customers are integrated into the cost analysis for the classified maintenance types.
- The long-run cost rate is minimized optimally, compared with other maintenance plans.
- The optimal process is updated online in the framework of Prognostics and Health Management (PHM).

The paper is organized as follows. In Section II, the assumptions for the studied system are listed, and the probability model of system types at each time step is established. In Section III, both the expected waiting time of suppliers and customers are provided. In Section IV, the overall cost in the proposed method is analyzed considering the cost of waiting time. In Section V, the optimal maintenance plan in the framework of the PHM is proposed. In Section VI, a numerical example is presented to demonstrate the proposed approach. In Section VII, conclusions are given.

II. System Assumptions and Model Description

A. System Assumptions

Three basic assumptions for the system are listed here.

- The system deteriorates with a monotone increasing degradation process. The damages at each time step are cumulative.
- The system fails when the degradation reaches the predetermined failure threshold.
- It is assumed that the system experiences a gamma degradation process. It follows three basic rules of gamma process [36]: 1) $X(0) = 0$; 2) the increments are s-independent; and 3) the increments follow a gamma distribution.

B. Degradation Process

The failure time T_F, which is the first passage time when the degradation exceeds the failure threshold X_F, is defined by $T_F = \inf(t \,|X(t) \geq X_F)$, where $X(t)$ is the system condition at time t . The cumulative density function (CDF) of failure time is $G_S(t; X_F)$, given as

$$G_S(t; X_F) = \Pr\{T_F < t\}. \tag{1}$$

The CDF of degradation is $G(x; t)$, expressed as

$$G(x; t) = \Pr\{X(t) < x\}. \tag{2}$$

When all of the system experiences monotone increasing degradation paths, it is obvious that $G(x; t) = 1 - G_S(t; x)$ as illustrated in Fig. 1.

The gamma process is used to model the damage accumulation process as one of the Levy processes. The increments of the degradation process at mutually exclusive discrete time intervals are positive, s-independent with each other, and follow the gamma distribution. The mathematical expression of the stationary gamma process is presented as follows, including a shape parameter αt and a parameter β. The probability density function (PDF) and CDF of the gamma distribution are respectively given as [20]

$$g(x; t) = \frac{1}{\Gamma(\alpha t)\, \beta^{\alpha t}} (x)^{\alpha t - 1} \exp\left(-\frac{x}{\beta}\right) \tag{3}$$

$$G(x; t) = \int_0^x g(x; t)\, dx = \frac{\gamma\left(\alpha t, \frac{x}{\beta}\right)}{\Gamma(\alpha t)} \tag{4}$$

where the complete gamma function is $\Gamma(\alpha t) = \int_0^\infty \xi^{\alpha t - 1} e^{-\xi} d\xi$, and the lower incomplete gamma function is $\gamma(\alpha t, x/\beta) = \int_0^{x/\beta} \xi^{\alpha t - 1} e^{-\xi} d\xi$. Then, the PDF of the increments is $g(x; \Delta t)$.

C. System Types

The systems are divided into five types by three thresholds. The time to schedule T_S, which is the first passage time when the degradation exceeds the scheduling threshold X_S, is defined by $T_S = \inf(t \,|X(t) \geq X_S)$. Similarly, the time to maintenance T_M is defined by $T_M = \inf(t \,|X(t) \geq X_M)$, where X_M is the maintenance threshold. We set $t = j\Delta t$, and lead time $T_L = L\Delta t$, where Δt is a unit time period. The system type is determined by the condition of degradation path, which includes not only the present condition $X(t)$, but also the past condition at time $X(t - \Delta t)$ and the future condition at time $X(t + T_L)$. The system condition in the past time $X(t - \Delta t)$ and the present time $X(t)$ are compared with the scheduling threshold X_S, in order to determine when the preparation should be started. The system condition in the future time $X(t + T_L)$ is compared with the maintenance threshold X_M and the failure threshold X_F to identify maintenance cost types. The system types are shown in Table I and in Fig. 2.

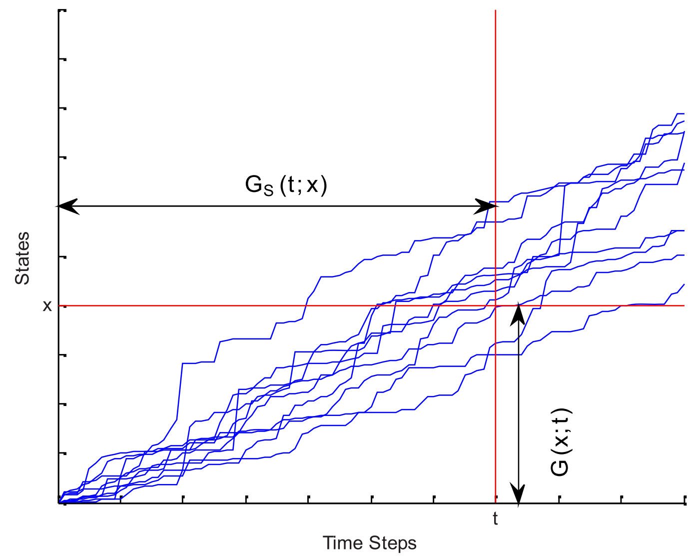

Fig. 1. CDF of the time and the degradation.

When the system experiences j time periods, each period results in a damage Y_j. Accumulated damage from time index i to time index j is X_i^j. In the following integral functions, the x stands for X_1^{j-1}, the y stands for Y_j, the z stands for X_{j+1}^{j+L}. When $X_1^{j-1} < X_S$ and $X_1^j \geq X_S$, j is the scheduling time index.

In type 1, j is the scheduling time index, and at time $j\Delta t + T_L$ the degradation is still lower than the maintenance threshold, which means the system still does not need the maintenance. The probability of type 1 is given by (5), shown at the bottom of the page. When the degradation conforms to a gamma process, the probability of type 1 is given by (6), shown at the bottom of the following page.

$$\begin{aligned} P_1(j) &= \Pr\left(Y_1 + Y_2 + \cdots + Y_{j-1} < X_S \leq Y_1 + Y_2 + \cdots + Y_j, Y_1 + Y_2 + \cdots + Y_{j+L} < X_M\right) \\ &= \Pr\left((Y_1 + Y_2 + \cdots + Y_{j-1} < X_S) \cap (X_S < Y_1 + Y_2 + \cdots + Y_j) \cap (X_S < Y_1 + Y_2 + \cdots + Y_{j+L} < X_M)\right) \\ &= \Pr\left(\left(X_1^{j-1} < X_S\right) \cap \left(Y_j > X_S - X_1^{j-1}\right) \cap \left(X_{j+1}^{j+L} < X_M - X_1^{j-1} - Y_j\right)\right) \end{aligned} \tag{5}$$

TABLE I
SYSTEM TYPES

Type Number for condition of degradation path	Condition at time $t-\Delta t$	Condition at time t	Condition at time $t+T_L$
Type 1	$X(t-\Delta t) < X_S$	$X(t) \geq X_S$	$X(t+T_L) < X_M$
Type 2	$X(t-\Delta t) < X_S$	$X(t) \geq X_S$	$X_M \leq X(t+T_L) < X_F$
Type 3	$X(t-\Delta t) < X_S$	$X(t) \geq X_S$	$X_F \leq X(t+T_L)$
Type 4	$X(t-\Delta t) < X_S$	$X(t) < X_S$	$\forall$
Type 5	$X(t-\Delta t) \geq X_S$	$\forall$	$\forall$

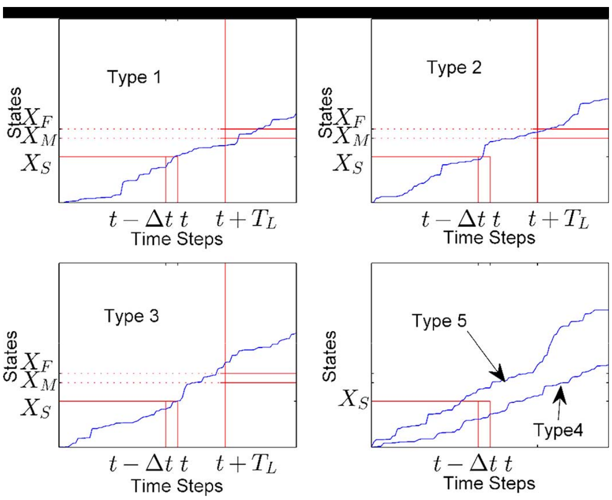

Fig. 2. System types.

In type 2, j is the scheduling time index, and at time $j\Delta t + T_L$ the degradation is higher than the maintenance threshold and lower than the failure threshold, which means it needs the maintenance immediately. The probability of the system in type 2 is given by (7), shown at the bottom of the page. When the degradation conforms to a gamma process, the probability of type 2 is given by (8), shown at the bottom of the page.

In type 3, j is the scheduling time index, and at time $j\Delta t + T_L$ the degradation is greater than the failure threshold, which means that the failure happens before maintenance resources are ready. The probability of type 3 is given by (9), shown at the bottom of the following page. When the degradation conforms to a gamma process, the probability of type 3 is given by (10), shown at the bottom of the following page.

Using (5), (7), and (9), it is obvious that

$$P_1(j)+P_2(j)+P_3(j) = \Pr\left(X_1^{j-1} < X_S \leq X_1^j\right) = G\left(X_S;(j-1)\Delta t\right) - G\left(X_S; j\Delta t\right). \quad (11)$$

$$\begin{aligned} P_1(j) &= \int_0^{X_S}\left(\int_{X_S-x}^{X_M-x}\left(\int_0^{X_M-x-y} g(z; L\Delta t)\,\mathrm{d}z\right) g(y;\Delta t)\,\mathrm{d}y\right) g(x;(j-1)\Delta t)\,\mathrm{d}x \\ &= \int_0^{X_S}\int_{X_S-x}^{X_M-x}\left(G\left(X_M-x-y; L\Delta t\right)\right) * g(y;\Delta t) * g(x;(j-1)\Delta t)\,\mathrm{d}y\mathrm{d}x \end{aligned} \quad (6)$$

$$\begin{aligned} P_2(j) &= \Pr\left(Y_1+Y_2+\cdots+Y_{j-1} < X_S < Y_1+Y_2+\cdots+Y_j, X_M \leq Y_1+Y_2+\cdots+Y_{j+L} < X_F\right) \\ &= \Pr\left(\left(X_1^{j-1} < X_S\right) \cap \left(Y_j > X_S - X_1^{j-1}\right) \cap \left(X_M - X_1^{j-1} - Y_j < X_{j+1}^{j+L} < X_F - X_1^{j-1} - Y\right)_j\right) \end{aligned} \quad (7)$$

$$\begin{aligned} P_2(j) &= \int_0^{X_S}\left(\int_{X_S-x}^{X_F-x}\left(\int_0^{X_F-x-y} g(z; L\Delta t)\,\mathrm{d}z\right) g(y;\Delta t)\,\mathrm{d}y\right) g(x;(j-1)\Delta t)\,\mathrm{d}x - P_1(j) \\ &= \int_0^{X_S}\int_{X_S-x}^{X_F-x} G\left(X_F-x-y; L\Delta t\right) * g(y;\Delta t) * g(x;(j-1)\Delta t)\,\mathrm{d}y\mathrm{d}x - P_1(j) \end{aligned} \quad (8)$$

Using (6), (8), and (11), the probability of type 3 is given by (12), shown at the bottom of the page.

In type 4, $j\Delta t$ is before the scheduling time. The probability of type 4 is

$$\begin{aligned} P_4(j) &= \Pr(Y_1+Y_2+\cdots+Y_{j-1} < Y_1+Y_2+\cdots+Y_j < X_S) \\ &= \Pr\left(X_1^j < X_S\right). \end{aligned} \tag{13}$$

When the degradation conforms to a gamma process, the probability of type 4 is

$$P_4(j) = G(X_S; j\Delta t). \tag{14}$$

In type 5, $j\Delta t$ is after the scheduling time. The probability of type 5 is

$$\begin{aligned} P_5(j) &= \Pr(X_S \le Y_1+Y_2+\cdots+Y_{j-1}) \\ &= 1 - \Pr\left(X_0^{j-1} > X_S\right). \end{aligned} \tag{15}$$

When the degradation conforms to a gamma process, the probability of type 5 is

$$\begin{aligned} P_5(j) &= 1 - \int_0^{X_S} g(y; (j-1)\Delta t)\, dy \\ &= 1 - G(X_S; (j-1)\Delta t). \end{aligned} \tag{16}$$

It is obtained that $P_1(j)+P_2(j)+P_3(j)+P_4(j)+P_5(j) = 1$ from (11), (14), and (16). It indicates that these five system types at time j contain all of the situations at that time. Also, system types 1–3 contain all of the maintenance cost types and it is obvious that $\sum_{j=1}^{\infty}(P_1(j)+P_2(j)+P_3(j)) = 1$. It implies that all degradation paths are calculated once in the system types 1–3.

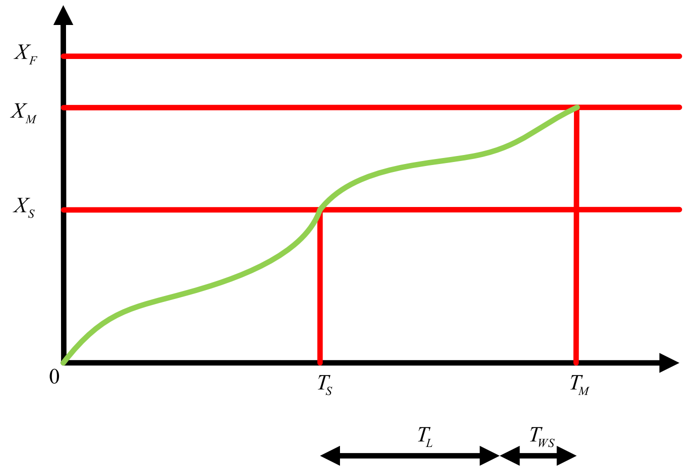

Fig. 3. Waiting time of suppliers.

III. Waiting Time

In time index j, when it contains the schedule time T_S, there is waiting time of suppliers in type 1, there is no waiting time in type 2, there is waiting time of customers in type 3. In types 4 and 5, time index j does not contain the schedule time T_S, and the waiting time is calculated in other time step.

A. Waiting Time of Suppliers

In type 1, though the resources are ready, the customer operates the devices until the system condition reaches the maintenance threshold and then performs the maintenance immediately, as shown in Fig. 3. In type 2 and 3, there is no waiting time of suppliers, so $E(T_{WS}(j)\,|\text{'type 2'}) = 0$ and $E(T_{WS}(j)\,|\text{'type 3'}) = 0$.

$$\begin{aligned} P_3(j) &= \Pr(Y_1+Y_2+\cdots+Y_{j-1} < X_S < Y_1+Y_2+\cdots+Y_j, X_F \le Y_1+Y_2+\cdots+Y_{j+L}) \\ &= \Pr\left(\left(X_1^{j-1} < X_S\right) \cap \left(Y_j > X_S - X_1^{j-1}\right) \cap \left(X_{j+1}^{j+L} \ge X_F - X_1^{j-1} - Y_j\right)\right). \end{aligned} \tag{9}$$

$$\begin{aligned} P_3(j) = &\int_0^{X_S}\left(\int_{X_S-x}^{X_F-x}\left(\int_{X_F-x-y}^{\infty} g(z; L\Delta t)\, \mathrm{d}z\right) g(y;\Delta t)\, \mathrm{d}y\right) g(x;(j-1)\Delta t)\, \mathrm{d}x \\ &+ \int_0^{X_S}\left(\int_{X_F-x}^{\infty}\left(\int_0^{\infty} g(z; L\Delta t)\, \mathrm{d}z\right) g(y;\Delta t)\, \mathrm{d}y\right) g(x;(j-1)\Delta t)\, \mathrm{d}x. \end{aligned} \tag{10}$$

$$\begin{aligned} P_3(j) = &G(X_S; (j-1)\Delta t) - G(X_S; j\Delta t) \\ &- \int_0^{X_S}\int_{X_S-x}^{X_F-x} G(X_F - x - y; L\Delta t) * g(y;\Delta t) * g(x;(j-1)\Delta t)\, \mathrm{d}y\mathrm{d}x \end{aligned} \tag{12}$$

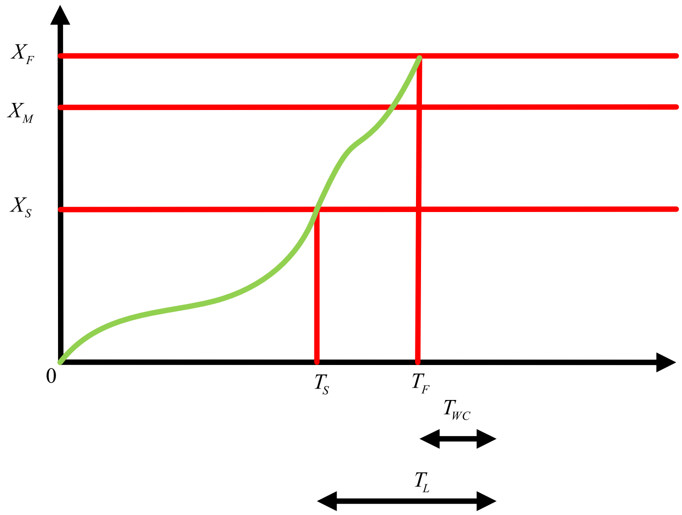

Fig. 4. Waiting time of customers.

When the schedule time is $j\Delta t$, the expected waiting time of suppliers is given by (17), shown at the bottom of the page.

B. Waiting Time of Customers

In type 3, the system fails before maintenance resources are ready, as shown in Fig. 4. It is a serious problem especially when the shutdown cost is expensive for customers, and the cost is dependent on the waiting time. In types 1 and 2, there is no waiting time of customers, so $E\left(T_{WC}\left(j\right)|\text{‘type }1'\right) = 0$ and $E\left(T_{WC}\left(j\right)|\text{‘type }2'\right) = 0$.

When the schedule time is $j\Delta t$, the expected waiting time of customers is given by (18), shown at the bottom of the page.

IV. Maintenance Policy

A. Maintenance Assumptions

The assumptions are listed as follows to model the optimization problem of maintenance policy.

- About the time. The unit time interval is Δt. Maintenance is immediately performed and the duration time is negligible. The time horizon is infinite. The lead time is a decision parameter and determined.
- About measurement. The degradation is estimated with measurement noise, and, in optimization, we regard the estimated degradation condition reflects the system condition perfectly in prognostics. Therefore, the false alarm rate and detection rate are not calculated in the optimization process of both thresholds.
- About maintenance. Implementing maintenance is technically feasible for changing the system condition to “as good as new”. When maintenance resources are ready, the system will be repaired immediately once the degradation cross the maintenance threshold. In this way, if the system fails when maintenance resources are ready, there will not be downtime.
- About cost. The maintenance cost of type 1, type 2, and type 3 are one-time expense in each life cycle. The waiting time cost and operation cost are fixed per unit time. Details of cost analysis are given in Section IV-B.

B. Cost Analysis

To minimize the long-run expected cost rate for maintenance is the objective. The long-run expected cost rate is defined as

$$CR = \frac{\text{Expected cost incurred within a cycle}}{\text{Expected length of a cycle}}. \tag{19}$$

$$\begin{aligned} E(T_{WS}\left(j\right)) &= E\left(T_{WS}\left(j\right)|\text{‘type }1'\right)P_1 + E\left(T_{WS}\left(j\right)|\text{‘type }2'\right)P_2 + E\left(T_{WS}\left(j\right)|\text{‘type }3'\right)P_3 \\ &= E\left(T_{WS}\left(j\right)|\text{‘type }1'\right)P_1 \\ &= \int_0^{X_S}\int_{X_S-x}^{X_M-x}\int_0^{X_M-x-y}\int_0^{\infty} tg\left(t;X_M-x-y-z\right)g\left(z;L\Delta t\right)g\left(y;\Delta t\right)g\left(x;\left(j-1\right)\Delta t\right)\mathrm{d}t\mathrm{d}z\mathrm{d}y\mathrm{d}x \\ &= \sum_{t=L}^{\infty}(t-L)\int_0^{X_S}\int_{X_S-x}^{X_M-x}\left(G\left(X_M-x-y;t-1\right)-G\left(X_M-x-y;t\right)\right)g\left(y;\Delta t\right)g\left(x;\left(j-1\right)\Delta t\right)\mathrm{d}y\mathrm{d}x \end{aligned} \tag{17}$$

$$\begin{aligned} E\left(T_{WC}\left(j\right)\right) &= E\left(T_{WC}\left(j\right)|\text{‘type }1'\right)P_1 + E\left(T_{WC}\left(j\right)|\text{‘type }2'\right)P_2 + E\left(T_{WC}\left(j\right)|\text{‘type }3'\right)P_3 \\ &= E\left(T_{WC}\left(j\right)|\text{‘type }3'\right)P_3 \\ &= \left(L-0\right)\int_0^{X_S}\left(1-G\left(X_F-x;1\right)\right)g\left(x;\left(j-1\right)\Delta t\right)\mathrm{d}x \\ &\quad + \sum_{t=1}^{L}(L-t)\int_0^{X_S}\int_{X_S-x}^{X_F-x}\left(G\left(X_F-x-y;t-1\right)-G\left(X_F-x-y;t\right)\right)g\left(y;\Delta t\right)g\left(x;\left(j-1\right)\Delta t\right)\mathrm{d}y\mathrm{d}x \end{aligned} \tag{18}$$

The total cost contains two parts: the maintenance cost and the waiting time cost.

The maintenance cost is divided into three kinds by types of degradation paths. In type 1, maintenance is carried out when the system condition just reaches the maintenance threshold. In type 2, though the resources are ready, the system degradation is higher than the maintenance threshold and lower than the failure threshold. In type 3, the maintenance is implemented after system fails. Hence, we assume $C_1 < C_2 < C_3$, where C_k is the maintenance cost for type k , $k = 1, 2, 3$.

The cost of waiting time of supplier and customer per time unit are C_{WS} and C_{WC} respectively. The expected waiting time at time j is given in Section III.

The expected cost rate $CR\left(X_S, X_M\right)$ is made of two parts: the maintenance cost rate, and the waiting time cost rate. The maintenance and the waiting time cost rate equal the corresponding expected cost divide by the expected time. In the first part, expected maintenance cost, is made up of the product of maintenance cost and probability for each type at each time unit $P_1\left(j\right)C_1 + P_2\left(j\right)C_2 + P_3\left(j\right)C_3$ and these types contain all possibilities $\sum_{j=1}^{\infty}\left(P_1\left(j\right) + P_2\left(j\right) + P_3\left(j\right)\right) = 1$. In the second part, the expected waiting time cost for suppliers and customers is $E\left(T_{WS}\left(j\right)\right)C_{WS} + E\left(T_{WC}\left(j\right)\right)C_{WC}$. During the waiting time of suppliers, the system is still working, and the T_{WS} is included into the useful time. During the waiting time of customers, the system is not working, and the T_{WC} is excluded. Then finally the expected useful time for each time unit, classified in the same way for the system type, is

$$\left(P_1\left(j\right)+P_2\left(j\right)+P_3\left(j\right)\right)\left(t+T_L\right)+E\left(T_{WS}\left(j\right)\right)-E\left(T_{WC}\left(j\right)\right).$$

Therefore, the expected cost rate of a renewal cycle is written as (20), shown at the bottom of the page.

The lead time T_L is a decision variable, decided by suppliers, whereas the lead threshold X_L is determined by the optimized results of the scheduling threshold and the maintenance threshold.

V. Optimal Maintenance Plan in PHM

In order to apply the approach to more general engineering problems, the optimal scheduling threshold and the maintenance threshold have to be updated dynamically to obtain the minimum cost rate based on the continuous online system condition estimation in the framework of PHM.

A. Inspection Process

Although the degradation could be measured, however, the imperfect inspection is one of the difficulties in system degradation condition estimation under the online monitoring. The random inspection error is assumed with normal distribution. Still the degradation process is assumed to be gamma process to be consistent with the above assumptions. While the stochastic process for the degradation process and the random distribution for the inspection process may vary from one to another, the proposed three thresholds based optimal maintenance plan relies on the online accurate estimation and relative plenty history data for the optimization.

Then, the inspection process $Y\left(t\right)$ contains two parts, the gamma process $X\left(t\right)$ and the Gaussian noise $\varphi \sim N\left(0, \sigma_{\varphi}^2\right)$.

B. Particle Filtering for the Estimation

The main steps for particle filtering are simply listed for reference.

The first step is to establish the model for the particle filtering. Then in this case it includes two parts: 1) the degradation condition transition $X\left(t\right) = f\left(X\left(t - \Delta t\right)\right)$ and 2) the continuous inspection process $Y\left(t\right) = X\left(t\right) + \varphi$.

The second step is the prediction and the updating. The prediction provides the prior probability distribution based on the history information with the degradation transition. Then in the updating step, the new observations are obtained and used to update the prior probability distribution, so as to estimate the system degradation condition.

The third step is the resampling process, with the aim to deal with the lack of the variability for particles, which is only requested when the variability is lower than the acceptable level. Here, the systematic resampling is applied for its easy implementation and sufficiently good performance.

Details about the method of particle filter are in [37]–[40].

C. Optimal Scheduling and the Maintenance Threshold

The whole process for optimization of the scheduling and the maintenance threshold in the framework of PHM is listed as follows.

Step 1) Collect the history information.
Step 2) Establish the degradation model and the inspection model
Step 3) Collect on-line data.
Step 4) Perform particle filter including: the prediction step, the updating step, and the resampling step.
Step 5) Reset the boundary of the scheduling and the maintenance threshold.
Step 6) Optimize the objective of the cost rate including the time before the last estimated time, and update the optimal scheduling and the maintenance threshold based on system condition.

$$CR\left(X_S, X_M\right) = C_0 + \frac{\sum\limits_{j=1}^{\infty}\left(P_1\left(j\right)C_1 + P_2\left(j\right)C_2 + P_3\left(j\right)C_3 + E\left(T_{WS}\left(j\right)\right)C_{WS} + E\left(T_{WC}\left(j\right)\right)C_{WC}\right)}{\sum\limits_{t=1}^{\infty}\left(\left(P_1\left(j\right) + P_2\left(j\right) + P_3\left(j\right)\right)\left(t + T_L\right) + E\left(T_{WS}\left(j\right)\right) - E\left(T_{WC}\left(j\right)\right)\right)} \tag{20}$$

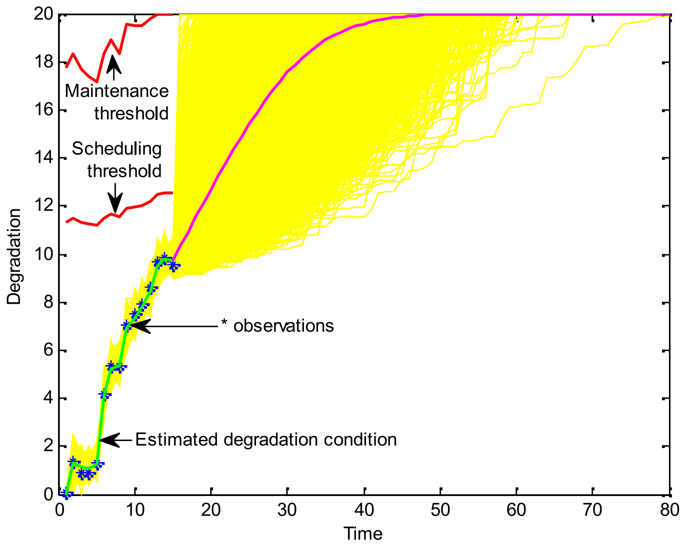

Fig. 5. Diagram of the proposed method in PHM.

Step 7) Compare the system condition with the optimal thresholds, and make decisions.

In this way, an online optimal maintenance scheme is established. A numerical example for optimization in PHM is illustrated to show the process of the optimization process for the three thresholds based maintenance plan. We use the data in Table I and show the results in Fig. 5. The example shows the observations and estimated system condition at each time point. In order to provide the optimal plan with the updated data, the scheduling threshold and maintenance threshold are both optimized based on the estimated degradation in each estimation step. Then we can make decisions based on these thresholds. In this way, the thresholds are dynamically updated with the online monitoring data under the framework of PHM.

The proposed optimization process has been coded in MATLAB software. The optimization tool is used with the constrained nonlinear minimization solver FMINCON by the SQP algorithm. Although our problem is linearly constrained, we take advantage of the SQP algorithm, which shows the strict feasibility to bounds and robustness to nondouble results. The proposed algorithm for the optimization process is listed as follows.

Step 1) Initialize all the parameters. Initialize the variable of scheduling threshold and the maintenance threshold in the feasible region.

Step 2) Create the objective function of the long-run expected cost rate by substituting (6), (8), (12), (17), and (18) into (20).

Step 3) Create the constraint functions $0 \leq X_S \leq X_M \leq X_F$.

Step 4) Update the Hessian Matrix.

Step 5) Solve the quadratic programming.

Step 6) Use line search on the merit function.

Step 7) If the tolerance of the objective function is larger than the specified value, go back to step 4).

Step 8) Output the scheduling threshold, the maintenance threshold, and the long-run expected cost rate.

TABLE II
DECISION PARAMETERS IN THE NUMERICAL EXAMPLE

Parameters	C_1	C_2	C_3	α	β	T_L	X_F
value	15	20	40	0.3	2	5	20

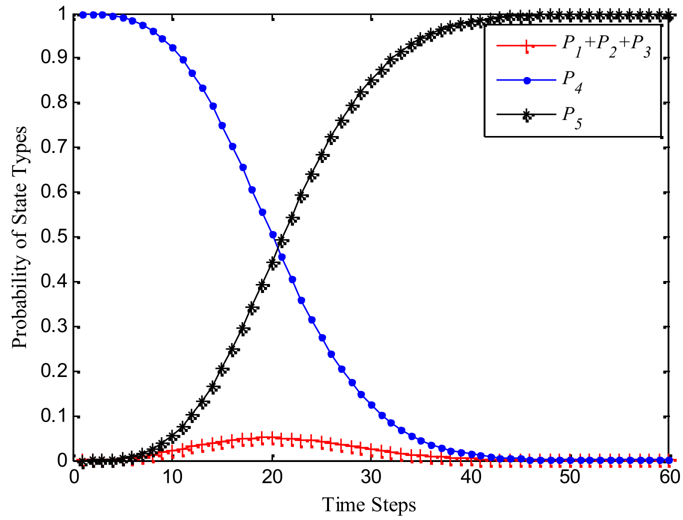

Fig. 6. Optimal P_4, P_5, and the sum of P_1, P_2, and P_3.

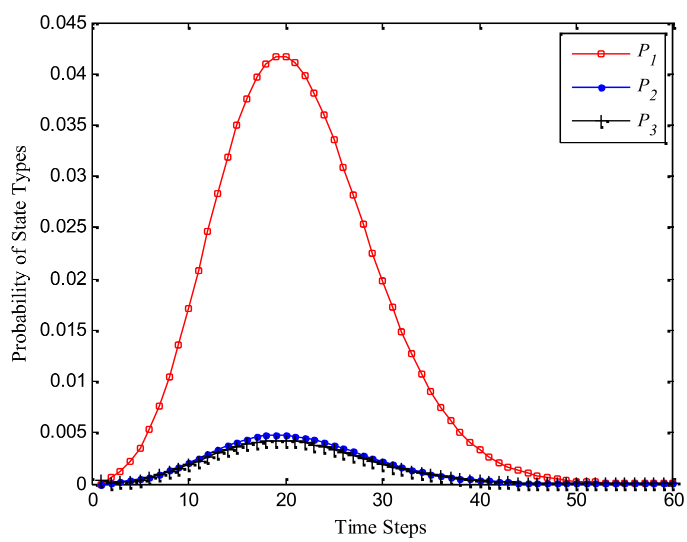

Fig. 7. Optimal P_1, P_2, and P_3.

VI. Numerical Study

A. Numerical Example

Here, we use a numerical example to demonstrate the optimal condition-based maintenance policy with a scheduling threshold and a maintenance threshold. The example follows all of the mentioned system assumptions and maintenance assumptions.

Initialize the parameters as listed in Table II. The tolerance of the objective in this example is 1e − 4, whereas the tolerances of the variables X_S X_M are not restricted.

System types are plotted with the optimized variables of X_S and X_M at each time step in Figs. 6–9.

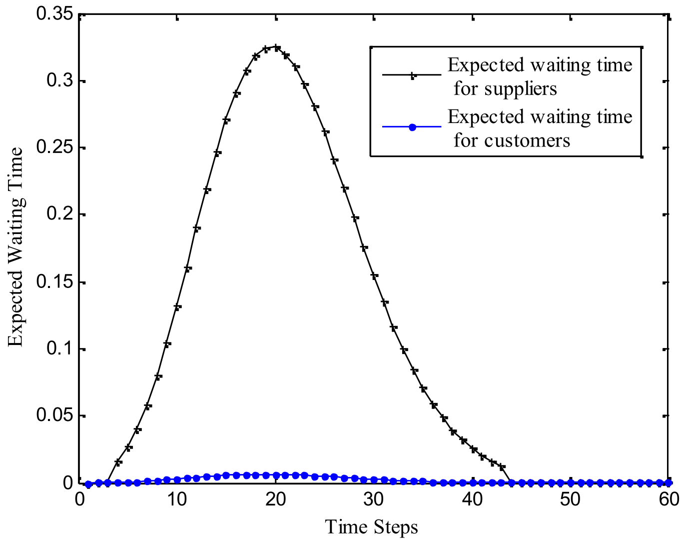

Fig. 8. Expected waiting time of suppliers and customers.

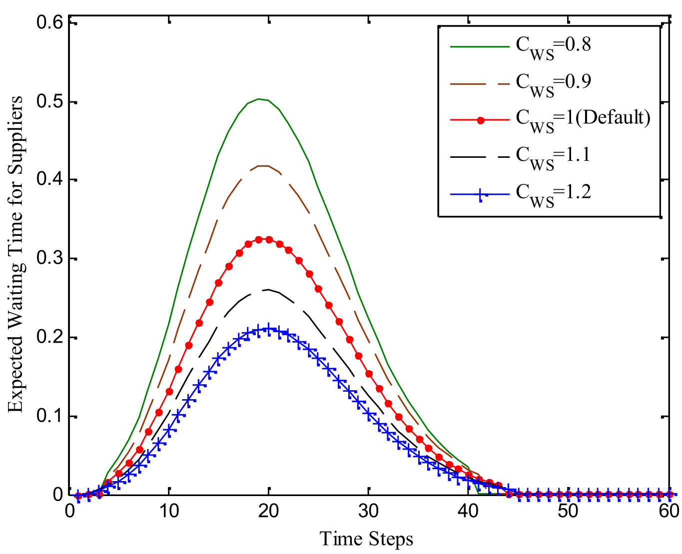

Fig. 9. Expected waiting time of suppliers depending on C_{WS}.

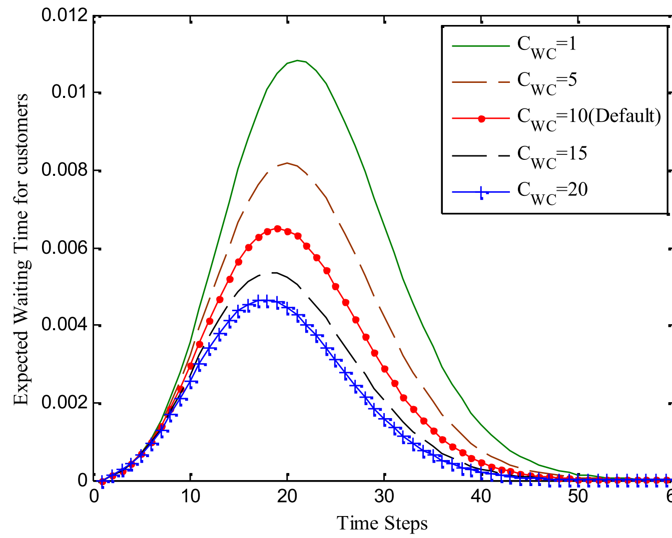

Fig. 10. Expected waiting time of customers depending on C_{WC}.

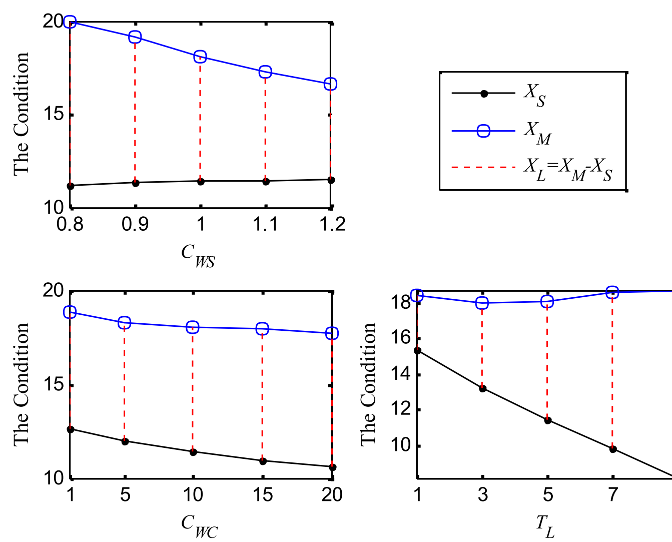

Fig. 11. Sensitivity analysis of optimal decision variables on C_{WS}, C_{WC}, and T_L.

In Fig. 6, the probability of system type 4 decreases with time, and the probability of system type 5 increases with time. It indicates the probability that the system condition is lower than the scheduling threshold is decreasing with time. Meanwhile, the time to schedule is increasing at first, and it is decreasing after a peak. It shows the distribution of scheduling time. In this case, the time to schedule is distributed and is decided by the degradation and the scheduling threshold.

In Fig. 7, when the degradation reaches the scheduling threshold, the probabilities of the three maintenance cost types are presented. In this case, the probability of type 1 is larger than type 2 and type 3, while the probabilities of types 2 and 3 are similar. The optimization results imply that we prefer the ability to perform maintenance immediately when the degradation reaches the maintenance threshold.

In Fig. 8, the expected waiting time of suppliers and customers are plotted. At each schedule time, the expected waiting time of suppliers is much higher than the expected waiting time of customers. Reasons lie in two points. First, the cost of waiting time of customers is much more than that of suppliers, so the

TABLE III

RESULTS OF THE NUMERICAL EXAMPLE OF THE PROPOSED METHOD

C_1	C_2	C_3	C_{WS}	C_{WC}	T_L	X_S	X_M	CR	$E(T_{WS})$	$E(T_{WC})$
15	20	40	0.8	10	5	11.1826	20.0000	0.7299	9.6532	0.1152
15	20	40	0.9	10	5	11.3488	19.1610	0.7559	8.0861	0.1243
15	**20**	**40**	**1**	**10**	**5**	**11.4082**	**18.0638**	**0.7776**	**6.3362**	**0.1278**
15	20	40	1.1	10	5	11.4595	17.2639	0.7956	5.0875	0.1308
15	20	40	1.2	10	5	11.5059	16.6409	0.8104	4.1350	0.1336
15	20	40	1	1	5	12.6429	18.8087	0.7325	5.6075	0.2248
15	20	40	1	5	5	11.9597	18.3029	0.7555	5.8755	0.1646
15	**20**	**40**	**1**	**10**	**5**	**11.4082**	**18.0638**	**0.7776**	**6.3362**	**0.1278**
15	20	40	1	15	5	10.9444	17.9038	0.7955	6.7951	0.1032
15	20	40	1	20	5	10.6044	17.7321	0.8106	7.0498	0.0882
15	20	40	1	10	1	15.3161	18.4329	0.6764	4.2699	0.0404
15	20	40	1	10	3	13.1990	17.9719	0.7333	5.1551	0.0882
15	**20**	**40**	**1**	**10**	**5**	**11.4082**	**18.0638**	**0.7776**	**6.3362**	**0.1278**
15	20	40	1	10	7	9.7758	18.5636	0.8156	7.8577	0.1662
15	20	40	1	10	9	8.1343	18.7071	0.8491	8.8338	0.1964

TABLE IV
RESULTS OF THE NUMERICAL EXAMPLE FOR COMPARISON

CASES	C_1	C_2	C_3	C_{WS}	C_{WC}	T_L	X_S	X_M	CR
PROPOSED	**15**	**20**	**40**	**1**	**10**	**5**	**11.4082**	**18.0638**	**0.7776**
CASE 1	15	20	40	1	10	5	11.6997	20.0000	0.7822
CASE 2	15	20	40	1	10	5	11.6898	11.6898	0.8804
CASE 3	15	20	40	1	10	5	11.5180	14.5180	0.8167
CASE 4	15	20	40	1	10	5	11. 5167	14. 5167	0.8167

corresponding costly time is reduced by optimization. Second, the failure are costly when maintenance resources are not ready.

From the results in Figs. 6–8, the optimal plan prefers to reduce the waiting time of customers, as well as to make the maintenance resources available before the time when the system condition reaches the maintenance threshold. It is generally consistent with the engineering practice.

Sensitivity analysis for the expected waiting time on the cost of the waiting time of customers and suppliers with the optimization results are plotted in Figs. 9 and 10, respectively. The peak of $E(T_{WS})$ decreases as the cost C_{WS} increases. In the same way, the peak of $E(T_{WC})$ decreases as the cost C_{WC} increases. It shows that the cost of the waiting time makes a significant impact on the optimal expected waiting time.

Sensitivity analysis of optimal decision variables on C_{WC} C_{WS} and T_L is plotted in Fig. 11. The C_{WS} has a great effect on X_M and a slight effect on X_S, whereas, the C_{WC} has a great effect on X_S and a slight effect on X_M. It implies the growth of cost of the waiting time of suppliers leads to reduction in optimal maintenance threshold and lead threshold. In the same way, the increased cost of the waiting time of customers results in low scheduling threshold and the long lead threshold. At the same time, the lead threshold X_L increase as the lead time T_L increase, with the decreasing X_S and the fluctuation of X_M.

From Figs. 9–11 and Table III, we can find out that: with the increasing of the cost of the waiting time of customers, the optimal scheduling threshold is declined and the expected waiting time of customers is also decreased; with the increasing of the cost of the waiting time of suppliers, the optimal maintenance threshold is declined and the expected waiting time of suppliers is also decreased; with the increasing of the lead time, the distance between the optimal scheduling threshold and the optimal maintenance threshold is increasing.

B. Compared With Other Maintenance Plan

Four cases are similar to our plan and are presented here to be compared with our method. They are given here.

Case 1) Tthe maintenance threshold equals the failure threshold.

Case 2) Tthe maintenance threshold equals the scheduling threshold.

Case 3) The scheduling threshold is a variable, and the maintenance threshold is based on the variable and the expected increasing value of the degradation during the lead time.

Case 4) The maintenance threshold is a variable, and the scheduling threshold is based on the variable and the expected increasing value of the degradation during the lead time.

Then we calculate the expected increasing value of the degradation during the lead time $\alpha\beta T_L$, based on the definition of the expectation of the gamma process. In case 3, we calculate the maintenance threshold as a function of the scheduling threshold by the equation $X_M = X_S + \alpha\beta T_L$; in case 4, we calculate the scheduling threshold as a function of maintenance threshold by the equation $X_S = X_M - \alpha\beta T_L$.

The optimization results for these four kinds of cases are listed in Table IV. As shown, the cost rate among the results shows that the proposed method of three thresholds based maintenance plan take advantage of two variable thresholds to obtain the optimal cost rate. The comparison indicates the role of our three thresholds maintenance plan in optimal decisions with lead time.

VII. CONCLUSION

In this study, a framework of condition-based maintenance with a scheduling threshold and a maintenance threshold is developed. Unlike existing models, system types are classified by the information from the past, the present and the potential future, compared with the scheduling threshold, the maintenance threshold and the failure threshold. The gamma process is used to demonstrate the three-threshold scheme. Moreover, within the scheme of the proposed system types, the expected waiting time of suppliers and for customers are presented. And the optimal maintenance plan with the objective of minimizing the long-run cost rate is given. Furthermore, the optimal maintenance plan is updated dynamically in the framework of PHM. Finally, a numerical example with sensitivity analysis shows that: 1) if the cost of waiting time of suppliers per unit time is increased, the optimum maintenance threshold is reduced; 2) if the cost of waiting time of suppliers per unit time is increased, the optimum scheduling threshold is reduced; and 3) if the lead time is increased, the optimum scheduling threshold is reduced and the lead threshold is extended. These trends are useful to optimize condition-based maintenance with the required lead time. Therefore, the proposed scheme improves the maintenance plan from preparation to execution with the online system monitoring.

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