Belief Universal Generating Function Analysis of Multi-State Systems Under Epistemic Uncertainty and Common Cause Failures

Jinhua Mi, Yan-Feng Li, Yu Liu, *Member, IEEE*, Yuan-Jian Yang, and Hong-Zhong Huang, *Member, IEEE*

***Abstract*—Because of the complexity of engineering systems, and the fact that insufficient data are only available to obtain the precise state probability of components, an extended universal generating function (UGF) based on belief function theory is introduced in this paper to conduct the reliability analysis of multi-state systems (MSSs) with epistemic uncertainty. The behavior of common cause failures (CCFs) is further incorporated, and the occurrence probability of CCFs is evaluated using a weighted impact vector method. A numerical example is used to illustrate how the proposed method works. In addition, a global optimization method is used to obtain the truth interval of the system reliability, and the results are compared with those obtained by using some existing methods. The case study shows that the belief UGF method can effectively avoid the interval expansion problem and the overestimation problem involved in the interval UGF method, and the proposed method can be used to provide a reliable way to evaluate the reliability of MSSs with interval data and CCFs.**

***Index Terms*—Universal generating function, multi-state system, epistemic uncertainty, belief function theory, common cause failure, α-factor method.**

Acronyms and Abbreviations

UGF	Universal Generating Function
MSS	Multi-State System
CCF	Common Cause Failure
IUGF	Interval Universal Generating Function
D-S	Dempster-Shafer
IDM	Imprecise Dirichlet Model
MGL	Multiple Greek Letters
BPA	Basic Probability Assignment
BUGF	Belief Universal Generating Function
MLE	Maximum Likelihood Estimation
SSI	Stress Strength Interference

Manuscript received April 26, 2014; revised August 03, 2014 and October 17, 2014; accepted March 31, 2015. Date of publication April 20, 2015; date of current version November 25, 2015. This work was supported by the National Natural Science Foundation of China under contract number U1330130. Associate Editor: G. Levitin.

The authors are with the Institute of Reliability Engineering, University of Electronic Science and Technology of China, Chengdu, Sichuan, 611731, China (e-mail: hzhuang@uestc.edu.cn).

Digital Object Identifier 10.1109/TR.2015.2419620

Notation

D	Frame of discernment
$m(\cdot)$	Mass function
$Bel(X)$	Belief function of event X
$Pel(X)$	Plausibility function of event X
$\mathbf{g}_j$	Performance state space of component j
$g_{j,i}$	ith state of component j
$[g]_{j,i}$	Focal element i after mapping by mass function
$\underline{p}_{j,i}$	Lower bound of ith state probability of component j
$\bar{p}_{j,i}$	Upper bound of ith state probability of component j
$[p_{i,j}]$	Interval state probability
Φ	System structure function
$perf_j$	State performance level of component j
$prob_j$	State probability of component j
$U_i(z)$	z function (UGF) of the component i
$U_j^B(z)$	BUGF of component j
$U_{G_i}^I(z)$	IUGF of component i
$[R_s]$	Interval-valued system reliability
Q_k	Failure frequency of k components
α_k	The k-th parameter of α-factor mode
V_{k}	Component degradation value
I_n	The weighted impact vector
$\bar{F}_k(i)$	The k-th element of weighted impact vector

I. Introduction

BECAUSE a system is designed to perform its intended task under given working conditions, it can usually perform the task with several levels of efficiency or performance. Such a system can be referred to as a multi-state system (MSS) [1], which has attracted much attention during the past few years. Different methods, such as extension Boolean models, stochastic processes, Monte Carlo simulations, and the universal generating function (UGF) method have been proposed to conduct the reliability analysis of MSSs [2]–[19]. Compared with the first three methods, which are only suitable for small scale MSSs due to the computational complexity, the UGF method has a high-computing speed, and is very easy to implement.

The UGF is an important method of modern discrete mathematics, which was first introduced by Ushakov [4], and was extended by Levitin *et al.* [1], [5]–[13] to evaluate the reliability and survivability of different types of MSSs. It is a powerful tool for the reliability modeling and assessment of large scale MSSs. Some extensions of traditional UGFs have been developed to handle the reliability analysis of complex systems under different conditions. Ding and Lisnianski [14], [15] developed a fuzzy UGF to extend the UGF with crisp sets, and gave a definition of fuzzy MSSs. Then, Liu and Huang [16] proposed a modified fuzzy MSS reliability assessment approach to address the uncertainty associated with state transition intersities. Li *et al.* [17] proposed a random fuzzy extension of the UGF method for MSS reliability assessment. Destercke and Sallak [2] presented an extension of the UGF method considering epistemic uncertainties based on belief function theory. Li [18], [19] proposed an interval UGF (IUGF) to analyze the reliability of MSSs when the available data about components are insufficient.

In practice, to ensure the high reliability and high safety of products, especially at the early stage of product design, designers have to face the situation when only insufficient data or imperfect information are available [20]. Uncertainty is a concern in aviation and aerospace industries because of the small samples, complex structure, and expensive cost of these products. There are various methods, such as Bayes theory, fuzzy set theory, Dempster-Shafer (D-S) evidential theory, information-gaps method, etc. [2], [15], [21] to cope with the uncertainty. Wang and Li [22] investigated the effects of uncertainty in both component reliability and load demand on system reliability for general MSSs. An imprecise Dirichlet model (IDM) was used by Troffaes [23] to represent epistemic uncertainty in the α-factor model. Zaman *et al.* [24] proposed a probabilistic approach to represent interval data for input variables in uncertainty analysis. A likelihood-based methodology was proposed to represent the epistemic uncertainty due to interval data or sparse point data by Sankararaman and Mahadevan [25].

Common cause failures (CCFs) are initiated by some common or shared fundamental causes. CCFs are s-dependent failures, which widely exist in engineering systems, such as mechanical systems and electric systems. For a system with CCFs, the reliability assessment, which is based on the assumption that the component failures are s-independent, may lead to errors [26], [27]. Considering CCFs in systems, many methods have been proposed to estimate and evaluate the probability of a common cause event. Examples include the basic parameter model, α-factor method, β-factor method, multiple Greek letters (MGL) model, etc. [28]. The β-factor method was introduced by Fleming [29], which is one of the simplest and the most commonly used quantitative methods for CCFs in engineering. However, this β-factor method is based on published statistics and engineering experience, can be only used as a rough estimate, and cannot be used to get an accurate result [30]. The α-factor model was developed by Mosleh and Siu [28], [31], which is a multi-parameter model that can be used to handle any redundancy level systems. The assessment of α-factor model parameters can be performed based on ratios of failure rates when there are no available statistical data [32]. With the progress of the understanding of the CCF occurring-mechanism, the United States Nuclear Regulatory Commission has developed a database for CCF parameters estimation based on the α-factor model and the MGL model [32], [33]. Over the past few years, the methods used for estimating the α-factor for CCF systems have been developed, and some extensions of α-factor have been made [34], [35].

In general, there exist extensive research efforts on using UGFs for the reliability analysis of MSSs subject to CCFs [7]–[13]. The interval mathematics or belief function in combination with UGFs has been used to model the uncertainty in MSSs [17]–[19]. To the best of our knowledge, none of the existing works has considered the effect of CCFs and uncertainty simultaneously in the reliability analysis of MSSs. In this paper, considering the complexity of the system, and insufficient available data, a method that integrates an extended UGF with the belief function theory is proposed for the reliability analysis of MSSs subject to epistemic uncertainty. Based on the aforementioned advantages of the α-factor method, a weighted impact vector method is employed to quantify the probability of CCFs in the system.

The remainder of this paper is organized as follows. In Section II, we present an extension of UGFs using belief function theory, and review the UGF and interval-valued UGFs. We employ the α-factor model and the weighted impact vector to calculate the occurrence probability of CCFs in Section III. Section IV modifies the form of belief UGFs (BUGF) and IUGFs when the CCFs in MSSs are considered. A numerical example is provided to illustrate the proposed method in Section V. Because the global optimization method can consider all the constraint conditions, it is used to obtain the true interval of the system reliability, and the results are compared with the BUGF and IUGF methods. Section VI concludes this paper.

II. UGF and Its Extension in MSSs Reliability Analysis

The UGF is an important method for the reliability analysis of MSS, which can clearly establish the relationship between state performances and probabilities of components and systems. This UGF method is flexible, and it has been widely applied to design different synthetic operators for different system structures and working conditions.

The basis of MSS reliability analysis is the state performance of components and their corresponding state probabilities. Lots of information and data are needed to determine the exact probability and performance level of each component state when the probabilistic methods are used to perform the reliability analysis of MSSs. In engineering practice, because of the complexity of systems, limited test samples, and insufficient data, the accurate values of state performance levels and state probabilities cannot be obtained; but we can get good upper and lower bounds of them. In this case, the probabilistic method is no longer applicable, and the non-probabilistic methods, such as interval theory, fuzzy theory, possibility theory, and belief function theory, have to be applied to the reliability analysis of MSSs.

A. Belief Function Theory

Belief function theory is an important theory for representing and manipulating epistemic uncertainties. It was first proposed by Dempster [36], [37], and then developed by Shafer [38]. The basic idea is to use a multivalued mapping method to obtain the upper and lower bounds of probabilities. Specifically, the basic probability assignment (BPA) function is used to obtain the posterior confidence interval, and the plausibility function and belief function are used as the upper and lower bounds of a confidence interval to describe the uncertainty of propositions. The basic notions and operations of belief function theory are explained as follows. For more details, refer to [39]–[42].

Belief function theory is defined on a discrete set called the frame of discernment D, which is composed of a series of s-independent and mutually exclusive discriminate assumptions. Moreover, D is a sample space of variable x, and thus contains all possible values of x. The belief function is based on the power set of D, and the power set contains 2^n elements when D contains n elements. A BPA is defined on the frame of discernment D to describe the differentiation of focal elements; that is, $m(X) : 2^D \to [0,1]$, which means the mapping from each element of 2^D to a number in [0, 1], and satisfies the following two conditions.

1) The BPA of null set $\emptyset$ is equal to zero; that is, $m(\emptyset) = 0$.
2) Normalization, such that $\sum_{X \in 2^D} m(X) = 1$.

The second condition means that the sum of all the BPA of focal elements in power set 2^D equals to 1. Each set X for which $m(X) > 0$ is referred to as a focal element. The mass $m(X)$ is the BPA of each focal element. For $\forall X \subseteq D, m(X)$ represents the precise belief degree of focal element X, and $m(D)$ is the degree of uncertainty and the ignorance of a proposition [43].

For any event $X \subseteq D$, the belief function and plausibility function could be defined from a mass function m as

$$Bel(X) = \sum_{\substack{Y \subseteq X \\ X \subseteq D}} m(Y), \tag{1}$$

$$Pl(X) = \sum_{Y \cap X \neq \emptyset} m(Y). \tag{2}$$

$Bel(X)$ is the total mass of information implying the occurrence of X, and it can be viewed as the lower bound on a set of probability measures. $Pl(X)$ measures the degree of event X consistent with information $m(Y)$, and it can be viewed as the upper bound. The value $Pl(X) - Bel(X)$ is a measure of information insufficient regarding the uncertainty of event X, which is the epistemic uncertainty about X [42], [44]. Then, the pair of functions $[Bel(X),\ Pl(X)]$ composes an integrated uncertainty interval of event X, which can be used to describe the uncertainty of focal element X. Fig. 1 shows the division of the belief degree of X in belief function theory.

When the interval length of $[Bel(X), Pl(X)]$ for each element in the frame of discernment D is equal to zero, which means $Pl(X) = 1 - Bel(X)$, then the belief function theory is the same as the Bayesian theory. $[Bel(X), Pl(X)] = [0,1]$ means a complete lack of information, and illustrates that this evidence is ignorant of event X

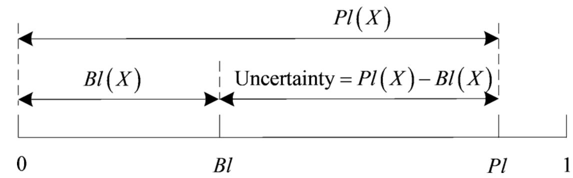

Fig. 1. The division of belief degree of X in belief function theory.

B. Extend UGF to Belief UGF

Suppose a component i has N states, the performance levels of these states are $X = \{x_1, x_2, \cdots, x_N\}$, and the corresponding probability of the component i at state x_j is $p_j = \Pr\{X = x_j\} (1 \leq j \leq N)$. The elements of $\{p_1, p_2, \cdots, p_N\}$ are the corresponding state probabilities of each state performance level. Then, the UGF of the variable X (corresponding to component i) can be expressed as

$$U_i(z) = \sum_{j=1}^{N} p_j z^{x_j}. \tag{3}$$

Assume that component j has m_j states, the performance state space can be expressed as $\mathbf{g}_j = \{g_{j,1}, g_{j,2}, \cdots, g_{j,m_j}\}$, and the corresponding intervals of the states' probabilities are $[\underline{p}_{j,1}, \bar{p}_{j,1}], \cdots, [\underline{p}_{j,m_j}, \bar{p}_{j,m_j}]$. Denoeux [45] proposed an efficient way to obtain a mass function from probability intervals, and this interval method has been mathematically proven in [44], [46]. When considering the computational efficiency, the following precise mass function with fewer focal elements is derived by a simple approximation method from intervals $[\underline{p}_{j,i}, \bar{p}_{j,i}]$ [2], [45], [46],

$$m(S) = \begin{cases} \underline{p}_{j,m} & (S = g_{j,m}) \\ 1 - \sum_{m=1}^{m_j} \underline{p}_{j,m} & \left(S = \{g_{j,1}, g_{j,2}, \cdots, g_{j,m_j}\}\right), \\ 0 & else \end{cases} \tag{4}$$

where S is a set containing all states of component j, and is a mass function on state space $\mathbf{g}_j$, which is the mapping from each element of state probability intervals to set [0, 1]. After the use of the mapping provided by the mass function $m(S)$, the focal elements of components j can be indicated as $[g]_{j,1}, [g]_{j,2}, \cdots, [g]_{j,k_j}$. For a system with n components, when the system performance level g_j is set-valued and expressed as $[g]_j, (1 \leq j \leq n, [g]_j \subset \mathbf{g}_j)$, the relationship of system structures before and after the mapping can be expressed as

$$\Phi\left([g]_1, \cdots, [g]_n\right) = \left\{ \Phi\left(g_1, \cdots, g_n\right) \middle| g_j \in [g]_j \right\}. \tag{5}$$

Let $m_{j,i}$ denote the mass of $[g]_{j,i} (1 \leq i \leq k_j, 1 \leq j \leq n)$, then the information about the state of each component j is transformed into the z function, which is the BUGF. The BUGF of component j with k_j performance states can be defined as

$$U_j^B(z) = \sum_{i=1}^{k_j} m_{j,i} z^{[g]_{j,i}}. \tag{6}$$

To obtain the BUGF of MSSs, using Ω to represent the compound operator of the BUGF, define (7) at the bottom of the page.

When system performance values are interval-valued, Φ can be obtained by interval algorithms in [18]. The BUGF of MSSs with n components is shown in (8) at the bottom of the page.

When the system performance requirement is w, the following two operators are employed to compute the plausibility and belief values of event X [2].

$$1_w^+\left(z^{\Phi}\right)=\begin{cases}1 & \Phi\left([g]_1,\cdots,[g]_n\right)\cap w\neq\emptyset\\ 0 & others\end{cases}, \tag{9}$$

$$1_w^-\left(z^{\Phi}\right)=\begin{cases}1 & \Phi\left([g]_1,\cdots,[g]_n\right)\subseteq w\\ 0 & others\end{cases}. \tag{10}$$

$\Phi([g]_1,\cdots,[g]_n)\cap w\neq\emptyset$ in (9) means that at least one element of the interval-valued performance Φ is above or equal to requirement w. $\Phi([g]_1,\cdots,[g]_n)\subseteq w$ as used in (10) means that all elements in Φ are above or equal to w. For instance, when the system space of a 3-state system is $\{1.5,2,\{1.5,2\}\}$, and $w=1.7$, then $1_w^+(z^{1.5})=0, 1_w^+(z^2)=1$, $1_w^+(z^{\{1.5,2\}})=1$, and $1_w^-(z^{1.5})=0, 1_w^-(z^2)=1$, $1_w^-(z^{\{1.5,2\}})=0$. While the state performance meets the requirement w, the upper bound $Pl(w)$ and lower bound $Bel(w)$ of the system reliability can be calculated by formulas (11) and (12) at the bottom of the page.

C. Interval UGF

For comparison, a non-probabilistic method called the IUGF, proposed by Li *et al.* [18], is employed in this paper. An interval number is defined as $[y]=[\underline{y},\bar{y}]=\{y|\underline{y}\leq y\leq\bar{y}\}$. For interval numbers $[x]=[\underline{x},\bar{x}]$ and $[y]$, the basic multiplication operation used in this paper can be defined as

$$[x]\cdot[y]=\left[\min\left\{\underline{x}\underline{y},\underline{x}\bar{y},\bar{x}\underline{y},\bar{x}\bar{y}\right\},\max\left\{\underline{x}\underline{y},\underline{x}\bar{y},\bar{x}\underline{y},\bar{x}\bar{y}\right\}\right], \tag{13}$$

when $\underline{x}\geq 0, \underline{y}\geq 0$. The multiplication rule can be simplified as

$$[x]\cdot[y]=\left[\underline{x}\underline{y},\bar{x}\bar{y}\right]. \tag{14}$$

Let $prob_i=\{[p_{i,1}],[p_{i,2}],\cdots,[p_{i,Ni}]\}$ denote the interval-valued state probabilities. Then the IUGF of component i is defined as

$$U_{G_i}^I(z)=\sum_{j=1}^{N_i}[p_{i,j}]\,z^{x_{ij}}. \tag{15}$$

According to the component state performance and the state probability, the system UGF can be derived from the system structure and the compound operation of the component UGF. When Ω is used to represent the compound operator, the system UGF can be expressed as

$$U_s^I(z)=\Omega\left(U_{G_1}^I(z),U_{G_2}^I(z),\cdots,U_{G_k}^I(z)\right)=\sum_{s=1}^{N_s}prob_s z^{perf_s} \tag{16}$$

where N_s is the number of system states, and $perf_s$ and $prob_s$ are the state performance and the corresponding state interval probability. When the system is composed of two components, and all state performances of the components are precise, then the compound operator Ω can be defined as

$$\Omega\left(U_{G_1}(z),U_{G_2}(z)\right)=\sum_{l=1}^{N_1}\sum_{k=1}^{N_2}[p_{1k}]\cdot[p_{2l}]z^{perf(x_{1k},x_{2l})}. \tag{17}$$

When these two components are in series, which means that the system performance level is equal to the minimum of that of the components, then $perf_1(x_{1k},x_{2l})=\min(x_{1k},x_{2l})$. When the system performance level is equal to the maximum of all components' performance levels, $perf_2(x_{1k},x_{2l})=$

$$\Omega_{\phi}\left(U_1^B(z),U_2^B(z)\right)=\sum_{i_1=1}^{k_1}\sum_{i_2=1}^{k_2}m_{1,i_1}\cdot m_{2,i_2}z^{\Phi\left([g]_{1,i_1},[g]_{n,i_2}\right)} \tag{7}$$

$$U_s^B(z)=\Omega_{\phi}\left(U_1^B(z),\cdots,U_n^B(z)\right)=\sum_{i_1=1}^{k_1}\cdots\sum_{i_n=1}^{k_n}\prod_{j=1}^{n}m_{j,i_j}z^{\Phi\left([g]_{1,i_1},\cdots,[g]_{n,i_n}\right)} \tag{8}$$

$$Pl(w)=1_w^+(U(z))=1_w^+\left(\sum_{i_1=1}^{k_1}\cdots\sum_{i_n=1}^{k_n}\prod_{j=1}^{n}m_{ji_j}z^{\Phi}\right)=\sum_{i_1=1}^{k_1}\cdots\sum_{i_n=1}^{k_n}\prod_{j=1}^{n}m_{ji_j}1_w^+, \tag{11}$$

$$Bel(w)=1_w^-(U(z))=1_w^-\left(\sum_{i_1=1}^{k_1}\cdots\sum_{i_n=1}^{k_n}\prod_{j=1}^{n}m_{ji_j}z^{\Phi}\right)=\sum_{i_1=1}^{k_1}\cdots\sum_{i_n=1}^{k_n}\prod_{j=1}^{n}m_{ji_j}1_w^- \tag{12}$$

$\max(x_{1k}, x_{2l})$. When the system performance level is equal to the sum of the components' performance levels, $perf_3(x_{1k}, x_{2l}) = x_{1k} + x_{2l}$.

For a demand level w, the system reliability $[R_s^I]$ is finally computed using (18) at the bottom of the page.

$$\left[R_s^I(w)\right] = P\{perf \geq w\} = \sum_{s=1}^{M_s} [prob_s] \cdot p\left(perf_s - w \geq 0\right) \tag{18}$$

III. Calculating the Probability of CCF Through α-Factor Model and Weighted Impact Vector

A. α-Factor Model

Based on the advantage introduced in Section I, the α-factor model is selected to analyze CCFs in this paper. The frequency of CCFs is determined by a series of component failure ratios and overall failure probability in the α-factor model. Considering a parallel system with 3 components A, B, and C, when considering the CCF, the failure probability of component A is composed of 4 parts: $P(A_I) = Q_1$, which is the probability of s-independent failure of A; $P(C_{AB}) = Q_2$, which is the CCF probability of components A and B; $P(C_{AC})$, which is the CCF probability of components A and C; and $P(C_{ABC}) = Q_3$, which is the failure probability of the whole set of three components caused by CCFs. The failure probability of component A is $P(A_T) = P(A_I) + P(C_{AB}) + P(C_{AC}) + P(C_{ABC})$. Assume that $P(A_I) = P(B_I) = P(C_I) = Q_1$, and $P(C_{AB}) = P(C_{AC}) = P(C_{BC}) = Q_2$. The component A failure probability is $Q_T = Q_1 + 2Q_2 + Q_3$ [28].

The α-factors are $\alpha_1 = Q_1/Q_{\mathrm{T}}, \alpha_2 = 2Q_2/Q_{\mathrm{T}}$, and $\alpha_3 = Q_3/Q_{\mathrm{T}}$. Therefore, the system failure probability Q_s can be written as (19) at the bottom of the page.

$$\begin{aligned} Q_s &= P(A_I) + P(B_I) + P(C_I) + P(C_{AB}) + P(C_{AC}) + P(C_{BC}) + P(C_{ABC}) \\ &= 3Q_1 + 3Q_2 + Q_3 \\ &= 3(\alpha_1 Q_T) + \frac{3}{2}(\alpha_2 Q_T) + (\alpha_3 Q_T) \end{aligned} \tag{19}$$

Therefore, for a system with m components, Q_T represents the total failure frequency of each component, and α_k is the fraction of the total frequency of failure events that occur in the system involving the failure of k components. For the staggered testing scheme, where only one component is tested in a test episode, and when there is a failure, the rest of the components will be tested. The α-factor is defined as [32], [33]

$$\alpha_k = \frac{\binom{m-1}{k-1} Q_k^{(m)}}{Q_t}, \tag{20}$$

with $\sum_{k=1}^{m} \alpha_k = 1$. Using the maximum likelihood estimation (MLE) method, the parameter of the α-factor model is derived as

$$\hat{\alpha}_k = \frac{n_k}{\sum\limits_{j=1}^{m} n_j}, \tag{21}$$

where $\hat{\alpha}_k$ is the k-th estimated parameter for the α-factor model, and n_j is the number of basic events when j components fail simultaneously, $(1 \leq j \leq m)$. Thus, n_k is the total number of basic events involving the failure of k similar components, and we use the following weighted impact vector method to get the n_k.

B. Weighted Impact Vector Method

According to the strict definition of CCFs, multiple redundant components will fail simultaneously when a CCF event occurs. However, in engineering, components often suffer degradation rather than direct failure. Here, we use the component degradation value V_{k} to describe the relationship between the redundant components and the probability of CCFs. The value V_{k} actually measures the severity of component degradation (the probability of component failure caused by its functional degradation). There are four categories of impact factors for component CCFs: the external environment; the interior component aging; the design, manufacturing, and installation quality; and human errors [47]. Only the first two categories are considered in this paper, and are elaborated as follows.

1) External Environment: CCFs occur when the harsh working environment of the component exceeds the design limitation, and the external environment impact factor begins to take effect on the component degradation factor. The stress strength interference (SSI) model is introduced to compute V_{k}. In this model, the strength of a component is defined as the ability to withstand environmental stress. The SSI model for environment stress and component strength is shown in Fig. 2. The resistance abilities to environmental stress of a large number of products will follows a normal distribution. When environmental stress meets or exceeds the resistance abilities of the product, the products with low resistance ability to environmental stress will fail. The failure probability of the component increases with the area of the overlapped range, so the area is defined as V_{k}.

2) Interior Component Aging: Let V_{k} denote the failure probability of a component due to its interior component aging.

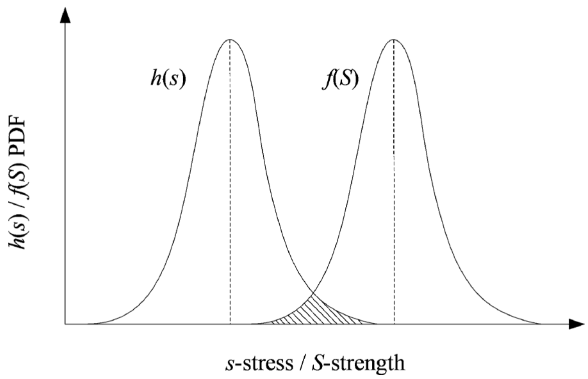

Fig. 2. Calculation of V_k based on SSI model.

TABLE I
IMPACT VECTOR ASSESSMENT OF COMPONENT DEGRADATIONS

Comp group size n	Elements of impact vector $I_n=(F_0, F_1, F_2, \cdots)$				
	F_0	F_1	F_2	F_3	…
2	$(1-V_1)$ $(1-V_2)$	$V_1(1-V_2)+$ $V_2(1-V_1)$	V_1V_2	—	—
3	$(1-V_1)$ $(1-V_2)$ $(1-V_3)$	$V_1(1-V_2)(1-V_3)+$ $V_2(1-V_1)(1-V_3)+$ $V_3(1-V_1)(1-V_2)$	$V_1V_2(1-V_3)+$ $V_1V_3(1-V_2)+$ $V_2V_3(1-V_1)$	$V_1V_2V_3$	—
…	—	—	—	—	—

The failure probability of components is calculated by classical life distributions, e.g., the exponential distribution is used for electronic components, and the Weibull distribution is used for mechanic components, and so on.

While the degradation value of the components is regarded as the failure probability, the average event impact vector for various elements can be calculated based on all the combination of the possible failures. The V_{k} based computation method for the weighted impact vector is shown in Table I, where I_n is the impact vector, and F_n are the elements of the impact vector which can be easily calculated for component groups of sizes 2 and 3 [32], [47].

Using the weighted impact vector, the number of basic events involving k components failing simultaneously can be calculated by

$$n_k = \sum_{i=1}^{m} w_i \bar{F}_k(i), \tag{22}$$

where m is the number of CCF categories considered in the CCF analysis, w_i is the weight of the i-th CCF cause category (determined by the industrial reliability database), and $\bar{F}_k(i)$ can be calculated by using the method in Table I.

C. Example

Suppose a system is comprised of two components: a controller, and a body part. The life distribution of the controller conforms to an exponential distribution with a parameter λ, and the life distribution of the body part obeys a Weibull distribution with two parameters α and β [47]. The parameters of these two components are listed in Table II. The component degradation value V_k, and the impact vector I_n are listed in Table II.

TABLE II
THE MODEL PARAMETERS AND RESULTS OF WEIGHTED IMPACT VECTOR METHOD

Impact factor	Parameters	W_i	V_1	V_2	Weighted impact vector
Exterior	Environmental $s\sim$ N(0,1) $S_1\sim$ N(3.1,1.5^2) $S_2\sim$ N(3.2, 0.7^2)	0.329	0.21	0.06	(0.743, 0.245, 0.0126)
Interior	λ_1=35; α_1=7, β_1=17 λ_2=46; α_2=7.3, β_2=15	0.671	0.12	0.1	(0.792, 0.0196, 0.012)

From (20) and (21), n_1, n_2, and α_2 are calculated as 0.0938, 0.0122, and 0.1151, respectively. Thus, the CCF probability α_c for this redundant system comprised of two components is 0.1151.

IV. INCORPORATING CCFS INTO BELIEF UGF OF MSS

To incorporate the impact of CCFs on the performance of MSSs, the UGF is modified in this section, and some assumptions used in the proposed method are made in the following.

1) The system is non-repairable.
2) The generalized strength of each component in the system is s-independent.
3) There are N types of common cause groups in the system, and each group (e.g., group j) is composed of r_j identical elements.

The failure rates of different common cause levels are only related to the number of failed components, and are irrelevant to specific components [19], [48]. Assume that all components of a common cause group will fail when any loads exceed the limit of components. When explosion occurs, the load is much larger than the strength of all components of a common cause group, and the identical elements in this group will fail at the same time.

In this paper, each common cause group has only two failure models. One is the s-independent component failure, and the other is the failure, of all components in the group at the same time or within a short time interval that can be neglected. Let $U_i(\mathrm{z})$ represent the UGF of the s-independent failure model of the common cause group, and α_c represent the occurrence probability of CCFs. The UGF of a subsystem with a CCF $U_i^C(\mathrm{z})$ can be thus described as [48]

$$U_i^C(z) = (1-\alpha_c)U_i(z) + \alpha_c z^{x_c}, \tag{23}$$

where x_c represents the output performance of a common cause group when a CCF occurs.

For a subsystem S comprised of k_s types of components, the UGF of system S is $U_s(z)$ when a CCF is absent. Assume that the system CCFs are caused by only one kind of external load, and the limit working stresses of k_s components meet $h_1 < h_2 < \cdots < h_{k_s}$. When all components of type i $(i = 1, \cdots, k_s)$ in the system fail at a probability of $C_{1,\ldots,i}$ due to a common cause, the UGF of system S is expressed as $U_{1,\ldots,i}^C(z)$, which means that all the components of type 1 to type i fail due to this

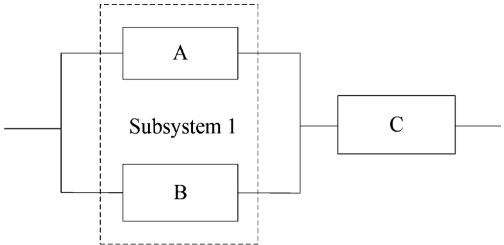

Fig. 3. A flow transmission system.

external load [19], [48]. Thus, the system UGF when considering CCF can be expressed as

$$U_s^C(z) = U_s(z)\left(1 - \sum_{i=1}^{k_s} C_{1,\cdots,i}\right) + \sum_{i=1}^{k_s} U_{1,\cdots,i}^C(z) C_{1,\cdots,i}. \tag{24}$$

Finally, the UGF of the entire system with CCFs can be calculated by the subsystems' UGFs with a reasonable combination operator Ω based on the system structure.

V. An Illustrative Example

In this example, we evaluate the reliability of a flow transmission system comprised of three pipes, as shown in Fig. 3 (the same system can also be found in [2], [18], [19]). All state performances of the components are precise, and the probability of each state is listed in Table III.

A. BUGF Method for MSSs With CCF

According to (4) and (6), the BUGF of each component is defined as

$$U_A^B(z) = 0.096z^0 + 0.095z^1 + 0.795z^{1.5} + 0.014z^{0,1,1.5}, \tag{25}$$

$$U_B^B(z) = 0.090z^0 + 0.195z^{1.5} + 0.695z^2 + 0.02z^{0,1.5,2}, \tag{26}$$

$$U_C^B(z) = 0.035z^0 + 0.958z^4 + 0.007z^{0,4}. \tag{27}$$

Because components A and B are in parallel, the performance level of subsystem 1 is equal to the sum of the performance of A and B. According to (8), the BUGF of subsystem 1 is expressed as (28) at the bottom of the page.

Subsystems 1 and C are in series; thus, the performance level of the overall system is equal to the minimum of subsystems 1 and C. Hence, the BUGF of the entire system is given as

$$U_s^B(z) = \Omega_{\min}\left(U_{sub1}^B(z), U_C^B(z)\right). \tag{29}$$

From (9) to (12), when the system performance requirement is $w = 1.5$, the upper bound $Pl(w)$, and lower bound $Bel(w)$ of the system reliability can be calculated by the (30) and (31) at the bottom of the page.

So the system reliability is $[R_s^B] = [Bel^B(w), Pl^B(w)] = [0.9253, 0.9484]$. For the system shown in Fig. 3, assume that components A and B are affected by some common cause events, and the occurrence probability of CCFs has been calculated using the method reported in Section III, which is $\alpha_c = 0.1151$. The BUGF $U_{sub1}^B(z)$ of subsystem 1 is known in the previous discussion. Then, according to (23), the BUGF of subsystem 1 with CCFs can be calculated as (32) at the bottom of the page.

The BUGF of the overall system considering CCFs is

$$U_s^{B,C}(z) = \Omega_{\min}\left(U_{sub1}^{B,C}(z), U_C^B(z)\right). \tag{33}$$

From (11) and (12), the upper bound $Pl(w)$, and lower bound $Bel(w)$ of the system reliability at $w = 1.5$ when considering CCFs can be calculated, and $[R_s^{B,C}] = [Bel^{B,C}(w = 1.5), Pl^{B,C}(w = 1.5)] = [0.8187, 0.8392]$.

$$\begin{aligned} U_{sub1}^B(z) = \Omega_{\text{sum}}\left(U_A^B(z), U_B^B(z)\right) = {} & 0.00864z^0 + 0.00855z^1 + 0.09027z^{1.5} + 0.06672z^2 + \\ & 0.018525z^{2.5} + 0.22105z^3 + 0.552525z^{3.5} + 0.00126z^{0,1,1.5} + 0.00192z^{0,1.5,2} + \\ & 0.0019z^{1,2.5,3} + 0.00273z^{1.5,2.5,3} + 0.0159z^{1.5,3,3.5} + 0.00973z^{2,3,3.5} + 0.00028z^{0,1,1.5,2,2.5,3,3.5} \end{aligned} \tag{28}$$

$$Pl^B(w = 1.5) = \sum_{i_1=1}^{k_1} \cdots \sum_{i_n=1}^{k_n} \prod_{j=1}^{n} m_{ji_j} 1_w^+ \left(z^{\Phi([g]_1,\cdots,[g]_n)\cap w \neq \emptyset}\right) = 0.9484, \tag{30}$$

$$Bel^B(w = 1.5) = \sum_{i_1=1}^{k_1} \cdots \sum_{i_n=1}^{k_n} \prod_{j=1}^{n} m_{ji_j} 1_w^- \left(z^{\Phi([g]_1,\cdots,[g]_n)\subseteq w}\right) = 0.9253 \tag{31}$$

$$\begin{aligned} U_{sub1}^{B,C}(z) = (1-\alpha_c) U_{sub1}^B(z) + \alpha_c z^0 = {} & 0.12275z^0 + 0.00757z^1 + 0.07988z^{1.5} + 0.05904z^2 + \\ & 0.01639z^{2.5} + 0.195607z^3 + 0.488929z^{3.5} + 0.001115z^{0,1,1.5} + 0.001699z^{0,1.5,2} + \\ & 0.00168z^{1,2.5,3} + 0.002416z^{1.5,2.5,3} + 0.01407z^{1.5,3,3.5} + 0.00861z^{2,3,3.5} + 0.000248z^{0,1,1.5,2,2.5,3,3.5} \end{aligned} \tag{32}$$

TABLE III
PARAMETERS OF THE FLOW TRANSMISSION SYSTEM

Comp i	x_{i1}	$[p_{i1}]$	x_{i2}	$[p_{i2}]$	x_{i3}	$[p_{i3}]$
A	0	[0.096,0.0102]	1	[0.095,0.105]	1.5	[0.795,0.805]
B	0	[0.090,0.110]	1.5	[0.195,0.205]	2	[0.695,0.705]
C	—	—	0	[0.035,0.042]	4	[0.958,0.965]

B. IUGF Method for MSSs With CCF

According to (15), the IUGF equations of the three components are given by (34)–(36) at the bottom of the page.

Similarly, the performance level of subsystem 1 is equal to the sum of A and B, because A and B are in parallel. According to (16) and (17), the IUGF of subsystem 1 is expressed as (37) at the bottom of the page.

Subsystem 1 and C are in series, and the performance level of the system is equal to the minimum of subsystem 1 and C, so the IUGF of the overall system is (38) at the bottom of the page.

From (18), the interval-valued reliability of the system $[R_s^I]$ at $\mathrm{w} = 1.5$ can be calculated as $[R_s^I(\mathrm{w} = 1.5)] = [0.909, 0.974]$.

Assume that components A and B are affected by a common cause event with the occurrence probability $\alpha_c = 0.1151$. According to (23), the IUGF of subsystem 1 with CCFs can be calculated as (39) at the bottom of the page.

The IUGF of the entire system considering CCFs is (40) at the bottom of the page.

From (18), the interval-valued reliability of the system at $w = 1.5$ when considering CCFs can be calculated as $[R_s^{I,C}(w = 1.5)] = [0.805, 0.862]$.

C. System Reliability Analysis Based on a Global Optimization Method

In this section, we use a global optimization method to calculate the reliability of the three-component flow transmission system. The reliability of subsystem 1 is given by $R_{sub1} = p_{A3} + p_{A2}p_{B3} + p_{A2}p_{B2} + p_{A1}p_{B2} + p_{A1}p_{B3}$. The upper and lower bounds of R_{sub1} can be expressed as two optimization propositions. 1) For the upper bound, the objective function is max R_{sub1}, and the constraint conditions are $p_{A1}+p_{A2}+p_{A3} = 1, p_{B1}+p_{B2}+p_{B3} = 1, 0.096 \leq p_{A1} \leq 0.102, 0.095 \leq p_{A2} \leq 0.105, 0.795 \leq p_{A3} \leq 0.805, 0.090 \leq p_{B1} \leq 0.110, 0.195 \leq p_{B2} \leq 0.205, 0.695 \leq p_{B3} \leq 0.705$. 2) For the lower bound, the objective function is min R_{sub1}, and the constraint conditions are identical to proposition 1. Using the MATLAB optimization toolbox, the reliability of subsystem 1 is obtained, and $[R_{sub1}^G] = [0.9774, 0.9825]$. It is obviously that the interval-valued reliability of component C is $[R_C] = [0.958, 0.965]$, then the system reliability $[R_s^G]$ is shown in (41) at the bottom of the page.

Considering the CCF in subsystem 1 with probability $\alpha_c = 0.1151$, the system reliability can be calculated by

$$\left[R_s^{G,C}\right] = \left(\left[R_{sub1}^C\right] - \alpha_c\right) \cdot \left[R_C\right] = [0.8261, 0.8370]. \quad (42)$$

The result of the system reliability obtained from the global optimization algorithm includes all the constraint conditions. Thus, the truth interval of the system reliability is obtained, and the result can be used as a reference in comparison with the other analytical methods' results. Table IV lists the results of the system reliabilities obtained by using the BUGF method,

$$U_A^I(z) = [0.096, 0.0102]\, z^0 + [0.095, 0.105]\, z^1 + [0.795, 0.805]\, z^{1.5}, \quad (34)$$
$$U_B^I(z) = [0.090, 0.110]\, z^0 + [0.195, 0.205]\, z^{1.5} + [0.695, 0.705]\, z^2, \quad (35)$$
$$U_C^I(z) = [0.035, 0.042]\, z^0 + [0.958, 0.965]\, z^4 \quad (36)$$

$$\begin{aligned} U_{sub1}^I(z) = \Omega_{sum}\left(U_A^I(z), U_B^I(z)\right) &= [0.009, 0.011]\, z^0 + [0.009, 0.012]\, z^1 + [0.090, 0.110]\, \mathrm{z}^{1.5} \\ &+ [0.067, 0.072]\, z^2 + [0.019, 0.022]\, z^{2.5} + [0.221, 0.239]\, z^3 + [0.553, 0.568]\, z^{3.5} \end{aligned} \quad (37)$$

$$\begin{aligned} U_s^I(z) = \Omega_{\min}\left(U_{sub1}^I(z), U_C^I(z)\right) &= [0.042, 0.054]\, z^0 + [0.082, 0.011]\, z^1 + [0.087, 0.106]\, \mathrm{z}^{1.5} \\ &+ [0.064, 0.069]\, z^2 + [0.018, 0.021]\, z^{2.5} + [0.212, 0.231]\, z^3 + [0.529, 0.548]\, z^{3.5} \end{aligned} \quad (38)$$

$$\begin{aligned} U_{sub1}^{I,C}(z) = (1-\alpha_c)\, U_{sub1}(z) + \alpha_c z^0 &= [0.123, 0.125]\, z^0 + [0.008, 0.010]\, z^1 + [0.080, 0.097]\, \mathrm{z}^{1.5} \\ &+ [0.059, 0.064]\, z^2 + [0.016, 0.019]\, z^{2.5} + [0.196, 0.212]\, z^3 + [0.490, 0.502]\, z^{3.5} \end{aligned} \quad (39)$$

$$\begin{aligned} U_s^{I,C}(z) = \Omega\left(U_{sub1}(z), U_C(z)\right) &= [0.152, 0.164]\, z^0 + [0.007, 0.010]\, z^1 + [0.077, 0.094]\, \mathrm{z}^{1.5} \\ &+ [0.057, 0.061] z^2 + [0.016, 0.018]\, z^{2.5} + [0.187, 0.204]\, z^3 + [0.468, 0.485]\, z^{3.5} \end{aligned} \quad (40)$$

$$\left[R_s^G\right] = \left[R_{sub1}^G\right] \cdot \left[R_C\right] = [0.9774, 0.9825]\, [0.958, 0.965] = [0.9363, 0.9481] \quad (41)$$

TABLE IV
RESULTS COMPARISON OF DIFFERENT METHODS

Methods	$[R_s(w=1.5)]$	$[R_s^c(w=1.5)]$
BUGF	[0.9253, 0.9484]	[0.8187,0.8392]
IUGF	[0.909, 0.974]	[0.805, 0.862]
Global optimization method	[0.9363, 0.9481]	[0.8261,0.8370]

the IUGF method, and the global optimization method, respectively.

From Table IV, see that $[R_s^G] \subseteq [R_s^B] \subseteq [R_s^I]$, which means that the range of system reliability obtained from the IUGF is greater than that obtained by using the global optimization method which is regarded as the true value. However, the upper and lower bounds of the reliability obtained by the BUGF method are much tighter than the results from the IUGF method. Furthermore, the computational procedure of the BUGF method is much easier, and the complexity mainly depends on the number of focal elements, which makes the computation complexity acceptable [2]. Comparing R_{s}^c with R_{s} in Table IV, it is obvious that the consideration of CCFs can decrease the system reliability greatly. This is to say, CCF can have remarkable effects on the system reliability. Therefore, it is necessary to consider this effect of CCF in the system reliability analysis.

VI. CONCLUSION

In this paper, belief function theory is applied to represent the uncertainty of components' state probabilities due to their capabilities in modeling imprecision and deficiency of knowledge. Based on belief function theory, an extended UGF method is introduced to perform the reliability analysis of MSSs when the component state probabilities are represented by interval values. The α-factor model is chosen to analyze CCFs. Considering the different kinds of impact factors for CCFs, a weighted impact vector method is employed to quantify the occurrence probability of CCFs. Finally, the CCF is incorporated into the BUGF and IUGF of MSSs using a simple formula. Because the global optimization algorithm considers all the constraint conditions, its result is regarded as the benchmark to compare and validate our proposed method with the existing IUGF method. Compared with the IUGF method, the BUGF method avoids the interval expansion problem, and the overestimation problem, of the interval UGF. The analysis of the example system shows that the BUGF method with CCFs can be easily extended and calculated.

The premise of the methods studied in this work is that the state performance of components and the corresponding state probabilities are known, and can be expressed by a non-probabilistic method (e.g., interval-valued method). In a practical engineering system, we usually can get some failure and maintenance data, but we may not be able to get the state probabilities directly. The determination of state performance measures and state probabilities is a problem that should be solved in future work. We will investigate how to estimate component reliability parameters, such as failure and repair rates, while considering epistemic uncertainty.

REFERENCES

[1] G. Levitin, *The Universal Generating Function in Reliability Analysis and Optimization*. Berlin, Germany: Springer-Verlag, 2005.

[2] S. Destercke and M. Sallak, "An extension of universal generating function in multi-state systems considering epistemic uncertainties," *IEEE Trans. Rel.*, vol. 62, no. 2, pp. 504–514, Jun. 2013.

[3] E. Zio, M. Marella, and L. Podofillini, "A Monte Carlo simulation approach to the availability assessment of multi-state systems with operational dependencies," *Rel. Eng. Syst. Safety*, vol. 92, no. 7, pp. 871–882, 2007.

[4] I. Ushakov, "A universal generating function," *Soviet J. Comput. Syst. Sci.*, vol. 5, pp. 118–129, 1986.

[5] G. Levitin and A. Lisnianski, "Importance and sensitivity analysis of multi state systems using the universal generating function method," *Rel. Eng. Syst. Safety*, vol. 65, no. 3, pp. 271–282, 1999.

[6] J. E. Ramirez-Marquez and G. Levitin, "Algorithm for estimating reliability confidence bounds of multi-state systems," *Rel. Eng. Syst. Safety*, vol. 93, no. 8, pp. 1231–1243, 2008.

[7] G. Levitin, "Optimal multilevel protection in series-parallel systems," *Rel. Eng. Syst. Safety*, vol. 81, no. 1, pp. 93–102, 2003.

[8] G. Levitin and S. Amari, "Three types of fault coverage in multi-state systems," in *Proc. 8th Int Conf. Reliability, Maintainability & Safety (ICRMS)*, Chengdu, China, 2009, pp. 122–127.

[9] G. Levitin and S. V. Amari, "Multi-state systems with multi-fault coverage," *Rel. Eng. Syst. Safety*, vol. 93, no. 11, pp. 1730–1739, 2008.

[10] G. Levitin and L. Xing, "Reliability and performance of multi-state systems with propagated failures having selective effect," *Rel. Eng. Syst. Safety*, vol. 95, no. 6, pp. 655–661, 2010.

[11] G. Levitin, "Optimal structure of multi-state systems with uncovered failures," *IEEE Trans. Rel.*, vol. 57, no. 1, pp. 140–148, Mar. 2008.

[12] G. Levitin, L. Xing, H. Ben Haim, and Y. Dai, "Reliability of series-parallel systems with random failure propagation time," *IEEE Trans. Rel.*, vol. 62, no. 3, pp. 637–647, Sep. 2013.

[13] G. Levitin, "Incorporating common-cause failures into series-parallel multi-state system analysis," *IEEE Trans. Rel.*, vol. 50, no. 4, pp. 380–388, Dec. 2001.

[14] Y. Ding and A. Lisnianski, "Fuzzy universal generating functions for multi-state system reliability assessment," *Fuzzy Sets Syst.*, vol. 159, no. 3, pp. 307–324, 2008.

[15] Y. Ding, M. J. Zuo, A. Lisnianski, and Z. Tian, "Fuzzy multi-state systems: General definitions, and performance assessment," *IEEE Trans. Rel.*, vol. 57, no. 4, pp. 589–594, Dec. 2008.

[16] Y. Liu and H. Z. Huang, "Reliability assessment for fuzzy multi-state systems," *Int. J. Syst. Sci.*, vol. 41, no. 4, pp. 365–379, 2010.

[17] Y. F. Li, Y. Ding, and E. Zio, "Random fuzzy extension of the universal generating function approach for the reliability assessment of multi-state systems under aleatory and epistemic uncertainties," *IEEE Trans. Rel.*, vol. 63, no. 1, pp. 13–25, Mar. 2014.

[18] C. Li, X. Chen, X. Yi, and J. Tao, "Interval-valued reliability analysis of multi-state systems," *IEEE Trans. Rel.*, vol. 60, no. 1, pp. 323–330, Mar. 2011.

[19] C. Li, "Research on reliability analysis and optimization based on the multi-state system theory," Ph.D. dissertation, National Univ. Defense Technology, Changsha, China, 2010.

[20] X. D. Zhang, "Multidisciplinary design optimization under uncertainties," Ph.D. dissertation, Univ. Electronic Science and Technology of China, Chengdu, China, 2011.

[21] Y. Ben-Haim, "Uncertainty, probability and information-gaps," *Rel. Eng. Syst. Safety*, vol. 85, no. 1, pp. 249–266, 2004.

[22] Y. Wang and L. Li, "Effects of uncertainty in both component reliability and load demand on multistate system reliability," *IEEE Trans. Syst., Man., Cybern. A*, vol. 42, no. 4, pp. 958–969, 2012.

[23] M. Troffaes, G. Walter, and D. Kelly, "A robust Bayesian approach to modeling epistemic uncertainty in common-cause failure models," *Rel. Eng. Syst. Safety*, vol. 125, pp. 13–21, 2014.

[24] K. Zaman *et al.*, "A probabilistic approach for representation of interval uncertainty," *Rel. Eng. Syst. Safety*, vol. 96, no. 1, pp. 117–130, 2011.

[25] S. Sankararaman and S. Mahadevan, "Likelihood-based representation of epistemic uncertainty due to sparse point data and/or interval data," *Rel. Eng. Syst. Safety*, vol. 96, no. 7, pp. 814–824, 2011.

[26] J. Zhou and L. Xie, "Common cause failure mechanism and risk probability quantitative estimation of multi-state system," *Chin. J. Mech. Eng.*, vol. 44, no. 10, pp. 77–82, 2008.

[27] J. Mi, Y. F. Li, H. Z. Huang, Y. Liu, and X. Zhang, “Reliability analysis of multi-state system with common cause failure based on Bayesian networks,” *Eksploatacja i Niezawodnosc—Maintenance and Reliability*, vol. 15, no. 2, pp. 169–175, 2013.

[28] X. Zheng, A. Yamaguchi, and T. Takata, “α -Decomposition for estimating parameters in common cause failure modeling based on causal inference,” *Rel. Eng. Syst. Safety*, vol. 116, pp. 20–27, 2013.

[29] K. N. Fleming, A Reliability Model for Common Mode Failures in Redundant Safety Systems, General Atomic Report, GA-13284, 1974.

[30] J. Braband, R. Vom Hövel, and H. Schäbe, “Probability of Failure on Demand—The Why and The How,” in *Computer Safety, Reliability, and Security*. Berlin, Germany: Springer, 2009, pp. 46–54.

[31] CCF Parameter Estimations, 2009 update, U.S. Nuclear Regulatory Commission, 2011.

[32] Rev 1_CCF Database and Analysis System Event Data Collection, Classification, and Coding. Washington, DC, USA, 2007, NUREG/CR-6268, Nuclear Regulatory Commission.

[33] Guidelines on Modeling Common-Cause Failures in PRA. Washington, DC, USA, 1998, NUREG/CR-5485, Nuclear Regulatory Commission.

[34] J. K. Vaurio, “Consistent mapping of common cause failure rates and alpha factors,” *Rel. Eng. Syst. Safety*, vol. 92, no. 5, pp. 628–645, 2007.

[35] V. Hassija, C. S. Kumar, and K. Velusamy, “A pragmatic approach to estimate alpha factors for common cause failure analysis,” *Ann. Nucl. Energy*, vol. 63, pp. 317–325, 2014.

[36] A. P. Dempster, “Upper and lower probabilities induced by a multivalued mapping,” *Ann. Math. Statist.*, vol. 38, no. 2, pp. 325–339, 1967.

[37] A. P. Dempster, “A generalization of Bayesian inference,” *J. Roy. Statist. Soc. B.*, vol. 30, no. 2, pp. 205–247, 1968.

[38] G. Shafer, *A Mathematical Theory of Evidence*. Princeton, NJ, USA: Princeton Univ. Press, 1976.

[39] J. P. Yang, H. Z. Huang, L. P. He, and D. Wen, “Failure mode and effects analysis of compressor blades of aeroengines using Dempster-Shafer evidence theory,” in *Proc. ASME 2011 Int. Design Engineering Technical Conf. & Computer and Information in Engineering Conf. (DETC)*, Washington, DC, USA, 2011, pp. 863–870.

[40] J. P. Yang, “Research on evidence theory and its reliability analysis methods and applications for complex systems,” Ph.D. dissertation, , Univ. Electronic Science and Technology of China, Chengdu, China, 2012.

[41] A. P. Dempster, “The Dempster-Shafer calculus for statisticians,” *Int. J. Approx. Reason.*, vol. 48, no. 2, pp. 365–377, 2008.

[42] F. Aguirre, M. Sallak, and W. Schön, “Construction of belief function from statistical data about reliability under epistemic uncerainty,” *IEEE Trans. Rel.*, vol. 62, no. 3, pp. 555–568, Sep. 2013.

[43] M. Beynon, D. Cosker, and D. Marshall, “An expert system for multicriteria decision making using Dempster-Shafer theory,” *Expert Syst. Appl.*, vol. 20, no. 4, pp. 357–367, 2001.

[44] J. W. Hall and J. Lawry, “Generation, combination and extension of random set approximations to coherent lower and upper probabilities,” *Rel. Eng. Syst. Safety*, vol. 85, no. 1, pp. 89–101, 2004.

[45] T. Denoeux, “Reasoning with imprecise belief structures,” *Int. J. Approx. Reason.*, vol. 20, no. 1, pp. 79–111, 1999.

[46] T. Denoeux, “Constructing belief functions from sample data using multinomial confidence regions,” *Int. J. Approx. Reason.*, vol. 42, no. 3, pp. 228–252, 2006.

[47] K. Wang, A. Xu, and H. Wang, “Research of common cause failure analysis of safety instrumented systems based on design diversity,” *China Safety Sci. J.*, vol. 23, no. 3, pp. 91–96, 2013.

[48] C. Li *et al.*, “Heterogeneous redundancy optimization for multi-state series—Parallel systems subject to common cause failures,” *Rel. Eng. Syst. Safety*, vol. 95, no. 3, pp. 202–207, 2010.

Jinhua Mi is currently a Ph.D. candidate in mechanical engineering at the University of Electronic Science and Technology of China.

Her research interests concern reliability analysis, reliability assessments of complex systems under uncertainties and common cause failures, and system reliability modeling.

Yan-Feng Li received his Ph.D. degree in mechatronics engineering from the University of Electronic Science and Technology of China.

He is a Lecturer in the School of Mechanical, Electronic, and Industrial Engineering, at the University of Electronic Science and Technology of China. He was a Visiting Pre-doctoral Fellow in the Department of Mechanical and Aerospace Engineering at Rutgers University, USA from 2011 to 2012. He has published over 30 peer-reviewed papers in international journals and conferences. His research interests include reliability modeling and analysis of complex systems, dynamic fault tree analysis, and Bayesian networks modeling and probabilistic inference.

Yu Liu received his Ph.D. degree in mechatronics engineering from the University of Electronic Science and Technology of China.

He is a Professor in the School of Mechanical, Electronic, and Industrial Engineering, at the University of Electronic Science and Technology of China. He was a Visiting Pre-doctoral Fellow in the Department of Mechanical Engineering at Northwestern University, USA from 2008 to 2010, and a Postdoctoral Research Fellow in the Department of Mechanical Engineering, at the University of Alberta, Canada from 2012 to 2013. He has published over 50 peer-reviewed papers in international journals and conferences. His research interests include system reliability modeling and analysis, maintenance decisions, prognostics and health management, and design under uncertainty.

Yuan-Jian Yang is currently a Ph.D. candidate in mechanical engineering at the University of Electronic Science and Technology of China.

His research interests include reliability testing, reliability analysis and evaluation of complex systems, and dynamic fault tree modeling.

Hong-Zhong Huang received a Ph.D. degree in reliability engineering from Shanghai Jiaotong University, China.

He is a Professor, and the Director of the Institute of Reliability Engineering, University of Electronic Science and Technology of China. He has held visiting appointments at several universities in the USA, Canada, and Asia. He has published 200 journal papers and 5 books in the fields of reliability engineering, optimization design, fuzzy sets theory, and product development. His current research interests include system reliability analysis, warranty, maintenance planning and optimization, and computational intelligence in product design.

Dr. Huang is an ISEAM Fellow, a technical committee member of ESRA, a Co-Editor-in-Chief of the *International Journal of Reliability and Applications*, and an Editorial Board Member of several international journals. He received the William A. J. Golomski Award from the Institute of Industrial Engineers in 2006, and the Best Paper Award of the ICFDM2008, ICMR2011, QR2MSE2013.