

Leveraging Degradation Testing and Condition Monitoring for Field Reliability Analysis With Time-Varying Operating Missions

Weiwen Peng, Yan-Feng Li, Yuan-Jian Yang, Jinhua Mi, and Hong-Zhong Huang, *Member, IEEE*

Abstract—Traditionally, degradation testing and condition monitoring are used separately to investigate field reliability. Barriers are naturally formed between these two types of methods due to condition-discrepancies between lab testing and field monitoring, as well as time-varying missions among product population groups. In this paper, a joint framework for field reliability analysis is presented by integrating degradation testing data as well as mission operating information with condition monitoring observations. A coherent modeling strategy is introduced for the information integration by gradually adopting random effects, dynamic covariates, and marker processes into a baseline stochastic degradation model. In detail, random effects are introduced to cope with the inherent unit-to-unit variation. Dynamic covariates are adopted to deal with the external condition heterogeneity. Marker processes are used to account for the time-varying missions. To facilitate information integration and reliability analysis, the Bayesian method is used to implement parameter estimation and degradation analysis. The reliability assessment of products' populations, degradation prediction, and residual life prediction of individual products are investigated. Finally, an illustrative example for field degradation analysis of oil debris in a lubrication system of a machine tool's spindle system is presented. The effectiveness of information integration and the capability of degradation inference are demonstrated through this example.

Index Terms—Field reliability, degradation model, random effect, dynamic covariate, Bayesian method.

ACRONYMS AND ABBREVIATIONS

RUL	Remaining useful life
ALT	Accelerated life test
ADT	Accelerated degradation test
PDF	Probability density function
CDF	Cumulative distribution function
MCMC	Markov chain Monte Carlo

Manuscript received April 01, 2014; revised October 27, 2014; accepted June 08, 2015. Date of publication June 19, 2015; date of current version November 25, 2015. This work was supported in part by the National Natural Science Foundation of China under the contract number 11272082 and in part the National Science and Technology Major Project of China under the contract number 2013ZX04013-011. Associate Editor: R. Yeh.

The authors are with the Institute of Reliability Engineering, University of Electronic Science and Technology of China, Chengdu, Sichuan, 611731, China (e-mail: weiwenpeng@hotmail.com; yanfengli@uestc.edu.cn; yuanjyang@hotmail.com; jinhuami@126.com; hzhuang@uestc.edu.cn).

Color versions of one or more of the figures in this paper are available online at <http://ieeexplore.ieee.org>.

Digital Object Identifier 10.1109/TR.2015.2443858

NOTATION

$Y(t)$	Degradation process of a product
$y(t)$	Degradation observation at time t
$\Delta y(t)$	Degradation increment
$\eta(t)$	Shape function of a gamma process
ω	Rate parameter of a gamma process
$g(x)$	PDF of a gamma distribution
$\Gamma(\bullet)$	Gamma function
D	Failure threshold of a degradation process
T	Failure time of a product
$f(y(t))$	PDF of a degradation observation
$F(t)$	CDF of a failure time
$R(t)$	Reliability function
$\Pr\{\bullet\}$	Probability
\mathbf{X}	Vector of accelerated variables
\mathbf{Z}	Vector of usage environment variables
m	Variable of mission type
$\tau(m)$	Mission duration
$S(m)$	Mission intensity
$\boldsymbol{\theta}^{test}$	Vector of parameters for a degradation testing model
$\boldsymbol{\theta}_\eta$	Model parameters for the shape function of a Gamma process
$\boldsymbol{\theta}^{ADT}$	Vector of parameters for an ADT model
$\boldsymbol{\theta}_X$	Model parameters associated with accelerated variables
$\boldsymbol{\theta}^{field}$	Vector of parameters for a field degradation model
$\boldsymbol{\theta}_M$	Model parameters associated with mission operating variables
$\boldsymbol{\theta}_Z$	Model parameters associated with usage environment variables
$\pi(\boldsymbol{\theta})$	Prior distribution
$L(\mathbf{Y} \boldsymbol{\theta})$	Likelihood function
$p(\boldsymbol{\theta} \mathbf{Y})$	Posterior distribution
\mathbf{Y}^{test}	Degradation testing data
\mathbf{Y}^{ADT}	ADT data
\mathbf{Y}^{field}	Field observation data

I. INTRODUCTION

FIELD reliability has long become a critical issue for both manufacturers and customers of modern products. For manufacturers, field reliability analysis is often used to estimate field returns within a given warranty period [1]. A precise estimation of field reliability is essential for the optimization of monetary reserves for warranty claims. In the long run, manufacturers will benefit from the field reliability analysis by eliminating potential weaknesses, and by delivering more competitive products. On the other hand, for customers, field reliability analysis is generally adopted to estimate the remaining useful life (RUL) of products [2]. A real-time prediction of RUL is one of the key factors for the optimization of system operation, and the planning of preventive maintenance. Under the support of real-time prediction of RUL, customers will obtain an excellent availability of products with cost-effective capital goods. Commonly, reliability tests are used by manufacturers to extrapolate products' reliabilities under field conditions (i.e. typical working conditions in the service stage of a product), while condition monitoring is used by customers to estimate products' RULs under unit-specific conditions. A gap is naturally formed between these two types of methods, as well as the corresponding reliability data when they are used separately.

Unfortunately, neither a precise estimation nor a real-time prediction can be achieved for the field reliability of complex products nowadays, if the degradation testing and condition monitoring are used separately. Take a machine tool as an example. Due to continually increasing competition in the machine tool industry, manufacturers of machine tools are under great pressure to deliver products with high reliability and availability under limited time. Accelerated life tests (ALT) and accelerated degradation tests (ADT) are used by manufacturers to investigate the field reliability of machine tools. However, due to differences of usage environments and operating missions between lab tests and field operations, large discrepancies are often found between the results of lab test inference and field failure analysis. This difference is large because the reliability of machine tools is mainly influenced by two groups of factors: the usage environment related factors such as temperature, moisture, and vibration; and the operating mission related factors such as work-piece material, cutting speed, and depth of cut. These factors are often dynamic or time-varying under field conditions. The misspecification or simplification of these factors generally leads to a biased inference of field reliability. On the other hand, for users of machine tools, the reduction of the total cost of ownership, and the improvement of availability, have become critical issues to make machine tools more profitable and productive. Condition monitoring and RUL prediction are adopted to investigate the field reliability of machine tools. Because machine tools are complex systems with multiple interactive components, indicators of system reliability can sometimes hardly be monitored. The indicators of system reliability for the machine tools generally include manufacturing precision, position precision, and oil debris. The observations of these indicators are often sparse because continuous monitoring of these indicators is either technically impractical or economically unaffordable. Most of these indicators are observed and measured discontinuously when the machine tools are idle, generating fragmented observations of

these indicators. In this situation, results of field reliability estimation using these sparse measurements under time-varying conditions are generally imprecise for further utilization.

Lab test methods for field reliability analysis are impacted by the uncertainty introduced by time-varying field conditions, while condition monitoring methods are challenged by the large variance resulting from fragmented field observations. It would be useful to combine these methods to investigate their correlations, and to mitigate their limitations. Ye *et al.* [3] raised the question about how heterogeneities in operating environments affect the predictions of field failures and the planning of lab tests. They introduced a model that linked the lab failure time distribution and field failure time distribution. Improved field reliability estimation and optimized ALT planning have been demonstrated through real life examples. Meeker *et al.* [4] presented a method for field reliability prediction based on the ALT results and field usage information. However, both methods are limited to field reliability analysis with lifetime data. Considering field reliability assessment based on degradation analysis, Liao and Elsayed [5] developed a method to relate ADT experiments to field applications by incorporating stochastic stresses into ADT models. However, only lab test data and field stress information can be integrated in their method. Recently, Liao and Tian [6] introduced a framework to integrate ADT models and Bayesian updating techniques for the RUL prediction of individual units under time-varying operating conditions. An ADT model was used by them to investigate degradation analysis with time-varying operating conditions. However, the effects of unit-specific usage environments and time-varying missions cannot be differentiated under their model. Unfortunately, in practical engineering, the field reliability of a product can be affected by many factors. Such as the example of machine tools described above, different factors contribute differently to field reliability. As a result, the inherent unit-to-unit variation, the external unit-specific usage environments, and time-varying missions should be treated separately in field degradation modeling. The models summarized above are not adequate to handle this challenging problem. It then motivates the research presented in this paper.

This paper presents an integrated framework for field reliability analysis with a Bayesian fusion strategy and a model coupling technique. Under this framework, the strengths of lab test methods and condition monitoring methods are synthesized. Degradation testing data and in situ mission operating information are integrated with condition monitoring observations to facilitate the reliability assessment of products' population and real-time RUL prediction for individual products. A joint model for multi-source information fusion is constructed by coupling the random effects, time-varying covariates, and marker processes into a degradation model. The effects of unit-to-unit variation, heterogeneous usage conditions, and time-varying missions are modeled coherently. The problems of fragmented degradation observations and simplified influence factors are handled properly. The remainder of this paper is organized as follows. Section II presents a literature review, and some discussion of field reliability analysis. The models for degradation modeling and information integration are presented step-by-step in Section III. Parameter estimation, population reliability estimation, and individual RUL prediction are described in Section IV. In Section V, an illustrative example is presented to demonstrate the proposed method.

Finally, Section VI concludes this paper, and highlights several points for future research.

II. RELATED WORKS

Degradation testing and condition monitoring are two types of methods for reliability analysis. In the past decades, various methods have been introduced to carry out field reliability estimation or RUL prediction or both. There are two classical lines of research that carry out the investigation of field reliability estimation and RUL prediction.

One research line follows the gradual development and improvement of degradation models. Various degradation models for different data types have been introduced to handle different engineering applications. These models include degradation path models [7], [8], Markovian models [9], [10], and stochastic process based models [11]–[22]. For the stochastic process based models, the models based on Wiener processes [11]–[15], gamma processes [16]–[18], and inverse Gaussian processes [19]–[22] have been studied to facilitate degradation based reliability analysis. Among the various degradation models, the paper by Lu and Meeker [7] has been recognized as one of the pioneering research works. They introduced a general degradation path model, which was composed by an actual degradation path function, and a random error term. They also derived the time-to-failure distribution based on this model, and introduced the methods for parameter estimation and reliability assessment. Many papers were published following their work. Comprehensive reviews about these degradation models for reliability assessment and RUL prediction were separately presented by Ye and Xie [23], and Si *et al.* [2]. In detail, the Wiener process and its extensions have been widely studied for degradation modeling. Ye *et al.* [14] proposed a degradation model using a random effects Wiener process with measurement errors. The heterogeneity of degradation rate among product population, together with the imperfectness of degradation measurements, were investigated in their model. In addition, Si *et al.* [13] proposed a nonlinear drift diffusion process for degradation modeling and RUL prediction based on a Wiener process. A time-space transformation was adopted by them to derive the analytical approximation of the failure time distribution. Wang *et al.* [15] proposed a generalized Wiener process model for degradation modeling. Various existing Wiener process degradation models were treated as their model's limiting cases. For the development of gamma process based degradation models, Lawless and Crowder [17] incorporated random effects and covariates into the gamma process for degradation modeling with unit-specific variability and explanatory variables. A closed form of the time to failure distribution was derived by them as well. Wang *et al.* [18] introduced a change-point gamma and Wiener process for a degradation process with change points or multi-phases. Real-time reliability assessment using this change-point degradation model was also studied. In general, stochastic process degradation models are often used due to the flexibility of these models for the incorporation of nonlinear degradation, unit-specific variability, and time-varying covariates. Among them, the Wiener process degradation models were the most used, because of their tractability and simplicity. These models were suitable for non-monotonic degradations. For monotonic degradations, the gamma process degradation models were

often used. However, the investigation of gamma process degradation models and their implementations for degradation analysis considering the inherent unit-to-unit variation, the external unit-specific usage environments, and time-varying mission operating heterogeneity have not been well studied yet.

The other line of research focuses on the integration of multi-source information for field reliability analysis. Padgett and Tomlinson [12] presented a method for lifetime inference by integrating degradation measures and failure times obtained through accelerated tests. Meeker *et al.* [4] introduced a use-rate model for field reliability analysis by integrating ALT data and field usage information. Hong and Meeker [24] further presented a method for field failure prediction by developing a cumulative exposure model to integrate dynamic usage information and failure time data collected in the field. All these methods were based on lifetime data analysis. Considering the field reliability analysis through degradation analysis, Gebraeel *et al.* [25] proposed a method for RUL prediction by incorporating information about the reliability characteristics of a product's population and real-time sensor information of specific products interested. Gebraeel and Pan [26] further extended the method to incorporate the real-time status of environmental conditions. Recently, Chen and Tsui [27] extended this information integration method to a more specific degradation situation. Within their work, degradation processes with change points, unit-specific variance, and correlated degradation prediction were modeled and investigated under the Bayesian information integration framework. Meanwhile, Liao and Tian [6] proposed a Bayesian framework for the RUL prediction of individual products by further extending the information integration method as mentioned above. In general, the information integration method for field reliability analysis is heading to a more specific and practical situation. Various types of information have been incorporated to facilitate field reliability estimation or RUL prediction or both. The inherent unit-to-unit variation and the time-varying usage environments are investigated. However, the methods summarized above were all based on the Wiener process degradation model due to its simplicity and tractability. Few works focused on other types of degradation models; a rare exception was the work presented by Wang *et al.* [28]. They proposed a Bayesian evaluation method by integrating ADT test data and field failure data, where a calibrating factor was incorporated to link the model for ADT and failure observations. Both Wiener process and gamma process degradation models were studied in their work. However, only lab test information and field observation were incorporated in their work. Information of field conditions can hardly be incorporated in their model, which makes the application of this method limited, especially for the situation where field reliability is affected greatly by field conditions and operating missions such as the situation of machine tools introduced above. Accordingly, the integration method for field reliability analysis by incorporating degradation tests, condition monitoring, and mission operating information deserve further study.

Based on the literature review presented above, the contributions of the proposed method lie in the following aspects. From the line of research about degradation modeling for field reliability analysis, other than introducing a specific model solely for degradation analysis with time varying operating missions, a group of degradation models raising from baseline

degradation, to lab degradation, and finally into field degradation, are constructed. A link between degradation test models and condition monitoring models is presented. It is implemented by gradually incorporating random effects, dynamic covariates, and marker processes into a baseline degradation model. The unit-to-unit variation and the effects of usage conditions and time-varying missions are modeled coherently. Moreover, a gamma process degradation model is investigated, and mathematically tractable conditional distributions for both RUL prediction and degradation inference are obtained. From the line of information integration for field reliability analysis, a coherent method for integrating degradation test, condition monitoring, and mission operating information, is presented. Together with the proposed model, the integration method is extended to the gamma process degradation model, which is suitable for monotonic degradation modeling. Moreover, information about usage environments and operation missions is treated separately in the proposed model. In detail, the information about usage environments is integrated through dynamic covariates, and the information about operation missions is incorporated through marker processes. It can facilitate the identification of the major impact factor to the degradation process. Especially for complex engineering systems, such as the machine tools introduced above, different factors affect the degradation process differently. It can also deliver a deeper understanding of the connection between lab tests and field observations, which in turn improve field reliability assessment, and RUL prediction for manufacturers, and customers respectively.

III. MODELS

To relate lab test data to condition monitoring data, the effects of unit-to-unit variability, usage conditions, and time-varying missions should be modeled coherently. In this section, the model proposed in this paper is constructed by gradually integrating random effects, dynamic covariates, and marker processes into a baseline degradation model.

A. Baseline Degradation Model

In this paper, we investigate the degradation process with continuous state and continuous time. For a product with degradation process $Y(t), t \geq 0$, given the degradation threshold D , the failure time of this product is defined as $T = \inf\{\tau \geq 0 : y(\tau) \geq D\}$. We consider the degradation processes with s -independent non-negative increments observed for many engineering systems today [23]. For illustration of the proposed method, the gamma process is used as the baseline degradation process model. A basic gamma process model is then defined for the degradation process $Y(t)$ with $Y(0) = 0$. It has the following properties.

- a) The degradation increments $\Delta y(t) = y(t + \Delta t) - y(t)$ are s -independent.

- b) The degradation increment $\Delta y(t)$ follows a gamma distribution $\text{Ga}(\eta(t + \Delta t) - \eta(t), \omega)$, where $\eta(t)$ is a monotone increasing function with $\eta(0) = 0$.

The probability density function (PDF) of a gamma distribution for a random variable $x \sim \text{Ga}(\kappa, \nu)$, $\kappa, \nu > 0$ with mean κ/ν and variance κ/ν^2 is

$$g(x|\kappa, \nu) = \frac{\nu^\kappa}{\Gamma(\kappa)} x^{\kappa-1} \exp(-\nu x), \quad (1)$$

where κ is the shape parameter, and ν is the inverse scale parameter.

The degradation process $Y(t)$ is described as $Y(t) \sim \text{Ga}(\eta(t), \omega)$. Its degradation increment is given as $\Delta Y(t) \sim \text{Ga}(\Delta\eta(t), \omega)$ with $\Delta\eta = \eta(t + \Delta t) - \eta(t)$. The PDF of the degradation observation $y(t)$ is obtained as

$$f(y(t)|\eta(t), \omega) = \frac{\omega^{\eta(t)}}{\Gamma(\eta(t))} y(t)^{\eta(t)-1} \exp(-\omega y(t)). \quad (2)$$

Because the failure time of this degradation process is defined as $T = \inf\{t \geq 0 : y(t) \geq D\}$, the cumulative distribution function (CDF) of the failure time is obtained as

$$F(t|\eta(t), \omega) = \Pr\{Y(t) \geq D|\eta(t), \omega\} = 1 - \frac{\Upsilon(\eta(t), \omega D)}{\Gamma(\eta(t))} \quad (3)$$

where $\Upsilon(\eta(t), \omega D) = \int_0^{\omega D} u^{\eta(t)-1} e^{-u} du$, which is a lower incomplete gamma function.

B. Degradation Model for Testing Data

Degradation tests or ADT are generally used to obtain lab test data, and to assess system reliability by mimicking normal field conditions and nominal operating missions. Commonly, these conditions and missions are well-controlled. Due to the uncertainty introduced in the design process or manufacturing process or both, there is unit-to-unit variation among products, which leads to the dispersion of degradation curves. It is common to introduce a unit-specific random effect into the baseline degradation model to account for the unit-to-unit variation. Two scenarios are considered in this paper: the normal degradation tests, and the accelerated degradation tests.

For the scenario of normal degradation tests, a random effect is incorporated in the basic degradation model by assuming that the inverse scale parameter follows a gamma distribution as in Lawless and Crowder [17], and Tsai *et al.* [29]. In this way, both the variance among the product's population and the variance within a product's degradation curve can be well modelled. That is, $Y(t) \sim \text{Ga}(\eta(t), \omega)$ with $\omega \sim \text{Ga}(\delta, \gamma)$. Then the PDF of the degradation observation is given as (4) at the bottom of the page.

It can be derived that $\delta y(t)/\gamma \eta(t)$ follows an F -distribution with parameters $2\eta(t)$, and 2δ [17], i.e., $\delta y(t)/\gamma \eta(t) \sim \mathcal{F}(2\eta(t), 2\delta)$. Accordingly, the reliability function is obtained as shown in (5) at the bottom of the next page, where

$$f(y(t)|\eta(t), \delta, \gamma) = \int_0^\infty g(y(t)|\eta(t), \omega) g(\omega|\delta, \gamma) d\omega = \frac{\Gamma(\eta(t) + \delta)}{\Gamma(\eta(t))\Gamma(\delta)} \frac{\gamma^\delta y(t)^{\eta(t)-1}}{(y(t) + \gamma)^{\eta(t)+\delta}} \quad (4)$$

$F_F(\delta D/\gamma\eta(t); 2\eta(t), 2\delta)$ is the CDF of a F -distribution with parameters $2\eta(t)$ and 2δ , $B_{incom}(D/(D+\gamma); \eta(t), \delta) = \int_0^{D/(D+\gamma)} u^{\eta(t)-1} (1-u)^{\delta-1} du$ is an incomplete beta function, and $B_{com}(\eta(t), \delta) = \Gamma(\eta(t))\Gamma(\delta)/\Gamma(\eta(t)+\delta)$ is a beta function.

As for the scenario of ADT, the baseline degradation model $Y(t) \sim \text{Ga}(\eta(t), \omega)$ is extended to an ADT model as $Y(t) \sim \text{Ga}(\eta(t, \mathbf{X}), \omega)$, where \mathbf{X} is a vector of accelerated variables. The accelerated variables \mathbf{X} are the stress factors used in the ADT, such as temperature-cycle, use-rate, voltage-stress, and so on [30]. The function $\eta(t, \mathbf{X})$ is obtained by incorporating the effects of accelerated variables into the shape function $\eta(t)$. The effects of accelerated variables are generally formulated by a parametric regression function, a Cox-proportional hazards model, or some other acceleration functions such as a power-law model, an Arrhenius reaction rate model, or an inverse-log model, to name a few. The random effect is incorporated in this ADT degradation model by assuming that the inverse scale parameter follows a gamma distribution. Similarly, the conditional PDF of the degradation observation and the conditional reliability function are obtained as shown in (6) and (7) at the bottom of the page.

C. Degradation Model for Field Observations

A degradation model for field observations is a model to describe the monitored degradation observations by considering the effects of usage conditions and time-varying missions. It is aimed to integrate condition monitoring data and mission operating information. Based on the engineering imperative presented by the case of machine tools introduced above, the information of field observations is categorized into three parts:

the monitored degradation observations, the usage environment information, and the operating mission information. This categorization is aimed to model the effects of usage environments and operating mission separately on the observed degradation. Because the usage environments are generally static or well-ordered situations, they are treated as non-stochastic covariates in this paper. Because the operating missions are generally unpredictable, and affected by many factors, they are modeled by marker processes in this paper.

The variables of usage environments \mathbf{Z} are generally measurements of conditions that affect the products' reliability, such as moisture, vibration, pressure, and so on [31]. The effects of these variables are introduced into the degradation model through the modification of the rate parameter ω [28]. The degradation process model is then given as $Y(t) \sim \text{Ga}(\eta(t, \mathbf{X}^{field}), \omega/\varepsilon(\mathbf{Z}))$, where $\varepsilon(\mathbf{Z})$ is formulated following the idea of ADT models, and \mathbf{X}^{field} is the accelerated variable in the field conditions if it is observed. As a result, the conditional PDF of the degradation observation and the conditional reliability function are obtained as shown in (8) and (9) at the bottom of the page.

For the incorporation of mission operating information, marker processes are used in this paper. By marker processes, we mean that the missions are characterized by various indexes, and these indexes are used as markers to differentiate different missions. In detail, the missions' type, duration, and intensity are taken into consideration. The information about these indexes is recorded and incorporated into the degradation model. The specification of mission operating information is originated from the fact that modern products and engineering systems can implement various missions. Each mission is characterized by a specific duration of time, and a particular stress intensity.

$$\begin{aligned} R(t|\eta(t), \delta, \gamma) &= \Pr\{Y(t) < D|\eta(t), \delta, \gamma\} \\ &= F_{\mathcal{F}}\left(\frac{\delta D}{\gamma\eta(t)}; 2\eta(t), 2\delta\right) = \frac{B_{incom}\left(\frac{D}{D+\gamma}; \eta(t), \delta\right)}{B_{com}(\eta(t), \delta)} \end{aligned} \quad (5)$$

$$f(y(t)|\eta(t, \mathbf{X}), \delta, \gamma) = \int_0^\infty g(y(t)|\eta(t, \mathbf{X}), \omega)g(\omega|\delta, \gamma)d\omega = \frac{\Gamma(\eta(t, \mathbf{X}) + \delta)}{\Gamma(\eta(t, \mathbf{X}))\Gamma(\delta)} \frac{\gamma^\delta y(t)^{\eta(t)-1}}{(y(t) + \gamma)^{\eta(t)+\delta}}, \quad (6)$$

$$R(t|\eta(t, \mathbf{X}), \delta, \gamma) = \Pr\{Y(t) < D|\eta(t, \mathbf{X}), \delta, \gamma\} = \frac{B_{incom}\left(\frac{D}{D+\gamma}; \eta(t, \mathbf{X}), \delta\right)}{B_{com}(\eta(t, \mathbf{X}), \delta)} \quad (7)$$

$$\begin{aligned} f(y(t)|\eta(t, \mathbf{X}^{field}), \varepsilon(\mathbf{Z}), \delta, \gamma) &= \int_0^\infty g(y(t)|\eta(t, \mathbf{X}^{field}), \omega/\varepsilon(\mathbf{Z}))g(\omega|\delta, \gamma)d\omega \\ &= \frac{\Gamma(\eta(t, \mathbf{X}^{field}) + \delta)}{\Gamma(\eta(t, \mathbf{X}^{field}))\Gamma(\delta)} \frac{(\gamma\varepsilon(\mathbf{Z}))^\delta y(t)^{\eta(t)-1}}{(y(t) + \gamma\varepsilon(\mathbf{Z}))^{\eta(t)+\delta}} \end{aligned} \quad (8)$$

$$\begin{aligned} R(t|\eta(t, \mathbf{X}^{field}), \varepsilon(\mathbf{Z}), \delta, \gamma) &= \Pr\{Y(t) < D|\eta(t, \mathbf{X}^{field}), \varepsilon(\mathbf{Z}), \delta, \gamma\} \\ &= \frac{B_{incom}\left(\frac{D}{D+\gamma\varepsilon(\mathbf{Z})}; \eta(t, \mathbf{X}^{field}), \delta\right)}{B_{com}(\eta(t, \mathbf{X}^{field}), \delta)} \end{aligned} \quad (9)$$

Different degradation levels or rates may be observed in different missions. Suppose there are M types of missions that can be carried out by a product. The marker process of mission type is a serial of numbers within the integers $1, \dots, M$. It acts as a marker to differentiate different mission types. For a specific mission m with $m = 1, \dots, M$, the marker processes of mission duration $\tau(m)$ and mission intensity $S(m)$ are serials of real numbers in their respective intervals. The processes $\tau(m)$ and $S(m)$ are used to describe the detailed information of mission operations experienced by the product within a series of missions. The degradation characteristics of a product are then differentiated mission by mission. By taking account of the marker processes, the degradation model of the product is given as a model of degradation increment within a mission as shown in (10) at the bottom of the page, where $\omega \sim \text{Ga}(\delta, \gamma)$, t_k is the specific time that the mission m starts, and $Y(t_k)$ is the degradation observation at time point t_k .

The key point for degradation modeling with mission operating information is to modify the shape function by incorporating the effect of mission intensity into the shape function as $\eta(t, S(m))$ or $\eta(t, S(m), \mathbf{X}^{field})$ if the accelerated variables are also presented in the field observations. Meanwhile, because the shape function is a function of both the operation time t and the mission type m , the characteristic of s -independent degradation increments is only valid within the time interval of a specific mission m . The degradation observations are accumulated by the degradation increments generated in all completed missions. In other word, it is impossible to derive a conditional PDF for the degradation observation at any time point as (4), (6), and (8). The only way to describe the degradation observation is through the degradation increments conditionally on the available mission operating information of a specific mission m . As a result, the conditional PDF of the degradation observation, and the conditional reliability of the product are obtained for a mission m as shown in (11) and (12) at the bottom of the page, where $\Delta\eta^{field}(t) = \eta(t, S(m), \mathbf{X}^{field}) - \eta(t_k, S(m), \mathbf{X}^{field})$.

IV. PARAMETER ESTIMATION, AND RELIABILITY INFERENCE

The models for degradation tests, condition monitoring, and mission operating information are presented above. A coherent framework is then constructed to integrate the information for field reliability inference in this section. The Bayesian method is adopted to implement parameter estimation and reliability inference. Four aspects are highlighted in this section: 1) derivation of the prior distribution, 2) formulization of the likelihood function, 3) construction of the joint posterior distribution, and 4) calculating or updating the field reliability inference.

A. Prior Distribution

The prior distribution is a representation of quantified prior information. Based on the degradation model presented in (4), (6), and (10), all model parameters are summarized as shown in (13) at the bottom of the page.

Due to the models for the degradation test, condition monitoring, and mission operating information being derived hierarchically, their model parameters are coupled together as shown in (13), where θ^{test} is included in θ^{ADT} , and θ^{ADT} is further included in θ^{field} . This approach can facilitate the combination of multiple-source information, and the updating of prior distributions. Two types of prior distributions are commonly adopted. The first type is the prior distributions derived from subjective prior information. The second type is the prior distributions transformed from analysis results of a previous data set.

Generally, two kinds of advantages are achieved for field reliability analysis by using these two types of prior distributions. For the prior distributions derived from subjective information, the incorporation of prior information can improve the precision of field reliability analysis, and further add value to the corresponding decision making. This kind of prior information is generally in the form of expert judgment which contains the engineer's experience and expertise. Wang and Zhang [32] have demonstrated the advantage of incorporating subjective expert

$$Y(t_k + \tau(m)) - Y(t_k) \sim \text{Ga}(\eta(t_k + \tau(m), S(m), \mathbf{X}^{field}) - \eta(t_k, S(m), \mathbf{X}^{field}), \omega/\varepsilon(\mathbf{Z})) \quad (10)$$

$$\begin{aligned} f(y(t)|y(t_k), m, \eta(t, S(m), \mathbf{X}^{field}), \varepsilon(\mathbf{Z}), \delta, \gamma) &= \int_0^\infty g(y(t) - y(t_k)|\Delta\eta^{field}, \omega/\varepsilon(\mathbf{Z}))g(\omega|\delta, \gamma)d\omega \\ &= \frac{\Gamma(\Delta\eta^{field} + \delta)}{\Gamma(\Delta\eta^{field})\Gamma(\delta)} \frac{(\gamma\varepsilon(\mathbf{Z}))^\delta (y(t) - y(t_k))^{\Delta\eta^{field} - 1}}{(y(t) - y(t_k) + \varepsilon(\mathbf{Z})\gamma)^{\Delta\eta^{field} + \delta}}, t_k < t \leq t_k + \tau(m), y(t) > y(t_k), \end{aligned} \quad (11)$$

$$\begin{aligned} R(t|y(t_k), m, \eta(t, S(m), \mathbf{X}^{field}), \varepsilon(\mathbf{Z}), \delta, \gamma) &= \Pr\{Y(t) < D|\eta(t, S(m), \mathbf{X}^{field}), \varepsilon(\mathbf{Z}), \delta, \gamma\} \\ &= \frac{B_{incom}\left(\frac{D - y(t_k)}{D - y(t_k) + \gamma\varepsilon(\mathbf{Z})}; \Delta\eta^{field}(t), \delta\right)}{B_{com}(\Delta\eta^{field}, \delta)}, t_k < t \leq t_k + \tau(m) \end{aligned} \quad (12)$$

$$\theta^{test} = (\theta_\eta, \delta, \gamma), \theta^{ADT} = (\theta^{test}, \theta_{\mathbf{X}}), \theta^{field} = (\theta^{ADT}, \theta_M, \theta_{\mathbf{Z}}) \quad (13)$$

judgment for residual life prediction. For specific techniques about the elicitation of experts' probability, and the derivation of the prior distribution, please refer to the work by O'Hagan *et al.* [33].

For the prior distributions transformed from the analysis results of the previous data set, a continual updating strategy can be formed for field reliability by making the posterior distributions of model parameters for the previous observed data as the prior distributions of model parameters for the present observed data. This kind of prior information is commonly in the form of a probability density function which contains all the information aggregated to the present time. Various works for field reliability analysis have utilized this advantage of prior distributions, including [5], [26], [28], and [34]. In this paper, the continual updating of model parameters is employed to gradually integrate degradation test data, ADT data, and condition monitoring data with mission operating information. A further demonstration of the prior distribution for the proposed model is presented later in an illustrative example.

B. Likelihood Function

A likelihood function is utilized to describe the information contained in degradation testing data, condition monitoring data, and mission operating data. It is constructed based on the PDF functions of degradation observations or degradation increments using the models derived above. Due to these models being derived hierarchically for different data types, the likelihood functions for these data are presented progressively in the following part for information integration.

Given degradation testing data \mathbf{Y}^{test} , suppose n^{test} units are tested, and the i th unit is observed at l_i^{test} different time points, where $i = 1, \dots, n^{test}$. Due to the degradation increment of degradation testing being modeled as

$\Delta Y(t) \sim \text{Ga}(\Delta\eta(t), \omega)$ with $\omega \sim \text{Ga}(\delta, \gamma)$, the likelihood function for the degradation testing data \mathbf{Y}^{test} is given as (14) at the bottom of the page, where $\Delta\eta(t)_{ij} = \eta(t_{ij}) - \eta(t_{i,j-1})$ and $\omega = \{\omega_1, \omega_2, \dots, \omega_{n^{test}}\}$ includes all the random effect parameters ω_i for the n^{test} units.

Given the ADT data \mathbf{Y}^{ADT} , suppose n^{ADT} units are tested at k^{ADT} different levels of accelerated stresses. Under the k th stress, there are n_k^{ADT} units under test, and the i_k th unit is observed at $l_{i,k}^{test}$ different time points, where $k = 1, \dots, k^{ADT}$, $i_k = 1, \dots, n_k^{ADT}$, and $\sum_{k=1}^{k^{ADT}} n_k^{ADT} = n^{ADT}$. Based on the degradation model for ADT data, which is given as $\Delta Y(t) \sim \text{Ga}(\Delta\eta(t, \mathbf{X}), \omega)$ with $\omega \sim \text{Ga}(\delta, \gamma)$, the likelihood function for the ADT data \mathbf{Y}^{ADT} is given as (15) at the bottom of the page, where $\Delta\eta(t, \mathbf{X}_k)_{ij} = \eta(t_{ij}, \mathbf{X}_k) - \eta(t_{i,j-1}, \mathbf{X}_k)$, and ω includes all the random effect parameters ω_i for the n^{ADT} units.

Given the condition monitoring data and mission operating information \mathbf{Y}^{field} , suppose that n^{field} units are monitored. For the i th unit, there are l_i^{field} degradation observations obtained from l_i^{field} missions. The mission type m , the mission duration $\tau(m)$, and the mission intensity $S(m)$ are recorded. The usage environments \mathbf{Z} and accelerated stresses \mathbf{X}^{field} are also monitored. Based on the degradation increment for the field observations presented in (10), the likelihood function for the condition monitoring data with the mission operating information is given as (16) at the bottom of the page, where $\Delta\eta(t, \mathbf{X}_i, m_{ij})_{ij} = \eta(t_{i,j-1} + \tau(m_{ij}), S(m_{ij}), \mathbf{X}_i^{field}) - \eta(t_{i,j-1}, S(m_{ij}), \mathbf{X}_i^{field})$.

Based on the likelihood functions derived above, the joint likelihood function for degradation testing, condition monitoring, and mission operating information is obtained as (17) at the bottom of the page.

From the joint likelihood function, we can find that all the information contained in the observed data is concentrated on the

$$L^{test}(\mathbf{Y}^{test}, \omega | \theta_\eta, \delta, \gamma) = \prod_{i=1}^{n^{test}} \prod_{j=2}^{l_i^{test}} g(\Delta y_{ij}^{test} | \Delta\eta(t)_{ij}, \omega_i) g(\omega_i | \delta, \gamma) \quad (14)$$

$$L^{ADT}(\mathbf{Y}^{ADT}, \omega | \theta^{test}, \theta_{\mathbf{X}}) = \prod_{k=1}^{k^{ADT}} \prod_{i=1}^{n_k^{ADT}} \prod_{j=2}^{l_{i,k}^{test}} g(\Delta y_{ij}^{ADT} | \Delta\eta(t, \mathbf{X}_k)_{ij}, \omega_i) g(\omega_i | \delta, \gamma) \quad (15)$$

$$L^{field}(\mathbf{Y}^{field}, \omega | \theta^{ADT}, \theta_M, \theta_{\mathbf{Z}}) = \prod_{i=1}^{n^{field}} \prod_{j=2}^{l_i^{field}} g(\Delta y_{ij}^{field} | \Delta\eta(t, \mathbf{X}_i, m_{ij})_{ij}, \omega_i / \varepsilon(\mathbf{Z})) g(\omega_i | \delta, \gamma) \quad (16)$$

$$L(\mathbf{Y}^{test}, \mathbf{Y}^{ADT}, \mathbf{Y}^{field}, \omega | \theta^{field}) = L^{test}(\mathbf{Y}^{test}, \omega | \theta_\eta, \delta, \gamma) \times L^{ADT}(\mathbf{Y}^{ADT}, \omega | \theta^{test}, \theta_{\mathbf{X}}) \times L^{field}(\mathbf{Y}^{field}, \omega | \theta^{ADT}, \theta_M, \theta_{\mathbf{Z}}) \quad (17)$$

parameters of the field degradation model, i.e., θ^{field} . A gradual integrating of information from degradation testing data, ADT data, and condition monitoring data with mission operating information is implemented through the coupling of model parameters and their likelihood functions.

C. Posterior Distribution

By combing prior distributions of model parameters with the likelihood function of reliability data through Bayes' theorem, the joint posterior distribution of model parameters is obtained. This joint posterior distribution is a description of the integrated results of degradation testing, condition monitoring, and mission operating information. Field reliability is then implemented based on this joint posterior distribution and the corresponding degradation models.

Based on the prior distribution of parameters, and the likelihood function presented in (17), the joint posterior distribution is obtained as (18) at the bottom of the page.

There is often no analytical solution to (18). The Markov chain Monte Carlo (MCMC) method is generally used to generate posterior samples of model parameters from this joint posterior distribution. These posterior samples are then used to implement reliability inference and degradation prediction. In this paper, the software WinBUGS [35] is used to facilitate the implementation of MCMC. For specific information about the modeling and calculation through WinBUGS, please refer to the works [36], [37].

D. Reliability Inference

When the parameters of degradation models introduced above are estimated, it is of interest to investigate the field reliability of a product through the degradation models. There are generally two kinds of field reliability that manufacturers and customers are concerned about. The first is the overall field reliability of the product population, which is related to the decision-making of product warranty. This type of field reliability is of particular importance for manufacturers of a product. The other is the particular field reliability of an individual product, which is related to the planning and optimization of preventive maintenance. This type of field reliability is of critical significance for users of a product. In the following part, both the field reliability of a product population, and the degradation

prediction and residual life prediction of individual products are presented.

For the field reliability of a product population, if all the products are used under a specific mission type m with a mission intensity $S(m)$, the conditional field reliability is given as (19) at the bottom of the page.

The mission types m and the mission intensity $S(m)$ that a product experienced in the field are generally hard to predict precisely. This difficulty is because the arrival of different mission types is influenced by various uncertain factors. In addition, the sequence of these mission types and their durations are generally unpredictable. As a result, (19) is generally used to estimate the field reliability of product population in some typical mission types and mission intensity.

Because (19) cannot be solved analytically, simulation based integration is adopted to facilitate the calculation. It is implemented by substituting the posterior samples of model parameters generated from $p(\theta^{field} | \mathbf{Y}^{field}, \mathbf{Y}^{ADT}, \mathbf{Y}^{test})$ through the MCMC method into the conditional reliability function $R(t | \eta(t, S(m), \mathbf{X}^{field}), \varepsilon(\mathbf{Z}), \delta, \gamma)$. By doing so, each posterior sample θ^{field} will generate an estimation of field reliability. A group of estimations of field reliability are then obtained using the posterior samples of model parameters. Statistics of these estimations form the estimations of the field reliability of the product population, which include kernel distribution, mean, variance, interval estimation, and so on.

For degradation prediction of the i th product under field conditions, the degradation prediction for a coming mission is given as follows, when the mission type m , mission duration $\tau(m)$, and mission intensity $S(m)$ for the coming mission are known. See (20) at the bottom of the next page, where $f(y(t_{k+1}) | \mathbf{Y}^{field}, \mathbf{Y}^{ADT}, \mathbf{Y}^{test}, y(t_k), m, S(m))$ is the PDF of degradation at the time point $t_{k+1} = t_k + \tau(m)$, $\Delta\eta^{field} = \eta(t_{k+1}, S(m), \mathbf{X}_i^{field}) - \eta(t_k, S(m), \mathbf{X}_i^{field})$, and $\theta = (\theta_\eta, \theta_{\mathbf{X}}, \theta_M, \theta_{\mathbf{Z}}, \omega_i)$.

Similarly, the field reliability of the i th product during the mission time of the coming mission is given as follows, when the mission type m , mission duration $\tau(m)$, and mission intensity $S(m)$ for the coming mission are known. See (21) at the bottom of the next page, where $R(t | \mathbf{Y}^{field}, \mathbf{Y}^{ADT}, \mathbf{Y}^{test}, y(t_k), m, S(m))$ is the reliability of the product during the mission duration of mission m with

$$p(\theta_\eta, \theta_{\mathbf{X}}, \theta_M, \theta_{\mathbf{Z}}, \omega, \delta, \gamma | \mathbf{Y}^{test}, \mathbf{Y}^{ADT}, \mathbf{Y}^{field}) \propto \pi(\theta_\eta, \theta_{\mathbf{X}}, \theta_M, \theta_{\mathbf{Z}}, \delta, \gamma) \times L(\mathbf{Y}^{test}, \mathbf{Y}^{ADT}, \mathbf{Y}^{field}, \omega | \theta^{field}) \quad (18)$$

$$\begin{aligned} & R(t | \mathbf{Y}^{field}, \mathbf{Y}^{ADT}, \mathbf{Y}^{test}, m, S(m)) \\ &= \int_{\theta^{field}} R(t | \eta(t, S(m), \mathbf{X}^{field}), \varepsilon(\mathbf{Z}), \delta, \gamma) p(\theta^{field} | \mathbf{Y}^{field}, \mathbf{Y}^{ADT}, \mathbf{Y}^{test}) d\theta^{field} \\ &= \int_{\theta^{field}} \frac{B_{incom}\left(\frac{D}{D+\gamma\varepsilon(\mathbf{Z})}; \eta(t, S(m), \mathbf{X}^{field}), \delta\right)}{B_{com}(\eta(t, S(m), \mathbf{X}^{field}), \delta)} p(\theta^{field} | \mathbf{Y}^{field}, \mathbf{Y}^{ADT}, \mathbf{Y}^{test}) d\theta^{field} \end{aligned} \quad (19)$$

$t_k < t \leq t_k + \tau(m)$, and $\Upsilon(a, b) = \int_0^b u^{a-1} e^{-u} du$ which is a lower incomplete gamma function.

Both (20) and (21) are indices derived for the coming mission of the product, which are derived on the condition that the mission type m , mission duration $\tau(m)$, and mission intensity $S(m)$ of the coming mission are known. Due to the unpredictability of the mission sequence that the product will endure in the remaining life, it is impractical to derive an estimation of RUL of this product. However, a preventive maintenance and an operation planning still can be made based on the prediction of the degradation and reliability of the product in the coming mission. This ability exists because there is a time interval between the present observation time point t_k and the predicted observation time point t_{k+1} . A decision can be drawn based on the comparison of the degradation level between the predicted degradation and the degradation threshold. It is of practical meaning for the management of the engineering systems or products that can fulfill multiple missions, yet the information of these missions is only available for the coming one or two missions. The field analysis of machine tools introduced in the introduction session is a good example. A further demonstration is also presented in the later example section.

The calculations of (20) and (21) are implemented through the incorporation of the simulation based integration, which are the same as the one used for (19). The posterior samples of model parameters generated from $p(\theta_\eta, \theta_X, \theta_M, \theta_Z, \omega_i | \mathbf{Y}^{field}, \mathbf{Y}^{ADT}, \mathbf{Y}^{test})$ using the MCMC method are substituted into the conditional PDF function $g(y(t_{k+1}) - y(t_k) | \Delta\eta^{field}, \omega_i/\varepsilon(\mathbf{Z}))$, and the conditional reliability function $1 - F(t | \Delta\eta^{field}, \omega_i/\varepsilon(\mathbf{Z}))$. Both of these two functions have analytical expressions as presented in (20), and (21). By doing so, a group of calculated samples of the conditional PDF and conditional reliability are then obtained using the posterior samples of model parameters. Degradation prediction and reliability inference for individual units during the coming mission are then simulated and summarized based on these calculated samples.

V. EXAMPLES

Take the spindle system of a machine tool as an example. The spindle system transmits the required energy, and rotates the tool (grinding, milling and drilling) precisely to implement high-precision machining. It exerts a great effect on the material removal rate and the final quality of machined parts. The spindle system is expected to possess high reliability and availability. Degradation testing and condition monitoring are implemented on the spindle system. The measurement and monitoring of the amount of debris in the lubricating oil is a possible way to monitor the deterioration of bearings and gears in the spindle system. In this section, degradation analysis of oil debris is used to illustrate the proposed method.

To avoid proprietary issues, degradation data are simulated using the estimated parameters from the original degradation data, and the units of values are omitted. Largely, however, the characteristics of the degradation observations and the application of the proposed methods are the same as with the original data.

A. Degradation Data

The degradation data of oil debris include the testing data and field observations. In detail, five spindle systems are tested by the manufacturers during the design and manufacturing processes. Testing data are recorded from the degradation tests, which include general degradation test data and ADT data. The degradation data collected by manufacturers are presented in Fig. 1. These tests are designed and implemented by professional reliability test engineers. We are not able to discuss more detail about the reliability tests. However, the accelerated degradation model is presented in the following part. The specific data for the normal degradation test, and ADT tests are respectively given in Tables I, and II.

These five systems are also monitored by the users during the usage processes. Field observations are obtained from condition

$$\begin{aligned}
 & f(y(t_{k+1}) | \mathbf{Y}^{field}, \mathbf{Y}^{ADT}, \mathbf{Y}^{test}, y(t_k), m, S(m)) \\
 &= \int_{\theta} g(y(t_{k+1}) - y(t_k) | \Delta\eta^{field}, \omega_i/\varepsilon(\mathbf{Z})) p(\theta_\eta, \theta_X, \theta_M, \theta_Z, \omega_i | \mathbf{Y}^{field}, \mathbf{Y}^{ADT}, \mathbf{Y}^{test}) d\theta \\
 &= \int_{\theta} \frac{(\omega_i/\varepsilon(\mathbf{Z}))^{\Delta\eta^{field}} \exp(-(y(t_{k+1}) - y(t_k))\omega_i/\varepsilon(\mathbf{Z}))}{\Gamma(\Delta\eta^{field}) (y(t_{k+1}) - y(t_k))^{1-\Delta\eta^{field}}} p(\theta_\eta, \theta_X, \theta_M, \theta_Z, \omega_i | \mathbf{Y}^{field}, \mathbf{Y}^{ADT}, \mathbf{Y}^{test}) d\theta \quad (20)
 \end{aligned}$$

$$\begin{aligned}
 & R(t | \mathbf{Y}^{field}, \mathbf{Y}^{ADT}, \mathbf{Y}^{test}, y(t_k), m, S(m)) \\
 &= \int_{\theta} (1 - F(t | \Delta\eta^{field}, \omega_i/\varepsilon(\mathbf{Z}))) p(\theta_\eta, \theta_X, \theta_M, \theta_Z, \omega_i | \mathbf{Y}^{field}, \mathbf{Y}^{ADT}, \mathbf{Y}^{test}) d\theta \\
 &= \int_{\theta} \frac{\Upsilon(\Delta\eta^{field}, (D - y(t_k))\omega_i/\varepsilon(\mathbf{Z}))}{\Gamma(\Delta\eta^{field})} p(\theta_\eta, \theta_X, \theta_M, \theta_Z, \omega_i | \mathbf{Y}^{field}, \mathbf{Y}^{ADT}, \mathbf{Y}^{test}) d\theta \quad (21)
 \end{aligned}$$

TABLE I
DEGRADATION DATA OBTAINED FROM NORMAL DEGRADATION TEST

Sample 1	Time	0	39	49	53	59	66	70	71	77	78
	Degradation	0	2.76	4.54	4.71	5.65	6.73	7.45	7.53	8.46	8.73
	Time	79	88	91	102	104	108	109	110	113	115
	Degradation	8.93	9.94	10.58	13.27	13.70	14.82	14.94	14.95	15.32	15.74
Sample 2	Time	0	6	10	13	23	24	40	54	57	64
	Degradation	0	0.02	0.12	0.12	0.41	0.41	0.76	1.09	1.20	1.52
	Time	66	75	80	81	95	98	108	109	113	120
	Degradation	1.57	1.71	2.08	2.12	3.05	3.16	3.58	3.65	3.84	4.24
Sample 3	Time	0	12	13	20	23	29	37	38	41	43
	Degradation	0	0.23	0.24	0.33	0.36	0.41	0.58	0.61	0.65	0.71
	Time	44	64	84	87	95	96	101	105	113	116
	Degradation	0.75	1.76	2.83	3.06	3.75	3.78	4.06	4.31	4.71	4.98
Sample 4	Time	0	8	9	17	20	23	27	28	30	38
	Degradation	0	0.25	0.36	0.59	0.60	0.84	0.93	0.93	0.96	1.28
	Time	55	57	60	76	79	92	107	108	112	117
	Degradation	2.21	2.40	2.51	3.70	3.93	4.63	5.68	5.73	5.98	6.57
Sample 5	Time	0	17	21	27	32	37	54	55	61	63
	Degradation	0	0.47	0.68	0.73	0.98	1.14	2.46	2.49	3.15	3.23
	Time	68	79	93	95	104	109	111	113	115	118
	Degradation	3.72	4.68	6.13	6.77	8.44	9.40	10.40	10.57	10.96	11.20

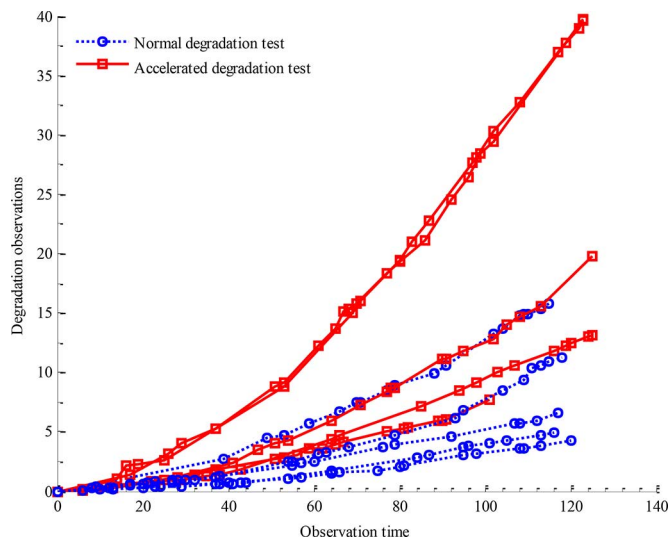


Fig. 1. Normal degradation test data, and ADT data.

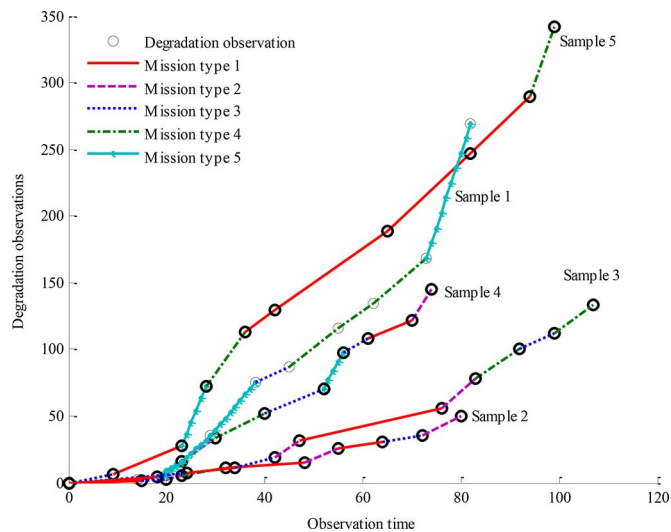


Fig. 2. Field degradation data, and mission type information.

monitoring and mission tracking, which include field degradation observations, and mission operating records. Degradation data and operating information collected by users are presented in Fig. 2.

The degradation observations are observed during the idle time of the machine tools when they finish specific missions. As presented in Fig. 2, there are five types of missions that

the machine tools fulfilled. The mission types, mission intensity, and mission durations were recorded by users of machine tools based on the operation sheets of these missions. In addition, the accelerated variables experienced by the machine tools under typical use conditions in the factories, and the environment information, were also recorded. The effect of these variables and the specific model are presented in the following part. The specific data for mission operating information, condition

TABLE II
ADT DATA WITH ACCELERATED VARIABLE PRESENTED IN THE BRACKET FOR EACH SAMPLE

	Time	0	6	14	16	19	25	53	69	71	80
Sample 1	Degradation	0	0.17	1.03	2.17	2.25	2.58	8.82	14.97	16.02	19.49
(50)	Time	83	87	97	98	102	108	117	119	122	123
	Degradation	21.02	22.80	27.63	28.08	30.28	32.79	37.01	37.68	38.97	39.59
Sample 2	Time	0	11	22	28	32	37	51	53	56	59
	Degradation	0	0.38	0.80	1.13	1.38	1.79	2.67	2.81	3.04	3.55
(60)	Time	62	65	67	77	81	82	89	90	91	101
	Degradation	3.61	3.97	4.12	5.02	5.25	5.35	5.94	5.97	6.08	7.70
Sample 3	Time	0	6	13	23	29	35	38	43	64	66
	Degradation	0	0.13	0.28	0.85	0.95	1.23	1.37	1.80	4.37	4.72
(70)	Time	85	94	98	103	107	116	119	120	124	125
	Degradation	7.16	8.51	9.19	10.07	10.60	11.85	12.19	12.42	12.98	13.14
Sample 4	Time	0	6	25	41	47	51	54	64	71	77
	Degradation	0	0.03	0.93	2.37	3.47	4.00	4.27	5.93	7.21	8.40
(80)	Time	78	79	90	91	95	102	105	108	113	125
	Degradation	8.65	8.70	11.09	11.14	11.82	12.81	13.99	14.67	15.58	19.74
Sample 5	Time	0	17	26	29	37	51	53	61	65	67
	Degradation	0	1.35	3.13	4.04	5.25	8.86	9.18	12.26	13.70	15.16
(90)	Time	68	70	77	80	86	92	96	99	102	123
	Degradation	15.33	15.82	18.30	19.35	21.15	24.53	26.45	28.39	29.43	39.76

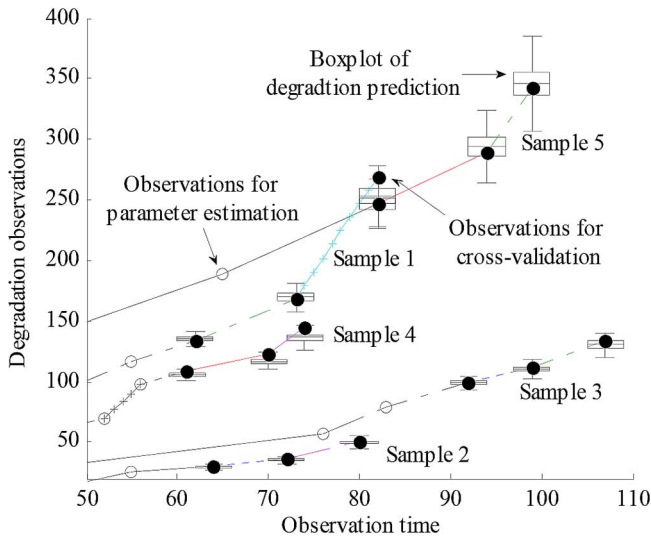


Fig. 3. Prediction of field degradations at three leave-one-out cross-validation points.

monitoring degradation data, and the values of accelerated variables and environment variables for machine tools are given in Table III.

B. Degradation Models

Based on the degradation models introduced in Section III, the models for degradation testing data and the model for condition monitoring information are chosen. These models are

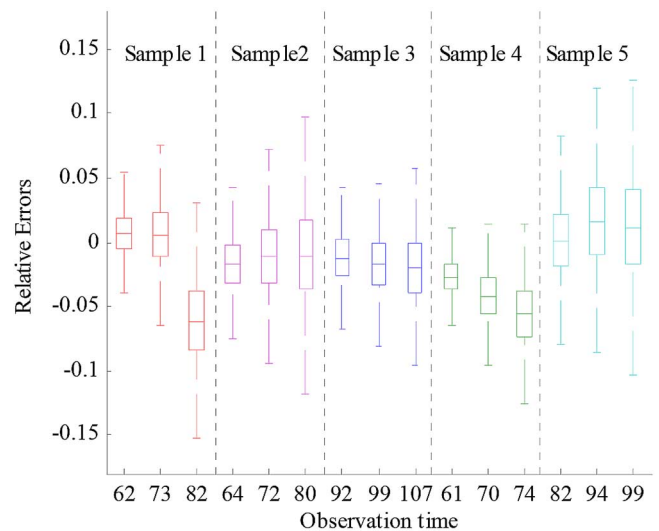


Fig. 4. Boxplot of relative errors of degradation predictions.

chosen by considering both the characteristic of degradation data presented in Fig. 1 and the subjective information of experts in that domain.

A gamma process with random effects is used as the degradation model for normal degradation tests. As the oil debris is often characterized as an increasing degradation rate, a power-law

TABLE III
DEGRADATION OBSERVATION AND MISSION OPERATING INFORMATION

	Mission Type	0	1	2	5	5	2	3	3	3	5
Sample 1	Intensity	0	1794	2202	3370	3362	2333	2700	2591	2601	3482
(53, 10)*	Duration	0	14	6	9	9	7	10	7	11	9
	Degradation	0	2.53	5.71	34.99	74.89	86.84	116.21	134.49	168.49	269.39
	Mission Type	0	1	2	4	3	1	4	1	2	4
Sample 2	Intensity	0	1824	2191	2928	2581	1841	3008	1866	2209	2932
(52, 20)	Duration	0	15	5	3	9	16	7	9	8	8
	Degradation	0	1.79	2.71	5.07	10.78	15.39	25.61	30.11	35.83	50.25
	Mission Type	0	2	1	2	4	1	4	3	2	3
Sample 3	Intensity	0	2174	1736	2280	3048	1851	2872	2575	2234	2566
(53, 30)	Duration	0	24	10	8	5	29	7	9	7	8
	Degradation	0	7.13	10.82	19.00	31.04	56.15	78.43	99.99	112.25	133.17
	Mission Type	0	1	5	4	3	2	5	2	1	4
Sample 4	Intensity	0	1917	3476	3027	2622	2294	3338	2188	1838	2967
(54, 40)	Duration	0	18	5	7	10	12	4	5	9	4
	Degradation	0	4.33	16.44	33.08	52.30	70.02	96.98	108.55	122.16	144.77
	Mission Type	0	2	1	5	3	1	1	1	1	3
Sample 5	Intensity	0	2191	1798	3324	2649	1808	1766	1737	1735	2536
(55, 50)	Duration	0	9	14	5	8	6	23	17	12	5
	Degradation	0	6.50	27.48	71.97	113.43	129.83	188.59	246.57	289.24	342.16

* Values of accelerated variables and environment variables are given in pair in the brackets.

function is used as the shape function in the degradation model as follows.

$$Y(t|\theta^{test}) \sim \text{Ga}(\eta(t), \omega), \eta(t) = \left(\frac{t}{b}\right)^a, \omega \sim \text{Ga}(\delta, \gamma) \quad (22)$$

where $\theta^{test} = (a, b, \delta, \gamma)$.

The degradation model for ADT data is formulated by incorporating the accelerated variables into the shape function. For the ADT of the oil debris, only one accelerated variable is used, and a linear accelerated model is adopted in the design of ADT tests. Here, we incorporate this accelerated model into the normal degradation test model. The model for ADT is

then given as (23) at the bottom of the page, where $\theta^{ADT} = (a, b, \delta, \gamma, \alpha)$, and X is the accelerated variable.

The degradation model for condition monitoring data and mission operating information are formulated by separately incorporating covariates into the shape function and scale parameters of the ADT model presented above. It is given as shown in (24) at the bottom of the page, where $\theta^{field} = (a, b, \delta, \gamma, \alpha, \beta, \nu)$, X^{field} is the accelerated variable observed under typical use conditions in the factories, $S_n = 1000$ is a nominal mission intensity designed for the machine tool, and Z is the environment variable.

As discussed in Section III, the effect of the mission intensity $S(m)$ is considered by modifying the shape function $\eta(t, X^{field})$ into $\eta(t, S(m), X^{field})$. It is implemented by introducing an exponential function $\exp(\beta(S(m) - S_n)/S_n)$

$$Y(t|\theta^{ADT}) \sim \text{Ga}(\eta(t, X), \omega), \eta(t, X) = \left(\frac{t}{b}\right)^a (1 + \alpha X), \omega \sim \text{Ga}(\delta, \gamma) \quad (23)$$

$$Y(t|\theta^{field}) \sim \text{Ga}(\eta(t, S(m), X^{field}), \omega/\varepsilon(Z)), \omega \sim \text{Ga}(\delta, \gamma)$$

$$\eta(t, S(m), X^{field}) = \left(\frac{t}{b} \exp\left(\beta \frac{S(m) - S_n}{S_n}\right)\right)^a (1 + \alpha X^{field}), \varepsilon(Z) = \exp(\nu Z) \quad (24)$$

TABLE IV
 STATISTICAL SUMMARY OF POSTERIOR SAMPLES OF MODEL PARAMETERS

Parameters	Mean	Stand Deviation	Confidence Interval	
			2.5%	97.5%
a	1.774	0.0263	1.725	1.827
b	9.522	0.614	8.414	10.76
α	0.0226	0.0020	0.0191	0.0265
β	0.655	0.0158	0.6237	0.6858
ν	0.0334	0.0013	0.0308	0.0361
δ	6.936	3.359	2.013	14.67
γ	0.5641	0.2865	0.1491	1.242
ω_1	5.175	0.5024	4.242	6.202
ω_2	20.21	1.973	16.55	24.27
ω_3	16.65	1.614	13.66	19.96
ω_4	14.21	1.396	11.63	17.06
ω_5	7.507	0.7544	6.106	9.059

into the shape function to directly affect the lifetime t . The effect of the mission intensity $S(m)$ is presented through its contrast with a nominal mission intensity S_n designed for the machine tool. In addition, the environment variable Z is considered by modifying the scale parameter ω into $\omega/\varepsilon(Z)$, which aims to adjust the variance of the degradation model.

C. Parameter Estimation and Reliability Inference

Following the procedure presented in Section IV, prior distributions for model parameters θ^{field} are derived. In this example, non-informative prior distributions are used for these parameters. In this way, the results of estimation and inference generated by the proposed method are mainly based on the integration of information presented above. In detail, according to the principle of indifference [38], s -uniform distributions with large intervals are assigned for the model parameters as prior distributions: $a \sim U(0, 100)$, $b \sim U(0, 1000)$, $\delta \sim U(0, 1000)$, $\gamma \sim U(0, 1000)$, $\alpha \sim U(0, 1000)$, $\beta \sim U(0, 100)$, and $\nu \sim U(0, 1000)$. $U(a_u, b_u)$ denotes a s -uniform distribution with

an interval (a_u, b_u) . The intervals of these prior distributions are chosen large enough to make these priors act as non-informative priors in the Bayesian analysis.

By combining the degradation models presented in (22), (23), and (24) with the general likelihood functions presented in Section IV, the joint likelihood function for the degradation data given above is obtained shown in (25) at the bottom of the page, where \mathbf{Y}^{field} , \mathbf{Y}^{ADT} , \mathbf{Y}^{test} are the data sets given above; and $\omega = \{\omega_1, \omega_2, \dots, \omega_5\}$, $\theta^{field} = (a, b, \delta, \gamma, \alpha, \beta, \nu)$, $\Delta y_{ij}^* = y_{ij}^* - y_{i,j-1}^*$, $\Delta \eta_{ij}^{test} = (t_{ij}/b)^a - (t_{i,j-1}/b)^a$, $\Delta \eta_{ij}^{ADT} = (1 + \alpha X_i)((t_{ij}/b)^a - (t_{i,j-1}/b)^a)$, and $\Delta \eta_{ij}^{field} = (1 + \alpha X_i^{field})(\eta(t_{j-1} + \tau(m_{ij}), S(m_{ij}), X_i^{field}) - \eta(t_{j-1}, S(m_{ij}), X_i^{field}))$.

According to Bayes' theorem, the posterior distribution for model parameters is obtained by combining the prior distributions given above with the joint likelihood function given in (25). It is given as (26) at the bottom of the page.

$$\begin{aligned}
 & L(\mathbf{Y}^{test}, \mathbf{Y}^{ADT}, \mathbf{Y}^{field}, \omega | \theta^{field}) \\
 &= \prod_{i=1}^5 L_i^{test}(\mathbf{Y}_i^{test} | a, b, \omega_i) L_i^{ADT}(\mathbf{Y}_i^{ADT} | a, b, \alpha, \omega_i) L_i^{field}(\mathbf{Y}_i^{field} | a, b, \alpha, \beta, \nu, \omega_i) g(\omega_i | \delta, \gamma) \\
 &= \prod_{i=1}^5 \prod_{j=2}^{20} g(\Delta y_{ij}^{test} | \Delta \eta_{ij}^{test}, \omega_i) \prod_{j=2}^{20} g(\Delta y_{ij}^{ADT} | \Delta \eta_{ij}^{ADT}, \omega_i) \prod_{j=1}^{10} g(\Delta y_{ij}^{field} | \Delta \eta_{ij}^{field}, \omega_i e^{-\nu Z_i}) g(\omega_i | \delta, \gamma) \quad (25)
 \end{aligned}$$

$$\begin{aligned}
 & p(a, b, \alpha, \beta, \nu, \delta, \gamma, \omega | \mathbf{Y}^{test}, \mathbf{Y}^{ADT}, \mathbf{Y}^{field}) \propto \pi(a, b, \alpha, \beta, \nu, \delta, \gamma) \times L(\mathbf{Y}^{test}, \mathbf{Y}^{ADT}, \mathbf{Y}^{field}, \omega | \theta^{field}) \\
 & \propto \prod_{i=1}^5 \prod_{j=2}^{20} g(\Delta y_{ij}^{test} | \Delta \eta_{ij}^{test}, \omega_i) \prod_{j=2}^{20} g(\Delta y_{ij}^{ADT} | \Delta \eta_{ij}^{ADT}, \omega_i) \prod_{j=1}^{10} g(\Delta y_{ij}^{field} | \Delta \eta_{ij}^{field}, \omega_i e^{-\nu Z_i}) g(\omega_i | \delta, \gamma) \quad (26)
 \end{aligned}$$

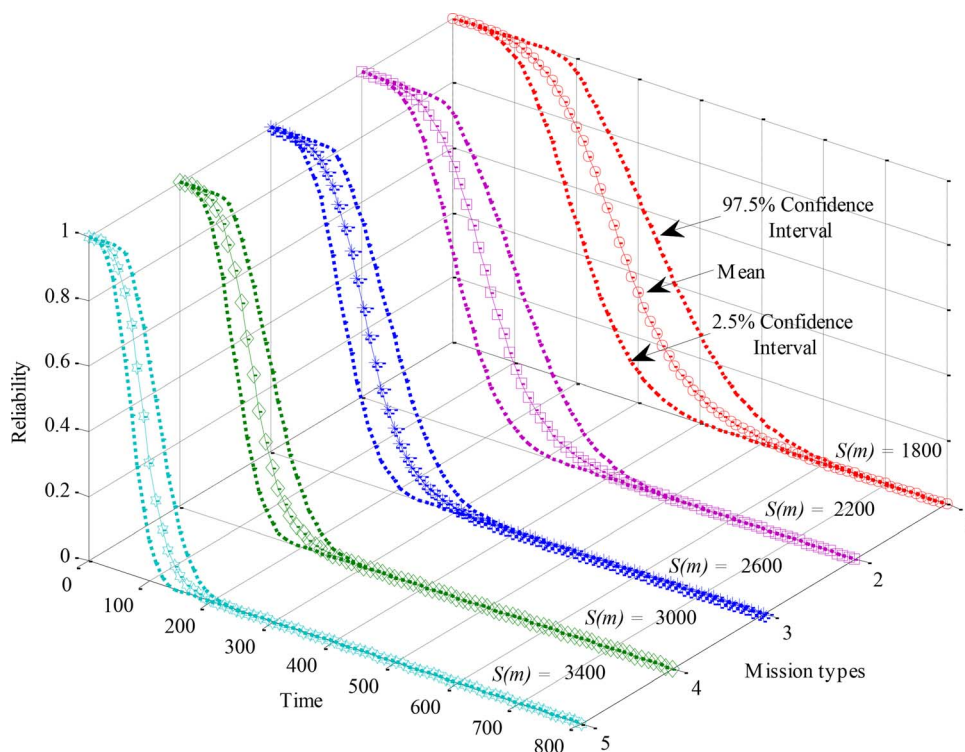


Fig. 5. Estimations of population reliability with different mission types.

Based on the joint posterior distribution, posterior samples of model parameters are simulated by implementing the MCMC through WinBUGS. In this example, 20,000 posterior samples are generated. A statistical summary of these posterior samples is presented in Table IV.

To verify the effectiveness of the proposed method, a leave-one-out cross-validation is implemented. We retain three latest degradation points of the field observations given in Table III, and carry out parameter estimation using the remaining data. The posterior samples generated by these data sets are then used to predict the degradations at these three leave-one-out time points. A pictorial description of the predicted degradations and the observed degradations at the three leave-one-out time points is presented in Fig. 3. The relative errors of these degradation predictions for each sample are also obtained as presented in Fig. 4. Both boxplots of the degradation predictions and the relative errors demonstrate high precision of the proposed method for field degradation prediction. We then proceed to the reliability estimation, degradation inference, and RUL prediction using the results of information integration.

By utilizing the simulation based method presented in Section IV, the reliability assessment of the machine tool's population is obtained based on the posterior samples of model parameters from (26). The degradation prediction, and the RUL prediction for individual machine tools are obtained as well based on these posterior samples of model parameters. For the manufacturers of machine tools, the conditional field reliability of the product population is calculated by assuming that the products are working under a specific mission type m with a mission intensity $S(m)$. Fig. 5 presents the estimations of population reliability with different mission types. The

corresponding mission intensity are 1800, 2200, 2600, 3000, and 3400 for mission types 1, 2, 3, 4, and 5.

From the estimations of population reliability, we can find out that the field reliability of machine tools can be greatly affected by the mission type and mission intensity fulfilled by the machine tools. A mission type with a large mission intensity, such as the mission type 5 with mission intensity 3400 shown in Fig. 5, generally leads to poor field reliability of the machine tool. Manufacturers can utilize these estimation results to facilitate decision making for the machine tools. A more specific estimation of field population reliability can be obtained based on the field degradation and posterior samples derived above, if more information about the process of mission types that a machine tool will experience is available.

For the user of a machine tool, if missions are assigned for this machine tool, degradation prediction can be obtained for the time points when the missions are completed. In addition, a prediction of RUL for individual machine tools under a specific mission type and mission intensity can be obtained as well. Fig. 6 presents degradation predictions for a machine tool under different mission types. This machine tool is the sample 1 presented in Fig. 2. The predictions of RUL under different mission types are also presented in Fig. 6.

The degradation predictions and the RULs vary with the mission types that the machine tool endured. The RULs changed from 110 to 23 with the mission type changing from mission type 1 to mission type 5. As a result, different strategies about operation management and preventive maintenance are going to be implemented on the machine tool if different mission types are carried out by the machine tool. In addition, based on the degradation model and the inference procedure described above, more specific predictions of field reliability for individual machine tools can be implemented if more specific

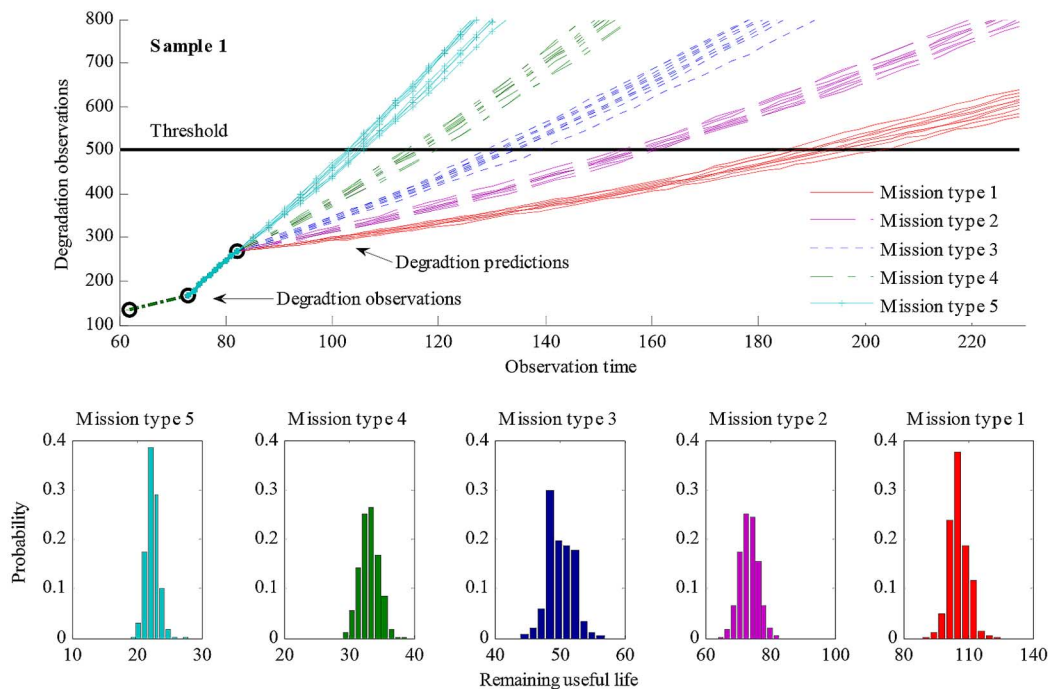


Fig. 6. Prediction of degradation and RUL of a machine tool with different mission types.

information about their respective missions in the future is available.

VI. CONCLUSION

This paper investigates field reliability analysis by leveraging degradation testing and condition monitoring data. Time-varying mission operating information is incorporated to facilitate the degradation analysis with degradation testing data and condition monitoring observations. The models for normal degradation testing data, ADT data, and degradation observations are derived progressively by separately incorporating random effects, dynamic covariates, and marker processes, into a baseline degradation model. The effects of inherent unit-to-unit variation, external condition heterogeneity, and time-varying missions are modelled coherently with these degradation models. Estimations of population reliability, and predictions of individual degradations and RULs, are implemented using a Bayesian information fusion strategy. Degradation analysis of machine tools' spindle systems is used to demonstrate the proposed method, where the degradations of oil debris in the lubrication systems of the machine tools' spindle systems are used. Within this example, the degradation inference capability of the proposed method is verified through leave-one-out cross-validation. In addition, by studying the degradation under different mission types, the effectiveness of incorporating mission operating information is demonstrated for both the reliability estimation of product population and the degradation prediction of individual products. Time-varying missions can exert significant influence in the degradation of the product under field conditions. Accordingly, the decision-making concerning product warranty for product manufacturers, and strategy-designing about preventive maintenance for system users, should consider the effect of time varying missions.

There are several points that deserve further investigation. One is the studying of a real-time estimation method for the proposed models and the integration of multi-source information. A study of degradation test planning and ADT planning by considering the effect of field condition heterogeneity and time-varying missions is also of interest for further investigation.

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Weiweng Peng is currently a Ph.D. candidate in mechanical engineering at the University of Electronic Science and Technology of China.

His research interests include degradation modeling, Bayesian reliability, and the reliability of complex systems.

Yan-Feng Li received his Ph.D. in mechatronics engineering from the University of Electronic Science and Technology of China in 2013.

His research interests include the reliability analysis and evaluation of complex systems, dynamic fault tree modelling, Bayesian networks modelling, and probabilistic inference.

Yuan-Jian Yang is currently a Ph.D. candidate in mechanical engineering at the University of Electronic Science and Technology of China.

His research interests include degradation modeling, and reliability assessment.

Jinhua Mi is currently a Ph.D. candidate in mechanical engineering at the University of Electronic Science and Technology of China.

Her research interests include reliability analysis, the evaluation of complex systems, and Bayesian networks modelling.

Hong-Zhong Huang (M'06) received his Ph.D. in reliability engineering from Shanghai Jiaotong University, China.

He is a Professor of the School of Mechanical, Electronic, and Industrial Engineering at the University of Electronic Science and Technology of China. He has held visiting appointments at several universities in the USA, Canada, and Asia. He has published 200 journal papers and 5 books in the fields of reliability engineering, optimization design, fuzzy sets theory, and product development. His current research interests include system reliability analysis, warranty, maintenance planning and optimization, and computational intelligence in product design.

Prof. Huang is an ISEAM Fellow, a technical committee member of ESRA, a Co-Editor-in-Chief of the *International Journal of Reliability and Applications*, and an Editorial Board Member of several international journals. He received the William A. J. Golomski Award from the Institute of Industrial Engineers in 2006, and the Best Paper Award of the ICFDM2008, ICMR2011, and QR2MSE2013.