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Importance identification for fault trees based on possibilistic information measurements

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Abstract. Subjective factors and nonlinear characteristics, inherent in the importance identification for a fault tree in the reliability and risk analysis, make it necessary for fuzzy (or possibilistic) approaches to accommodate the quantificational assessment of epistemic uncertainty in a practical problem when data and information are very limited. After investigating the intuitive interpretations, possibilistic information semantics, measure-theoretic terms and entropy-like models, a new axiomatic index of importance measure for fault trees is proposed based upon possibilistic information entropy, which adopts the possibilistic assumption in place of the probabilistic one. An example of the fault tree is provided along with the concordance analysis and other discussions. The more conservative numerical results of importance rankings that involve more choices could be viewed as "soft" fault identification under a certain expected value. Finally, possible extension to the evidence space and further research directions are discussed.

Keywords: Importance ranking, information measurement, possibilistic entropy, epistemic uncertainty, fault tree

Notations and terminologies:

- $\emptyset$ – empty set
- $\wp(A)$ – a collection of subsets of A (power set of A)
- U – a universe of discourse (universal set)
- Ω – sample space
- A – a set element of U
- F – a fuzzy subset of U
- C – fuzzy complement of a set A
- $U(\cdot)$ – U-uncertainty
- $p(\cdot)$ – probability measure
- Ψ – topology of probability
- $\pi(\cdot)$, $P_{oss}(\cdot)$ – possibility measure
- Φ – topology of possibility
- Π – possibility distribution
- P – probability distribution
- r – possibility distribution function
- $\mu_A(x)$ – membership function
- $S(p)$ – Shannon entropy
- $H(p)$ – Hartley measure
- $|A|$ – the cardinality of a finite set A
- m – a basic probability assignment (a mass function)
- $\mu_A(x)$ – membership function
- $N_{ec}(\cdot)$ – necessity measure
- $B_{el}(\cdot)$ – belief measure
- $P_l(\cdot)$ – plausibility measure
- $N(m)$ – measure of non-specificity
- AU – aggregate uncertainty measure
- GH – generalized Hartley measure
- GS – generalized Shannon measure
- **E** – state space of fault mode
- **F** – attribute space of fault possibility

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1. Introduction

With an ever-increasing tendency of taking uncertainty factors into account in engineering design, fault trees provide logical and diagrammatic structures and have been widely used in reliability modeling and risk analysis, especially for large-scale and complex systems, e.g. nuclear, chemical and transportation sectors. Design and maintenance of such complex systems are associated with common practice of risk-informed decision making.

In the qualitative and quantitative analysis procedures for a fault tree, importance measures which provide significance-ranking of the component failures in evaluating how much a basic event (or cut set) contributes to the top event (or cut sets) must be defined. An important measure, acting as sensitivity or significance analysis in essence [27], is a function of system structure, component life (distribution), and operation time. Such ranking or prioritizing of basic events are valuable for reliability improvement, quality assurance, safety categorizations, or other similar activities. Therefore, there is a need for identifying the importance of failures of components or other basic events, which will probably result in a severe accident in testing and diagnosis maintenance, daily configuration control, and design improvement of complex systems [38].

Different types of importance measures contain different information and expressions, thus they have their own uses. With respect to the different objects and demands, the categories of importance measure include structural importance, probability importance (Birnbaum importance), criticality importance, and Fussell-Vesely importance, among others. From the perspective of applications, importance measures can be classified as risk importance, safety importance, and uncertainty importance [4].

In the quantitative evaluation of fault trees, special attention needs to be given to the lack of pertinent failure-rate data. Common reasons for the lack of statistical data include the following:

– The increased complexity of large-scale systems;
– The collection of statistical data being difficult and/or costly;
– The rarity of failures in some highly reliable systems; and
– Data being imprecise or unavailable under various testing conditions.

Considering uncertainties in reliability and risk analysis, a distinction is commonly made between aleatory (stochastic) uncertainty and epistemic (subjective) uncertainty, respectively. Aleatory uncertainty derives from natural variability of the physical world and reflects the inherent randomness in nature, whereas epistemic uncertainty arises from human's imperfect knowledge of understanding the physical world and the lack of the ability of modeling the physical world. Hence, how to treat and quantify different uncertainties associated with a risk-informed decision-making has become a challenging task. In engineering safety assessment, probability is a predominant tool being used to measure uncertainties [17, 18].

Conventional approaches for analyzing an index of importance, which is based on the probability theory and the binary-state assumption (perfectly functioning or completely failed), have manifested their deficiencies when the collected data is insufficient and reliability indices are imprecisely represented. To circumvent these difficulties, some theoretical improvements and practical explorations have been achieved to date in place of probabilistic methodology [27]. The pioneering work on fuzzy fault tree analysis (fuzzy FTA) belongs to Tanaka et al. [37], who treated imprecise probabilities of basic events as trapezoidal fuzzy numbers and defined an index function analogous to the importance measure for ranking the effectiveness of each basic event. Later on, Huang et al. [20] presented a model of posbist fault tree analysis based on the context of possibility measures. Li et al. [28] describes the quantification vagueness making use of triangular fuzzy membership functions Fuzzy importance used in fuzzy FTA was originally investigated by Furuta and Shiraishi [14]. They proposed a kind of importance measure by means of the max/min fuzzy operator and fuzzy integrals other than Tanaka's approach. Liang and Wang [29] proposed another importance measure, named fuzzy importance index, which is calculated by ranking of triangular fuzzy numbers. Suresh et al. [36] put forward a simpler fuzzy importance measure (FIM), which is based on the Euclidean distance approach. Such work was followed by that of Guimarêes and Ebecken [15]. Pan and Yun [31] assumed that the gates in fault tree are modeled as a fuzzy relation, and then proposed a procedure to obtain possibility of the top event and a new importance index of each component, especially for large-scale and complex systems. Another consideration of the importance measure is the trade-off between the improvement and cost while improving the system reliability. Cho and Yum [6] proposed a two-stage approach to achieve computational efficiency in allusion to complexity for large-sized FTA.

It should be noted that the aforementioned works have one common feature: a certain specific distribution of model inputs is required in the analysis of basic events or logic gates.

More recently, imprecise probability theory has been introduced into establishment of several new importance measures. Hall [16] has explored three different uncertainty-based sensitivity indices for imprecise information, addressing the intractability that the model inputs are expressed as intervals or sets of intervals without particular distribution over the intervals. Borgonovo [3] has introduced a global sensitivity analysis method with the moment independent uncertainty measure and has then discussed its application. Very recently, Contini and Matuzas [6] have described new algorithms for determining the importance of initiating and enabling events for both coherent and non-coherent fault trees. Huang et al. [21] presented a priority ordering method to improve the efficiency when dealing with large and complex systems taking advantage of BDD. Yang et al. [41] proposed an epistemic importance to describe the effect of ignorance degree of event on the basis of evidence theory. It is demonstrated that the research angle has been expanded a) from the first-order sensitivity measure to the second-order sensitivity measure [43], b) from the component-level (basic-event level) to gate-event level [6], c) from the variance-based sensitivity indices to the relative entropy-based sensitivity indices [16], and d) from the probabilistic importance measure to interval-valued importance measure [7, 16], and e) from the moment correlated measure to the moment independent measure [3]. More holistic information and overviews can be found in [1, 2, 13, 23].

Considering uncertainty inherent involved in some practical situation, we found that the aforementioned measures indeed encountered epistemic uncertainty, which is relevant to subjective information deficiency. It implies that the completeness and preciseness of the probabilistic information, as a consequence of the additivity axiom, is often violated in practice for the sake of missing data or conflicting evidence. The reported approaches to importance analysis, however, rarely consider non-probabilistic information measurements and do not work well on the essence of non-specificity characteristic. In fact, the possibility, by which a low-probability event can occur, is not always low [40], especially for the catastrophic failure in nuclear power plants. As nuclear power plants are designed based on the defense-in-depth principle, a large accident will be probably the result of combinations of multiple basic events (or cut sets), thus uncertainty propagation should be taken into account in risk and reliability analysis process. Moreover, subjective assessment and linguistic information about expert heuristics are substantially imprecise and may leads to either over-estimated or under-estimated results. Furthermore, in constructing the importance measure in a fault tree, there are some nonlinear and imprecise factors to be considered. Taking the value of a structural or probability importance for example, it depends on not only the location of a certain basic event in a minimal cut (or path) sets (MCS/MPS), but also on the frequency of this basic event occurred in such MCS (or MPS) [27]. The non-linearity, which often occurs between location and frequency, is also one of the difficulties in the importance identification.

As to we know about existing approaches, no matter theoretical explorations of the general measure of information and practical applications in various engineering backgrounds, limited work that incorporates and alleviates the above mentioned difficulties in terms of possibilistic uncertainty was found. There is no clear consensus on how to make meaningful possibilistic semantic and information measurement in the importance quantification. That is the reason for us to introduce a new importance measure for fault trees.

To accomplish this, we start with an integrated angle of measure grounds based on possibility-probability transformations (Section 2) and operational methods based on epistemic uncertainty quantifications (Section 3). A new possibilistic uncertainty-based importance measure in the foundation of the modeling principles and the properties of the possiibilistic entropy is introduced in Section 4, followed by a case study of a nuclear power plant to illustrate the possibilistic approach for intuition constraint condition in comparison with the existent ones in Section 5. A further discussion and brief conclusion are given in Section 6.

2. Possibilistic measure and representation of epistemic uncertainty

2.1. Fuzzy-measure representations of epistemic uncertainty

We note that in the presence of epistemic uncertainty, importance indexes in fault tree, which are used to measure the level of reliability and risk, are usually defined for parameters or for basic events in the context of fuzzy measures. The probabilistic model of conventional importance measures represents uncertainty-based information in the foundation of classical Lebesgue measure, which is the emergence of nonlinear integrals such as Sugeno integral and associated measures. Indeed, a fuzzy measure, first coined by Sugeno, specifies the degree to which an arbitrary element of the universal set belongs to the individual crisp subsets. Hence, when data is scarce, the concept of fuzzy measure provides us with a useful framework to represent different types of uncertainty. These representations are within various special classes of measures along with the corresponding properties [9, 39], as illustrated in Fig. 1.

In the traditional probabilistic approach to representation of epistemic uncertainty, the uncertainty in importance identification is characterized by a sequence of probability distributions. We consider one of alternative approaches to represent and characterize epistemic uncertainty in risk assessments, including interval or imprecise probability, possibility theory, and evidence theory. Specifically, it has been suggested that a possibilistic representation of epistemic uncertainty may be more appropriate when sufficient statistical data are not available due to the lack of repetitive tests.

Given a universal set U and assuming the power set $\Phi = 2^U$, a possibility measure, $P_{oss}(\cdot)$ is a mapping from Φ to [0, 1] and uniquely associated with a possibility distribution function $r(u)$ for each $A \in 2^U$ such that [40]

$$P_{oss}(A) \equiv \pi(A) = \sup_{u \in A} r(u) \tag{1}$$

And the property of possibilistic normalization $\sup_{u \in U} r(u) = 1$ should be satisfied. Since $r(u)$ is actually non-distributive compared with a probability distribution function, it seems more precise to call it the basic possibility function. Actually $r(u)$ estimates the consistency of the statement $u \in A$ and encodes a preference relation about the possible values of the variable that takes values in U. In this regard, the possibility acts as a subjective measure characterizing the extent to which, on one side, a person believes that an event can occur, or on the other side, the available evidence indicates that an event can occur. As one of the two dual monotone measures, necessity measure, N_{ec} is then defined as

$$N_{ec}(A) = 1 - P_{oss}\left(\bar{A}\right) \tag{2}$$

And has the properties as shown in Table 1.

Table 1
A comparison of measures and properties between possibility and probability

Possibility	Probability
Distribution Π, measure π	Distribution P, Probability p
$\sup_{u \in U} \pi(u) = 1$	$\sum_{u \in U} p(u) = 1$
$\forall (A, B) \in \wp(U)$	$\forall (A, B) \in \wp(U)$
$P_{oss}(A) = \pi(A) = \sup_{u \in A} \Pi(u)$	$P_{ro}(A) = p(A) = \sum_{u \in A} P(u)$
$\pi(A \cup B) = \max(\pi(A), \pi(B))$	$p(A \cup B) = p(A) + p(B)$, when A, B is disjoint
$N_{ec}(A \cap B) = \min(N_{ec}(A), N_{ec}(B))$	$p(A \cap B) \leq p(A) \cdot p(B)$
$\begin{cases} \max\left(\pi(A), \pi\left(\bar{A}\right)\right) = 1 \\ \pi(A) + \pi\left(\bar{A}\right) \geq 1 \\ N_{ec}(A) + N_{ec}\left(\bar{A}\right) \leq 1 \end{cases}$	$p(A) + p\left(\bar{A}\right) = 1$
$\forall u \in U, \Pi(x) = 1$	$\forall u \in U, P(x) = 1 \big/ \lvert U \rvert$

2.2. Intuitive interpretations and probability-possibility transformations

A theory based on possibility and necessity measures is called *possibility theory* [40]. Equation (1) also coincides with the well-known *fuzzy-set interpretation of possibility theory* proposed by Zadeh [40], which constructed the natural connection between membership degrees and possibility degrees with set-valued functions [30]. For example, let us consider a simple proposition of " $P \triangleq x$ is A (or $x \in A$)" to implement importance analysis in a fault tree. The truth of such proposition reveals the degree of evidence x (e.g. the unavailability of the component or basic event), which supports set A (e.g. a certain undesired accident). Consider set A is a crisp set (or a fuzzy set) with respect to uncertainty-based information obtained from the fault tree considered, and we can interpret the meaning of the proposition as shown in Fig. 2.

From Fig. 2 (a) and (b), we note that when set A is a crisp set, Equation (1) and Equation (2) can be integrated as a dual measure with intuitive interpretations as follows

$$\begin{cases} Truth_{\text{optimistic}} = P_{oss}(A) = \sup_{\forall x \in A} \pi(x) \\ Truth_{\text{pessibimistic}} = N_{ec}(A) = 1 - \sup_{\forall x \notin A} \pi(x) \end{cases} \tag{3}$$

Similarly, from Fig. 2 (c) when set A is a fuzzy set, Equation (1) and Equation (2) can be integrated

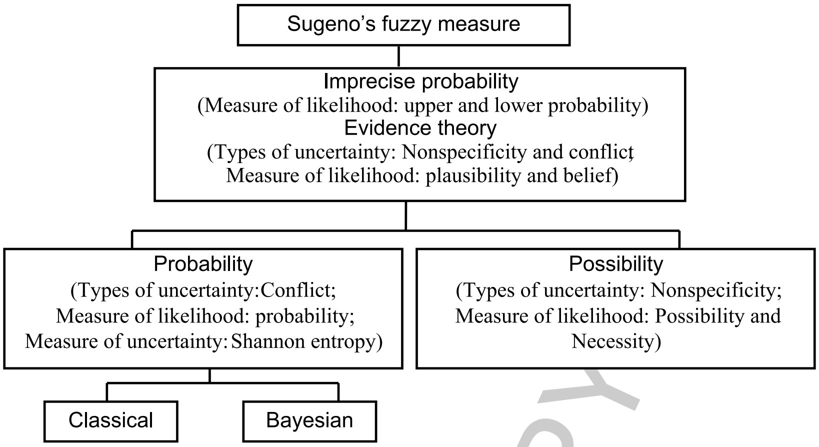

Fig. 1. A measure-background of different representations of epistemic uncertainty.

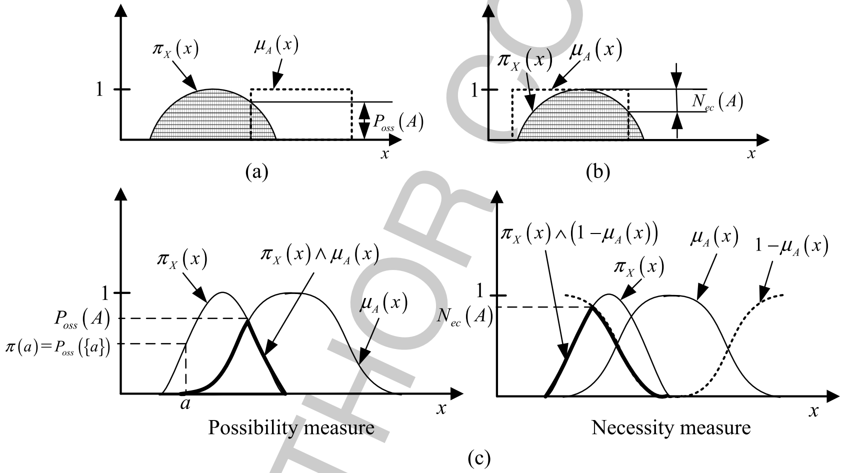

Fig. 2. Intuitive interpretations of dual measures when *A* is a crisp (or fuzzy) set.

from both the optimistic and pessimistic viewpoints as follows

$$\begin{cases} Truth_{\text{optimistic}} = P_{oss}(A) \\ \quad = \sup_{\forall x} (\pi(x) \wedge \mu_A(x)) \\ Truth_{\text{pessimistic}} = N_{ec}(A) \\ = 1 - \sup_{\forall x} [\pi(x) \wedge (1 - \mu_A(x))] \end{cases} \tag{4}$$

Owing to the above mathematical framework for managing uncertain knowledge, possibility theory represents, propagates, and integrates imprecise or incomplete information for reasoning and decision-making in reliability and risk assessments by possibility distributions. Moreover, the associated possibilistic representation of epistemic uncertainty can be transformed into a probabilistic representation, with reference to a specific fault tree. Currently the main transformations include [8, 11, 24, 40]:

– Zadeh's consistency principle;
– Dubois and Prade's transformation from a histogram;
– Transformation based on maximal specificity;
– Klir's transformation based on uncertainty invariance; and
– Yamada's transformation based on evidence theory.

For example, let (**F**, *m*) be the body of evidence as defined in evidence theory, then $E_p = (\mathrm{F}^p, m_p)$ and $E_\pi = (\mathrm{F}^\pi, m_\pi)$ are bodies of evidence to define probability and possibility distribution, respectively. So

Table 2
Probability-possibility transformations based on evidence theory

Possibility types	Given principles	Transformation expressions
T_1: Ordinal scale	Probabilistic order preservation principle	$F_k^{\pi} = \overset{k}{\underset{h=1}{\cup}} U_h, \forall\, k = 1, \ldots, K$
T_2: Ratio scale (1)	Consistency principle and probabilistic order preservation principle	$\pi(u_i) = \begin{cases} \sum_{k=q_j}^{n} p(u_k), & (q_j \le i \le r_j,\ j = 1, \ldots, m) \\ \sum_{k=i}^{n} p(u_k), & \text{otherwise} \end{cases}$
T_3: Ratio scale (2)	Equidistribution principle	$p(u_i) = m_p(\{u_i\}) = \sum_{h=k}^{K} m_{\pi}\left(F_h^{\pi}\right)/\lvert F_h^{\pi}\rvert, \forall u_i \in G_k^{\pi} = F_k^{\pi} - F_{k-1}^{\pi}$

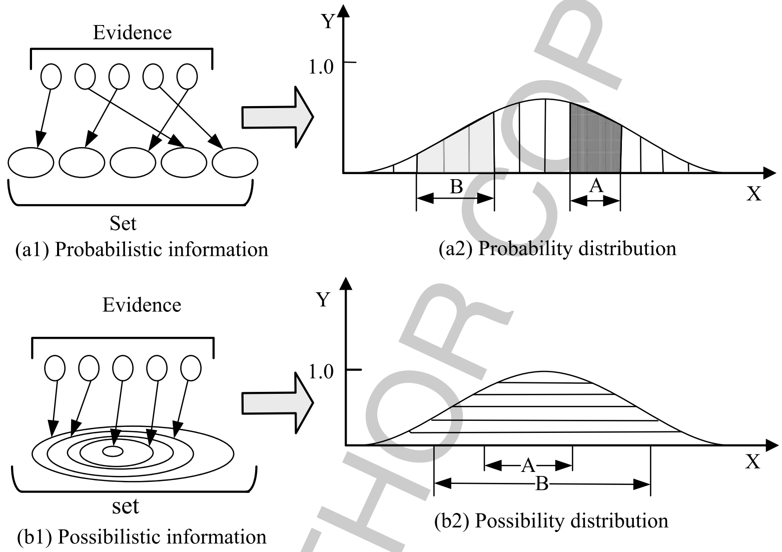

Fig. 3. Different information patterns with respect to corresponding quantifications.

the transformation between $p(u_i)$ and $\pi(u_i)$ can be substituted for the transformation E_p and E_{π}, where $m_p = (\{u_i\}) = p(u_i)$ and the focal elements are described as $F^P = \left\{\{u_i\}\,|u_i \in U_1 \cup U_2 \ldots \cup U_{Kp}\right\}$, as follows in Table 2.

3. Possibilistic entropy and quantification of epistemic uncertainty

3.1. Information-measure quantifications of epistemic uncertainty

In real world, what we concern much more is the amount of subjective information rather than statistical information only. Considerable efforts have been devoted to measurement methodologies of the information recognized by subjective perceptions, linguistic opinions, as well as information of single signal, alterable sources, and channels [32]. In the practice of reliability and risk analysis, when the information pertaining to an uncertainty quantity of interest is expressed only as a set of possible values that the quantity might take from, we look upon the information as being imprecise. Generally, we classify the information of evidence x in a fault tree as the following two patterns, whose information quantifications are illustrated in Fig. 3.

From Fig. 3 (a1) and (a2), we see that conflicting information could be quantified in the probability context, while from Fig. 3 (b1) and (b2), possibility measurements suit to quantify consonant information.

The attempt to quantify uncertainty has lead to an uncertainty function for each likelihood distribution. This agrees with the informal properties of uncertainty to some extent [25]. All major theories of uncertainty

Table 3
A summary of information quantifications for different uncertainties

Type	Name	Formula
Classical	Hartley information	$H(A) = \log_2 \|A\|$
	Shannon entropy	$S(p) = -\sum_{i=1}^{n} p_i \log_2 p_i$
General	U-uncertainty	$U(\Pi) = \sum_{i=1}^{n} (\pi_i - \pi_{i+1}) \log_2 i$
Vagueness	Measure of fuzziness	$f_C(A) = \|X\| - \sum_{u \in U} \|\mu_A(u) - C(\mu_A(u))\|$
	Measure of nonspecificity	$V(m) = \sum_{A \in F} m(A) \log_2 \|A\|$
Ambiguity	Measure of dissonance	$E(m) = -\sum_{A \in F} m(A) \log_2 P_l(A)$
	Measure of confusion	$C(m) = -\sum_{A \in F} m(A) \log_2 B_{el}(A)$

permit defining some numerical formulations for quantifying the information of uncertainty inherent in information sources, i.e. measures of uncertainty, as shown in Table 3.

In view of two types of uncertainty as possibilistic uncertainty and probabilistic uncertainty respectively, two classical measures of uncertainty have been correspondingly established successively as *Hartley measure* (or *Hartley information*) and *Shannon measure* (or *Shannon entropy*) [10, 11]. The former one applies to uncertainty conceived in terms of classical set theory and intends to capture the lack of specificity inherent in a finite set A of possible alternatives, by a set function $H(A)$ defined as (the cardinality $|A|$ is determined by the numbers of elements in A); whereas the latter one, *Shannon entropy*, qualifies uncertainty formalized in terms of precise probability theory and characterizes the amount of information by the distribution $p(\cdot)$, which is the most popularly used measure of uncertainty at present. Moreover, Shannon entropy is exemplified as the difficulty of discerning the outcome more likely eventuated. Provided that the case where $p_j = 1/n$ means the maximal uncertainty, the total evidential claim which conflicts with any alternative one is exhibited by $\left(\log_2 p(u)\right)$ and can be extended on a different scale. The Shannon entropy indicates the weighted average of conflict among evidential claims as the expected value of information provided, expressed by $p(u)$. However, the limitation of classical Shannon information theory lies in its unsuitability for intuitive information. This is due to that it stresses the transfer of information, but does not question the meaning of signals and the interpretations by observers.

According to the classification of uncertainty by Klir [24–26] from the viewpoint of information semantics, as seen in Table 3, the Shannon entropy measures the uncertainty emerging from randomness or conflict; whereas the Hartley measure captures that aspects of uncertainty characterized by the term non-specificity (or possibilistic ignorance). The degree of Hartley information is indeed the degree of non-specificity. The notion of non-specificity means our inability to distinguish which of several possible alternatives is the true one in a certain situation. Indeed the larger the set of possible alternatives is, the less specific is the characterization and vice versa.

With the rapid-growing uncertainty theories, the concept of information measure has been pushed further under the framework of generalized information theory [24]. In accordance with the two classical types of uncertainty and their corresponding information models, these new trends of extensions can be obtained as *Shannon entropy-like model* and *Hartley entropy-like model*, respectively. Their common properties of axiomatic requirements such as additivity, sub-additivity, expansibility, symmetry/consistency and continuity etc, expressed in a generic form [25, 26], make it credible to consider importance measures, transforming from one theory (e.g. probability theory) to another one (e.g. possibility theory) as request.

Comparing with the Shannon-like measures, which are only justified on intuitive grounds without satisfying the sub-additivity property, Hartley-like measure has been discovered in the framework of bodies of evidence, focusing on the derivation of functions satisfying proper axioms by means of suitable techniques. More details and constructive results could be found in references [24, 39]. In this paper, our focus is the possibilitic entropy deriving from the so-called Hartley measure and its extension, which requires a well-founded underlying theory of uncertainty-based information for our discussions.

3.2. Semantics interpretations and axiomatic and operational entropy models

The treatment of information measures may be twofold. First, the *operational approach* is based on coding-theoretic meaning, which represents the ideal rate of a compression code. Second, the *axiomatic one* is relevant to the viewpoint of verifying some properties belong to an adequate information measure by a specific functional solution, concerning about the perspective of uncertainty management. When it comes to possibility framework, we will exhibit such two entropy models proposed by the two approaches.

Given a basic possibility function r on the domain of discourse $U = \{u_1, u_2, \ldots, u_n\}$, it is assumed that the n-tuple function of possibility (possibility profile) $\mathbf{r} = \{r_1, r_2, \ldots, r_n\}$ and subsets $A_i = \{u_1, u_2, \ldots, u_i\}$ are both recorded in nested structures, i.e. $1 = r_1 \geq r_2 \geq \ldots \geq r_n$ and $A_i \subseteq A_{i+1}$ ($A_0 = \emptyset$ and $A_n = U$). Consider a mapping $m : 2^U \rightarrow [0, 1]$ defined on the power set 2^U, satisfying $m(\emptyset) = 0$ and $\sum_{A \in 2^U} m(A) = 1$. Here $m(A) \neq 0$ only when $A = A_i$. Let $m(A_i) = m_i$, then the n-tuple $\mathbf{m} = \{m_1, m_2, \ldots, m_n\}$ is regarded as the sets of *basic probability assignments* (BPA) [24]. Such consonant belief structures provide the measure-ground for possbilistic importance and then the ordered possibility distributions could be denoted as

$$r_i = m_i + m_{i+1} + \ldots + m_n \tag{5}$$

Solving them for m_i, we arrives at

$$m_i = r_i - r_{i+1} \tag{6}$$

Owing to this, the measure of *non-specificity* as shown in Table 3 can thus be preferably developed into the *generalized Hartley measure* (*GH*) in the general evidence space [24], denoting a weighted average of Hartley measure of all focal elements ($A \in 2^U$ and $m(A) > 0$), i.e.

$$GH(m) = \sum_{A \in 2^U} m(A) \log_2 |A| \tag{7}$$

Whose essential properties of additivity and subadditivity have been validated at present.

In the case of possibilistic uncertainty, the *GH* functional can be expressed in various forms and is usually called U-uncertainty [24], only through broadening the notion of possibility and as a possibilistic counterpart of the Shannon entropy [19]. According to Equations in Table 3, we obtain the weighted expression of U-uncertainty by the formula

$$U(\mathbf{r}) = \sum_{i=1}^{n} (r_i - r_{i+1}) \log_2 |A_i| \tag{8}$$

Essentially, it is a class of continuous and non-decreasing uncertainty measures with monotonicity, with similarity of the entropy-like function to a certain extent. In order to make the expected value of information meaningful, there exists $\sum_{i=1}^{n} (r_i - r_{i+1}) = 1$ ($r_{n+1} = 0$).

In addition to the axiomatic entropy-like model, the study of possibilistic information entropy $H_\varepsilon(\Pi)$ [33] as an operational solution to the corresponding information-coding problem enriches the contents of possibilistic information theory. Given information sources Π with the length of coding n, let $R_n(\cdot)$ denotes a compression rate of optimal codes, the function of possibilistic entropy can be written as

$$\begin{aligned} H_\varepsilon(\Pi) &= \lim_{n \rightarrow \infty} R_n(\pi, \varepsilon) \\ &= \log |\{a_i : \pi_i(a) > \varepsilon\}| \quad (0 \leq \varepsilon < 1) \end{aligned} \tag{9}$$

$H_\varepsilon(\Pi)$ settles both the problems of source coding (i.e. information compression to smaller sizes) and those of channel coding (i.e. information protection from transmission at a higher rate). Moreover, analogous to the probabilistic formulation of information entropy $H_\varepsilon(P)$, $H_\varepsilon(\Pi)$ is precisely equal to the Shannon entropy when $\varepsilon > 0$, or the Hartley measure when $\varepsilon = 0$. In fact, by means of replacing probabilities by possibilities and *independence* by *non-interactivity*, the possibilistic model of data transmission is more likely to be a "soft" coding with mathematically vague constraints, e.g. linguistic descriptions with practical meaning [34, 35]. At the same time, when investigating possibilistic coding based on distortion measure, we found that the typical ideas in traditional probabilistic approach indeed inspired the improvement for possibilistic information theory, which adopts the asymptotic point of view induced by the Shannon theory and attempts to provide finite-length code constructions afterwards. In a strict sense, the operational approach to possibilistic analogues of entropy is also meaningful from the viewpoint of axiomatic foundations [33].

Without loss of generality, due to the definition of the cardinality-based function [24], it follows directly that $|A_i| = i$ and $\log_2 |A_1| = \log_2 1 = 0$. Equation (8) can thus be further simplified as

$$U(\mathbf{r}) = \sum_{i=2}^{n} (r_i - r_{i+1}) \log_2 i \tag{10}$$

Here $U(\mathbf{r})$ preserves the ordering of $\mathbf{r}$ on the same set in the range of $[0, \log_2 |U|]$. In this sense, $U(\mathbf{r})$ is interpreted in semantics as exactly a rate of transmission of the number of possible events (or acceptable choices), where $\log_2 i$ represents the feasibility of transmitting the sent information and $(r_i - r_{i+1})$ denotes the fractions of the population that have to transmit precisely the event (or choice) u_i [32].

The branching theorem is essential among all the axiomatic requirements for obtaining a unique measure of possibilistic uncertainty. The uniqueness of $U(\mathbf{r})$ has been well established after Klir and Mariano [26] formulated the possibilistic branching requirement on axiomatic grounds.

4. Construction of possibilistic entropy-based measures of importance

4.1. Model framework and principles of matrix-based representation

With the aim of exhibiting the evolvement and connection of the above-mentioned axiomatic and operational approach for measures of uncertainty, we can integrate them with the measure-theoretic modeling framework and information principles proposed in previous sections. We start with the relevant notations [22] to present how an information entropy-based measure for importance identification in a fault tree can be constructed with respect to epistemic uncertainty. Let

a. $X = \{X_i\}$ $(i = 1, 2, \ldots, k)$ be the set of the basic and initiating events, which constitute unique, cut sets;
b. $Y = \{Y_j\}$ $(j = 1, 2, \ldots, m)$ be the set of cut sets, which contribute to the top event;
c. Y_{Top} be the algebraic expression of top event with the following composite of these events,

$$Y_{Top} = \sum_{j=1}^{m} X_1^{\alpha_{1j}} X_2^{\alpha_{2j}} \ldots X_k^{\alpha_{kj}} \tag{11}$$

Where $Y_{Top}(X_i)$ may be either the frequency or the probability of the top event (a component). The index α_{ij} is assigned as

$$\alpha_{ij} = \begin{cases} 1, & \text{the}i\text{th event is contained} \\ & \text{in the } j\text{th cut set} \\ 0, & \text{otherwise} \end{cases} \tag{12}$$

d. $\mathbf{T} = [\alpha_{ij}]_{k\times m}$ $(i = 1, 2, \ldots, k;\quad j = 1, 2, \ldots, m)$ be the indicator matrix for the top event derived from Equation (16), where the elements α_{ij} is the characteristic value as denoted in Equation (12). m is the number of cut sets in the top event and k is the total number of basic events. Thus, the column sums in $\mathbf{T}$ give the order of each cut set, at the same time, the row sums in $\mathbf{T}$ give the occurrence times of each event. Furthermore, the characteristic value α_{ij} is more likely to be the frequency status of a certain fault during some statistical time period. In this sense, the vector $\mathbf{E} = \{Y,\ P_{oss}(Y)\}$ can be taken on as the state space of fault mode and $\mathbf{F} = \{X,\ P_{oss}(X)\}$ can be the attribute space of fault possibility. $\mathbf{T}$ is thus used as an observation matrix for fault status.

Here the matrix-based representation $\mathbf{T}$ captures all relevant information stated in the top event in terms of a computationally general structure for measurement and is convenient for understanding the relation between unique basic events and their contributed cut sets [22].

4.2. Model descriptions of importance on basis of axiomatic entropy model

The measure for importance identification that we now put forth is mainly based on the possibilistic entropy in terms of axiomatic approach, which is also connected to operational one in an extended sense. The default order of cut sets is assumed to be the possibilistic preferential orderings (weak constraints). We assume the acceptable choices $A_i \in U$, listed in the order of their preference $A_1 \succ A_2 \succ \ldots \succ A_n$, correspond to the magnitude of their possibilisties [24, 32]. Then the relevant definitions are constructed as follows.

Definition 1. When a basic or initiating event X_i occurs, the uncertainty for possible combinations of cut sets which include X_i (e.g. nonlinearity and insufficient priori data) is reflected and quantified by the locally conditional possibilistic entropy as follows:

$$H(Y/X_i) = \sum_{j=1}^{m} \left[\pi(Y_j/X_i) - \pi(Y_{j+1}/X_i)\right] \log_2 j \tag{13}$$

Here $\pi(Y_{m+1}/X_i) = 0$, and $\alpha_i \cdot \sum_{j=1}^{m} [\pi(Y_j/X_i) - \pi(Y_{j+1}/X_i)] = 1$ should be satisfied to make the definition feasible.

Definition 2. Informed by the locally conditional entropy $H(Y/X_i)$ and state-feature matrix

$\mathbf{T} = \{\mathbf{E}, \mathbf{F}\} = \left[\alpha_{ij}\right]_{k\times m}$, the average amount of information of fault characteristics according to unique cut sets can be evaluated by the average conditionally possibilistic entropy as follows,

$$H\left(Y/X\right) = \sum_{i=1}^{k} \pi\left(X_i\right) \cdot H\left(Y/X_i\right) \tag{14}$$

Which reveals the degree of coherence between $X = \{X_i\}$ and $Y = \{Y_j\}$. Generally the more amount of information that fault characteristics possesses, the less entropy that the corresponding fault mode holds. Here $\pi\left(X_i\right)$ can be transformed from $p\left(X_i\right)$ according to the *preference preservation principle* [24] and Zadeh's *consistency principle* [40] in the following formulation. If $p\left(X_i\right)$ is also in a degressive sequence,

$$\pi\left(X_i\right) = i \bullet p\left(X_i\right) + \sum_{l=i+1}^{k} p\left(X_l\right) \tag{15}$$

By normalization, it follows that

$$\begin{aligned}\pi\left(X_i\right) &= \frac{p\left(X_i\right)}{\sum\limits_{i=1}^{k}\left[\max\limits_{l=i}^{k} p\left(X_l\right)\right]} \\ &= \frac{\sum\limits_{j=1}^{m}\alpha_{ij}}{\sum\limits_{i=1}^{k}\left(\max\limits_{l=i}^{k}\alpha_l\right)} = \frac{\sum\limits_{j=1}^{m}\alpha_{ij}}{\sum\limits_{j=1}^{m}\sum\limits_{i=1}^{k}\left(\max\limits_{l=i}^{k}\alpha_{lj}\right)}\end{aligned} \tag{16}$$

Definition 3. The possibilistic information-based importance measure with regard to X_i is then defined as

$$S_{T_i} = \frac{H\left(Y/X\right)}{H\left(Y/X_i\right)} \quad (i = 1, 2, \ldots, k) \tag{17}$$

The larger the index S_{T_i} is, the more influential the event X_i is. Applying Equation (13) and Equation (14) into it, we get

$$S_{T_i} = \frac{\sum\limits_{i=1}^{k}\pi\left(X_i\right)\cdot H\left(Y/X_i\right)}{H\left(Y/X_i\right)} = \frac{\sum\limits_{i=1}^{k}\left\{\pi\left(X_i\right)\cdot\sum\limits_{j=1}^{m}\left[\pi\left(Y_j/X_i\right) - \pi\left(Y_{j+1}/X_i\right)\right]\log_2 j\right\}}{\sum\limits_{j=1}^{m}\left[\pi\left(Y_j/X_i\right) - \pi\left(Y_{j+1}/X_i\right)\right]\log_2 j} \tag{18}$$

Among them, there is

$$\begin{aligned}\pi\left(Y_j/X_i\right) &= \frac{p\left(Y_j/X_i\right)}{\sum\limits_{i=1}^{k}\left[\max\limits_{l=i}^{k} p\left(Y_j/X_i\right)\right]} \\ &= \frac{\alpha_{ij}}{\sum\limits_{j=1}^{m}\left(\max\limits_{h=j}^{m}\alpha_{ih}\right)}\end{aligned} \tag{19}$$

5. A case study of importance in FTA

5.1. Case descriptions and computations of importance measures

To illustrate the application of the proposed method, an importance identification for a fault tree model of a specific nuclear power accidents prediction is presented here. As required by the possibilistic uncertainty, the matrix-based risk assessment model put forward by Iman [22] is utilized endowing the elements with preferential orderings information. According to the defense-in-depth design principle, a large accident in a nuclear power plant, for example the accidents in a high-level nuclear waste repository, will be the result of combinations of multiple basic events (or cut sets). Provided that there are 7 basic events leading up to performance lapse, even critical failure of automatic transmission in nuclear waste repositories, which are illustrated as follows. X_1 stands for release without tank rupture, X_2 stands for release with tank rupture, X_3 stands for oil seal leak, X_4 for oil seal failure undetected, X_5 for pressure too high, X_6 for oil temperature above limit and X_7 stands for electronics fault, respectively. Referring to [22], the final risk equation of the top event is represented as follows,

$$\begin{aligned}Y_{Top} = {} & X_1X_3X_5 + X_1X_3X_6 \\ & + X_1X_4X_5 + X_1X_4X_6 \\ & + X_2X_3X_4 + X_2X_3X_5 \\ & + X_2X_4X_5 + X_2X_5X_6 \\ & + X_2X_4X_7 + X_2X_6X_7\end{aligned} \tag{20}$$

which consists of $m = 10$ cut sets and $k = 7$ events for top events.

In accordance with parameters meanings explained in Section 4.2, the investigated matrix is written as

Table 4
Results for the proposed importance measure with respect to the basic events

Issues	X_1	X_2	X_3	X_4	X_5	X_6	X_7
$\pi(X_i)$	0.1333	0.2000	0.1333	0.1667	0.1667	0.1333	0.0667
$H\left(Y/X_i\right)$	0.5000	0.2203	0.3962	0.3429	0.2526	0.4399	0.1610
S_{T_i}	0.6644	1.5079	0.8384	0.9688	1.3151	0.7551	2.0633

Table 5
A comparison of importance rankings

Approaches	X_1	X_2	X_3	X_4	X_5	X_6	X_7
I	7	2	5	4	3	6	1
II	6	1	7	4	2	3	5
III	6	3	7	4	2	1	5

Table 6
SSCC results among the different approaches

SSCC	X_1	X_2	X_3	X_4	X_5	X_6	X_7
Approach I	0.1429	1.5929	0.5095	0.7595	1.0929	0.3095	2.5929
Approach II	0.3095	2.5929	0.1429	0.7595	1.5929	1.0929	0.5095
Approach III	0.3095	1.0929	0.1429	0.7595	1.5929	2.5929	0.5095

$$T = \left[t_{ij}\right]_{k\times m} = \begin{pmatrix} 1\,1\,1\,1\,0\,0\,0\,0\,0\,0 \\ 0\,0\,0\,0\,1\,1\,1\,1\,1\,1 \\ 1\,1\,0\,0\,1\,1\,0\,0\,0\,0 \\ 0\,0\,1\,1\,1\,0\,1\,0\,1\,0 \\ 1\,0\,1\,0\,0\,1\,1\,1\,0\,0 \\ 0\,1\,0\,1\,0\,0\,0\,1\,0\,1 \\ 0\,0\,0\,0\,0\,0\,0\,0\,1\,1 \end{pmatrix}_{7\times 10} \tag{21}$$

Applying Equation (16) and Equation (19), such matrixes as $[\pi(X_i)]_{7\times 1}$ and $\left[\pi\left(Y_j/X_i\right)\right]_{7\times 10}$ can be achieved, respectively. After calculating the locally conditional possibilistic entropy referred as Equation (13), we obtain the average possibilistic entropy $H\left(Y/X\right) = 0.3322$ by Equation (14). Thus the *possibilistic entropy-based measures of importance* with regard to each basic event are achieved in turn, as presented in Table 4.

Table 5 shows the ranking results by the proposed measures in this paper (Approach I) in comparison with those of Iman's (Approach II) [22], which presents the example of FTA first, and in comparison with the newly improved one named as moment independent sensitivity indicator by Borgonovo (Approach III) [3], which reuses the same examples. Note that the value of uncertainty importance measure in Approach II is actually

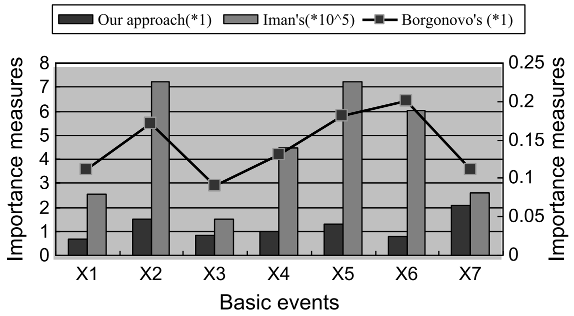

Fig. 4. A comparative illustration of the importance rankings.

Table 7
The SSCC matrix for the three importance measures presented in Table 3

Correlation ratios	Approach I	Approach II	Approach III
Approach I	1	0.6614	0.6946
Approach II	0.6614	1	0.4895
Approach III	0.6946	0.4895	1

(2.54E-5, 7.23E-5, 1.51E-5, 4.46E-5, 7.20E-5, 6.05E-5, 2.60E-5) and that in Approach III is (0.11, 0.17, 0.09, 0.13, 0.18, 0.20, 0.11).

This comparison is then illustrated explicitly as shown in Fig. 4.

5.2. Results Analysis

(1) Concordance Analysis. In order to measure the agreement among the three sets of ranking results by different approaches, we then compute the Savage Score Correlation Coefficients (SSCC) [5] which are more sensitive to concordance on the top ranks and are defined as follows,

$$S_i = \sum_{j=i}^{k} \left(1/j\right), \tag{22}$$

where i is the rank assigned to the ith order input factor for the sample sets made of k factors. The computation of SSCC involving $k = 7$ factors (events) gives the following results, as shown in Table 6.

Furthermore, the classical correlation ratios among the obtained SSCC are investigated as the following matrix shown in Table 7. One takes notice that the ranking results of Approach I in this discussion are in a higher agreement with Borgonovo's approach III, and then with Iman's approach II.

(2) Difference Discussions. We then turn our attention to the relatively concordant Approach I and

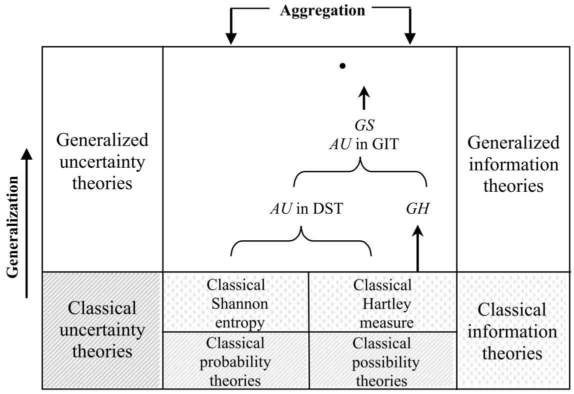

Fig. 5. The research perspective of an expanded framework for measurements of importance.

Approach III. The main difference from these two sets of results lies in the evaluation of the most relevant parameter. By possibilistic entropy-based measure of Approach I X_7 is the most influential event, and then followed by X_2 and X_5 in turn. Meanwhile by moment independent sensitivity indicator of Approach III, such case is located as X_6, then followed by X_5 and X_2.

The main reason related to such differences lie in the possibilistic nature, and the premises of nested structure (see Section 3.1) as well as the frequency property. Different from probabilistic uncertainty, the possibilistic one is more likely to be handled in an ordinal setting other than a ratio value. In the framework of possibility theory, the marginal and joint distribution models based on the rules of maximum and minimum are also distinct from those in the framework of probability theory.

In addition, the curve of Approach I (see Fig. 4) is much smoother than those other two, which implies that the importance measure of possibilistic one produces more conservative results, in comparison with that of probabilistic one. The faults identified by Approach I may involve more choices as a "soft" identification under a certain expected level. In this sense, possibilistic uncertainty acts as utility functions in decision theory to some extent.

5.3. Discussions

(1) Future research. With the rapid development of possibility theory, Dempster-Shafer theory of evidence (DST) and imprecise probability theory, which are all involved in the generalized information theory (GIT) [24], the formulation of uncertainty functions, are not limited to a certain axiomatic definition. The work also paves the way to further research in an expanded framework. As for the extensions of information measures, the first line of research concerns the construction of a general measure of information based on the Shannon entropy models. A second line of research is the exploration of Shannon entropy-like model or Hartley entropy-like model in the spirit of a general measure of information. Moreover, aggregation of the discussed information measures for total uncertainty is also in consideration. The research perspective is demonstrated in Fig. 5.

In fact information-based uncertainty and uncertainty-based information can be viewed as complementary due to the dual notions of uncertainty and information, and they can be expressed in two expanded frameworks named as *generalized uncertainty theory* (GUT) and *generalized information theory* (GIT) respectively. The former framework, drawn from fuzzy set theory is centered on a generalized constraint on values of various given variables, and is based on the notions of granularity and linguistic variables. The latter is characterized by gradually generalizing non-additive measures of various types and then, fuzzifying these measures further. It is predicted that the categories of generalized constraints manifested in GUT can thus guide the research in GIT.

(2) Extension to evidence space and connection with the belief function. The compatibility of DST with some special theories makes the current approach applicable to the evidence space, connecting with corresponding belief function. In general, we can say that the classical probability theory and possibility theory are subsets of the DST, respectively in accordance with the singletons and consonant focal elements. As a kind of upper probability, the possibility measure acts as not only a generalization of the probability measure, but also a certain plausibility measure [24, 25]. Such interrelationship is a mathematical superiority to applications of importance analysis involving various types of uncertainty.

Since the *U*-uncertainty is well established as a *GH*, its further generalization to DST is then theoretically fairly straightforward and is based on the following simple facts. First of all, the expression of Equation (8) is just like a weighted average of Hartley measure for all focal subsets. This view is consistent with the view of a body of evidence as a convex combination of sets. In second point, the weights as $(r_i - r_{i+1})$ are values of the BPA function m_i. The functional in Equation (7) is thus viewed as $GH(m)$ by satisfying the expansibility requirement. No matter what the permutation values of m_i are, for subsets with equal cardinalities,

$GH(m)$ holds invariant in accord with the symmetry requirement in DST.

Furthermore, when all focal subsets are singletons, the BPA function m_i actually acts as a probability distribution on U. Hence $GH(m) = 0$ is derived for the probabilistic bodies of evidence, which explains the bound condition is then $0 \leq GH(m) \leq \log_2 |U|$. On the other hand, when all focal subsets are nested or consonant, P_l and B_{el} in DST are the same functions as P_{oss} and N_{ec} in possibility theory respectively by possibilistic bodies of evidence. It follows from these facts that the importance measure discussed under possibilistic uncertainty could be investigated in evidence space by a specific belief function. When handling a mixture of input parameters due to incomplete data, the range of each input can be described in a specific confidence interval $[B_{el}(A), P_l(A)]$ [12], which means more information based on empirical knowledge may be elicited in an assessment. As a consequence, these generalizations play a part in such developing levels of uncertainty modeling as "Formalization - Calculus - Measurement - Methodology" [24].

6. Conclusions

This paper discusses the challenges confronted during the representation and treatment of epistemic uncertainties in reliability and safety assessment to overcome the deficiency in handling imprecise information encountered by probabilistic importance measures by introducing possibilistic measurements in risk-informed decision-making. According to the measurability of possibilistic uncertainty-based measures and feasibility of generalized information measures, an importance measure of fault tree based on possibilistic information entropy is proposed and its construction and implementation are discussed, addressing the following points:

- The fuzzy-measure theoretical ground integrating intuitive interpretations of epistemic uncertainty with possibilistic measurements;
- The information-measure quantification approaches by means of the axiomatic treatment and operational representation;
- The bipolar semantic interpretations and dual uncertainty measures in the modeling framework and construction principles;
- A comprehensive case study along with the concordance analysis and difference discussions.

The numerical example of importance computation in a fault tree indicates that from the viewpoint of intuitive backgrounds and information semantics, possibilistic uncertainty can be regarded as the expected value of information identifying the acceptability of events in the range of fuzzy domain. In this respect, the possibility assignments are of close relationship of the preferential orderings and are similar to utility functions used in decision theory.

The proposed method in this paper is applicable to situations wherein events (or propositions) are imprecise in nature, or necessary information is insufficient for complex or fuzzy systems and relations. Moreover the proposed measure for importance identification is not only aiding in identifying the weakness in a system, but also assists in system control, test, equipment diagnosis, and optimal configuration.

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