Fatigue Life Estimation Considering Damaging and Strengthening of Low amplitude Loads under Different Load Sequences Using Fuzzy Sets Approach

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ABSTRACT: In this study, based on the Miner rule, a new linear damage accumulation rule is proposed to consider the strengthening and damaging of low amplitude loads with different sequences using fuzzy sets theory. This model improves the application of the traditional Miner rule, by considering not only the damaging and strengthening of low amplitude loads, but also the load sequence effects. To apply the proposed model, the law of selecting membership functions for different load spectra is found, and different membership functions are investigated to show the important influence on estimating fatigue life. Applicability of the method is validated by comparing with the experimental data. It is also found that the predicted fatigue life by the proposed method is more accurate and reliable than that by the traditional ones.

KEY WORDS: fatigue, low amplitude loading strengthening, Miner rule, life prediction, membership functions, load sequence, fuzzy sets.

INTRODUCTION

In most engineering structures and parts, the fatigue damage is one of the main forms of failure (Stephen et al., 2000). A reliable lifetime prediction is particularly important in the design, safety assessments, and optimization of engineering materials and structures. With the accumulation of fatigue damage, some accidents will occur. Therefore, it is important to formulate

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a method to evaluate the fatigue damage accumulation in order to predict the fatigue life, the residual strength, etc. Current fatigue analyses of metallic structures undergoing variable amplitude load histories, including many engineering components operating in aviation, power generating, and other industries, are mostly based on linear cumulative damage concepts, as proposed by Palmgren and Miner (Schijve, 2001).

Damage accumulation in materials is very important, but very challenging to characterize in a meaningful and reliable manner. Until now, various theories and models have been proposed to predict the life of fatigue loaded structures (Golos and Ellyin, 1988; Cheng and Plumtree, 1998; Shang and Yao, 1999; Svensson, 2002; Masahiro, 2005; Oller et al., 2005; Chen et al., 2007; Kreiser et al., 2007; Scott-Emuakpor et al., 2008; Grammenoudis et al., 2009a, b; Makkonen, 2009; Zhu and Huang, 2010; Zhu et al., 2010). In general, fatigue damage cumulative theories can be classified into two categories: (1) linear damage cumulative theories and (2) nonlinear damage cumulative theories. The linear damage rule, also called the Palmgren–Miner rule (just the Miner rule for short), is commonly used in analyzing cumulative fatigue damage. However, this rule also has some drawbacks for its empirical nature (Schijve, 2001). The evidenced errors can be observed by applying the rule because the effects of loading sequence and load interaction are ignored. The rule also neglects damages induced by stresses below the fatigue limit. Some design procedures have been proposed to overcome the difficulty by introducing the concept of a two-slope *S–N* curve. The generation of fatigue damage is a procedure that material properties are continuously deteriorated under cyclic loading and the damage variables dependent on the size of stress and strain. Based on these characteristics of fatigue damage, recently, some nonlinear damage cumulative theories are presented and the typical examples include ductility exhaustion models (Cheng and Plumtree, 1998; Zhu et al., 2010), continuum damage mechanics approaches (Oller et al. 2005; Shang and Yao, 1999; Grammenoudis et al., 2009a, b), and energy-based damage methods (Golos and Ellyin, 1988; Scott-Emuakpor et al., 2008; Zhu and Huang, 2010). Several attempts have been made to propose more reliable fatigue damage rules. Recently, according to crack arrest theory and using a fracture mechanics approach, Svensson (2002) extended the Miner rule to the case for the decrease of a fatigue limit with the increase of damage. Based on energy principle and a cumulative damage parameter that accounts for the progressive damage accumulation dictated by the path of load history, Kreiser et al. (2007) developed a nonlinear damage accumulation model. Lately, using statistical methods, Makkonen (2009) proposed a new method for constructing design curves for finding the crack initiation life for any material. It estimated the total fatigue life by calculating and summing up the stable crack growth time and the estimated initiation life.

Among many developed damage models, the Miner rule is one of the most efficient models due to its simplicity. Comprehensive reviews of the subject about fatigue damage models have been conducted by Fatemi and Yang (1998), Schijve (2003), and Besson (2010). However, new propositions are limited to very specific conditions (e.g., certain load sequences or load interaction) hindering the adoption of those rules in many design practices.

The objectives of this study are to develop a new accumulation damage model based on the Miner rule to investigate the damage induced by stresses below the fatigue limit and study the load sequence effects. Special attention will be paid on the strengthening and damaging effects of low amplitude loads below the fatigue limit. It is fuzzy when the stress that the structural components subjected to is slightly lower than the fatigue limit. It should be emphasized that this study has the advantage of reflecting the influence of load sequence on the fatigue life. The strengthening and damaging under low amplitude loads below the fatigue limit can be taken into account for the whole process of fatigue damage through using fuzzy sets method. The model will be used to predict the life of specimens subjected to various multilevel and two-stress level tests and analyze the corresponding damage evolution.

LINEAR DAMAGE ACCUMULATION RULE

Fatigue is a damage accumulation process in which material property deteriorates continuously under the loading. For fatigue damage problem, the Miner rule was proposed based on the following key assumptions:

1. The rate of damage accumulation remains constant over each loading cycle.
2. The damage occurs and accumulates only when the stress is higher than the fatigue limit.
3. The failure of component is assumed to occur when cumulative damage reaches the unity.

According to the above assumptions, fatigue life prediction under constant amplitude block loading can be predicted by the Miner rule:

$$D = \sum_{i=1}^{k} \frac{n_i}{N_i} \tag{1}$$

However, most metallic materials exhibit more complex behaviors than the ones modeled by a linear damage rule. Therefore, the linear damage rule sometimes gives inaccurate prediction results due to the following shortcomings. First, it neglects the damage induced by stresses below the fatigue limit. In fact, the strengthening and damaging under low amplitude loads below the fatigue limit are not properly considered in this theory. The experimental results in Lu and Zheng (2008, 2009a, b), Sinclair (1952), and Makajima et al. (2007) have shown that the damage of loads (including low amplitude loads nearby the fatigue limit) is one of the main reasons for prediction errors. Second, it is not sensitive to the load sequence by the linear accumulation theorem of fatigue damage. The influence of load sequence on fatigue life is ignored. Finally, it is an accumulation theorem of fatigue damage based on the determinate theory. It cannot consider the factors of indeterminacy.

In this article, an effort will be made to get a better life prediction capability and applicability of the proposed Miner rule by considering the damage induced by stresses below the fatigue limit and the effects of load sequence and load interaction.

THE PROPOSED FUZZY FATIGUE LIFE PREDICTION MODEL

The Linear Damage Rule under the Low Amplitude Strengthening Loading

It has been well known that the fatigue limit for some metals may be improved by low amplitude loads, followed by a process of gradually increasing the amplitude of applied alternating stress in small increments – a procedure ordinarily called coaxing (Lu and Zheng, 2008). This phenomenon is also called the effect of low amplitude strengthening loading, which leads to an increase in fatigue strength and fatigue life of metallic materials and structures exhibiting the strengthening effect by applying proper low amplitude loads below the fatigue limit and proper relative strengthening cycles. The strengthening effect of material under low amplitude loads should be applied in lightweight design and reliability evaluation (Nakagawa and Ikai, 1979).

The fatigue experiments show that the characteristic of material and fatigue life of components can be obviously increased or prolonged by strengthening under low amplitude loads below the fatigue limit (Sinclair, 1952; Makajima et al., 2007; Lu and Zheng, 2008, 2009b). However, in the Miner rule, not only the damage induced by any stress below the fatigue limit but also the strengthening of low amplitude loads below the fatigue limit is not properly considered. For many components subjected to loads below the fatigue limit, the use of indiscriminate often leads to nonconservative situations. Therefore, the strengthening and damaging of low amplitude loads nearby the fatigue limit need to be investigated properly to evaluate the fatigue damage, accurately predict the fatigue life, and fully exploit the fatigue limit of material.

It has been verified that some metallic materials exhibit highly nonlinear fatigue damage evolution with load dependency (Pereira et al., 2008). Therefore, depending on load history, the Miner rule can underestimate or overestimate the fatigue life due to the load interaction and load sequence effects. For most metallic materials, the mechanism of low amplitude loading strengthening effects could be explained by micro-plastic deformation or work hardening, in which the strengthening effect in loading spectrum could be considered (Sinclair, 1952). In the actual engineering, the components are operated in the process of continuous cyclic hardening and softening. This process depends upon the loading stress and the cycles under the current stress. In this section, the effects of low amplitude strengthening loading below the fatigue limit and load interaction are characterized by a strengthening function based on the damage caused by the low amplitude loads.

According to the characteristic of load strengthening, Lu and Zheng (2009a) divided the loads into three regions, which are the damage region, strengthening region, and useless region corresponding to the stress for $\sigma > \sigma_U$, $\sigma_L \leq \sigma \leq \sigma_U$ and $\sigma < \sigma_L$, respectively. In fact, the stress slightly lower than the fatigue limit applied on the structural components would introduce not just only the strengthening effects as proposed in Lu and Zheng (2009a), but also the cumulative damage. Therefore, the load regions can be suggested as (Lu and Zheng, 2009a): the region $\sigma_L \leq \sigma \leq \sigma_U$ is the strengthening and low amplitude damaging region, $\sigma > \sigma_U$ the damaging region, and $\sigma < \sigma_L$ the useless region. The modified load regions and their strengthening and damaging in different cycle ratios are shown in Figure 1.

According to Figure 1, in the strengthening and damaging regions, the strength of material or components can be increased when the strengthening cycles are enough. When the load is less than σ_L, the small loads can be ignored for the load spectrum analysis. In damaging region, experimental results show that the residual strength of material is not monotonically decreased. In earlier life, the residual strength will be increased and then decreased until fractured (Nakagawa, 1983).

Based on the mechanism analysis outlined above, the following facts will be considered from the tests: (1) the strengthening effect of low amplitude loads is determined not just by the current loading stress, but also the number of applied loading cycles; (2) these effects could not exhibit if the relative low amplitude loads are too small or even negligible; (3) these effects were based on the damage caused by low amplitude loads blow

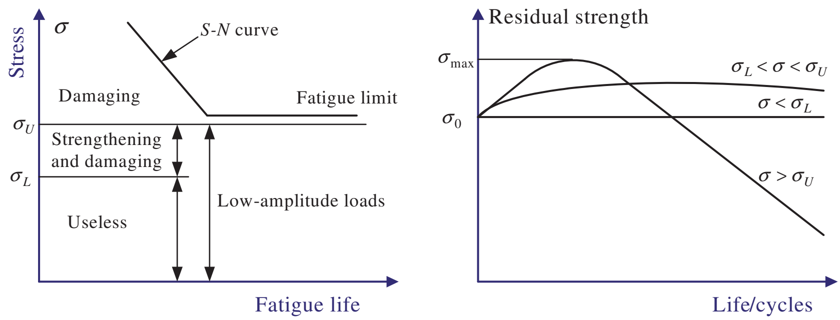

Figure 1. *Load region, its strengthening and damaging in different cycle ratios.*

the fatigue limit. In the study, the Miner rule is modified by considering low amplitude loading strengthening. In order to calculate these effects, a strengthening function is introduced into the cumulative damage as

$$S(\sigma_i, n_i) = \exp(-m'\sigma_i) \tag{2}$$

where m' is the strengthening coefficient related to material properties and obtained from fatigue experimental data. The strengthening functions, which exhibit the relationship among, fatigue life after strengthening under low amplitude loads, the strengthening loads and the strengthening cycles, are acquired.

For the stress within $\sigma < \sigma_L$, the corresponding value of strengthening function equals 1. Under the low amplitude loading strengthening, we assume that damages caused by the current cycle can be calculated by multiplication of damage caused by the previous cycle and strengthening function. The strengthening and damaging caused by the r–k level stress amplitude blocks can be deduced based on the above described behavior. The strengthening and damaging of material after applied n_i cycles at σ_i may be obtained by the following formula as

$$\frac{\exp(m'\sigma_i)}{m'\sigma_i}[1 - \exp(-m'\sigma_i n_i)]\frac{1}{N_i}, \; k+1 \le i \le r \tag{3}$$

where N_i is the number of cycles to failure under the solely application of $\sigma_{\max,i}$ of block i. According to the power exponent form of S–N curves for materials, $1/N_i$ corresponds to the damage caused by the first cycle of block i.

Similarly, using these concepts and the assumptions given, an expression for estimating the number of cycles to failure is developed by considering low amplitude loading strengthening and damaging for repeated blocks of many different stress levels as

$$\sum_{i=1}^{k}\frac{n_i}{N_i}+\sum_{i=k+1}^{r}\frac{\exp(m'\sigma_i)}{m'\sigma_i}[1-\exp(-m'\sigma_i n_i)]\frac{1}{N_i}=1 \tag{4}$$

Equation (4) can be used to estimate the cumulative damage for components or materials under the multilevel loading. It should be noted that there are no strengthening effects if the strengthening cycle is equal to zero.

The Linear Damage Rule Considering Load Sequences

The Miner rule ignores the influence of load sequence on fatigue life. In fact, the cumulative damage depends not only upon the current applied stress level, but also upon its loading history. Generally, the fatigue life prediction is on the dangerous side by the Miner rule under high–low (H–L) loading sequences, and the fatigue life prediction is on the safe side by the Miner rule under low–high (L–H) loading sequences.

According to the traditional Miner rule, there is a determinate fatigue limit whether the component bears any type of loads. From the key assumptions of Miner rule (Frost et al., 1999; Liu et al., 2006), it does not reflect the rule of fatigue phenomenon. The reason that traditional Miner rule does not consider the load sequence is that the fatigue limit is assumed to be determinate. Practically, the fatigue limit of the component is indeterminate, and the load sequence has a strong influence on the fatigue limit. Therefore, the fatigue damage caused by the stress nearby the fatigue limit under different load sequences needs to be further investigated.

The Miner rule is a linear accumulation theorem of fatigue damage which is assumed to be absolutely equivalent and objective. It can be characterized by Figure 2.

The damage function for the Miner rule which indicates that damage occurs and accumulates only when the stress is higher than the fatigue limit is expressed as

$$\mu(\sigma)=\begin{cases}1, & \sigma\geq\sigma_{-1}\\ 0, & \sigma<\sigma_{-1}\end{cases} \tag{5}$$

Experimental evidence under completely reversed loading condition often indicates that $D<1$ for H–L loading sequences, and $D>1$ for L–H loading sequences. In this section, the influence of load sequence on fatigue limit will be discussed and the Miner rule is modified to reflect the influence of load sequence on fatigue life.

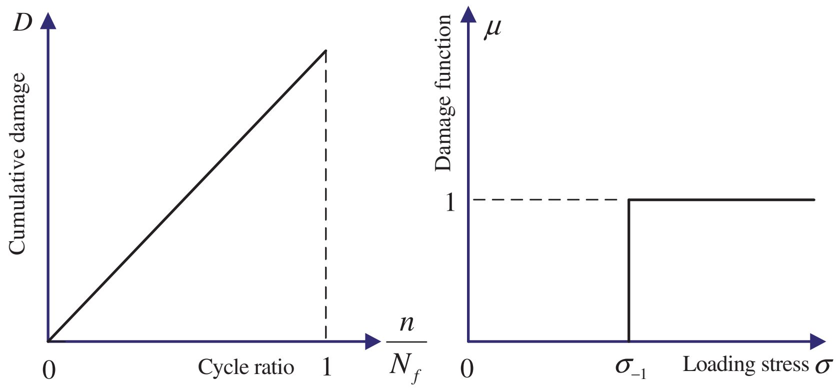

Figure 2. *Miner rule and its damage function.*

As outlined above, if $D > 1$, the strengthening effects are exhibited and the linear cumulative damage is decelerated for the load sequence effect. Otherwise, the linear cumulative damage is accelerated. It is easy to explain why the linear cumulative damage is not equal to 1 in variable amplitude load spectrum if the strengthening or damage effects of low amplitude loads below the fatigue limit are rationally considered under different load sequences.

The assumption of the Miner rule is that the fatigue limit is a material constant. However, recent studies (Lu and Zheng, 2008, 2009a, b; Wang and Nie, 2008) have shown that the fatigue limit of material is not simply a material constant which depends not only upon the materials, but also upon the sequences of load spectrum while its value can be influenced by different load sequences. Hence, it is inappropriate to consider the fatigue limit as a material constant in the Miner rule, which is also supported by the experimental results.

The indeterminacy of fatigue limit results in the indeterminacy of fatigue damage under different stress levels. To reflect the influence of load sequence on the fatigue life, the indeterminacy of fatigue limit under different load sequences must be considered. In general, the fatigue life prediction is not effective by using the traditional Miner rule when the loads are in the sequence from high to low. The reason is that the fatigue limit of the component is reduced because of the damage for high-amplitude load. If the original fatigue limit is used to calculate for all the sequences, the damage caused by some stress lower than the original fatigue limit could be ignored. However, it does not agree with the rule of fatigue phenomenon. In fact, the low amplitude loads may become damaging loads in next cycles.

Therefore, the fatigue damage caused by some stresses within the section below the determinate fatigue limit should be extended. In that way, the damage caused by the stress lower than the original fatigue limit must be considered for the load sequence effects. And the cumulative damage is calculated by adding the extra damage caused by some stresses within the falling interval of fatigue limit on the basis of Miner rule.

When the loads are in L–H loading sequences, we usually get $D > 1$. The reason is that the fatigue limit of the component increases in virtue of the low amplitude load for the low amplitude loading strengthening effect. If the original fatigue limit is used for calculation, the damage caused by the stress higher than the fatigue limit is over predicted. Therefore, the fatigue damage caused by some stresses within the section above the determinate fatigue limit should be extended. Consequently, the damage caused by the stress within the rising interval of fatigue limit is over predicted. And the cumulative damage is calculated by subtracting the extra damage caused by some stresses within the rising interval of fatigue limit on the basis of Miner rule.

The indeterminacy of fatigue limit is reflected in the following two aspects. The first one is the indeterminacy of original fatigue limit. For the different specimen with same material, the original fatigue limit is different and tends to become dispersed. The second one is the indeterminacy of fatigue limit under different loading sequences. Under the H–L loading sequences, the fatigue limit of specimen tends to decrease which corresponds to the left moving of σ_{-1}. For L–H loading sequences, the fatigue limit of specimen tends to increase which corresponds to the right moving of σ_{-1}. Due to the load sequence effect, the fatigue limit tends to be variable instead of a material constant. The change of fatigue limit under different loading sequences is shown in Figure 3.

From Figure 3, we can get

$$(\sigma_{-1})_{\mathrm{H-L}} = \sigma_{-1} - \Delta\sigma_{-} \tag{6}$$

$$(\sigma_{-1})_{\mathrm{L-H}} = \sigma_{-1} + \Delta\sigma_{+} \tag{7}$$

According to the experimental results (Makajima et al., 2007; Lu and Zheng, 2008, 2009a), $\Delta\sigma_{-}$ and $\Delta\sigma_{+}$ are influenced remarkably by the damage extent of materials caused by the first level stress in the loading history, which includes the first level stress amplitude and its relative cycles. $\Delta\sigma_{+}$ caused by the strengthening effects of low amplitude loads below the fatigue limit will be weakened in program loading if the relative cycles of low amplitude loads are too small. The strengthening effects will be increased with the increase of low amplitude loading cycles. With increasing cycles, the strengthening effects of low amplitude loads make the damage of material,

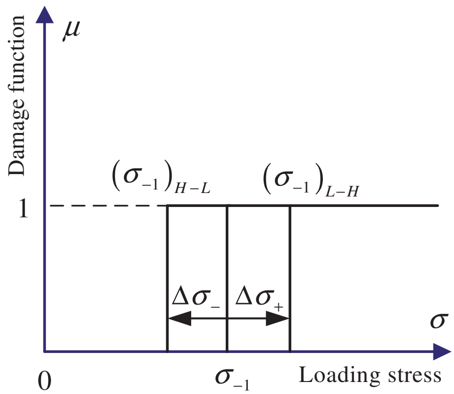

Figure 3. *The change of fatigue limit under different loading sequences.*

caused by the high amplitude load, partly be restored in every program loading. The whole cumulative damage is decelerated and the fatigue life will be prolonged in program loading. In this case, the fatigue damage will be overestimated and residual life will be underestimated if the strengthening effects were not considered. $\Delta\sigma_{-}$ caused by the accelerating damage of high amplitude loads will also be weakened in program loading due to small cycles of high amplitude loads. In this case, the linear cumulative damage is accelerated and the fatigue limit of component becomes smaller than its original fatigue limit. If the accelerating damage effects were not considered, the fatigue damage will be underestimated and residual life will be overestimated.

As mentioned above, the fatigue limit under different load sequences can be expressed as a interval variable, which tends to decrease from the original fatigue limit to the lower bound of fatigue limit under H–L loading sequences, and increase from the original fatigue limit to the upper bound of fatigue limit under L–H loading sequences

$$[\sigma_{-1}] = \begin{cases} \left[\sigma_{-1} \overset{-}{\longrightarrow} (\sigma_{-1})_{\mathrm{H-L}}\right], \ \mathrm{H-L\,loading\,sequences} \\ \left[\sigma_{-1} \overset{+}{\longrightarrow} (\sigma_{-1})_{\mathrm{L-H}}\right], \ \mathrm{L-H\,loading\,sequences} \end{cases} \tag{8}$$

where $(\sigma_{-1})_{\mathrm{H-L}} \leq \sigma_{-1} \leq (\sigma_{-1})_{\mathrm{L-H}}$. The lower and upper bound of fatigue limit under different loading sequences can be determined by experiments or practical experience. For the standard material specimen of mild steel, the maximum growth rates of fatigue limit are about 28% after strengthening by coaxing effect and about 25% for cast iron (Lu and Zheng, 2008).

According to Equations (6)–(8), the Miner rule can be used to predict the fatigue life based on the proper selection of fatigue limit under the relevant loading sequence. In next section, the fuzzy sets method are used to analyze the fuzziness of fatigue damage caused by the stress nearby the fatigue limit under different load sequences and low amplitude loading strengthening effects.

The Proposed Linear Damage Rule Considering Damaging and Strengthening of Low Amplitude Loads under Different Load Sequences

According to the shortcomings of traditional Miner rule, the fuzziness of fatigue damage caused by the stress nearby the fatigue limit and low amplitude loading strengthening damage needs to be considered. The indeterminacy of fatigue limit leads to the indeterminacy of fatigue damage under different stress levels. The indeterminacy is a fuzziness of the fatigue damage, which can be described by the membership grade in the fuzzy set theory. In the 'fuzzy zone' nearby the fatigue limit, there is no need to give the absolute conclusion of bringing damage or not, but to describe how much the degree of the damage is (Liu et al., 2006; Wang and Nie, 2008; Zhu et al., 2009). In this section, the fuzzy factors are taken into account and the new approach is given.

Some uncertainties, especially those involving descriptive and linguistic variables, cannot be handled satisfactorily in the deterministic method and the fuzzy set theory is more appropriate. Specifically, the applicability of fuzzy set theory can be extended to the situations which involve subjective uncertainty, or for which data are insufficient for statistical calculation. So, the fatigue damage caused by one cycle of loading stress is expressed by a fuzzy set $\tilde{D}$. The damage of component caused by one cycle under σ_i is denoted by $D(\sigma_i)$, and the corresponding D_{-1} under σ_{-1}. A fuzzy variable has values which can be expressed in natural language and defined by membership functions (MFs). Therefore, we can use the fuzzy set method to describe the fatigue as follows:

For the stresses below the fatigue limit, the MFs of $\tilde{D}$ are defined as

$$\mu_{\tilde{D}}(\sigma_i) = D(\sigma_i)/D_{-1}, \text{ when } \sigma_i \in [0, \sigma_{-1}) \tag{9}$$

For the stresses higher than the fatigue limit, the damage occurs and accumulates inevitably into the component as

$$\mu_{\tilde{D}}(\sigma_i) = 1, \text{ when } \sigma_i \in [\sigma_{-1}, \sigma_M] \tag{10}$$

Combining Equations (9) and (10) gives

$$\mu_{\tilde{D}}(\sigma_i) = \begin{cases} \dfrac{D(\sigma_i)}{D_{-1}}, & \sigma_i \in [0, \sigma_{-1}) \\ 1, & \sigma_i \in [\sigma_{-1}, \sigma_M] \end{cases} \tag{11}$$

where $0 \leq \mu_{\tilde{D}}(\sigma_i) \leq 1$.

According to the traditional Miner rule, the damage in one cycle under σ_i is given by

$$D(\sigma_i) = \frac{1}{N_i} \tag{12}$$

Due to the same critical value of cumulative damage to failure, we can easily get

$$D(\sigma_i)N_i = D_{-1}N_0 = 1 \tag{13}$$

Based on the above described behavior, Equations (12) and (13), the fuzzy model of S–N curve can be given by the following formula

$$N_i = \begin{cases} N_0/\mu_{\tilde{D}}(\sigma_i), & \sigma_i \in [0, \sigma_{-1}) \\ N_0(\sigma_{-1}/\sigma_i)^m, & \sigma_i \in [\sigma_{-1}, \sigma_M] \end{cases} \tag{14}$$

Substituting Equation (14) into Equation (4) leads to

$$\sum_{i=1}^{k} \frac{n_i}{N_i} + \sum_{i=k+1}^{r} \frac{\exp(m'\sigma_i)}{m'\sigma_i}[1 - \exp(-m'\sigma_i n_i)] \frac{\mu_{\tilde{D}}(\sigma_i)}{N_0} = 1 \tag{15}$$

Comparing Equation (15) with Equation (1), it can be easily noted that the expression of Equation (15) is the proposed Miner rule under low amplitude loading strengthening and considering the load sequence effects using fuzzy set theory. The former part of Equation (15) which corresponds to the traditional Miner rule in Equation (1) can be written as

$$D_1 = \sum_{i=1}^{k} \frac{n_i}{N_i} \tag{16}$$

The latter part of Equation (15) corresponds to the accumulative damage considering the low amplitude loading strengthening and load sequence effects, which can be described by

$$\Delta D = \sum_{i=k+1}^{r} \frac{\exp(m'\sigma_i)}{m'\sigma_i}[1 - \exp(-m'\sigma_i n_i)] \frac{\mu_{\tilde{D}}(\sigma_i)}{N_0} \tag{17}$$

In the fuzzy rules, the fuzzy information cannot be analyzed and dealt with until the fuzziness is quantitatively described by the MFs.

Eliciting MFs from data is one of the fundamental issues associated with the application of fuzzy set theory. Currently, there are many known ways of obtaining MFs. The existing MFs can be classified in the following way (Dombi, 1990): heuristic MFs, MFs based on reliability concerns with respect to the particular problem, MFs based on more theoretical demand, and MFs and control. Actually, the heuristic methods use predefined shapes for MFs and have been used successfully in mechanical engineering (Huang, 2000; Liu et al., 2006; Wang and Nie, 2008; Zhu et al., 2009).

The shape of such functions may vary in infinite ways. Still the shape selection cannot be arbitrary but it should be believable. Dombi's (1990) review of heuristic functions found the following common properties for constructing the MFs: (1) all MFs are continuous; (2) all MFs map an interval $[a, b]$ to $[0, 1]$, $\mu[a, b] \to [0, 1]$; (3) the MFs are either monotonically increasing or monotonically decreasing or both increasing and decreasing; (4) the monotonous MFs on the whole interval are either (a) convex functions, or (b) concave functions, or (c) there exists a point c in the interval $[a, b]$ such that $[a, c]$ is convex and $[c, b]$ is concave (called S-shaped functions); (5) boundary condition, monotonically increasing functions have the property $\mu(a) = 0$, $\mu(b) = 1$, while monotonically decreasing functions have the property $\mu(a) = 1$, $\mu(b) = 0$; (6) very important is the linear form or linearization of the MFs; (7) $\mu(x)$ is a rational function of polynomials,

$$\mu(x) = \frac{a_0 x^n + a_1 x^{n-1} + \cdots + a_n}{A_0 x^m + A_1 x^{m-1} + \cdots + A_m} (m \neq 0)$$

By choosing different parameters $a_0, a_1, \ldots, a_n$, $A_0, A_1, \ldots, A_m$, and n, m, the MFs can be piecewise linear functions (linearly increasing, linearly decreasing or a combination of these) or piecewise monotonic functions (a piecewise smooth transition between non-membership and full-membership regions, the smooth transition may be described by functions such as x^2, $\sin(x)$, and $\exp(x)$). In summary, the obtained MFs should meet the following fundamental requirement:

1. on a theoretical basis;
2. easy to calculate and fit to the problem;
3. described by only a few parameters and with parameters that are meaningful;
4. with a linearized form for the applications;

In this article, a proper function needs to be selected as the MF of fatigue damage according to their characteristics. Due to the rule of fatigue phenomenon, increasing MFs should be selected to analyze the fuzziness of fatigue damage to the components. From Equation (11), $\mu_{\tilde{D}}(\sigma_i)$ is an increasing continuous function with its value between 0 and 1 for the stresses lower than the fatigue limit. For the stresses $\sigma > \sigma_U$, the value of $\mu_{\tilde{D}}(\sigma_i)$ is always identical with 1. For the stresses $\sigma < \sigma_L$, it is useless small loads which can be omitted load spectrum analysis. That is to say if $\sigma_i \in [0, \sigma_L)$, then $\mu_{\tilde{D}}(\sigma_i) = 0$.

In order to determine an approximate expression for the fuzziness of fatigue damage, some common distribution functions are often used as the MFs. In practice, some increasing heuristic MFs frequently used in mechanical engineering are shown as follows (Dombi, 1990; Huang, 2000; Liu et al., 2006; Wang and Nie, 2008; Zhu et al., 2009):

1. Increasing half-trapezoidal shaped MF

$$\mu_{\tilde{D}}(\sigma_i) = \begin{cases} 0, & \sigma_i < \sigma_L \\ \dfrac{\sigma_i - \sigma_L}{\sigma_{-1} - \sigma_L}, & \sigma_L \le \sigma_i < \sigma_{-1} \\ 1, & \sigma_i \ge \sigma_{-1} \end{cases} \tag{18}$$

2. Increasing half-parabolic shaped MF

$$\mu_{\tilde{D}}(\sigma_i) = \begin{cases} 0, & \sigma_i < \sigma_L \\ \left(\dfrac{\sigma_i - \sigma_L}{\sigma_{-1} - \sigma_L}\right)^2, & \sigma_L \le \sigma_i < \sigma_{-1} \\ 1, & \sigma_i \ge \sigma_{-1} \end{cases} \tag{19}$$

3. Increasing half-square root shaped MF

$$\mu_{\tilde{D}}(\sigma_i) = \begin{cases} 0, & \sigma_i < \sigma_L \\ \left(\dfrac{\sigma_i - \sigma_L}{\sigma_{-1} - \sigma_L}\right)^{\frac{1}{2}}, & \sigma_L \le \sigma_i < \sigma_{-1} \\ 1, & \sigma_i \ge \sigma_{-1} \end{cases} \tag{20}$$

4. Increasing half-Haibach shaped MF

Based on the fatigue life curve model for the stress below the fatigue limit proposed by Haibach (Li and Otto, 1987), namely, $\sigma^{2m-1}N = C$, the MF is defined as

$$\mu_{\tilde{D}}(\sigma_i) = \begin{cases} 0, & \sigma_i < \sigma_L \\ \left(\dfrac{\sigma_i - \sigma_L}{\sigma_{-1} - \sigma_L}\right)^{2m-1}, & \sigma_L \le \sigma_i < \sigma_{-1} \\ 1, & \sigma_i \ge \sigma_{-1} \end{cases} \tag{21}$$

5. Increasing half-normal shaped MF

$$\mu_{\tilde{D}}(\sigma_i) = \begin{cases} \exp\left[-\left(\dfrac{\sigma_i - \sigma_{-1}}{\sigma_c}\right)^2\right], & \sigma_i < \sigma_{-1} \\ 1, & \sigma_i \ge \sigma_{-1} \end{cases} \tag{22}$$

The corresponding MFs of the traditional Miner rule are shown in Equation (5) and Figure 2. According to the low amplitude loading strengthening, the load range with strengthening effects is about from 75% fatigue limit to 95% fatigue limit when the low amplitude loads are performed alone (Lu and Zheng, 2009a). In Equations (18)–(21), the lower bound of strengthening load below the fatigue limit is defined as $\sigma_L = \alpha\sigma_{-1}$, α is the lower bound coefficient of strengthening load, $\alpha = 0.65 - 0.95$. In Equation (22) σ_c is given as $\sigma_c = 0.05\sigma_{-1}$. The corresponding lower bound of strengthening load below the fatigue limit under different load sequences can be written as

$$\sigma_L = \begin{cases} \alpha(\sigma_{-1})_{H-L}, & \text{H} - \text{L loading sequences} \\ \alpha(\sigma_{-1})_{L-H}, & \text{L} - \text{H loading sequences} \end{cases} \tag{23}$$

Moreover, all these combined with the linear damage rule considering load sequences discussed earlier, the proposed Miner rule under low amplitude loading strengthening can be characterized in Figure 4. As mentioned above, by taking the increasing half-trapezoidal shaped MF, the damage function under different load sequences can be given in Figure 4.

It is worth noting that MFs are determined empirically. These functions are determined based on the continuous correction through practice and feedback. The construction of MFs is often possible if the expert knowledge exists. In addition, the MFs of a fuzzy set are based on statistical data. The accuracy of fuzzy fatigue life prediction depends upon the choice of MF and determination of its related parameters. For the actual components, whether

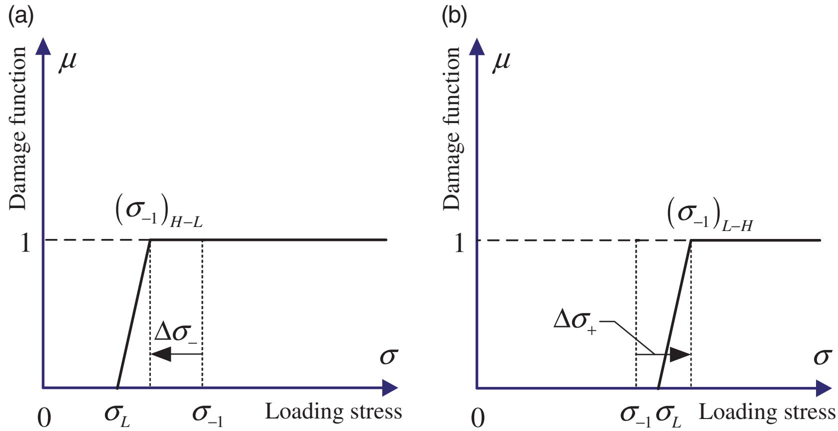

Figure 4. *the proposed Miner rule under low amplitude loading strengthening and different load sequences (using increasing half-trapezoidal shaped MFs).*

the proposed model can accurately predict the fatigue life or not will be evaluated and verified by analyses of fatigue life prediction with experimental data reported in the literature in section 'Validation of the proposed linear damage rule'.

VALIDATION OF THE PROPOSED LINEAR DAMAGE RULE

In this section, the validity and accuracy of the proposed method are evaluated through a comparison of predicted and observed fatigue life distributions for metallic specimen subjected to various multilevel and two-stress level uniaxial fatigue loadings.

Multilevel Stress Loading under H–L Loading Sequence

In order to verify the descriptive ability of Equation (15) under multilevel loading, experimental data available in Li and Otto (1987) are used and the details of test conditions are reported in Li and Otto (1987). We will discuss the only assessments of data with the new model below. Under the multilevel loading, the data comprised by cumulative fatigue damage test 1 (CFD1) and its real fatigue life is $N_{\mathrm{real}\,1} = 2.0 \times 10^6$. For these tests, the details of mechanical properties of the materials are as follows: the fatigue limit $\sigma_{-1} = 173.5\,\mathrm{MPa}$, the test parameter $m = 5.1$, and $N_0 = 2 \times 10^6$. Test parameters and results are given in Table 1.

Table 1. Results of CFD1 test.

Stress level	Stress amplitudes, σ_i (MPa)	n_i (cycles)	N_i (cycles)	n_i/N_i
1	505	4	9.00×10^3	0.0004
2	475	32	1.16×10^4	0.0028
3	423	560	2.10×10^4	0.0267
4	362	5440	4.70×10^4	0.1158
5	287	40,000	1.55×10^5	0.2580
6	212	184,000	8.70×10^5	0.2110
7	137	560,000	$\to\infty$	0
8	63	1,210,000	$\to\infty$	0

Using the traditional Miner rule, the fatigue life of the sample in CFD1 is $N_{\mathrm{CFD1}} = 3.2536 \times 10^6$ cycles, the error of life prediction is $\delta_{\mathrm{CFD1}} = 62.68$ %. Under the H–L loading sequences of CFD1 test, the lower bound coefficient of strengthening load is $\alpha = 0.65$, then get $\sigma_L = 95.859$ MPa. It can be experimentally determined from two-step fatigue tests that $(\sigma_{-1})_{\mathrm{H-L}} = (0.85 - 0.9)\sigma_{-1}$ (Sinclair, 1952; Nakagawa and Ikai 1979; Nakagawa, 1983; Lu and Zheng, 2009a). Based on the pre- and post-test analyses of material properties, the value of strengthening coefficient m' was obtained from fatigue test results under block loading of low amplitude loads, and $m' = 1.21 \times 10^{-7}$ for CFD1 test.

Using the fuzzy Miner rule in Equation (15) and Figure 4, the predicted fatigue life of CFD1 test under different MFs is shown in Table 2. Moreover, to reflect the capability of this new model, the test data are also assessed by the nonlinear damage cumulative model proposed in Shang and Yao (1999). The nonlinear damage cumulative model can be applied to multi-level loading condition through sequential calculation to get the fatigue damage cumulative formula (Shang and Yao, 1999), and the cyclic lives predicted are listed in Table 2.

In the above experiments, the loads are in the sequence from high to low. As shown in the results, an adequate agreement is achieved by the proposed Miner rule. For the CFD1 test, the fatigue life prediction by traditional Miner rule is not effective and the error of prediction reaches 62.68%. Besides, the error of prediction for the nonlinear damage cumulative model (Shang and Yao, 1999) is about 14.24%. However, by the proposed Miner rule, the prediction error can reduce significantly no matter what kind of MF is chosen. In our application, the best results were obtained by using the increasing half-trapezoidal and square root shaped MFs, the prediction error for CFD1 test can be decreased to 0.52% and 3.90%. Comparing with

Table 2. Results of CFD1 using different MFs.

No.	MF	Load sequence	D_1	ΔD	$N_{predicted}/10^6$ (cycles)	Error of prediction, δ (%)
1	Traditional Miner rule	H–L	0.6147	0	3.2536	62.68
2	Nonlinear damage cumulative model (Shang and Yao, 1999)		0.8754	0	2.28471	14.24
3	Increasing half-trapezoidal shaped		0.6147	0.38012	2.01041	0.52
4	Increasing half-parabolic shaped			0.30275	2.17995	8.99
5	Increasing half-square root shaped			0.42593	1.92191	3.90
6	Increasing half-Haibach shaped			0.06154	2.95752	47.87
7	Increasing half-normal shaped			0.06344	2.94924	47.46

the prediction errors of those methods under different MFs, results indicate that the proposed model has a better predictability than the Miner rule under H–L loading sequence. Under some MFs, the proposed Miner rule has a better effect of life prediction than the nonlinear damage cumulative model (Shang and Yao, 1999).

Two-Stress Level Loading under L–H Loading Sequence

Similarly, all the two-stress level results of Shang and Yao (1998, 1999) under L–H sequence are compared to predictions from our model and, for comparison, predictions by the Miner rule and the nonlinear damage cumulative model (Shang and Yao, 1999). The mechanical properties are as follows: Normalized 45 steel, the fatigue limit $\sigma_{-1} = 280.8\,\text{MPa}$, the test parameter $m = 2.3$, and $N_0 = 5 \times 10^5$. Test parameters and results are presented in Table 3.

Under the L–H loading sequences of tests I and II, the minimum strengthening load is $\sigma_L = 209.9\,\text{MPa}$. Similarly, the fatigue limit $(\sigma_{-1})_{\text{L-H}}$ can be estimated as $(1.15 - 1.25)\sigma_{-1}$ (Sinclair, 1952; Lu and Zheng, 2008, 2009a, b). Based on the pre- and post-test analyses of material properties, the value of strengthening coefficient m' was obtained from fatigue test results in Table 3, and $m' = 5.14083 \times 10^{-8}$ for the tests I and II of normalized 45 steel. Comparisons of experimental results and predicted data using

Table 3. Results of two-stress level tests under L–H sequence for normalized 45 steel.

Tests	Stress level	Stress amplitudes, σ_i (MPa)	n_i (cycles)	N_i (cycles)	n_i/N_i
I	1	284.39	250,000	500,000	0.50
	2	331.5	38,900	50,000	0.7780
II	1	284.39	375,000	500,000	0.75
	2	331.5	43,400	50,000	0.8680

Table 4. Results of Test I using different MFs for normalized 45 steel.

No.	MF	Load sequence	D_1	ΔD	$N_{predicted}/10^5$ (cycles)	Error of prediction, δ (%)
1	Traditional Miner rule	L–H	1.278	0	2.26056	21.75
2	Nonlinear damage cumulative model (Shang and Yao, 1999)		1.10	0	2.62636	9.09
3	Increasing half-trapezoidal shaped		0.7780	0.20735	2.93195	1.48
4	Increasing half-parabolic shaped			0.13666	3.15855	9.33
5	Increasing half-square root shaped			0.25540	2.79562	3.23
6	Increasing half-Haibach shaped			0.07014	3.40627	17.90
7	Increasing half-normal shaped			0.00110	3.70812	28.35

Equation (15) and nonlinear damage cumulative model (Shang and Yao, 1999) are presented in Tables 4 and 5. As shown by the results, the predicted data are in good agreement with the experimental results.

In summary, by comparison with a large reasonably good predictive capability, the capability of the proposed model equals or exceeds that of the traditional Miner rule. It should be noted that the proposed model under the increasing half-trapezoidal, parabolic and square root shaped MFs is more precise to predict fatigue life for the L–H loading sequence than other MFs. Moreover, compared with the nonlinear damage cumulative model (Shang and Yao, 1999), the proposed Miner rule gives a better life prediction under increasing half-trapezoidal shaped MFs. Due to the low amplitude loading strengthening effects, the traditional Miner rule gives a conservative life

Table 5. Results of Test II using different MFs for normalized 45 steel.

No.	MF	Load sequence	D_1	ΔD	$N_{predicted}/10^5$ (cycles)	Error of prediction, δ (%)
1	Traditional Miner rule	L–H	1.619	0	2.58431	38.23
2	Nonlinear damage cumulative model (Shang and Yao, 1999)		1.11	0	3.76937	9.91
3	Increasing half-trapezoidal shaped		0.8680	0.20735	3.89082	7.01
4	Increasing half-parabolic shaped			0.13666	4.16459	0.46
5	Increasing half-square root shaped			0.25540	3.72441	10.98
6	Increasing half-Haibach shaped			0.07014	4.45988	6.59
7	Increasing half-normal-shaped			0.00110	4.81417	15.06

prediction. As shown in Table 3, the strengthening effects are increased with increasing the relative cycles of low amplitude loads in two-stress level loadings. The prediction errors of proposed method under increasing half-parabolic and Haibach shaped MFs are decreased monotonously with increasing the cycles of low amplitude loads. So, these MFs should be chosen within the proposed Miner rule when there are remarkable increasing strengthening effects of low amplitude loads.

According to the proposed Miner rule, the cumulative damage considering the low amplitude loading strengthening and load sequence effects ΔD is not equal to zero. By choosing different parameters (such as σ_L) and MFs, the precision of life prediction can be improved. To a certain extent, the low amplitude loading strengthening effects are influenced by the load sequences in program loading. Through the tests mentioned above, it is clear that the load spectrums include small loads below the fatigue limit, which meet the demands of practicability of the proposed model in actual engineering structures.

According to the applicable conditions of the proposed Miner rule, it is valid for most metallic materials, such as carbon steels, cast irons, and alloy steels. Traditionally, the fatigue endurance limit is defined as the amplitude below which there appears to be no number of cycles that will cause failure under 10^7 cycles. For some materials under very high cycle fatigue (VHCF) regime, if the *S–N* curve of some materials is not flat, the stress level corresponding to some arbitrarily chosen value of N, say 10^8 cycles, is defined as the conditional fatigue limit. For different materials under VHCF, the conditional fatigue limit can be determined by choosing a different value of N, e.g., 5×10^8 cycles for Aluminum Alloy LC4CS (Yao and Guo, 2007), which also can be determined from the experiment results and material properties (Liu et al., 2010). Based on the conditional fatigue limit σ_{-1} at some chosen N cycles under VHCF, similarly, we can use our proposed Miner rule to estimate the fatigue life of components.

Choosing an optimal MF plays a pivotal role in the life prediction using the proposed Miner rule. Under fatigue loading, determination of MF depends on stress amplitude, low amplitude stress levels below fatigue limit and the relative cycles of low amplitude loads. Based on this, a rational MF can be selected to describe the fuzziness of the problem in engineering. It can be concluded from Tables 2, 4, and 5 that the new model presented has a better capability of life prediction under different load sequences and MFs than the Miner rule. Besides, to examine the application of this model in other cases such as changes in mechanical properties of engineering components and different materials under VHCF regime after damaging and strengthening under low amplitude loads below the fatigue limit will be further evaluated.

CONCLUSIONS

Based on the Miner rule, this study refers not only to the damaging and strengthening effect of low amplitude loads below the fatigue limit, but also the effects of load sequence and load interaction. Compared with the traditional Miner rule, results of those fatigue tests clearly indicate that the proposed Miner rule gives more accurate and reliable predictions on fatigue lives. It also provides a better life prediction under the appropriate MFs than the nonlinear damage cumulative model (Shang and Yao, 1999). The main conclusions are as follows:

1. The strengthening and damaging of low amplitude loads below the fatigue limit are investigated within the Miner rule.
2. Experiments show that the load sequence effect has a strong influence on the fatigue life. However, the traditional Miner rule cannot consider the load sequence effect. The proposed method can solve this problem by considering the fatigue limit to be a variable instead of a simple material constant.
3. To improve the accuracy of life prediction using the proposed model, different MFs are investigated and the determination of MFs and its related parameters are optimized by comparison of prediction errors.

NOMENCLATURE

D = Accumulated fatigue damage
σ_i = Loading stress
k = Number of constant amplitude blocks
n_i, N_i = Load cycles and cycles to failure at σ_i
σ_L = Minimum strengthening stress
σ_U = Maximum strengthening stress
$\sigma_{\max}$ = Maximum residual strength
σ, σ_0 = Load variable and the original strength
m' = Strengthening coefficient
σ_{-1} = Fatigue limit
$(\sigma_{-1})_{\text{H-L}}$ = Fatigue limit of component under H–L loading sequences
$(\sigma_{-1})_{\text{L-H}}$ = Fatigue limit of component under L–H loading sequences
$\Delta\sigma_-$ = Negative increment of fatigue limit for H–L loading sequences
$\Delta\sigma_+$ = Positive increment of fatigue limit for L–H loading sequences
σ_M = Maximum loading stress amplitude
N_0 = Fatigue life of component or material under σ_{-1}
MFs = Membership functions

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