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A modified nonlinear fatigue damage accumulation model

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Abstract

This paper presents a modified nonlinear fatigue damage accumulation model accounting for load interaction effects. The original model is based on physical property degradation of materials, from which the load interaction effects are ignored. However, the load interaction effects have a significant influence on the fatigue life. In the study, by analyzing five damage models, a load interaction parameter is obtained and added to the original model. Experimental work is then carried out to verify the modified model of four categories of experimental data from smooth and notch specimens under two-level stress loading. Moreover, comparison is made among the results calculated by the test data, the Miner's rule, the original model, and the modified model.

Keywords

Fatigue, damage accumulation, load interaction, life prediction, fracture

Introduction

Fatigue is a failure mode where the materials' damage accumulation and property degraded under cyclic loading, and finally lead to fracture. Damage is an important parameter to describe the change of material state in the process of fatigue failure, and it plays a key role in life prediction of mechanical components and structures.

Until now, lots of deterministic damage accumulation models have been developed, which can be mainly classified into two categories (Yang et al., 2003): linear damage cumulative theories and nonlinear damage cumulative theories. Among these fatigue damage accumulation models, the linear damage accumulation theory, also known as Palmgren–Miner's rule (Miner, 1945), is commonly

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used in analyzing cumulative fatigue damage due to its relative simplicity, close approximation to reality, and widespread knowledge and utilization. The Miner's rule can be expressed as

$$D = \sum_{i=1}^{n} \frac{n_i}{N_i} \tag{1}$$

But there are some shortcomings for the Miner's rule.

(1) It fails to consider load history.
(2) Cumulative damage has no relationship with load sequence effects.
(3) Effects of load interaction are not taken into account.

Generally, the result of life prediction by Miner's rule is nonconservative. Therefore, to address the above-mentioned disadvantages of Miner's rule, the nonlinear cumulative damage theories were suggested. However, until now, there is not a comprehensive model that can take all these factors into consideration (Fatemi and Yang, 1998).

Under complex loading, either small cycles or large cycles are dependent on loading interaction, sequence, or memory effects. Although a lot of work has been done in this area (Bui-Quoc et al., 1976; Corten and Dolan, 1956; Freudenthal and Heller, 1959; Morrow, 1986; Zhang et al., 1991), there is still no clear explanation to explain the reduced fatigue lives due to these interactions. In some fatigue damage accumulation models, loading interaction effects are assumed to be insignificant, although experimental data show that the fatigue life will be significantly shorter than predicted one (Teledyne CAE, 1979). There are explanations that can be used to explain this phenomenon (Morrow, 1986).

(1) Large strain usually results in the initial microcracks earlier in life, and then these microcracks propagate under the small cycles, which will cause more damage sooner than that under constant amplitude loading conditions.
(2) For the specimen, the major cycles will cause a roughening of the surface in the procedure of cyclic plastic deformation (Watson et al., 1973), providing more crack initiation sites under the smaller cycles.
(3) Damage cannot be accumulated linearly (Manson and Halford, 1980).

In order to improve the fatigue life prediction, Corten and Dolan (1956) proposed a model called the Corten–Dolan model. In this model, load interaction effects are introduced, and this theory is based on the modification of the *S–N* curve. In addition, there is a reference point, which is corresponding to the highest level in the load history. Then, Freudenthal and Heller (1959) present another interaction type of cumulative damage theory, which is also based on the modification of the *S–N* curve. The difference between the Corten–Dolan model and Freudenthal–Heller model is the reference point. For the Freudenthal–Heller model, the reference point is chosen at the stress level accordant with a fatigue life of 10^3–10^4 cycles. Bui-Quoc et al. (1976) proposed a series of hybrid models; in these models, both the effects of mean stress/strain and the effects of temperature and strain rates (Zhang et al., 1991) are taken into account. Recently, Morrow (1986) proposed a nonlinear plastic work interaction damage model by introducing an interactive factor into Miner's rule, which gives a more accurate fatigue life prediction.

The aim of this paper is to represent the load interaction effects using a parameter and considering that in what form the load interaction parameter can be introduced into an original model, so it can get a better result of the life prediction.

Theories accounting for load interaction effects

In general, the nonlinear cumulative damage theories can be divided into six categories (Zhu et al., 2012).

(1) Damage theories based on the physical property degradation of materials (Zhu et al., 2013).
(2) Damage curve approaches.
(3) Continuum damage mechanics approaches.
(4) Damage theories based on energy (Zhu and Huang, 2010; Zhu et al., 2012).
(5) Damage theories accounting for load interaction effects.
(6) Damage theories based on thermodynamic entropy.

There is no clear boundary that exists among these models.

The crack formation life should be related to the applied maximum stress in the loading spectrum (Hua and Fernando, 1996). The load interaction effects can be used to explain the phenomenon that the crack growth increment in a given cycle under variable amplitude loading will be different from the increment under constant amplitude loading (Skorupa, 1998). Usually, if a specimen is loaded under a multilevel loading condition, the crack length at the end of the first stress level depends on both the first loading condition and the subsequent stress level (Bui-Quoc, 1982). For instance, the high–low loading sequence often leads to a shorter lifetime than the low–high loading sequence (Adam et al., 1994), whereas the opposite result is obtained for some composite materials (Schaff and Davidson, 1997). In Van Paepegem and Degrieck (2002), the authors concluded that there is no universal statement that the specimen under high–low loading sequence is more or less damaged than low–high loading sequence. But, in actuality, the load interaction effects lead to the change of the damage accumulation during the remaining life. The load interaction effects depend on the transitions of stress levels and their number of appearance. The larger the difference between the stress amplitudes, the stronger damaging effects it results (Freudenthal and Heller, 1959). Therefore, the load interaction effects must be considered under variable amplitude loading.

In this section, this paper will obtain a load interaction parameter by studying and analyzing five damage models accounting for load interaction effects. They are the Corten–Dolan model, the Freudenthal–Heller approach, the Morrow's plastic work interaction rule, the Carpinteri's model, and the V. Dattoma's model.

Corten–Dolan model

For the Corten–Dolan model (Corten and Dolan, 1956), the definition of fatigue damage is

$$D_i = m_i r_i n_i^{a_i} \tag{2}$$

where m_i is the number of damage nuclei, r_i is the damage evolution rate, and a_i is the constant.

The failure criterion is given as

$$1 = \sum_{i=1}^{p} \frac{n_i}{N_1 \left(\frac{\sigma_{\max}}{\sigma_i}\right)^d} \tag{3}$$

Here, the exponent d can be considered as the material's sensitivity to stress load history.

For constant amplitude stresses σ_1 and σ_2, the damage is a constant when the material or structure gets failure as

$$D_f = m_1 r_1 n_1^{a_1} = m_2 r_2 n_2^{a_2} \tag{4}$$

For two different alternative stress amplitudes, and assuming that total load cycles are n, the specimen load cycles are an ($a < 1$) under amplitude stresses σ_1 and load cycles are $(1–a)n$ under amplitude stresses σ_2. Due to the interaction effects of these two stress amplitudes, the crack nuclear will continue to expand under low amplitude stress which produced the high stress amplitude. Assuming that $\sigma_1 > \sigma_2$, in this case $m_2 = m_1$ then according to equation (4), we can get

$$D_f = m_1 r_1 n_1^{a_1} = m_2 r_2 n_2^{a_2} \tag{5}$$

Equation (5) can be simplified as

$$n_2^A R^{1/a_1} = n_1 \tag{6}$$

In equation (6), $R = r_2/r_1$, $A = a_2/a_1$.

Experimental work had been done by Corten and Dolan to investigate the relationship between the parameter R^{1/a_1} and magnitude of two stress levels in a spectrum. Also, when $a_1 = a_2$, according to the experimental work, Corten and Dolan concluded that

$$R^{1/a} = \left(\frac{\sigma_2}{\sigma_1}\right)^d \tag{7}$$

For multiple-level stress loading, we have

$$R^{1/a} = \left(\frac{\sigma_i}{\sigma_{\max}}\right)^d \tag{8}$$

According to the relationship between the stress amplitudes and the parameter $R^{1/a}$, as in equation (6), the interactions between the two stresses σ_1 and σ_2 play an important role to determine the value of n_2.

Freudenthal–Heller model

For the Freudenthal–Heller model (Freudenthal and Heller, 1959), the definition of fatigue damage is

$$D = \sum_{i=1}^{p} \frac{n_i \omega_i}{N_i} \tag{9}$$

The failure criterion is

$$1 = \frac{\overline{N}}{\sum_i^p n_i (\sigma_i/\sigma_m)^d} \tag{10}$$

Although this theory is another interaction type of cumulative damage theory, it is interesting to note that the expression of Freudenthal–Heller model has a similar form as the Corten–Dolan model. The difference between them is the selection of the reference point. In Freudenthal–Heller approach, the parameter $(\sigma_i/\sigma_m)^d$ is used to describe the effects of load interaction, usually it is called the interaction factor. For this model, it connected the fatigue damage with the slip striations. The stress amplitudes will determine the distance between the striations, and the interaction effects between different stress amplitudes will produce an acceleration of the damage accumulation. More detailed information about this approach can be found in Teledyne CAE (1979).

Morrow's plastic work interaction rule

Morrow's (1986) rule is a nonlinear accumulative damage model which accounts for load interaction effects. According to the plastic work interaction of Morrow's rule, if the specimen is under a variable amplitude stress condition, the fatigue damage caused by the stress of amplitude σ_i is

$$D_i = \frac{n_i}{N_i}\left(\frac{\sigma_i}{\sigma_{\max}}\right)^f \tag{11}$$

The total damage under variable amplitude loading spectrum can be calculated as

$$D = \sum \frac{n_i}{N_i}\left(\frac{\sigma_i}{\sigma_{\max}}\right)^f \tag{12}$$

The exponent f is considered as material's sensitivity to variable amplitude stress history, and it can be expressed as $f = (c + e + 1)/c$ in which c and e are fatigue strength and ductility exponents. The parameter $(\sigma_i/\sigma_{\max})^f$ indicates that the maximum loading in the loading spectrum has an influence on the damage which is caused by other loadings. This model suggests that the parameter $(\sigma_i/\sigma_{\max})^f$ modifies the slope of the S–N curve in a way similar to the Corten–Dolan approach.

Carpinteri's model

Carpinteri et al. (2003) proposed an accumulative damage model by combining with the rainflow counting method. In this model, the fatigue damage caused by one reversal is given as

$$D_i = \begin{cases} \dfrac{1}{2N_l}\left(\dfrac{\sigma_i}{\sigma_l}\right)^b & \sigma_i \geq c\sigma_l \\ 0 & \sigma_i < c\sigma_l \end{cases} \tag{13}$$

In this model, there is also a parameter which has a form as the ratio of different series of stress with exponent.

V. Dattoma's model

Dattoma et al. (2006) proposed an accumulative damage model by considering the effects due to load interactions and to different sequences of load application. In this model, the fatigue damage function is given as

$$D = 1 - \left[1 - \left(\frac{n}{N_f}\right)^{1/(1-\alpha)}\right]^{1/(1+\beta)} \tag{14}$$

In this damage evolution model, D is chosen to be equal to 1 if the applied stress is less than the fatigue limit, and it is a monotonically decreasing function of stress which can take the effects of load interaction into account for multilevel loading.

Generally, under complex loading, the larger cycles and smaller cycles have a relationship with loading interaction, sequence, or memory effects. Although a lot of work has been done in this area, there is still no clear explanation that can be used to explain reduced fatigue lives due to these factors. In this section, by studying and analyzing five damage models accounting for load interaction effects, it is concluded that under complex loading, the ratio of different series of stress level can be used to consider the load interaction effects. Based on this idea, a load interaction parameter is added to an original model in the following section.

The modified model

The original model

The original model is based on the physical property degradation of the materials. The decline of the elastic modulus and yield strength during the fatigue failure process is mainly related to the crack initiation and bearing area loss of specimens caused by crack growth under cyclic loading. The plastic degradation process of materials can be attributed to the reduction of the amount of movable dislocations and the formation process of dislocation movement barriers. The decline of material toughness is the performance of the deterioration of strength and plasticity during the process of fatigue failure.

In the original model (Ye and Wang, 2001), the fatigue damage is defined as

$$D_n = -\frac{D_{(N_f-1)}}{\ln N_f}\ln\left(1 - \frac{n}{N_f}\right) \tag{15}$$

Or

$$D_n = -\frac{D_{(N_f-1)}}{\ln\frac{1}{2}\left(\sigma_a/\sigma_f\right)^{1/h}}\ln\left[1 - 2\left(\sigma_a/\sigma_f\right)^{-1/h}n\right] \tag{16}$$

For the original model, it has a good physical basis, and the final damage expression contains only one parameter without any other parameters, so it has the advantages of simple form, but it does not take the interaction effects between the loading stresses into consideration. As aforementioned, the load interaction effects will produce stronger damaging effects to the life predictions, and in most cases, the predicted life by this model is nonconservative.

Next, we introduce the interaction factor ($\sigma_i/\sigma_{\max}$) into equation (15), and the modified model is shown as

$$D_n = -\frac{D_{(N_f-1)}}{\ln N_f}\ln\left(1-\frac{n}{N_f}\right)\cdot\frac{\sigma_i}{\sigma_{\max}} \tag{17}$$

In the previous section, for Corten–Dolan model and Morrow's plastic work interaction rule, from the micro perspective, it is concluded that the load interaction effects under cyclic loading including several stresses are caused by the number of damage nuclei. The number of damage nuclei for the higher stress is larger than it for the lower stress (Schijve, 2001). Meanwhile, the number of damage nuclei produced by the highest stress amplitude under complex loading will lead to a change of the damage accumulation at the lower stress amplitude. Moreover, the greater the difference between the loading amplitudes is, the more obvious the load interaction effects are (Kaechele, 1963). From the macro perspective, a relationship between the load amplitudes can be used to represent the load interaction effects which are caused by the number of damage nuclei. Therefore, if the specimen is loaded under multilevel cyclic loading, through introducing an interaction factor into the fatigue damage model, we chose the maximum loading stress as a reference point. Similar with the Corten–Dolan model and Morrow's plastic work interaction rule, we introduced an interaction factor in a form as $\sigma_i/\sigma_{\max}$ into a fatigue damage model.

For a fatigue cumulative damage model, it must answer three questions quantitatively: (a) For the material or structure, how much fatigue damage is caused by one stress level; (b) how to cumulate fatigue damage properly under multiple stress levels; and (c) when the material or structure gets failure, how much the critical damage is (Xia and Yao, 2013). In order to verify its application, the predictions made by the modified model should be compared with the test results under variable amplitude. For the modified model, the damage caused by the first loading stress is

$$D_1 = -\frac{D_{(N_{f1}-1)}}{\ln N_{f1}}\ln\left(1-\frac{n}{N_{f1}}\right)\cdot\frac{\sigma_1}{\sigma_1} = -\frac{D_{(N_{f1}-1)}}{\ln N_{f1}}\ln\left(1-\frac{n}{N_{f1}}\right) \tag{18}$$

In this case, there is only one-stage cyclic loading, which means that the interaction between the different load amplitude does not exist, so the modified model can be simplified as the original model.

For two levels loading, supposing that the specimen is first loaded at a stress σ_1 for n_1 cycles, then at a stress σ_2 for n'_2 cycles until it fails. By using the equivalence of fatigue damage, the residual circulation ratio under the second level stress σ_2 is

$$\frac{n'_2}{N_{f2}} = \left(1-\frac{n_1}{N_{f1}}\right)^{\frac{D_{(N_{f2}-1)}}{D_{(N_{f1}-1)}}\cdot\frac{\ln N_{f2}}{\ln N_{f1}}\cdot\frac{\sigma_1}{\sigma_2}} \tag{19}$$

More detailed information about the equivalence of fatigue damage method can be found in Chen et al. (2006).

In equation (19), $D_{(N_{f2}-1)}/D_{(N_{f1}-1)}$ can be expressed in a approximated form as

$$\frac{D_{(N_{f2}-1)}}{D_{(N_{f1}-1)}} \approx 1$$

So equation (19) can be simplified as

$$\frac{n_2'}{N_{f2}} = \left(1 - \frac{n_1}{N_{f1}}\right)^{\frac{\ln N_{f2}}{\ln N_{f1}} \cdot \frac{\sigma_1}{\sigma_2}} \tag{20}$$

By using the equivalence of fatigue damage, we can convert the multilevel load to two-level load to obtain fatigue damage accumulation formula under multilevel stress loading. According to equation (17), the damage caused by the first loading stress is

$$D_1 = -\frac{D_{(N_{f1}-1)}}{\ln N_{f1}} \ln\left(1 - \frac{n}{N_{f1}}\right) \tag{21}$$

According to equations (18) and (20), the cumulative cycle ratio under two-level stress loading is

$$\frac{n_2'}{N_{f2}} + \frac{n_2}{N_{f2}} = \left(1 - \frac{n_1}{N_{f1}}\right)^{\frac{\ln N_{f2}}{\ln N_{f1}} \cdot \frac{\sigma_1}{\sigma_2}} + \frac{n_2}{N_{f2}} \tag{22}$$

If there is a three-level stress loading, similarly, by using the equivalence of fatigue damage and according to equations (17) and (22), we can get the residual circulation ratio under σ_3 as

$$\frac{n_3'}{N_{f3}} = \left\{1 - \left[\left(1 - \frac{n_1}{N_{f1}}\right)^{\frac{\ln N_{f2}}{\ln N_{f1}} \cdot \frac{\sigma_1}{\sigma_2}} + \frac{n_2}{N_{f2}}\right]\right\}^{\frac{\ln N_{f3}}{\ln N_{f2}} \cdot \frac{\sigma_2}{\sigma_3}} \tag{23}$$

Therefore, the cumulative cycle ratio under three-level stress loading is

$$\frac{n_3'}{N_{f3}} + \frac{n_3}{N_{f3}} = \left\{1 - \left[\left(1 - \frac{n_1}{N_{f1}}\right)^{\frac{\ln N_{f2}}{\ln N_{f1}} \cdot \frac{\sigma_1}{\sigma_2}} + \frac{n_2}{N_{f2}}\right]\right\}^{\frac{\ln N_{f3}}{\ln N_{f2}} \cdot \frac{\sigma_2}{\sigma_3}} + \frac{n_3}{N_{f3}} \tag{24}$$

Similarly, we can get the formula for damage accumulation under variable loading conditions, suppose that

$$Y_{i-1} = \frac{n_{i-1}'}{N_{f(i-1)}} + \frac{n_{i-1}}{N_{f(i-1)}} \tag{25}$$

where Y_{i-1} represents the cumulative cycle ratio under $(i-1)$-level stress loading and $\frac{n_{i-1}'}{N_{f(i-1)}}$ represents the residual circulation ratio under σ_{i-1}.

By repeating the above steps, we have

$$\frac{n_i'}{N_{fi}} = \{1 - Y_{i-1}]\}^{\frac{\ln N_{fi}}{\ln N_{f(i-1)}} \cdot \frac{\sigma_{i-1}}{\sigma_i}} \tag{26}$$

Then the cumulative cycle ratio under i-level stress loading is

$$\frac{n_i}{N_{fi}} + \frac{n_i'}{N_{fi}} = \{1 - Y_{i-1}]\}^{\frac{\ln N_{fi}}{\ln N_{f(i-1)}} \cdot \frac{\sigma_{i-1}}{\sigma_i}} + \frac{n_i}{N_{fi}} \tag{27}$$

Therefore, when sequence of multilevel stress loading is known, the remaining life of the component or structure can be predicted by equation (27).

For metallic materials, according to the physical meaning of the continuum fatigue damage, it should meet the following conditions for a reasonable fatigue damage model (Chaboche and Lesne, 1988).

(1) Fatigue damage is irreversibly processed of energy consumption, along with the cyclic loading, the damage is monotone increased as

$$\frac{\partial D_n}{\partial n} > 0 \tag{28}$$

For the modified model, we have

$$\frac{\partial D_n}{\partial n} = \frac{D_{(N_f-1)}}{\left[\left(N_f - n\right) \ln N_f\right]} \cdot \frac{\sigma_i}{\sigma_{\max}} > 0 \tag{29}$$

(2) The damage value produced by a small load should be less than that by a large load for in same number of cycles

$$\frac{\partial^2 D_n}{\partial n \partial \sigma} > 0 \tag{30}$$

According to equations (16) and (30), we have

$$\begin{aligned} \frac{\partial^2 D_n}{\partial n \partial \sigma} = & \left\{ \frac{\left(1 - D_{(N_f-1)}\right)}{\sigma_a\left[\left(N_f - n\right) \ln N_f\right]} - \frac{1}{b} \cdot \frac{D_{(N_f-1)}\left[N_f \ln N_f + \left(N_f - n\right)\right]}{\sigma_a\left[\left(N_f - n\right) \ln N_f\right]^2} \right\} \cdot \frac{\sigma_i}{\sigma_{\max}} \\ & + \frac{D_{(N_f-1)}}{\left[\left(N_f - n\right) \ln N_f\right]} \cdot \frac{1}{\sigma_{\max}} > 0 \end{aligned} \tag{31}$$

The result of equations (29) and (31) indicates that the modified model is reasonable in theory.

Verification of the modified model and discussion

In this section, predicted results by the modified model are compared with the test data, Miner's rule, and the original model. Five categories of experimental data from smooth and notch specimens of normalized 45 steel and 16Mn steel (Shang and Yao, 1998; Yao, 2003) are used to verify the modified model under two-level stress loading. The notch specimens have a shape of ring groove with the root radius being 0.25 mm and the notch depth being 0.25 mm. The specimens were under a given number of cycles loading for a defined stress level. In order to compare the efficiency of the modified model, test data are also assessed by the Miner's rule and the original model. For normalized 45 steel under tensile test, the high–low loading spectrum is 331.463–284.4 MPa and low–high loading spectrum is 284.4–331.463 MPa, and when $\sigma = 331.463$ MPa, the fatigue life of normalized 45 steel is $N = 5 \times 10^4$, when $\sigma = 284.4$ MPa, fatigue life of normalized 45 steel is $N = 5 \times 10^5$. For smooth normalized 16Mn steel under tensile test, the high–low loading spectrum is 562.9–372.65 MPa and low–high loading spectrum is 372.65–392.3 MPa, and when $\sigma = 562.9$ MPa, the fatigue life of the 16Mn steel is $N = 3968$; when $\sigma = 372.65$ MPa, the fatigue life of 16Mn steel is $N = 78730$; and when $\sigma = 392.3$ MPa, the fatigue life of 16Mn steel is $N = 78723$. For notch normalized 16Mn steel under tensile test, the high–low loading spectrum is 294.2–166.71 MPa and low–high loading spectrum is 166.71–294.2 MPa, when $\sigma = 294.2$ MPa, the fatigue life of 16Mn steel is $N = 55400$, and when $\sigma = 166.71$ MPa, the fatigue life of 16Mn steel is $N = 1,040,000$. For smooth normalized 16Mn steel under rotating bending stress, the high–low loading spectrum is 394–345 MPa and 366–324 MPa, the low–high loading spectrum is 345–394 MPa. When $\sigma = 324$ MPa, the fatigue life of 16Mn steel is $N = 1,370,200$; when $\sigma = 345$ MPa, the fatigue life of 16Mn steel is $N = 402,200$; when $\sigma = 366$ MPa, the fatigue life of 16Mn steel is $N = 199,700$; and when $\sigma = 394$ MPa, the fatigue life of 16Mn steel is $N = 93,500$.

A comparison of experimental data and predicted results using equations (1), (6), and (19) is listed in Tables 1 to 5. According to Tables 1 to 5, it should be noted that among these three models, Miner's rule has the simplest form and is easiest to be used for calculation. But because of its linearity, it has the maximum life prediction error. For the original model, since it has taken the load sequence effects into account, the results of life prediction under high–low loading spectrum are different from the results under low–high loading spectrum. For these two types of materials, using the original model, the total damage value is larger than 1 for low–high load sequences and less than 1 for high–low load sequences. Comparing with Miner's rule, the predicted results produced by the original model have been improved. Then, through taking the load interaction effects into consideration and using the modified model, once again the precision of prediction results is improved, and the larger the ratio of the two-stage load amplitudes, the more obvious is the improved precision on life prediction. Above all, by introducing an interaction factor into the original model, we can see that the modified model gives a better life prediction under two-level stress loading (refer to Appendix 2 for Tables 1 to 5).

In the modified model, the load interaction effects have been used to explain the strong deviation of life predictions. In this paper we proposed a nonlinear fatigue damage model for life prediction under variable loading conditions. However, application of the proposed model to life prediction for different materials and multiaxial loading also needs further study.

Conclusions

The main purpose of this paper is to find a parameter to represent the load interaction effects and then add it to an original model to improve life prediction result. The main work that has been done is summarized as follows:

(1) According to Corten–Dolan model, Freudenthal–Heller approach, Morrow's rule, V. Dattoma's model, and V. Dattoma's model, this paper introduces a load interaction parameter.
(2) A nonlinear cumulative damage model based on physical property degradation of materials is modified, and the expression of the modified model under multilevel load spectrums is derived.

(3) By using the two-level cyclic fatigue test data of normalized 45 steel and 16Mn steel, a comparison of experimental data and predicted results is made. It is found that the modified model gives a better life prediction than Miner's rule and the original model.

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Appendix 1

Notation

a	material constant
d	material parameter
D	damage variable
$D_{(N_f-1)}$	the critical value of the damage variable
f	Morrow's plastic work interaction exponent
h	fatigue strength exponent
m	number of damaged nuclei
n_i	number of cycles at a given stress amplitude
N_i	number of cycles to failure at a given stress amplitude
$\overline{N}$	the number of cycles to failure at load stress $\overline{\sigma}$
p	stress level series
r	damage evolution rate
α	function in the damage model
β	coefficients of the damage model

σ_a applied stress amplitude
σ_f fatigue strength coefficient
σ_i the ith load stress level
σ_m stress level corresponding to a fatigue life of $10^3 - 10^4$ cycles
$\sigma_{\max}$ the maximum stress of multilevel alternating load stress
ω_i interaction factor

Appendix 2

Table 1. Comparison between lives predicted by test data, the Miner rule, the original model, and the new model for smooth normalized 45 steel under two-level tensile stress loading.

Two-stress leveltest (MPa)	Load sequences	n_1	$\frac{n_1}{N_{f1}}$	n_2	Test data $\frac{n_2}{N_{f2}}$	Miner rule $\frac{n_2}{N_{f2}}$	Original model $\frac{n_2}{N_{f2}}$	New model $\frac{n_2}{N_{f2}}$
331.46–284.4	High–low	500	0.010	423,700	0.8474	0.990	0.988	0.986
		12,500	0.250	250,400	0.5008	0.750	0.705	0.665
		25,000	0.500	168,300	0.3366	0.500	0.431	0.375
		37,500	0.750	64,500	0.1290	0.250	0.186	0.141
284.4–331.46	Low–high	125,000	0.250	37,900	0.7580	0.750	0.788	0.816
		250,000	0.500	38,900	0.7780	0.500	0.565	0.613

Table 2. Comparison between lives predicted by test data, the Miner rule, the original model, and the new model for notch normalized 45 steel under two-level tensile stress loading.

Two-stress level test (MPa)	Load sequences	n_1	$\frac{n_1}{N_{f1}}$	n_2	Test data $\frac{n_2}{N_{f2}}$	Miner rule $\frac{n_2}{N_{f2}}$	Original model $\frac{n_2}{N_{f2}}$	New model $\frac{n_2}{N_{f2}}$
284.4–331.46	Low–high	125,000	0.25	55,500	1.110	0.750	0.788	0.816
		250,000	0.50	58,000	1.160	0.500	0.565	0.612
		500,000	0.75	24,600	0.492	0.250	0.319	0.375
331.46–284.4	High–low	5000	0.10	421,500	0.843	0.900	0.880	0.862
		12,500	0.25	337,700	0.6754	0.750	0.705	0.666
		25,000	0.50	234,500	0.4690	0.500	0.431	0.375

Table 3. Comparison between lives predicted by test data, the Miner rule, the original model, and the new model for smooth normalized 16Mn steel under two-level tensile stress loading.

Two-stress level test (MPa)	Load sequences	n_1	$\frac{n_1}{N_{f1}}$	n_2	Test data $\frac{n_2}{N_{f2}}$	Miner rule $\frac{n_2}{N_{f2}}$	Original model $\frac{n_2}{N_{f2}}$	New model $\frac{n_2}{N_{f2}}$
562.9–392.3	High–low	2	0.0005	73,600	0.9352	0.9995	0.9993	0.9989
		200	0.0504	59,400	0.7548	0.9496	0.9321	0.9040
		1000	0.2520	56,300	0.7154	0.7480	0.6736	0.5672
		1700	0.4284	47,600	0.5411	0.6176	0.5191	0.3903
372.65–392.3	Low–high	38,900	0.1450	75,500	0.9590	0.8550	0.8550	0.8617
		64,400	0.2400	62,800	0.7980	0.7600	0.7600	0.7705
		116,000	0.4330	62,900	0.7990	0.5670	0.5670	0.5833

Table 4. Comparison between lives predicted by test data, the Miner rule, the original model, and the new model for notch normalized 16Mn steel under two-level tensile stress loading.

Two-stress level test (MPa)	Load sequences	n_1	$\frac{n_1}{N_{f1}}$	n_2	Test data $\frac{n_2}{N_{f2}}$	Miner rule $\frac{n_2}{N_{f2}}$	Original model $\frac{n_2}{N_{f2}}$	New model $\frac{n_2}{N_{f2}}$
166.71–294.2	Low–high	260,000	0.25	52,500	0.9492	0.75	0.7971	0.8794
		520,000	0.50	37,900	0.6852	0.50	0.5791	0.7338
		780,000	0.75	18,500	0.3345	0.25	0.3353	0.5384
294.2–166.71	High–low	13,800	0.25	497,400	0.4788	0.75	0.6943	0.5252
		27,700	0.50	343,900	0.3310	0.50	0.4151	0.2119

Table 5. Comparison between lives predicted by test data, the Miner rule, the original model, and the new model for notch normalized 16Mn steel under two-level rotating bending stress loading.

Two-stress level test (MPa)	Load sequences	n_1	$\frac{n_1}{N_{f1}}$	n_2	Test data $\frac{n_2}{N_{f2}}$	Miner rule $\frac{n_2}{N_{f2}}$	Original model $\frac{n_2}{N_{f2}}$	New model $\frac{n_2}{N_{f2}}$
394–345	High–low	9350	0.1000	269,500	0.6701	0.9000	0.8880	0.8731
		19,700	0.2107	236,100	0.5870	0.7893	0.7658	0.7373
		46,750	0.5000	159,300	0.3961	0.5000	0.4577	0.4096
		56,100	0.6000	74,000	0.1840	0.4000	0.3559	0.3073
366–324	High–low	39,940	0.2000	995,670	0.7267	0.8000	0.7723	0.7469
		79,880	0.4000	545,300	0.3980	0.6000	0.5535	0.5127
		109,800	0.5498	448,780	0.3275	0.4502	0.3969	0.3521
		139,790	0.7000	306,000	0.2233	0.3000	0.2481	0.2071
345–394	Low–high	72,400	0.1800	96,733	1.0346	0.8200	0.8386	0.8572
		181,000	0.4500	82,867	0.8863	0.5500	0.5885	0.6286
		197,100	0.4900	80,970	0.8660	0.5100	0.5503	0.5928
		233,300	0.5800	59,750	0.6390	0.4200	0.4633	0.5098